EFFECTS OF AN OPTIMIZED CARSHARING-SYSTEM DESIGN ON DEMAND: A JOINT OPTIMIZATION-SIMULATION APPROACH

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ABSTRACT

We present an integrated simulation-optimization approach to study whether a changed carsharing-system design can lead to more active carsharing (CS) users. We present a MIP model to optimize the CS supply. To evaluate the supply, we integrate our optimization model with an evaluation model based on the MATSim multi-agent transport simulation. This simulation yields a demand estimation, which we then—after linearizing—feed back to the optimization. We demonstrate the feasibility of our approach by presenting data on the demand structure for Stockholm city.

Keywords: carsharing, mutli-agent simulation, mixed-integer programming

1. INTRODUCTION

The number of privately owned cars, and with them all of their negative side effects, still increases: in Sweden the number of private cars in use increased by about 12% over the last 9 years from 4.447 to 4.986 million from 2012 to 2021 (Trafikanalys, 2022). This contradicts climate goals for reduced greenhouse gas (GHG) emissions (Johansson, 2015), and many municipalities try to give incentives for using other modes of transportation. One such alternative is carsharing (CS)—promoted by the European Commission as an efficient measure to accelerate the transition to low-carbon emissions in Europe. Currently, only about 1% of the population use CS (Trafikverket, 2012).

CS offers a myriad of advantages: access to flexible-type private cars without the fixed costs and responsibilities of ownership; reduced transport costs, hence, mobility advantage of a car for households that did not have access to a car before; reduction of the number of privately owned vehicles (e.g., one CS car was shown to substitute 5 privately owned vehicles in Sweden (Indebetou & Börefelt, 2014) and even 9 to 13 cars in North America (Martin, et al., 2010)); newer and more environmentally friendly cars than in the private-vehicle fleet (Martin, et al., 2010); statistically significantly reduced household GHG emissions (Chen & Kockelman, 2016) (Martin & Shaheen, 2011); and, inter alia, reduced congestion, improved air quality, and freed urban space currently allocated to parking lots and roadways.

Two operating models for CS systems exist: round-trip systems and one-way systems (and with free-floating CS a variant of the latter). Trips in the former and latter system end at the same or non-coinciding locations, respectively. Even though there is a documented demand for one-way systems, these are difficult to implement because one-way trips result in severe imbalances of vehicle availability (de Almeida Correia & Antunes, 2012). The majority of today's CS systems offers round-trips. However, these systems cannot reflect all travel patterns, e.g., travel to work, to a railway station or airport are practically prohibited by a resulting high fee for long-term use—a desired effect as carsharing cars should not stand idle for many hours.

To fully take advantage of this environmentally friendly transportation mode, we need to attract a wider customer base (and account for the increased demand in strategic urban planning). Hence, we study whether a changed CS-system design can lead to more active CS users: by adapting station locations to demand and by studying a mixed system of round and one-way trips (to reflect a larger number of trip types). In this paper, we concentrate on the approach for the first problem. We use a mixed-integer-programming (MIP) formulation to alter the CS-system design. Such an optimization of the CS supply requires assumptions about CS demand and network performance. Moreover, the MIP does not enable us to evaluate how the changed system performs. Thus, we integrate our optimization model with an evaluation model based on the MATSim multi-agent transport simulation. Addressing CS-supply optimization and system-impact estimation jointly by iterating between supply optimization and demand estimation improves the realism of the assumptions made when solving each problem individually.

In the remainder of this section, we present related work. In Section 2, we detail our approach based on a combination of simulation and optimization: the integration of the mode CS into MATSim in Subsection 2.1, the linearization of the output demand in Subsection 2.2, and our optimization model in Subsection 2.3. In Section 3, we give computational results for the simulation-generated demand for our case study of Stockholm, and we conclude in Section 4.

Related Work. A lot of research on CS has been devoted to empirical studies. The largest was performed in 2008 by Martin and Shaheen: the authors conducted a survey of 6281 CS users in North America, members of various CS organizations (CSOs) (Martin, et al., 2010) (Martin & Shaheen, 2011) (Martin & Shaheen, 2011). This study gave detailed insights in the behavioral change of new CS users, the effect on car holding, GHG emissions, and conditions that facilitate decisions to become a CSO member.

Various further studies have been performed, many of these also included optimization based on mathematical models, e.g., optimizing locations using a heuristic to solve a non-linear integer program (Kumar & Bierlaire, 2012); optimizing depot location in a one-way system, using a MIP formulation (de Almeida Correia & Antunes, 2012). A 2011 study in Nijmegen (Martens, et al., 2011) examined the effects of increased supply on the demand in contrast to the usually considered incremental growth of CS systems. The handling of imbalances in one-way systems has been studied in various studies: Operator-based relocation strategies have been researched using, e.g., a queueing-based transportation model (Barth & Todd, 1999), heuristic algorithms (Duron, et al., 2000), stochastic MIPs (Miller-Hooks & Nair, 2011), and MIPs (de Almeida Correia & Antunes, 2012); price incentives for users have been considered using discrete event systems (Di Febbraro, et al., 2012) and design science techniques (Brendel, et al., 2017). Some studies also considered vehicle relocation together with fleet and personnel dimensioning as well as trip pricing, using a mixed-integer convex programming model (Xu, et al., 2018). Moreover, mixes of user-and operator-based strategies using MIPs have been considered (Saade & Doig, 2016). Relocation has also been considered for CS with electrical vehicles—with the added requirement of charging the cars (Boyacı, et al., 2015) (Ait-Ouahmed, et al., 2018) (Folkestad, et al., 2020) (Hellem, et al., 2021).

Modeling and estimating carsharing demand has not been studied in detail: (Rodier & Shaheen, 2004) identified this as a relevant question; LeVine forecasts carsharing usage for different CS system types for London, but his model cannot capture changes of mobility patterns in response to the CS supply (Le Vine, 2012); (Kortum & Machemehl, 2012) estimate demand in free-floating CS. Demand uncertainty has been integrated in an optimization approach by (Lu, et al., 2017). Ciari et al. describe how carsharing demand can be modeled in MATSim (Ciari, et al., 2016). Simulation has even been used for optimizing a CS system (one-way), but demand has been considered as a fixed input (Repoux, et al., 2015).

To the best of our knowledge, the only combined simulation-optimization approach was presented by (Zhou, et al., 2017): they used historical CS reservation data to estimate latent demand in a roundtrip system and optimized the expected profit deciding on an assignment of vehicles to existing stations. Based on the demand and supply, the simulator stochastically evaluates the supply performance.

2. JOINT SIMULATION-OPTIMIZATION APPROACH

In this section, we describe the components of our approach. As described in the introduction, we iterate between a MIP model for the optimization of the CS-system design and a multi-agent transport simulation to evaluate the demand. With the MIP model, we decide on station locations and the number of available cars per station. This CS system is the input for the simulation, which estimates the demand for the given system design, see Figure 1. While we can directly use the optimization output as input for the simulation, we need to linearize the demand output by MATSim to obtain a feasible input for the optimization. We overlay the city (here Stockholm city) with a grid and per grid cell *C* we have one candidate location for a CS station.

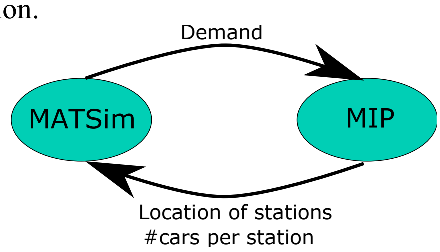

Figure 1 Overview of the integration of simulation and optimization.

2.1 MATSim

Our detailed evaluation model is based on the MATSim multi-agent transport simulation (www.matsim.org, (Horni, et al., 2016)). It allows to trace individual (synthetic) travelers throughout their entire simulated days, including experiences of travel and activity participation. Multi-model trip chaining, within-day dynamics, and individual level attributes are hence naturally captured. (Ronald, et al., 2015) elaborate on the advantages of agent-based simulation for the analysis of demand-responsive transportation services. Inserting CS as an additional mode into MATSim allows to analyze who, where, and even why (in terms of available transport mode attributes such as price) CS is used (or not).

MATSim operates on a population of synthetic travelers. It iterates through many simulated days, where in each day (i) some travelers get to revisit their all-day travel plan (basically a sequence of trips, annotated with departure time, route and mode information), (ii) all travelers get to execute their plans in the transport system (a particle-discrete multimodal network flow simulation allowing for congestion effects), (iii) all travelers collect information about the performance (delay etc.) of the most recently executed travel plan as a basis for plan choice in the upcoming iteration. A non-standard feature of our model system is that the simulation attempts to approximate a deterministic user equilibrium rather than pursuing the usual (and less well understood) "relaxation" approach of MATSim (Flötteröd, 2022) implying relatively fast convergence to a relatively noise-free near-solution point.

We implement a two-way CS system in an existing model system for the Greater Stockholm, where MATSim instances for road traffic ((Canella, et al., 2016), (Canella, et al., 2017), (Berglund, et al., 2014)) and public transport (Flötteröd, 2020) are already available. The implementation is a much-simplified version of the the readily available MATSim plugin for carsharing (the "carsharing contrib", see https://github.com/matsim-org/matsim-libs/tree/14.x/contribs/carsharing). The major simplifications are (i) assumption of infinite station capacity, meaning that there always is a vehicle available, and (ii) no within-day replanning, meaning that travelers who decide to use CS do not revisit that choice during a simulated day. Without getting into technical detail, these two simplifications are mutually consistent: since travelers opting for CS always receive access to a vehicle, there is no need for a within-day fallback option of changing travel mode because no CS vehicle is available.

The underlying travel demand model for Stockholm is tour-based, implying that the choice of a shared car is made per tour. Given that each tour starts and ends at home, a two-way CS system is the most natural CS instance to consider. We assume here that travelers always use the CS station nearest to their home. A CS tour is then composed of the following episodes: (i) leave home and walk to the nearest CS station, (ii) rental starts, (iii) drive with CS vehicle to destination and park there, (iv) activity participation at destination, (v) drive from activity location back to CS station, (vi) rental ends, (vii) walk back to home location. The utility of a CS tour is composed of regular walk utility, time utility of regular car usage, plus monetarized utility of the car rental fee (100SEK per hour plus 2SEK per kilometer).

2.2 Generating Input for the Optimization: Linearization of the Demand

In our MIP model, see Subsection 2.3, we use a vector $x = (x_i)_{i \in C}$ with $x_i = 1$iff a station is placed in cell i. Given a population of travelers (n=1,...,N), the expected number of users of station i in a given configuration x is given as $d_i(x) = P_n(i|x)$. However, in our optimization model, we do not only need to know the expected number of users in the complete time frame, but the expected number of users for every time interval $[t_k, t_l)$—we need to assign cars over time. Hence, we are interested in $d_i(x, t_k, t_l)$. To be able to feed this demand to the MIP (which requires linear dependence on our variables x_i), we assume a linearization:

$$d_i(x, t_k, t_l) = \sum_{j \in C} \alpha_{i,j}^{k,l} \cdot x_j + \beta_i^{k,l}$$

Thus, from the simulation, we need to produce the coefficients $\alpha_{i,j}^{k,l}$ and $\beta_i^{k,l}$. These are computed at the end of each iterated MATSim simulation. Given a current station configuration, random variations thereof are created (adding and/or removing a few stations). For each variation, each agent is asked to re-evalute its daily travel plan, including the option of switching to a different station (if a newly opened station is advantageous or if the previously used station is closed). This computation is done given fixed travel times, meaning that it can be done in a computationally efficient manner and without interdependence between agents. If the new travel plan of an agent is more advantageous than the originally chosen one, it is checked if a plan switch would imply a change of station usage, and the alpha and beta coefficients are updated accordingly.

2.3 MIP Model

In this subsection, we describe our MIP model for optimizing the CSO's profit. We let C denote the set of all cells in our grid. The binary variable x_i (for all cells i) indicates whether we place a station in cell i or not. We discretize time and let T denote the set of all discrete time points. We present the variables and parameters used in our model in Table 1 and Table 2, respectively.

Table 1 Decision variables

x_i	Station in cell i turned on/off: binary variable
n_i	Number of cars at station in cell i at start of the day: integer variable
n_i^t	Number of cars at station in cell i at time t; integer variable
$y_i^{t,t'}$	Number of cars assigned to users from station in cell i for the tme interval [t, t'); integer variable
Δ_i	$=\|x_i - x_i^{\text{old}}\|$

Table 2 Parameters

x_i^{old}	Vector of the x_i from the previous optimization iteration (does not exist for the first iteration)
S_i	Maximum capacity of station in cell i, defined for a subset $C' \subseteq C$
M	Large constant ($M \gg \max_{i \in C} \sum_{t \in T} \sum_{t' > t} d_i\{x, t, t'\})$)
c_m	Maintenance cost for a car used for one time period
c_u	User fee for one time period
c_{ms}	Maintenance cost for a used station
R	Cost of depreciation for one vehicle for T
$r_i^{t,t'}$	= c_u (t'-t); revenue for CSO from a user trip for the time interval [t,t')
$c_i^{t,t'}$	= c_m (t'-t); cost for CSO for a user trip for the time interval [t,t')
$\Delta_{\max}$	Upper bound on the Hamming distance between the current and the previous solution vector
$S_{\max}$	Upper bound on the number of stations that might be placed

$$max \sum_{i \in C} \sum_{t_1 \in T} \sum_{t_2 \in T, t_2 < t_1} \left(r_i^{t_1,t_2} - c_i^{t_1,t_2}\right) \cdot y_i^{t_1,t_2} - \sum_{i \in C} x_i \cdot c_{ms} - \sum_{i \in C} R \cdot n_i \quad (1)$$

$$y_i^{t_k,t_l} \le d_i(x, t_k, t_l) \qquad \forall i \in C, \forall t_k < t_l \in T \quad (2)$$

$$y_i^{t_k,t_l} \leq x_i \cdot M \quad \forall i \in C, \forall t_k < t_l \in T \tag{3}$$
$$n_i \leq x_i \cdot M \quad \forall i \in C, \forall t_k < t_l \in T \tag{4}$$
$$n_i^t = n_i - \sum_{t_k<t} \sum_{t_l>t} y_i^{t_k,t_l} \quad \forall i \in C, \forall t \in T \tag{5}$$
$$n_i^t \geq \sum_{t_k>t} y_i^{t,t_k} \quad \forall i \in C, \forall t \in T \tag{6}$$
$$n_i^t \leq S_i \quad \forall i \in C', \forall t \in T \tag{7}$$
$$n_i \leq S_i \quad \forall i \in C', \forall t \in T \tag{8}$$
$$\sum_{i \in C} x_i \leq S_{\max} \tag{9}$$
$$\Delta_i \geq x_i - x_i^{\text{old}} \quad \forall i \in C \tag{10}$$
$$\Delta_i \geq x_i^{\text{old}} - x_i \quad \forall i \in C \tag{11}$$
$$\Delta_{\max} \geq \sum_{i \in C} \Delta_i \tag{12}$$

$$y_i^{t_k,t_l} \in Z^+ \ \forall i \in C, \forall t_k < t_l \in T; n_i \ t \in Z^+ \ \forall i \in C, \forall t \in T; n_i \in Z^+ \ \forall i \in C; \Delta_i \in \{0,1\} \ \forall i \in C; x_i \in \{0,1\} \ \forall i \in C$$

Equation (2) ensures that we at most fulfil the demand from any station i for any time interval $[t_k, t_\ell)$, however, we can choose to not fulfil the complete demand in case this does not increase the CSO's profit. Equation (3) ensures that we can only fulfil demand from station i for any time interval if station i is opened. Analogously, Equation (4) enforces that we can only place vehicles at station i if station i is opened. With Equation (5) we bookkeep the number of cars at station i: at time point t, we have the number of cars from the start of the day minus those that left before t and will return after t. Equation (6) enforces that we can only fulfil the demand with existing cars. Equations (7) and (8), given only for a subset C' of cells, ensure an upper bound on the available parking spots at a station in these cells. In case the CSO wants to enforce an upper bound on the total number of open stations, we enforce this using Equation (9).

Equations (10)-(12) are used in all but the first iteration: we aim to limit the deviation of station locations from one iteration to another (such that our linearization still represents the situation sufficiently well). We define the variable Δ_i as the absolute value of $x_i - x_i^{\text{old}}$ and use these variables in Equation (12) to upper bound the Hamming distance between the consecutive solution vectors $\Delta_{\max}$.

3. COMPUTATIONAL EXPERIMENTS: CASE STUDY STOCKHOLM

The model introduced in Section 2.1 is used to investigate CS usage in the Greater Stockholm region. After adding CS, mode-specific alternative specific constants were recalibrated to recover observed modal shares, including approx. 1% of CS usage. In this section, we summarize the simulation results.

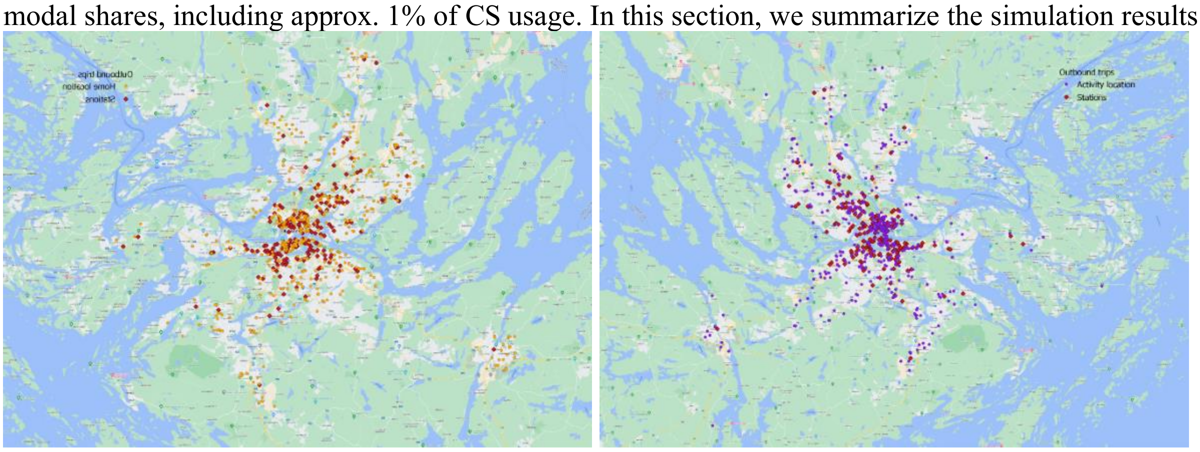

Figure 2: Spatial distribution of CS users' home locations, used stations, and travel destinations

Travelers access stations that are nearby their home locations (Fig. 2 left) compared to their destinations (2. right), which is what one would expect. Figure 3 (left) indicates that CS users have relatively high incomes, which is compatible with the observation that employed persons are overrepresented among CS users (Figure 3 right) and consistent with survey data (Martin & Shaheen, 2011). Gender plays no identifiable role in CS choice, neither does (somewhat surprisingly) car ownership.

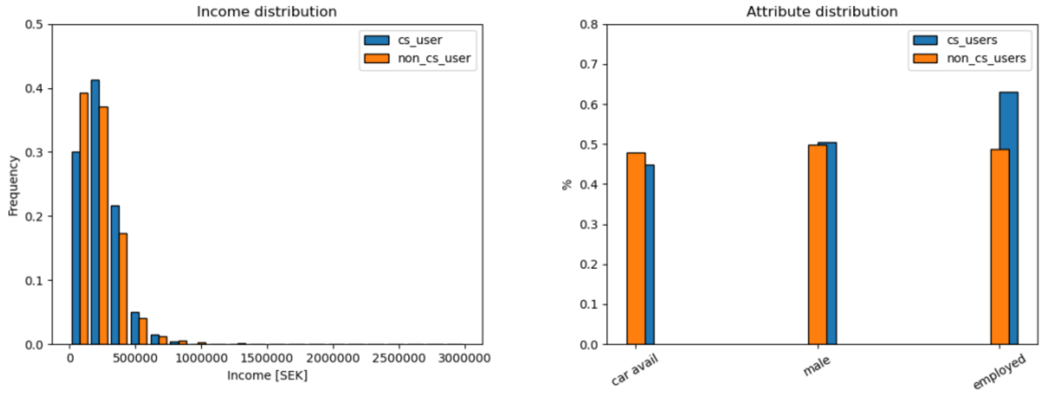

Figure 3: Socio-demographics of CS users vs. non-users

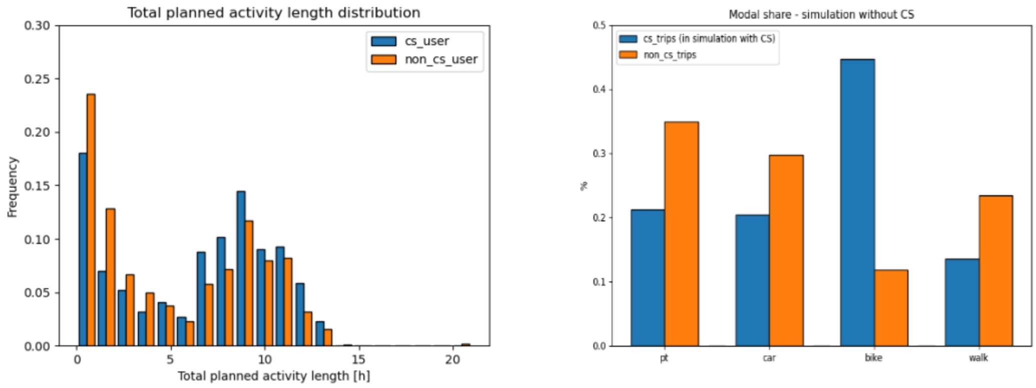

Figure 4: Activity participation of (non) CS users (left); modes replaced by CS if available (right)

Figure 4 (left) indicates that CS users tend to spend more time participating in activities than other persons; again an observation that is compatible with relatively high employment in that group. Finally, Figure 4 (right) shows in orange the modal share over the entire population (except CS) and in blue the modal share of all CS users in a scenario where CS is not available—i.e., the distribution according to which CS attracts travelers from other modes. The observation that the present model that CS mainly draws from is biking is, however, to be taken with some care; the behavioral model is only coarsely calibrated and does not account for attitudinal aspects such as and health and environmental benefits of biking over CS.

4. SUMMARY AND OUTLOOK

We presented one of the first combined simulation-optimization approaches for the design of a CS system. We used a MIP formulation to alter the CS-system design: the station locations. This optimization necessitates information on CS demand and network performance. Hence, we integrated our MIP model with an evaluation model based on MATSim: this enables us to generate the demand and to evaluate the performance of an altered CS system. We showed how we achieve this coupling in an iterative approach—by iterating between supply optimization and demand estimation we improve the realism of the assumptions made when solving each problem individually.

We presented first computational results for the case study of Greater Stockholm to verify our model assumption. In future work, we will present the case-study results for the integrated approach.

While our approach successfully yields realistic assumptions in comparison to separate models, we only change the station locations in a round-trip system. However, as detailed in Section 1, the integration of (some) one-way trips into a round-trip system would represent a larger share of all trip types and hence, potentially yields more active CS users. We plan to study this mixed system in the future.

5. ACKNOWLEDGMENTS

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