k-Transmitter Watchman Routes (and Some Guarding Problems)

Christiane Schmidt

NYU Geometry Seminar, December 06, 2022

LINKÖPING UNIVERSITY

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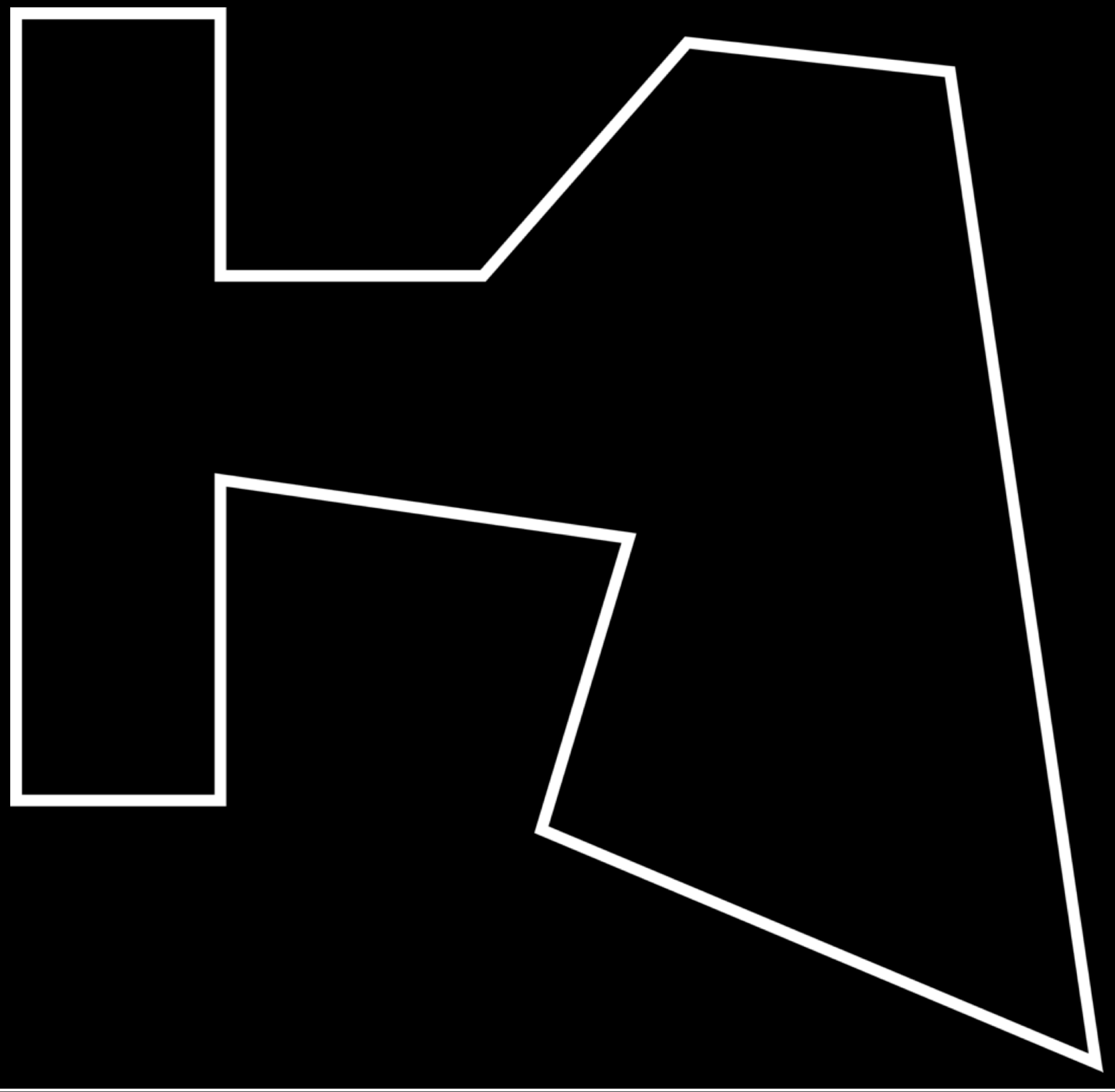
Agenda

- *k*-Transmitters
- The Watchman Route Problem (WRP)
- *k*-Transmitter Watchman Routes
- Open Problem: *k*-Transmitters
- Outlook

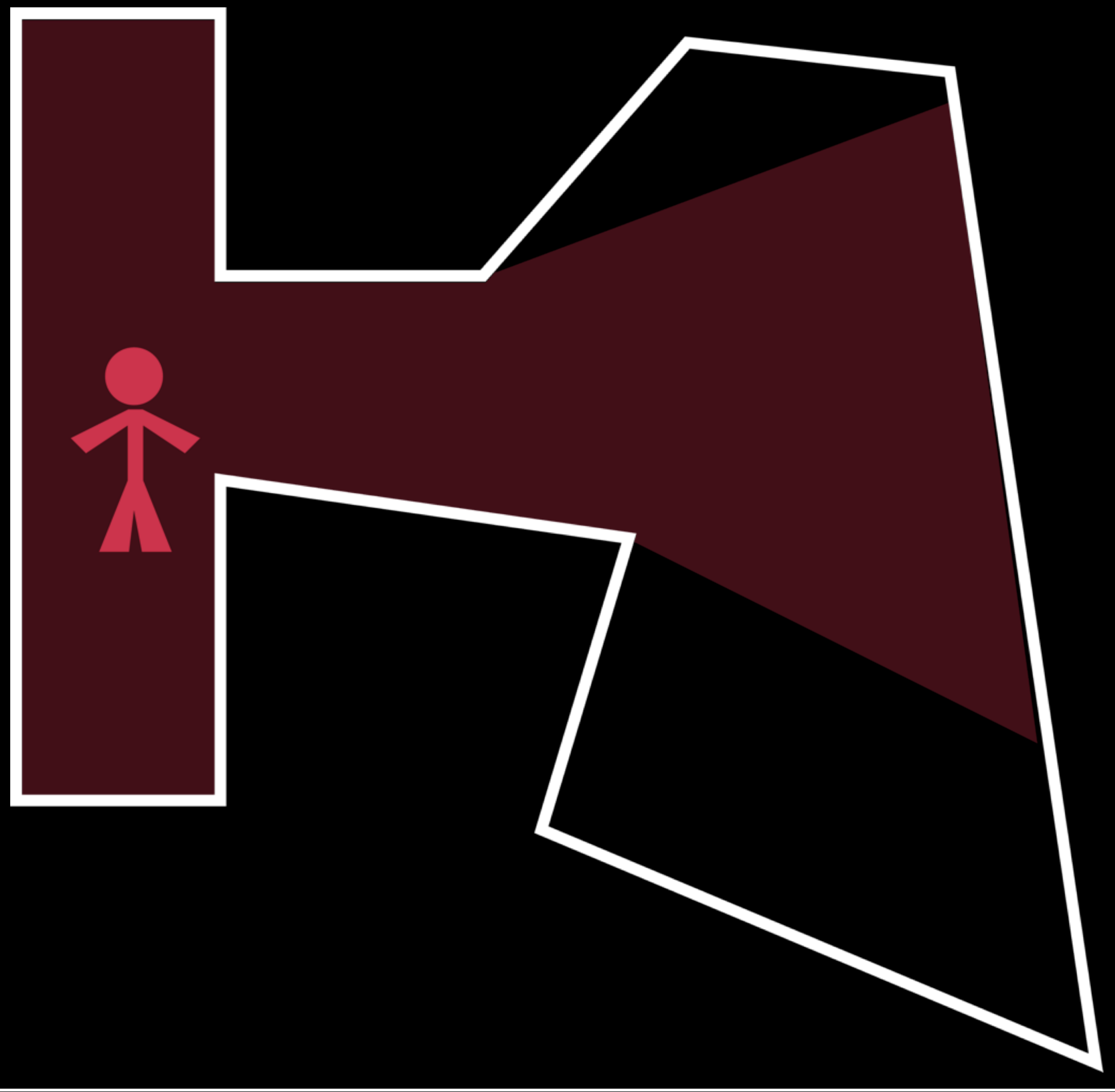
The Art Gallery Problem (AGP)



Given: Polygon P

How many guards do we need to monitor P?

The Art Gallery Problem (AGP)



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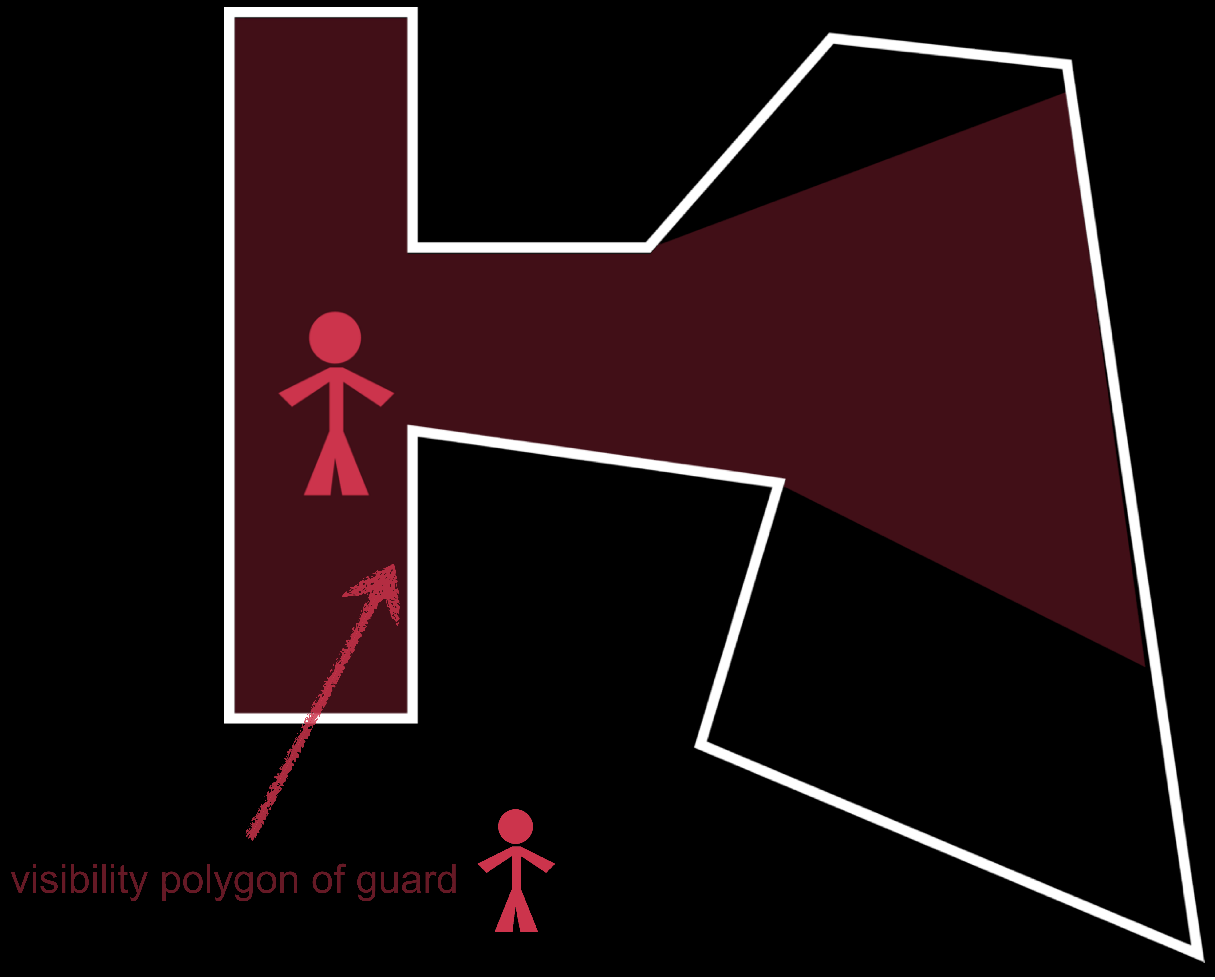

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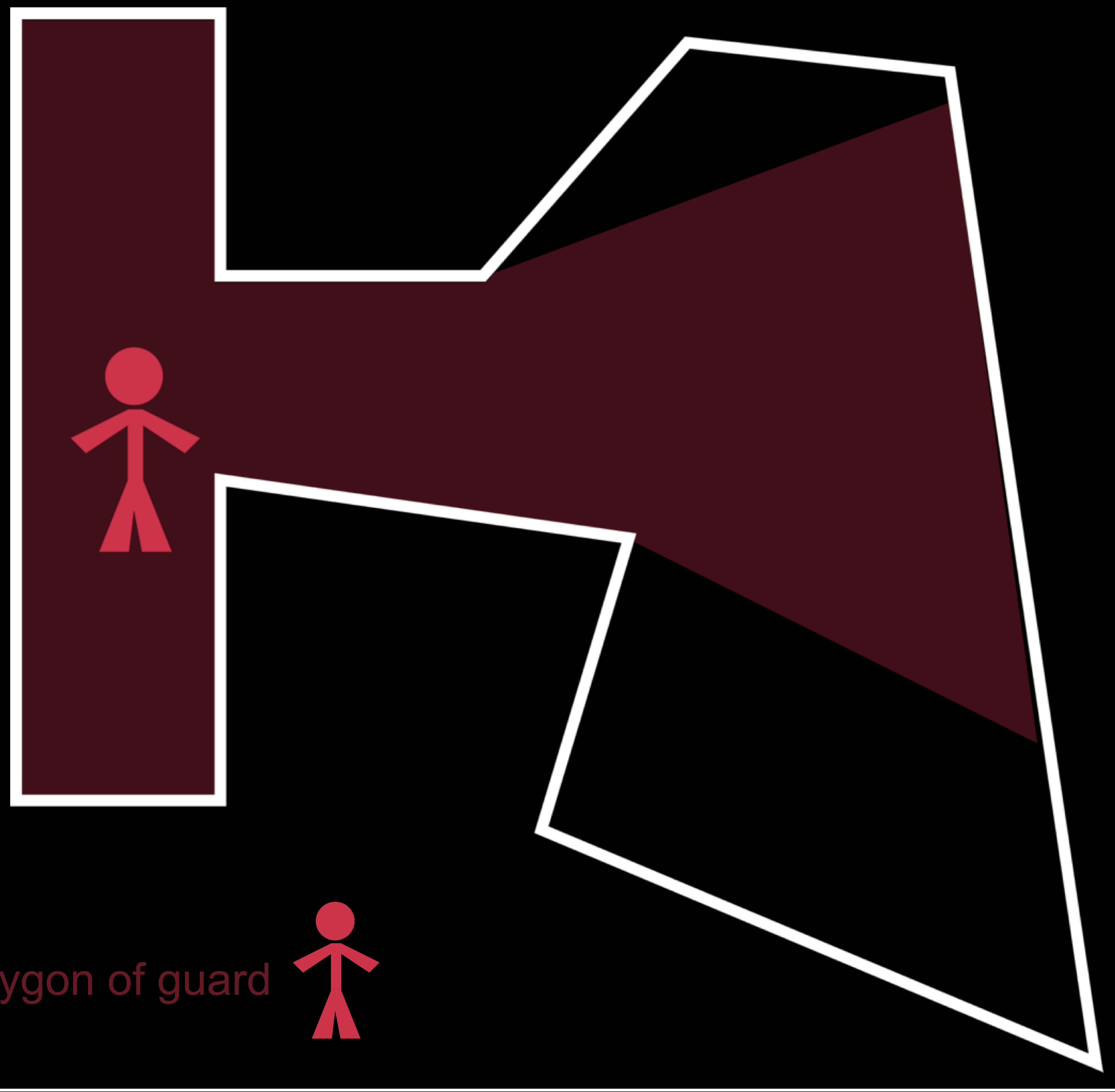

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visibility polygon of guard

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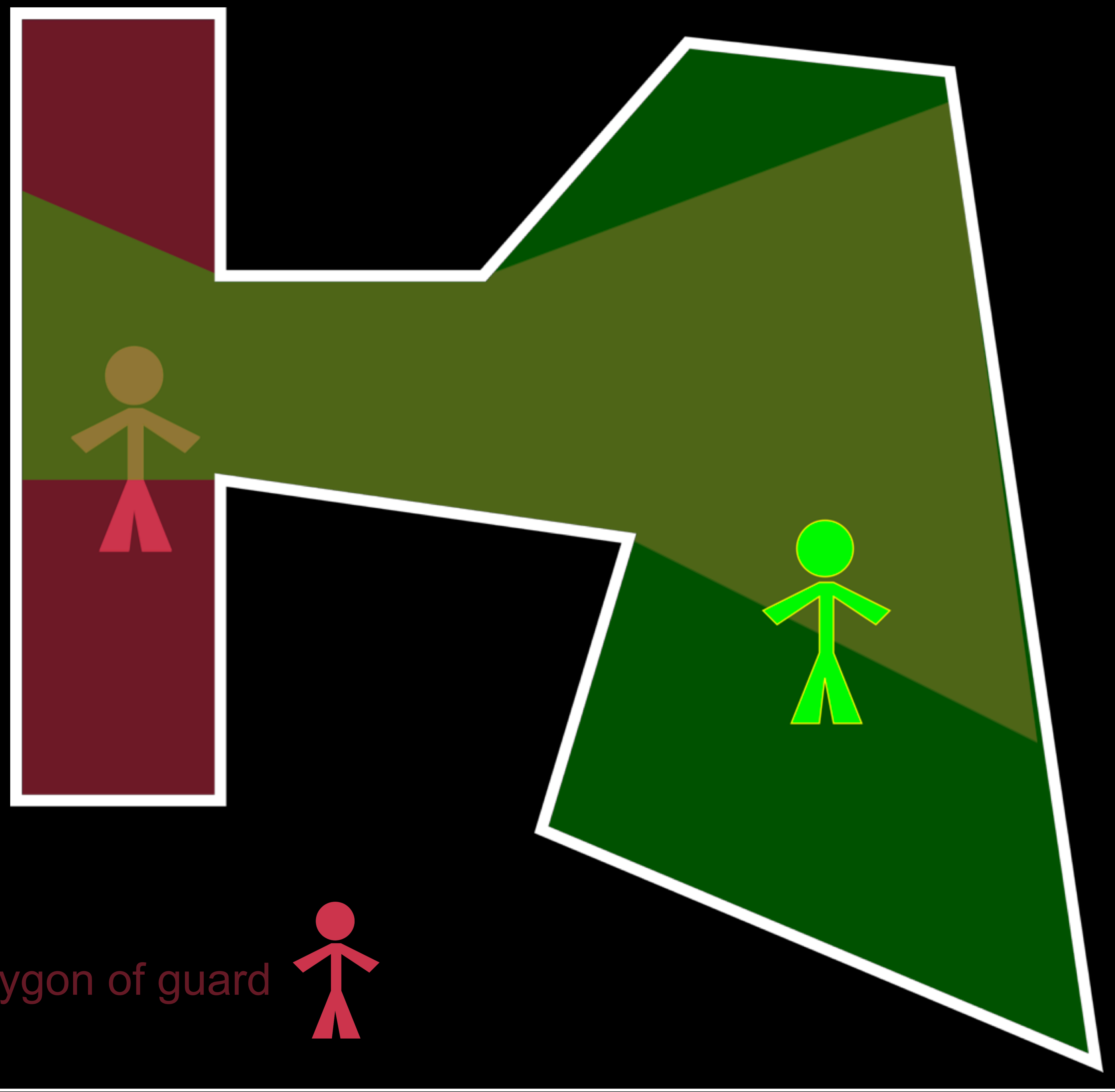

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k-Transmitters

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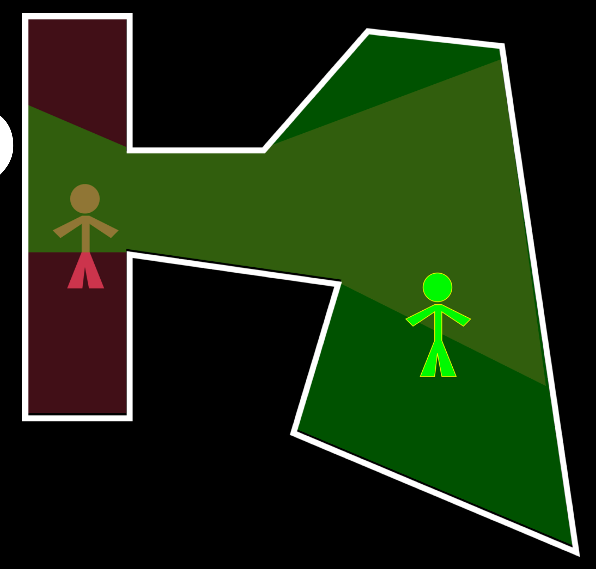

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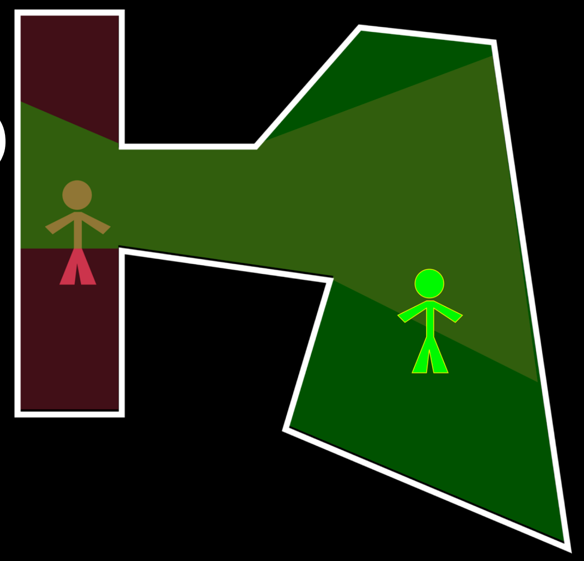

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- Capabilities of the guards
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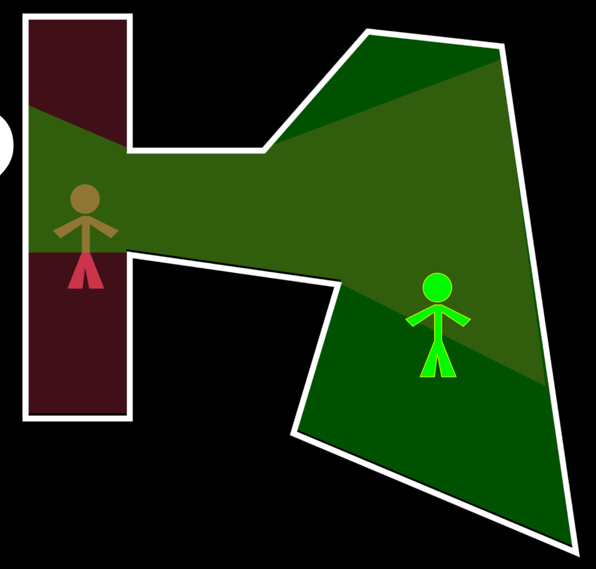

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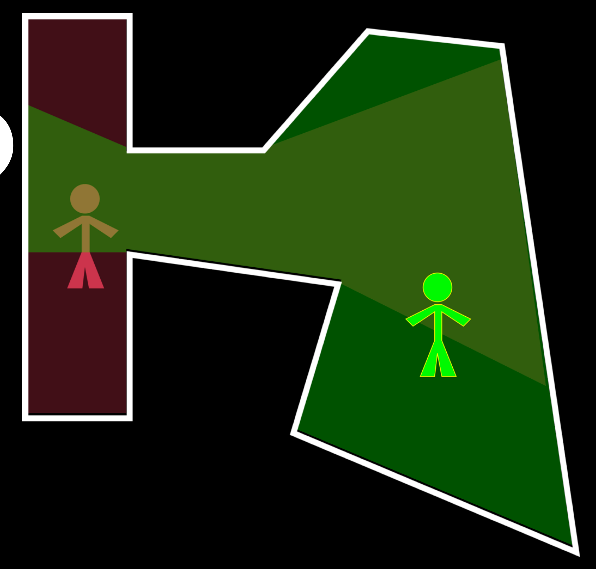

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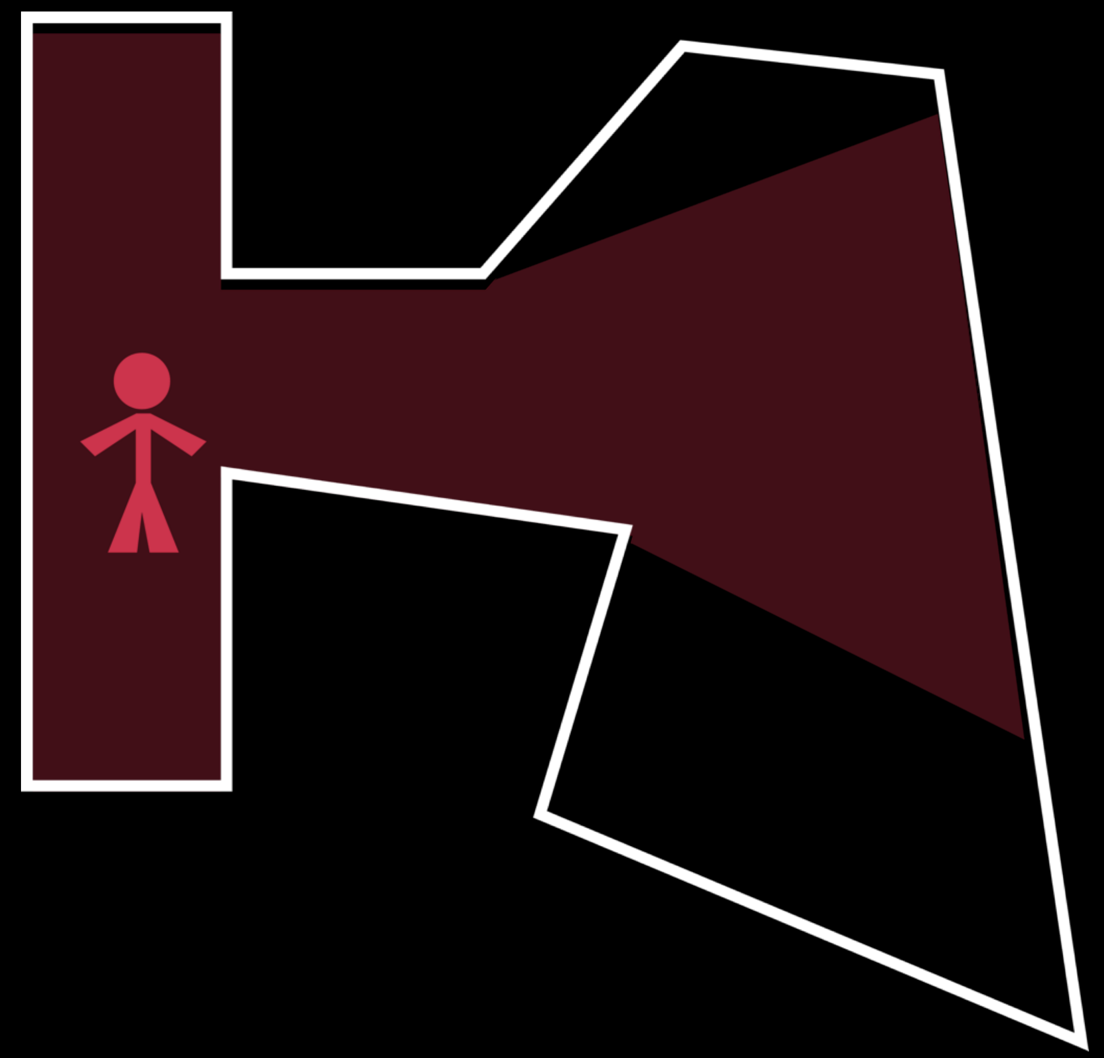

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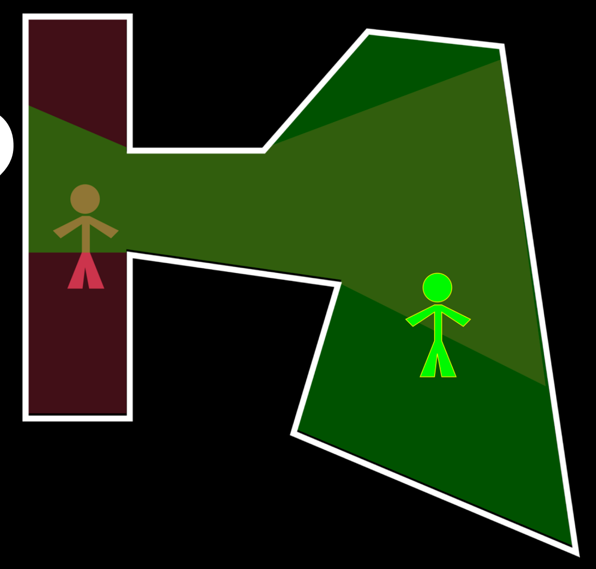

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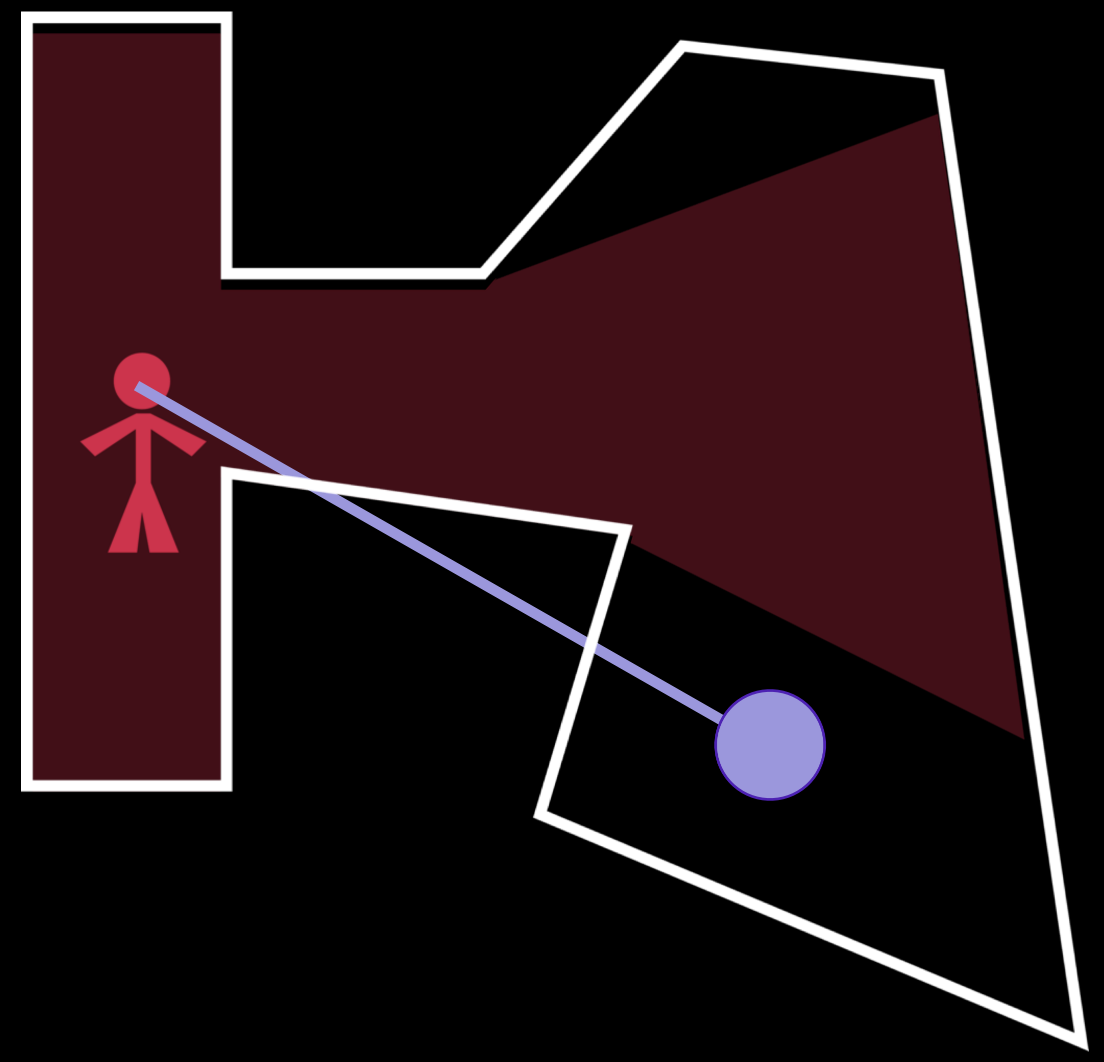

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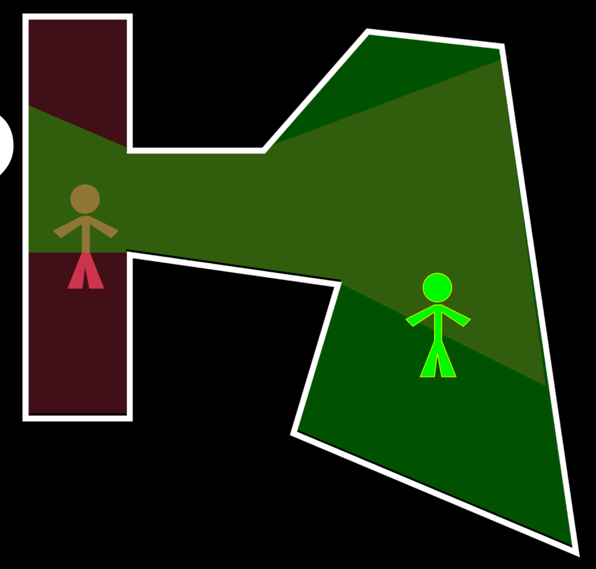

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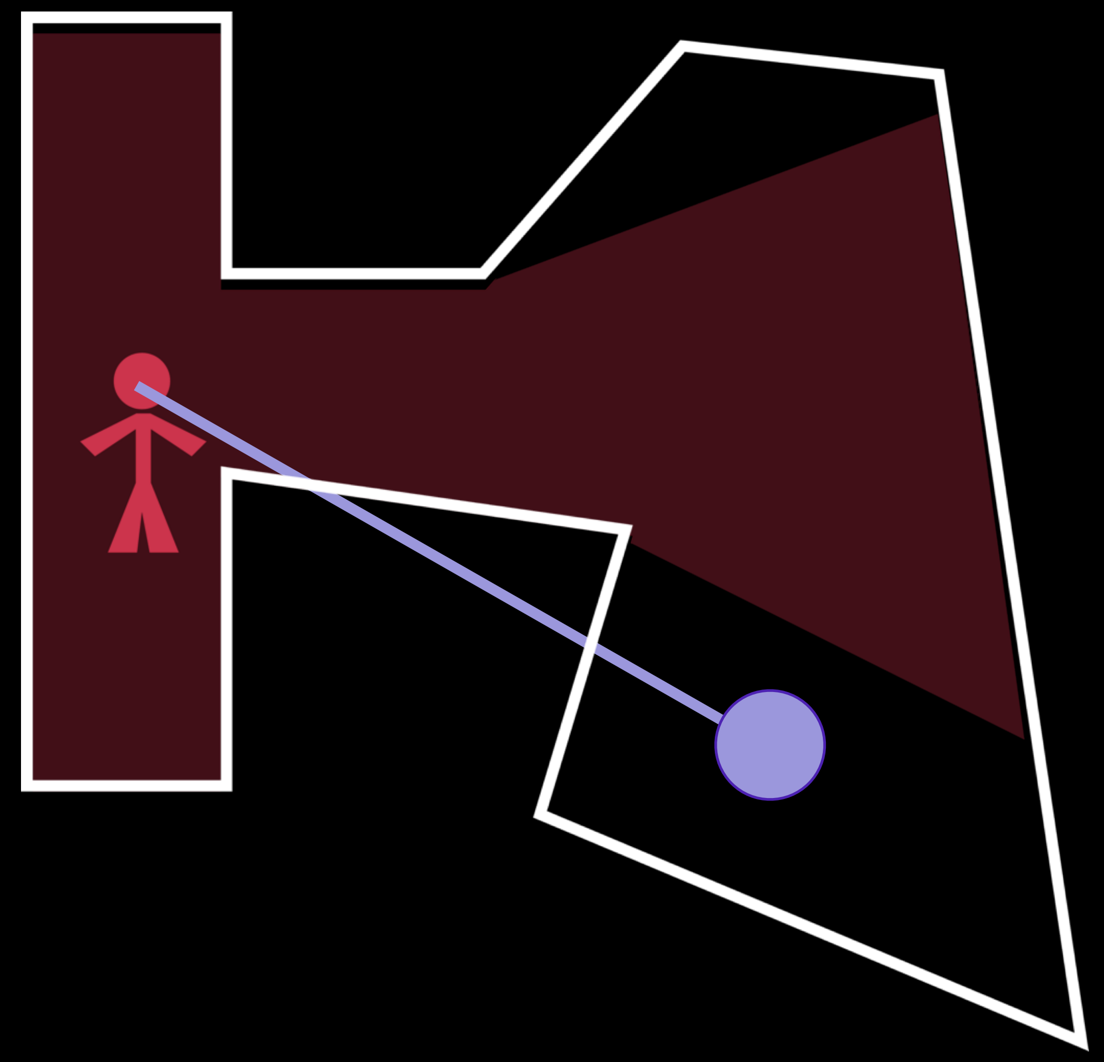

Line crosses at most 2 walls
⇒visible from the 2-transmitter

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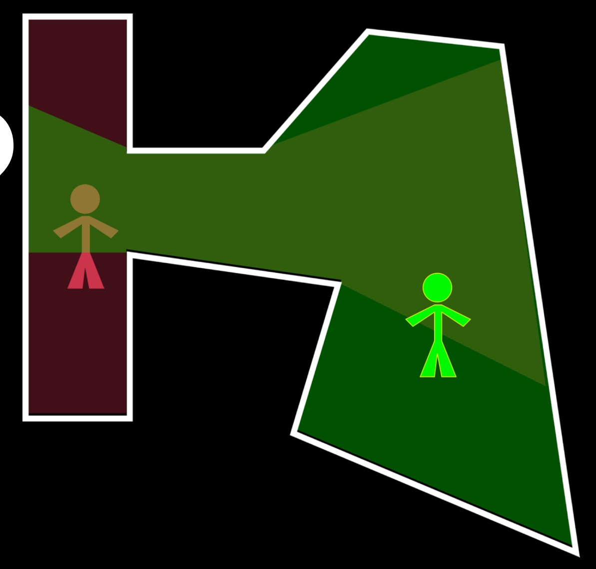

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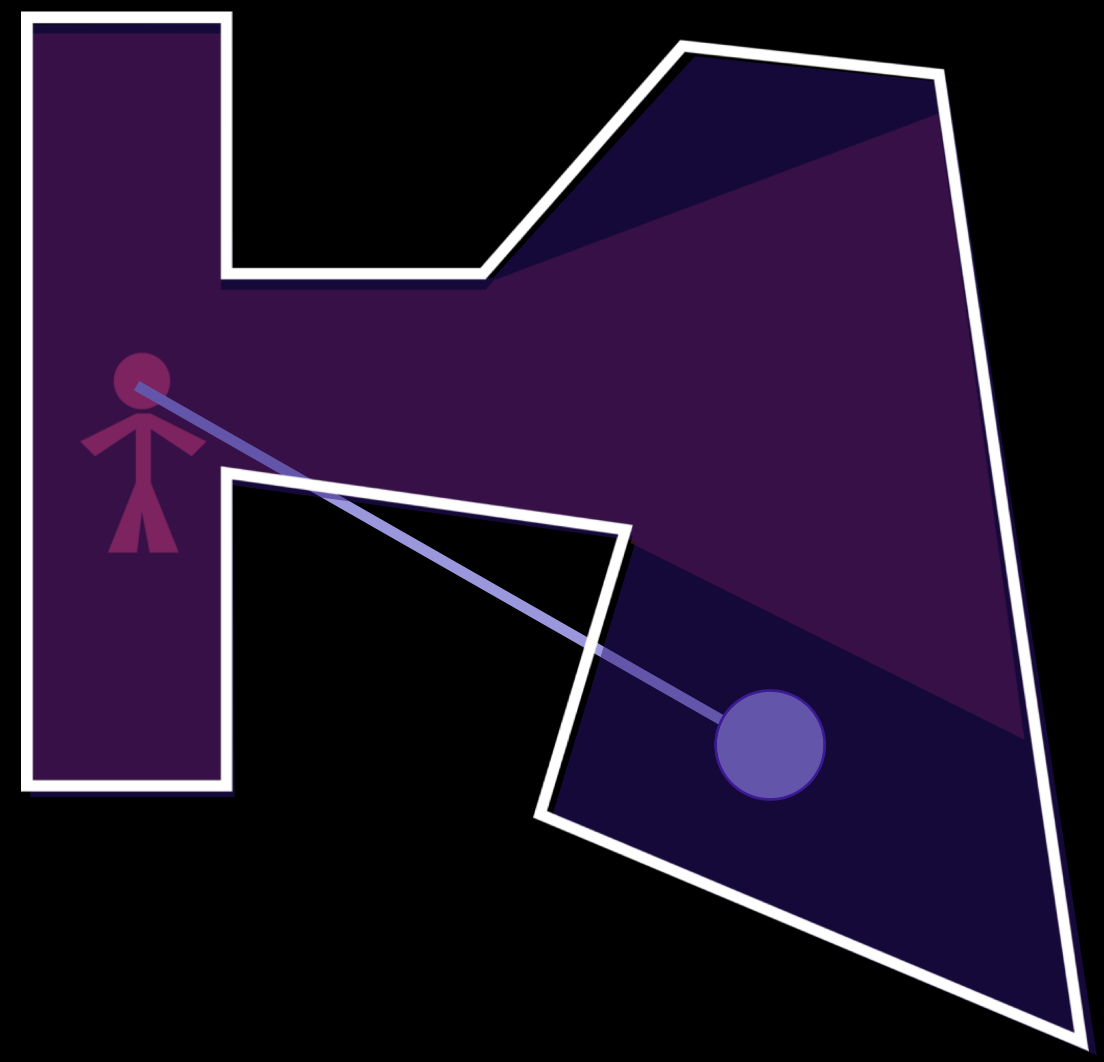

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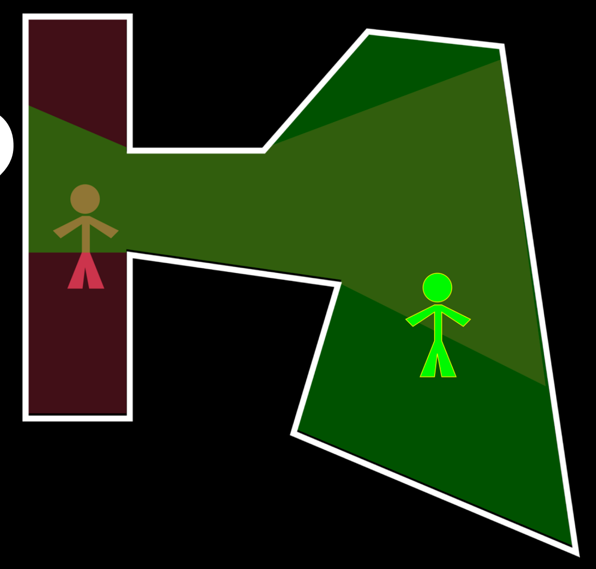

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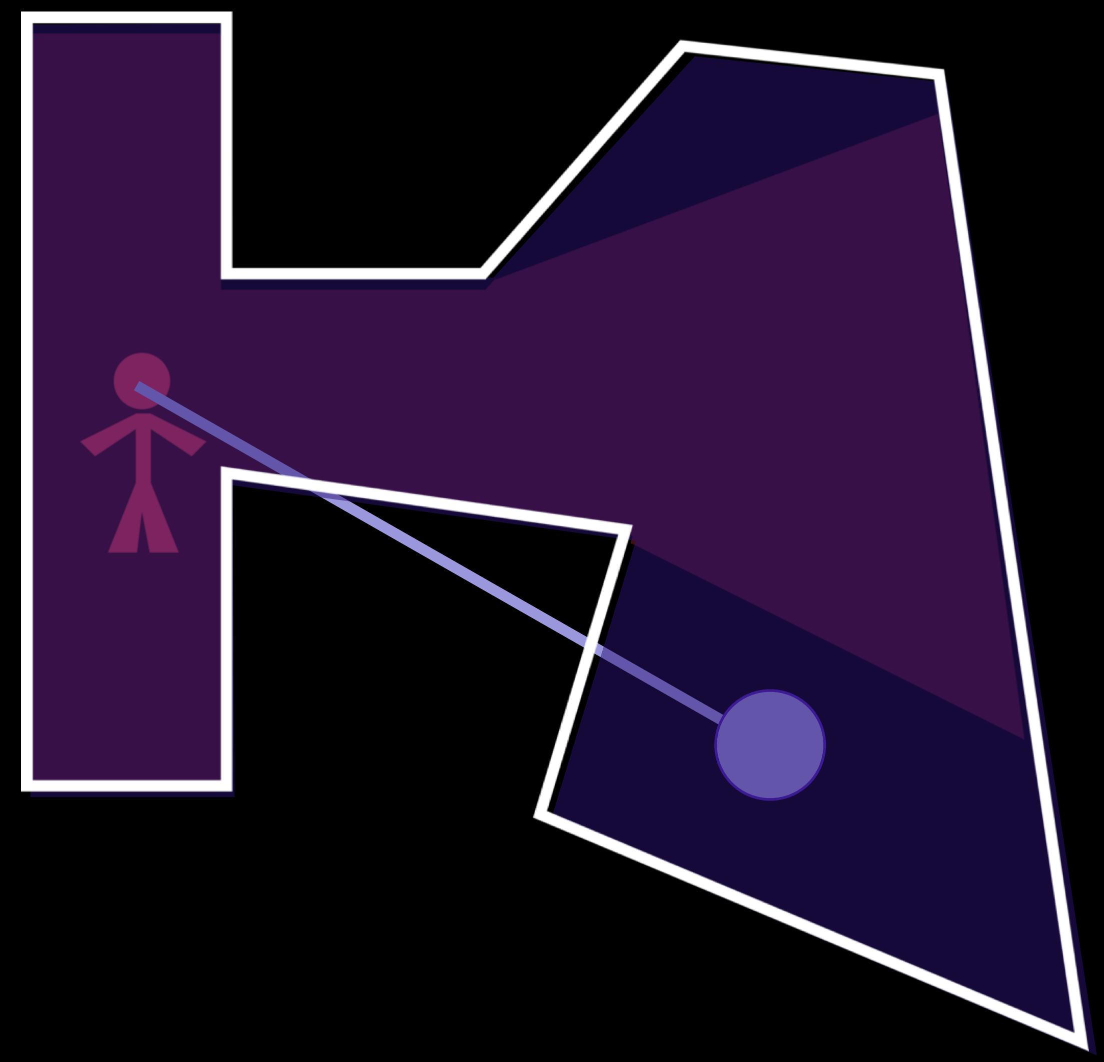

Line crosses at most 2 walls
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Formally: a point p is **2(k)-visible** from a point q, if the straight line connection pq intersects P in at most two **(k)** connected components.

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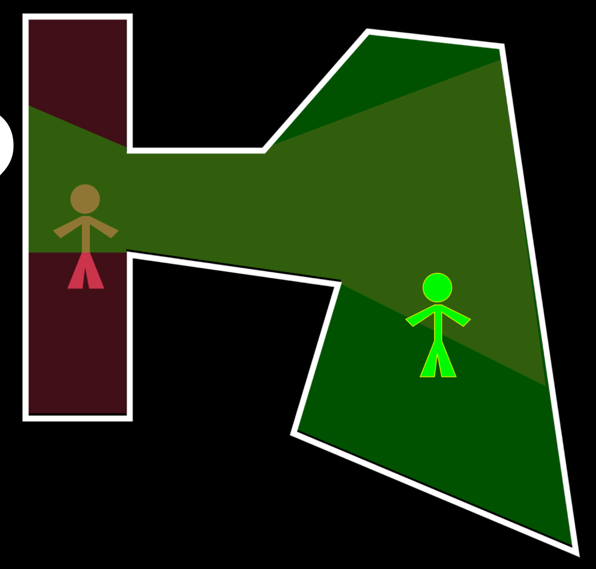

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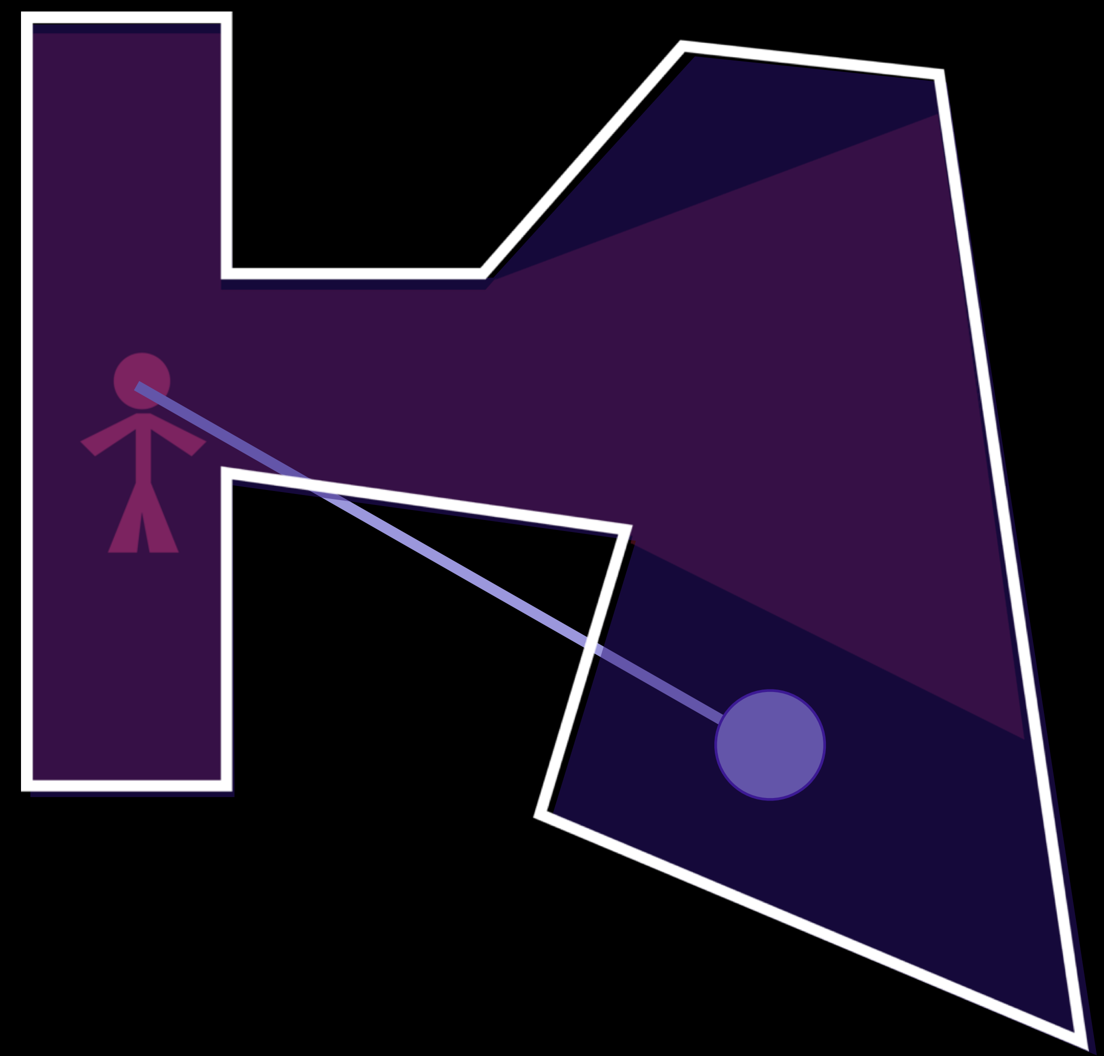

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Formally: a point *p* is **2(k)-visible** from a point *q*, if the straight line connection *pq* intersects *P* in at most two **(k)** connected components.

2VR(p) = set of points in P, 2-visible from p
*k*VR(*p*) = set of points in P, *k*-visible from *p*

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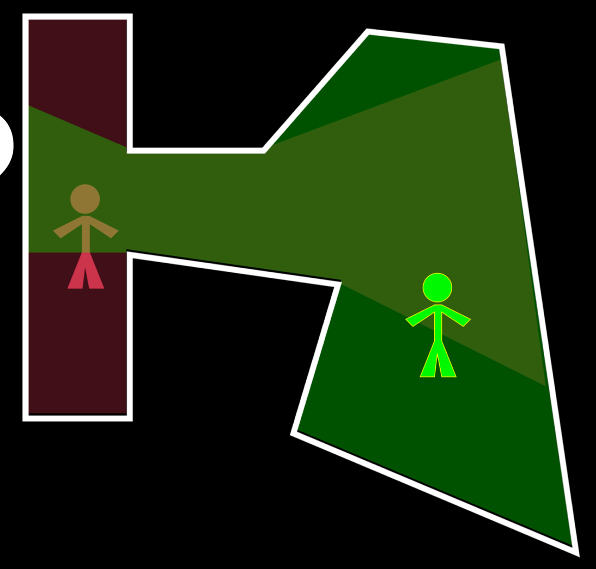

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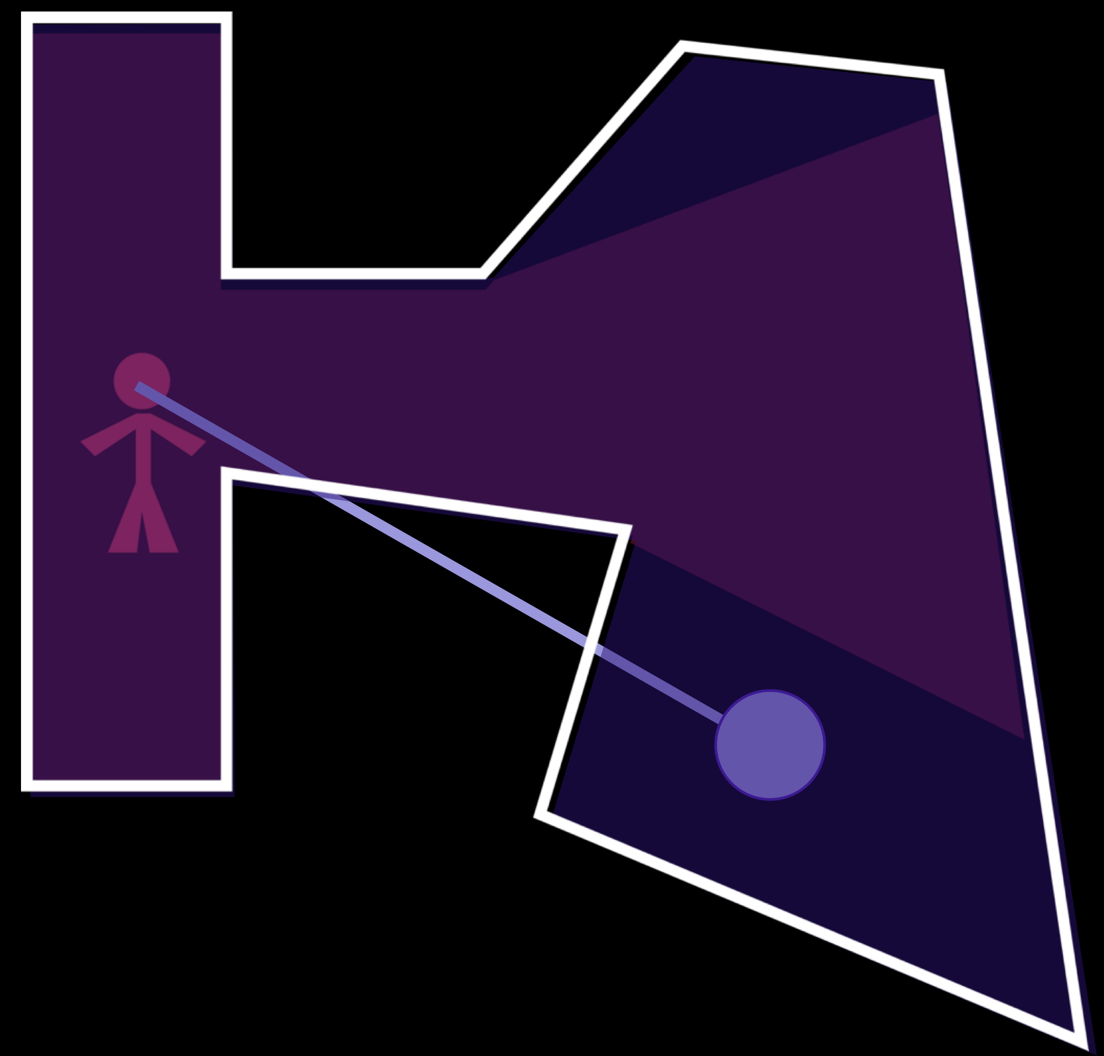

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Stationary:

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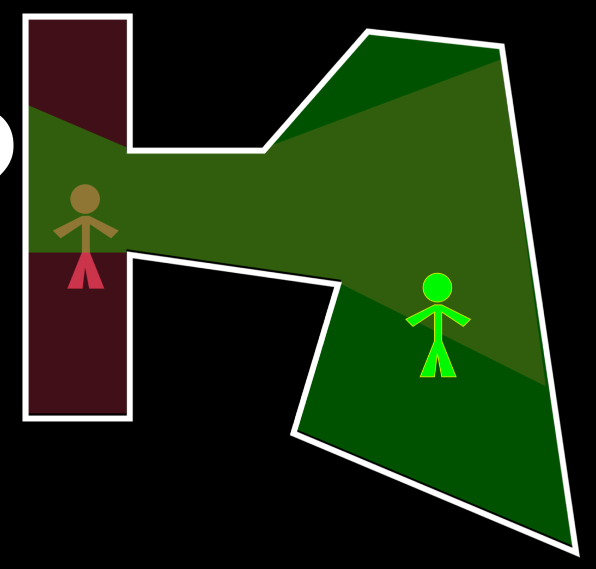

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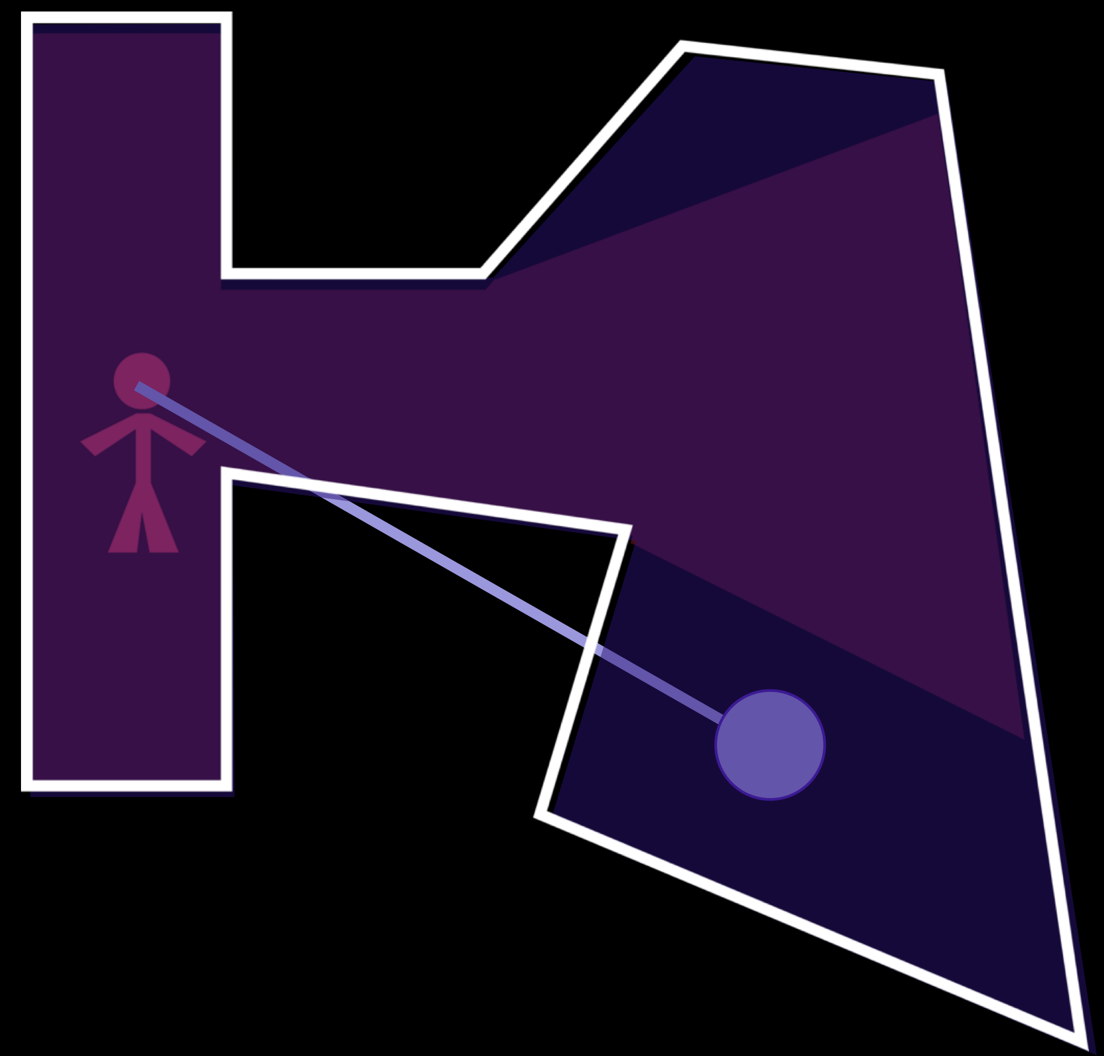

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Stationary:

A set C is a *2-transmitter cover*: $2\mathrm{VR}(C) = \cup_{p \in C} 2\mathrm{VR}(p) = P$

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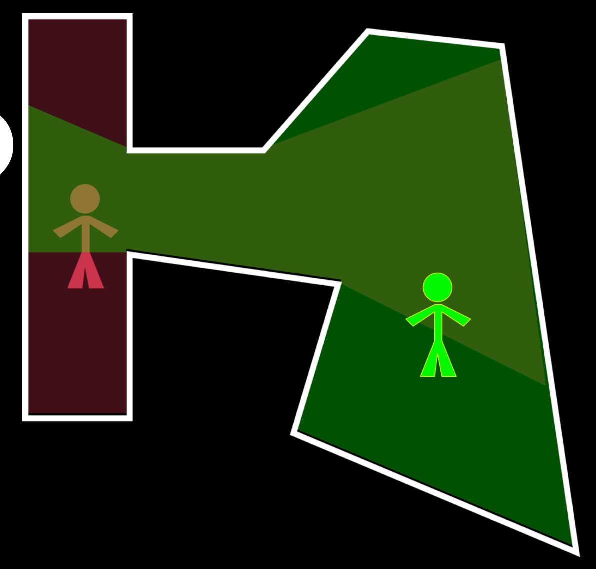

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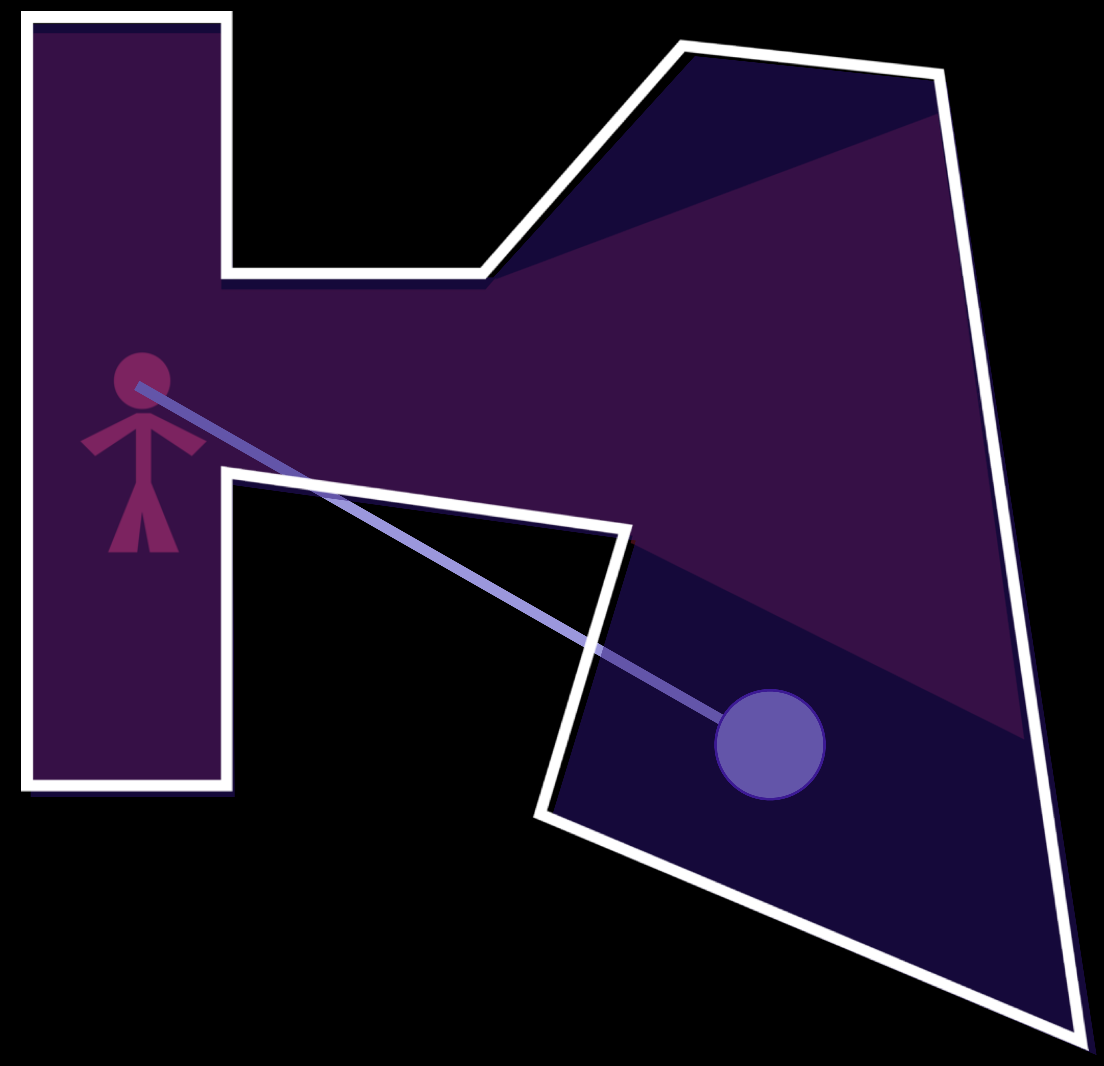

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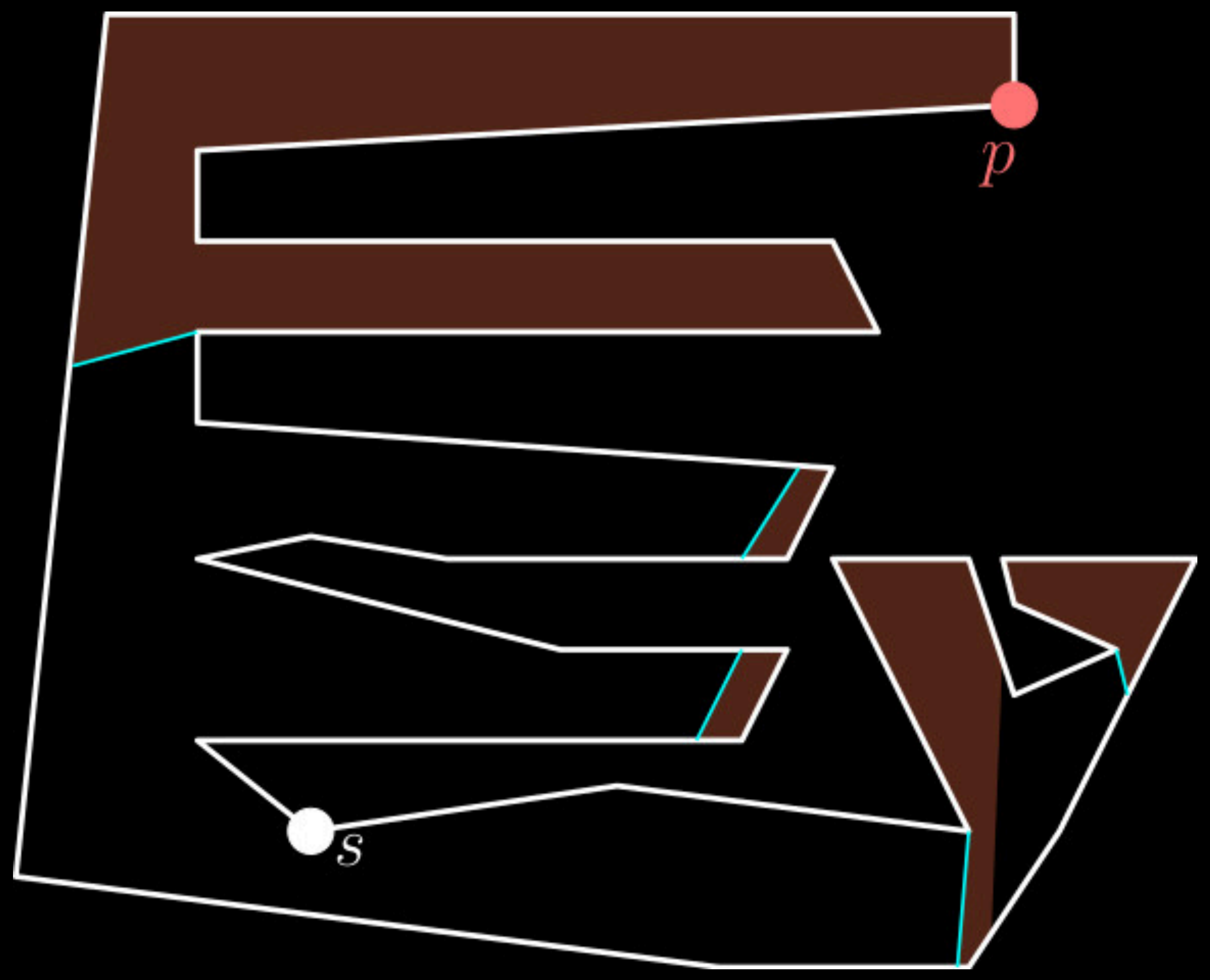
A set C is a *2-transmitter cover*: $2\mathrm{VR}(C) = \cup_{p \in C} 2\mathrm{VR}(p) = P$
A set *C* is a *k*-transmitter cover: $k\mathrm{VR}(C) = \cup_{p \in C} k\mathrm{VR}(p) = P$

k-/2-Transmitter



2VR(p)/kVR(p) can have O(n) connected components.

k-Transmitters

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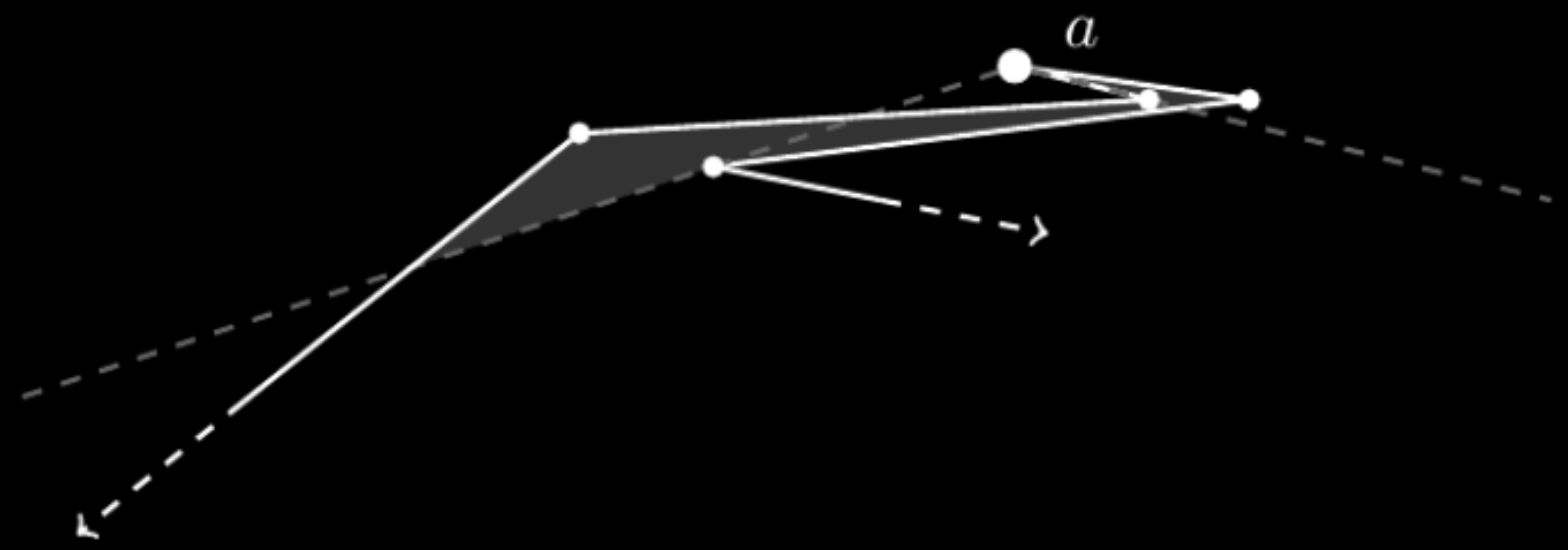

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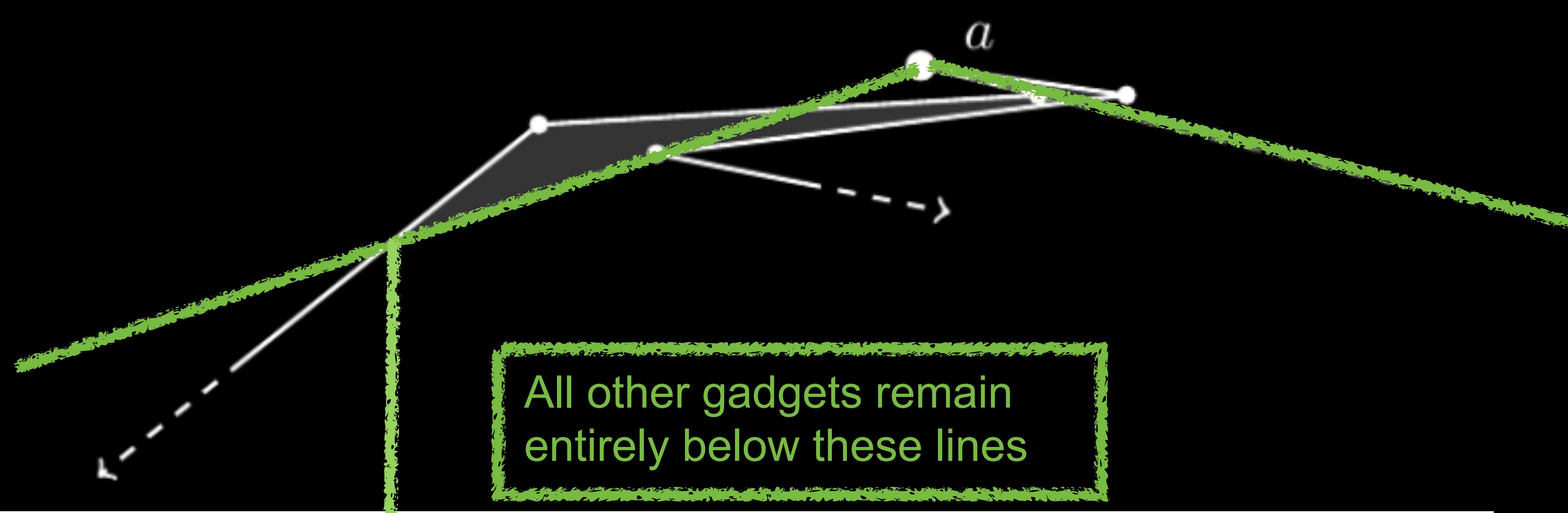

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- Minimum 2-/k-transmitter cover:

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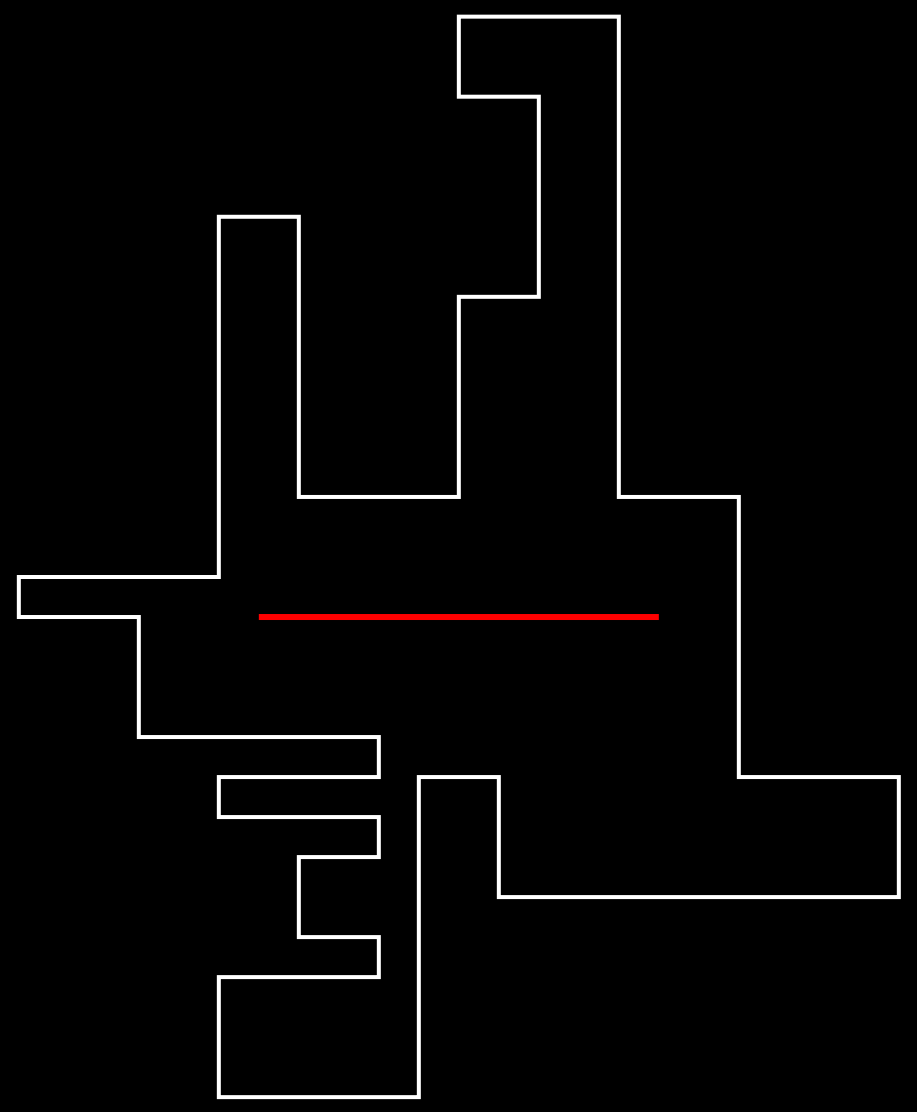
- "Art Gallery Theorems"
  - AFFHUV2018: tight bounds for monotone and monotone orthogonal polygons ($\lceil \frac{n-2}{2k+3} \rceil$ k-transmitters are sometimes necessary and always sufficient to cover a monotone n-gon)
  - BBBDDDFHILMSSU2010:
    - Bounds for line segments in the plane
    - Lower bound of $\lfloor \frac{n}{6} \rfloor$ 2-transmitters to cover a simple n-gon
  - CFIL**S**2018:
    - Upper and lower bounds for # edge 2-transmitters in simple, monotone, orthogonal, orthogonal monotone polygons
    - Lower bound of $\lfloor \frac{n}{5} \rfloor$ 2-transmitters to cover a simple n-gon
- Minimum 2-/k-transmitter cover:
  - CFIL**S**2018: NP-hard to compute point 2-transmitter/point k-transmitter/edge 2-transmitter cover in simple polygon, point 2-transmitter also for orthogonal polygons

AFFHUV2018: Oswin Aichholzer, Ruy Fabila-Monroy, David Flores-Peñaloza, Thomas Hackl, Jorge Urrutia, and Birgit Vogtenhuber. Modem illumination of monotone polygons.

BBBDDDFHILMSSU2010: Brad Ballinger, Nadia Benbernou, Prosenjit Bose, Mirela Damian, ErikD. Demaine, Vida Dujmovic, Robin Flatland, Ferran Hurtado, John Iacono, Anna Lubiw, Pat Morin, Vera Sacristán, Diane Souvaine, and Ryuhei Uehara. Coverage with k-transmitters in the presence of obstacles.

CFIL**S**2018: Sarah Cannon, Thomas G. Fai, Justin Iwerks, Undine Leopold, and Christiane Schmidt. Combinatorics and complexity of guarding polygons with edge and point 2-transmitters.

k-Transmitters



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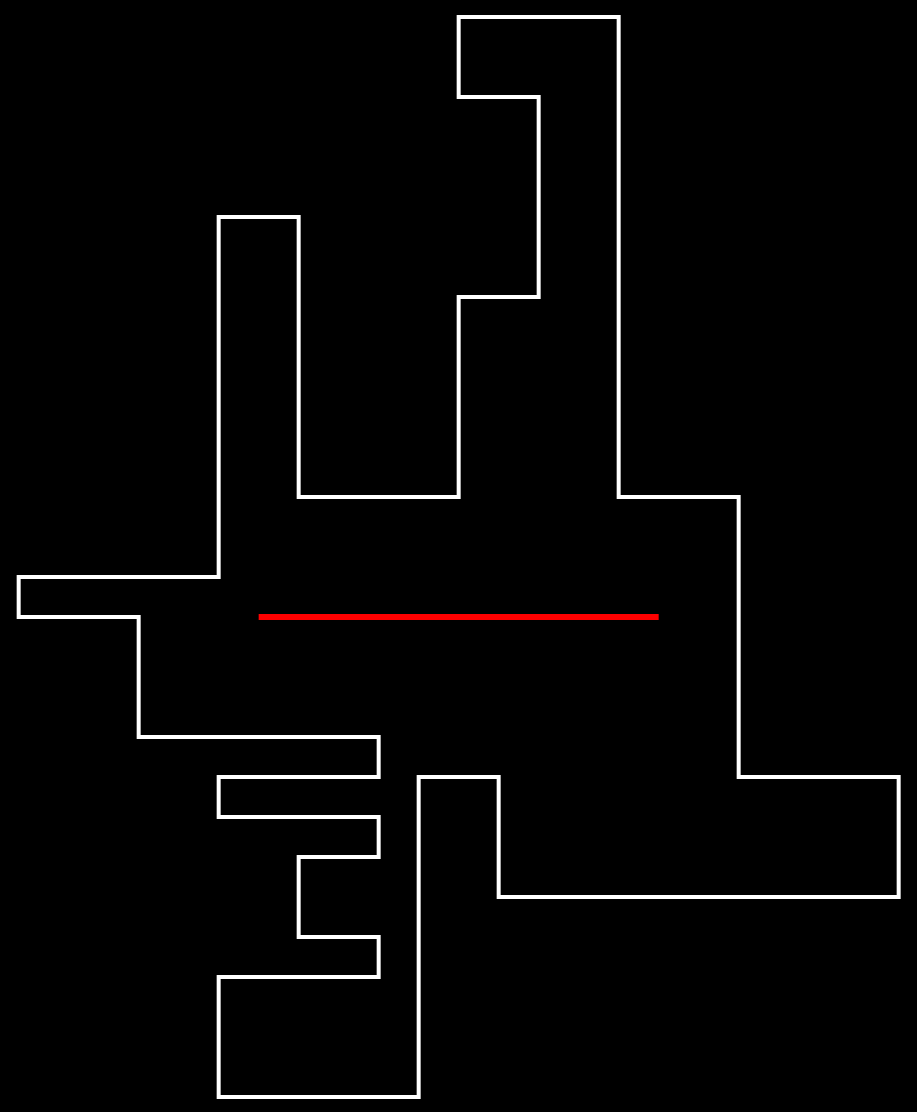

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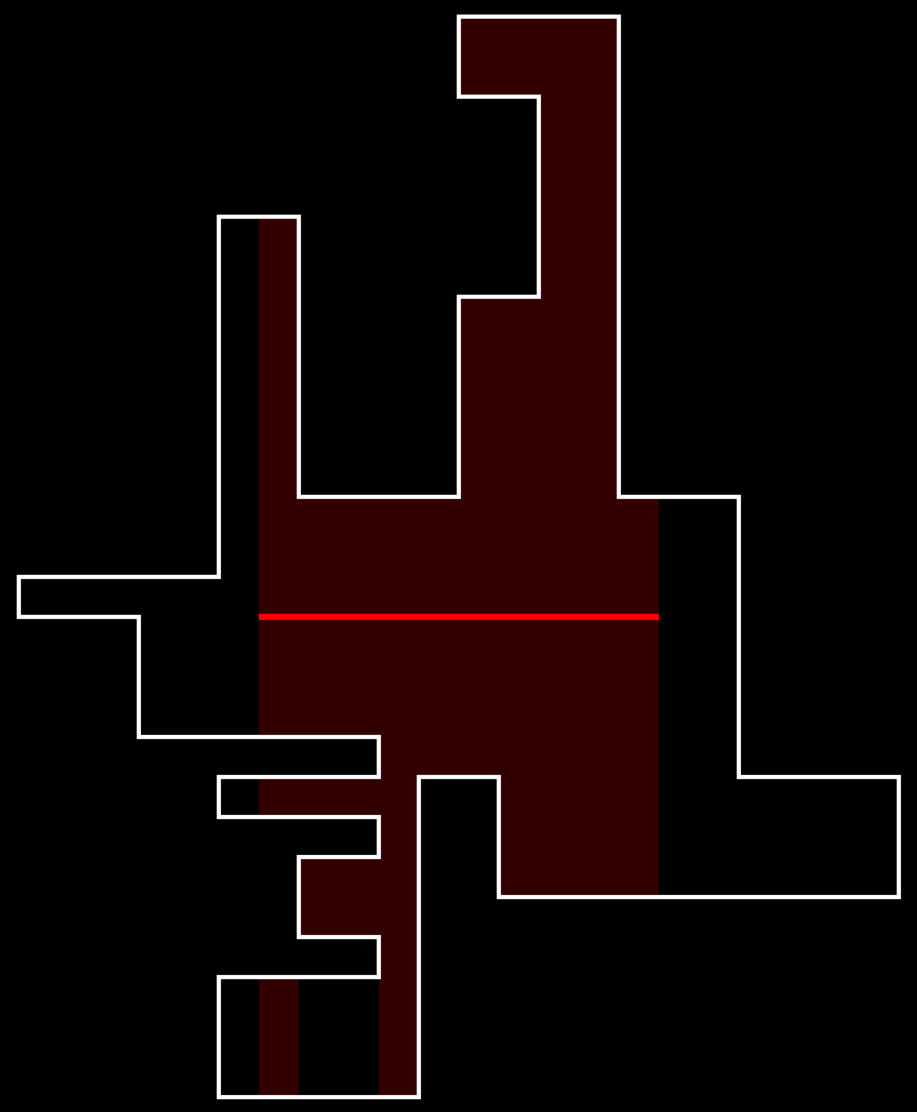

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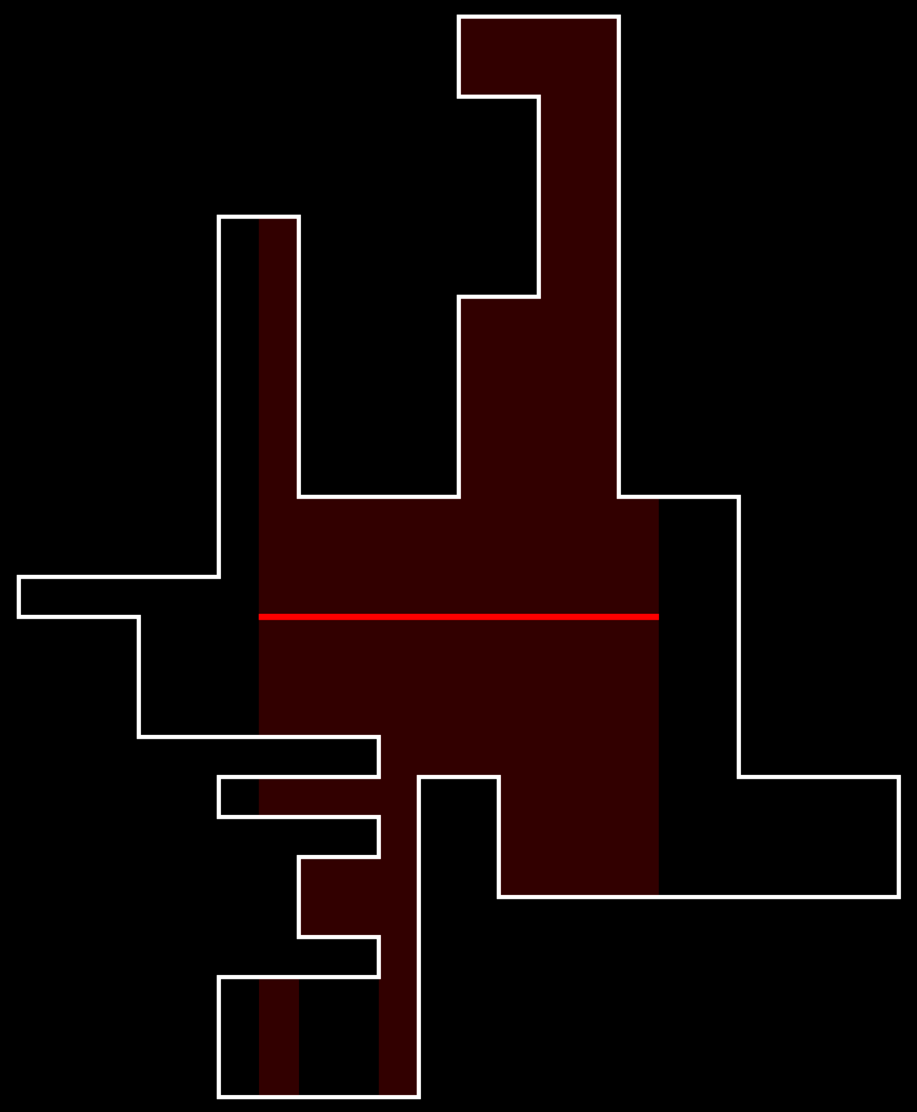

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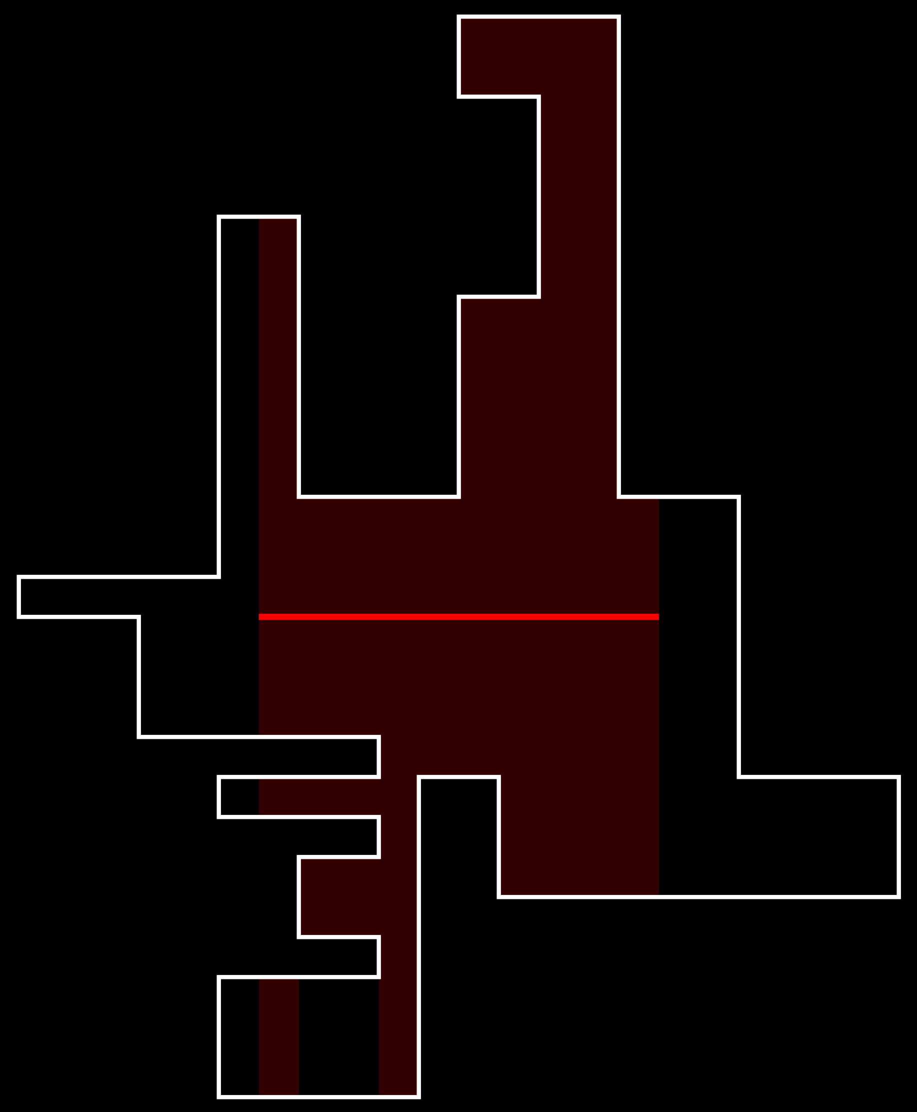

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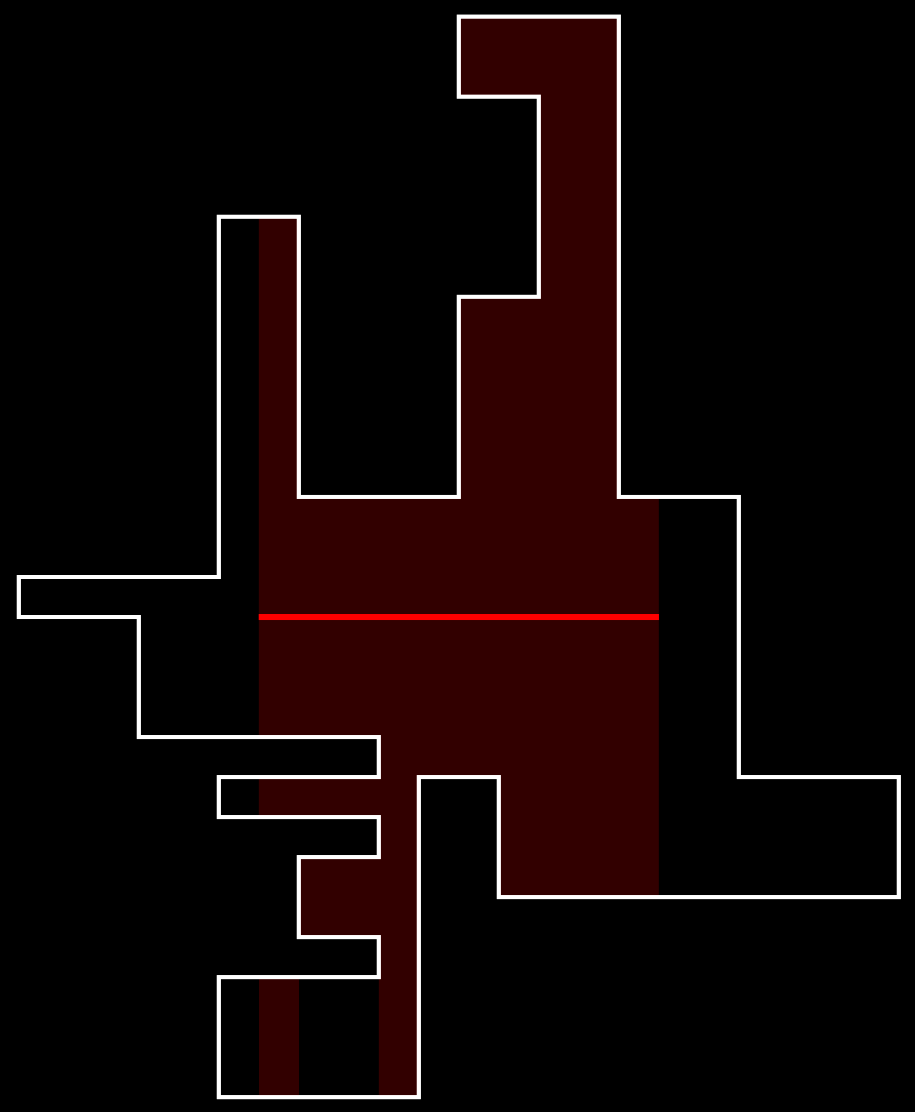
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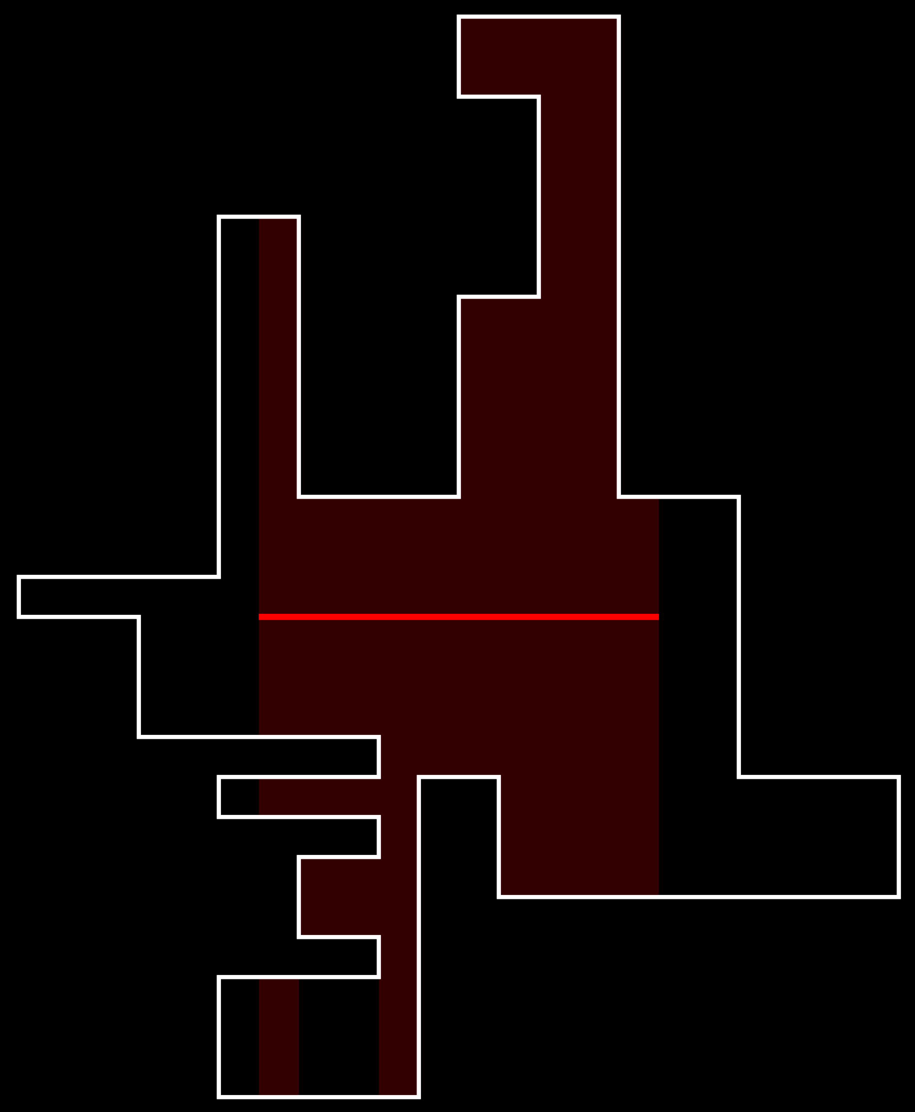

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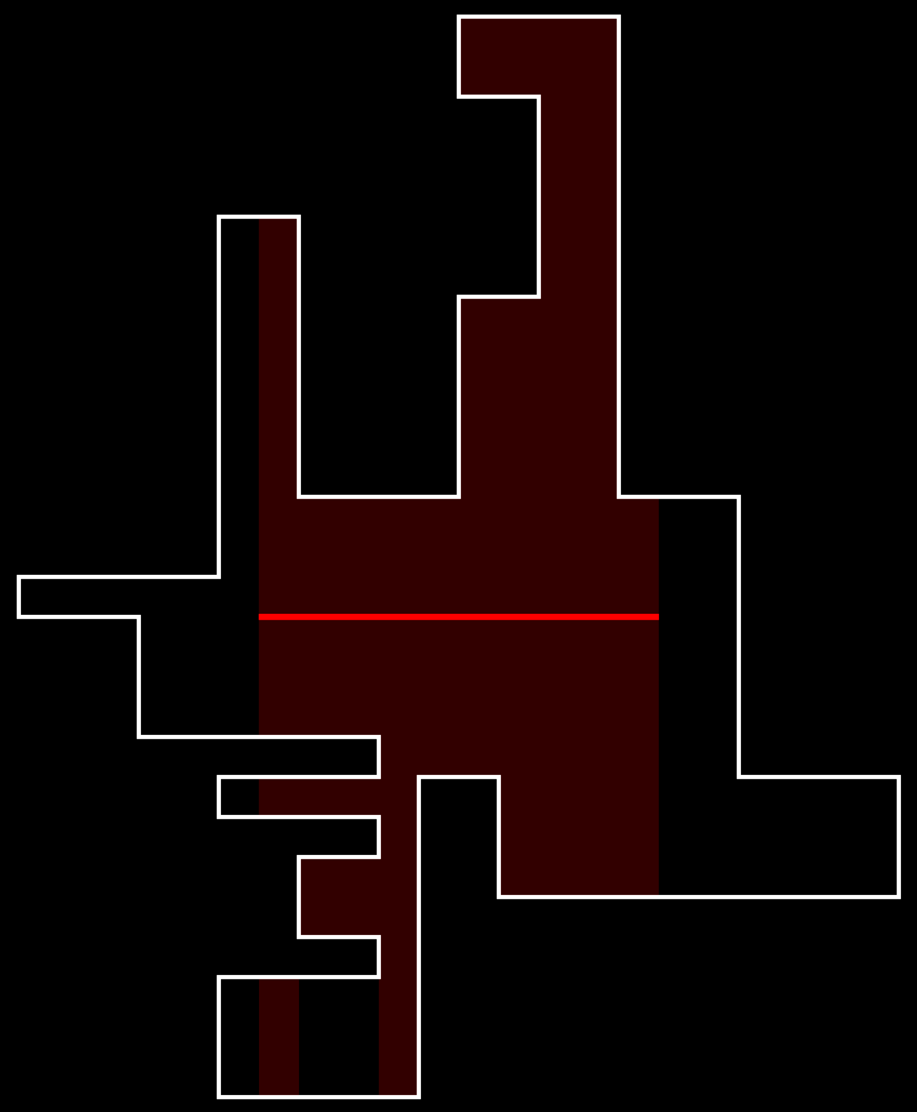

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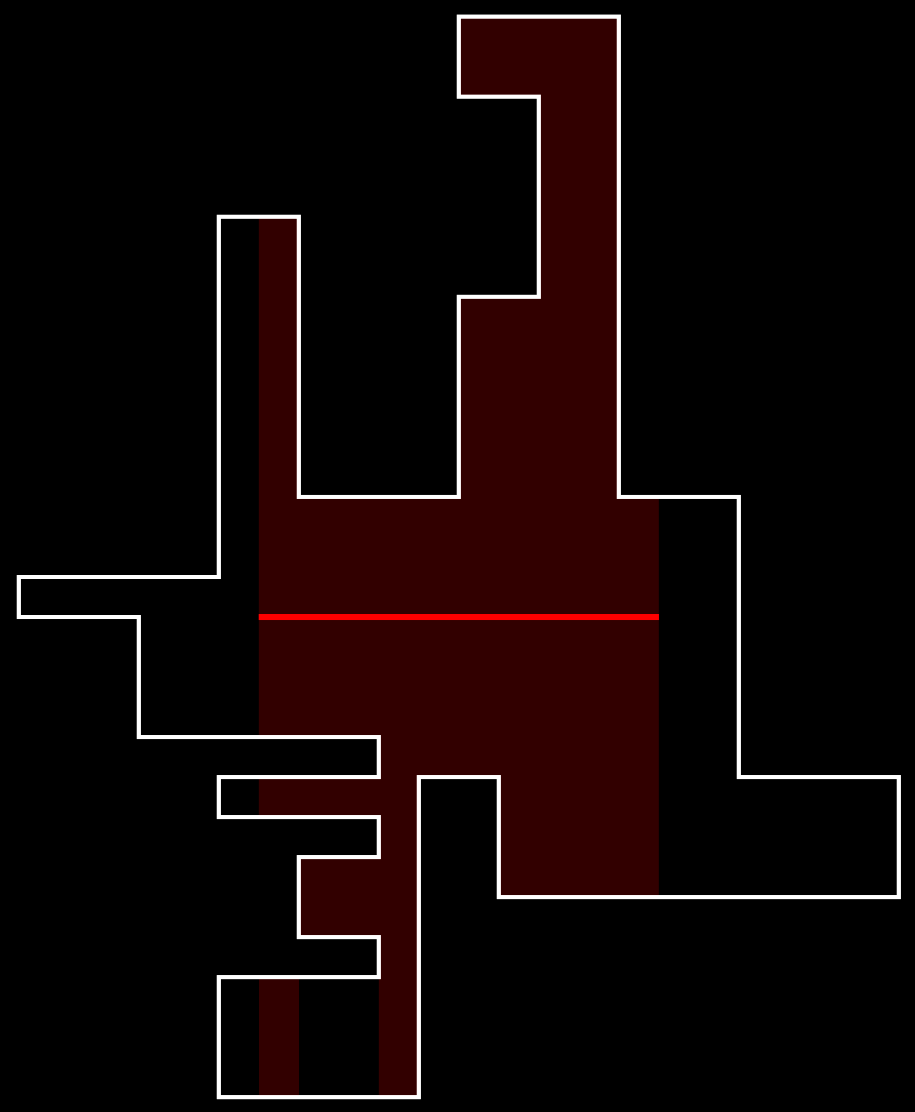

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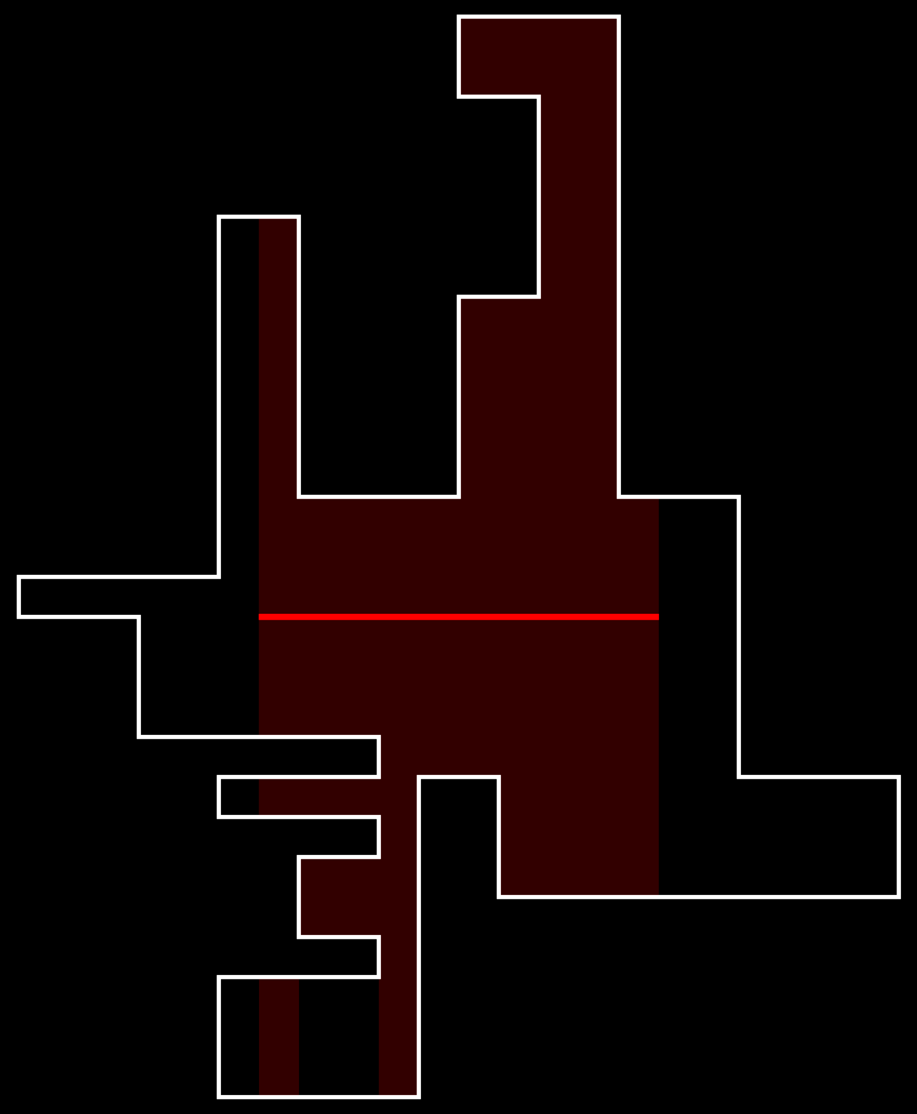

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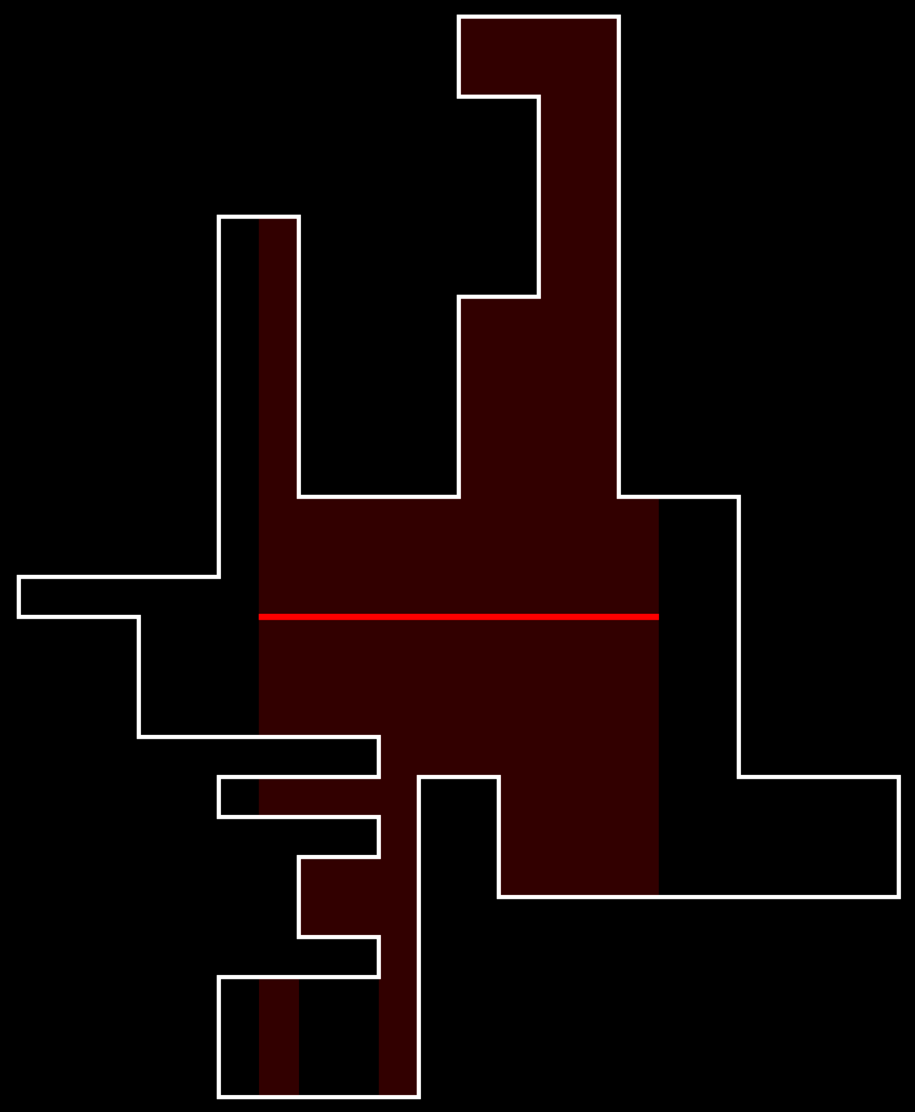

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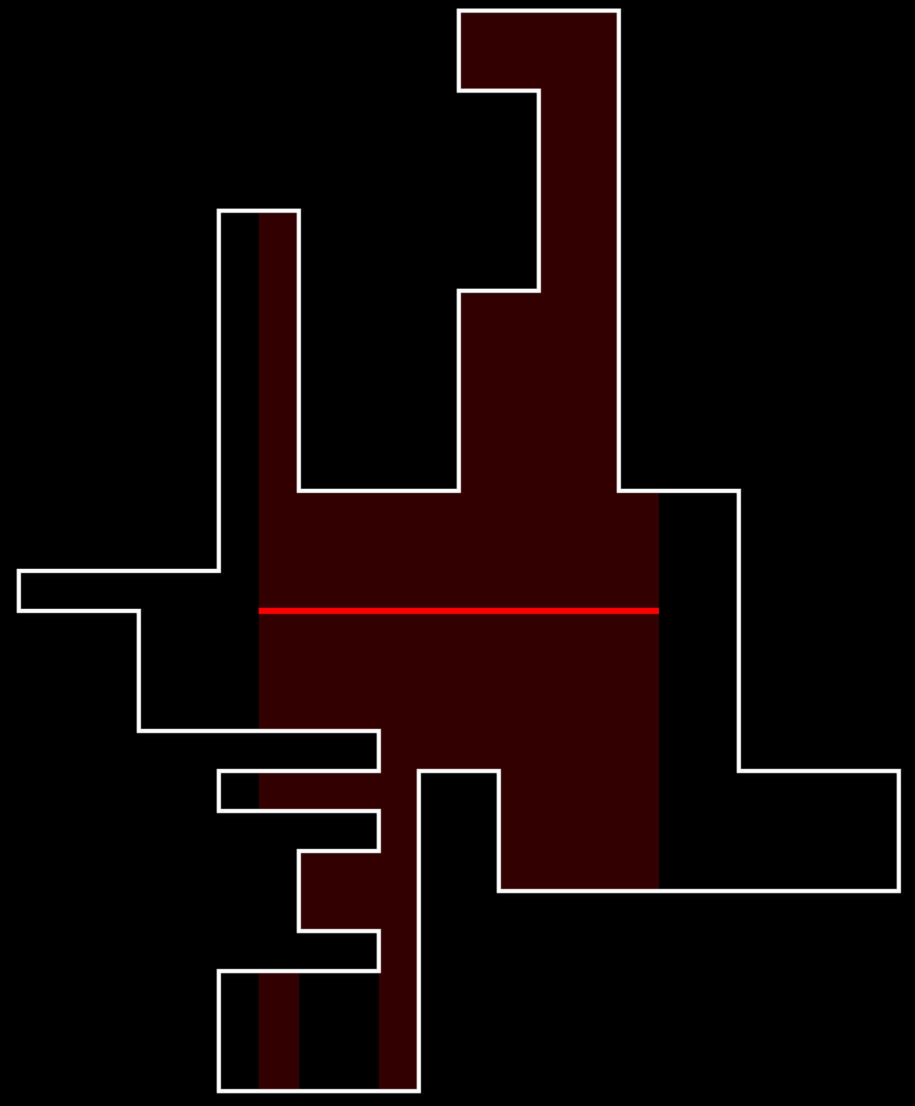

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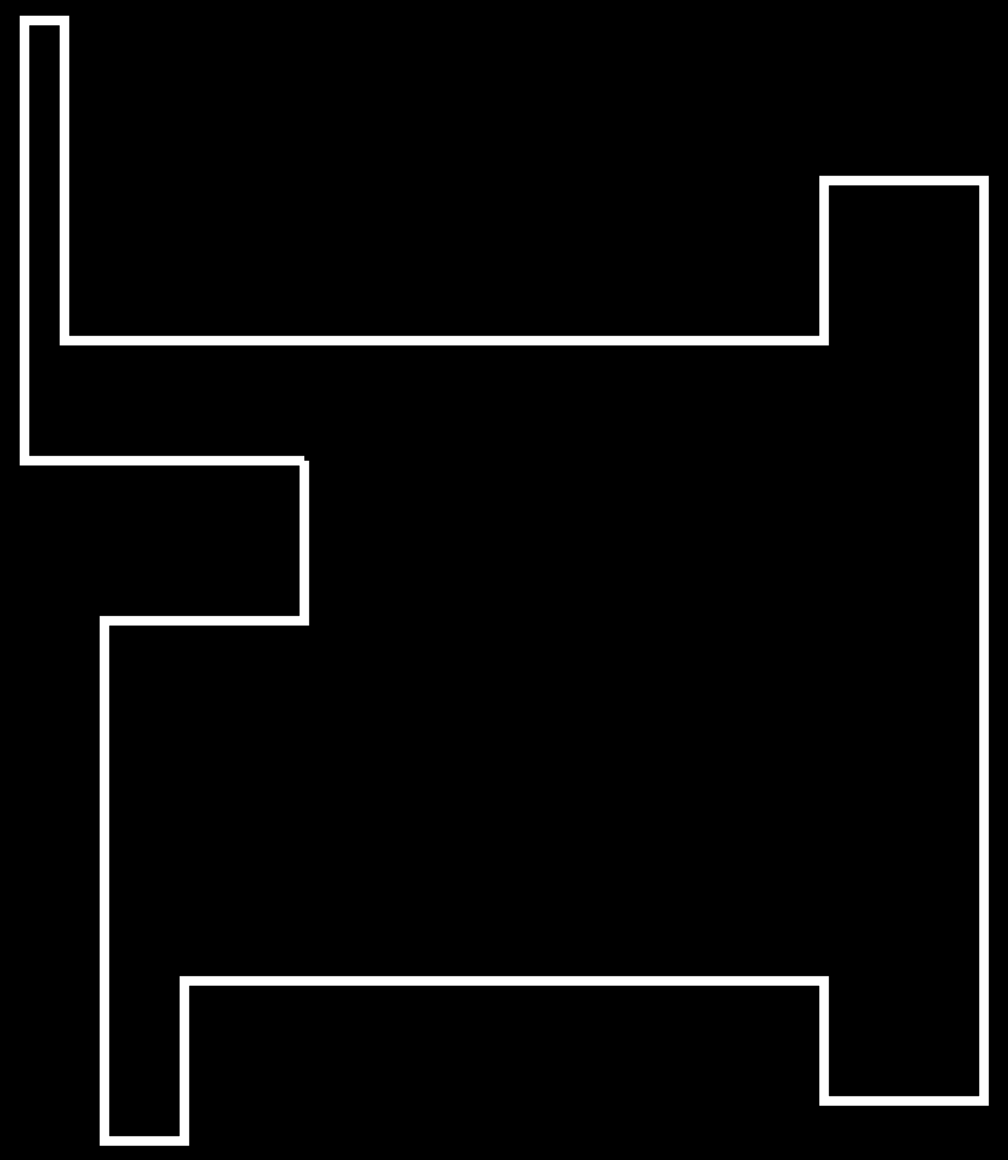

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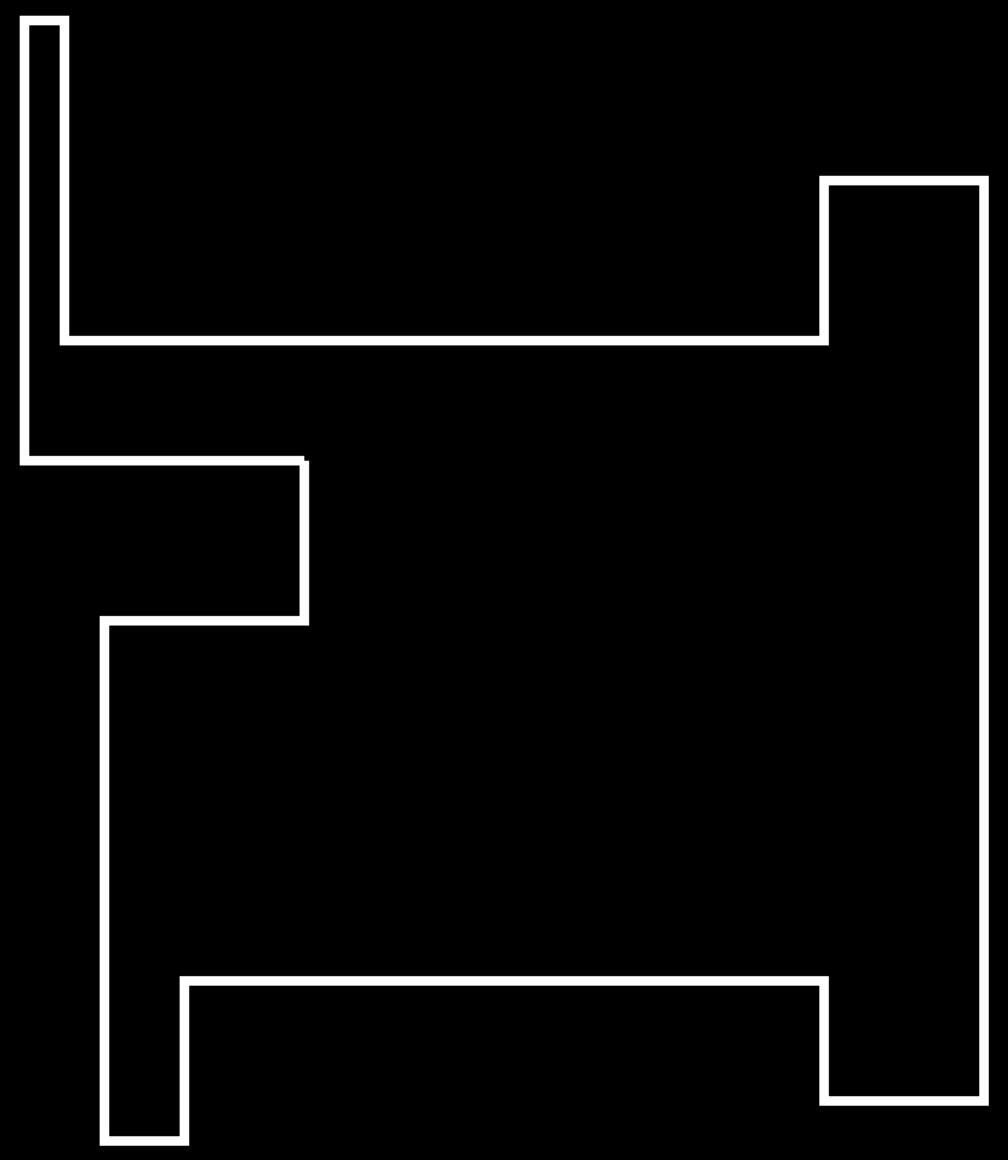

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What is the shortest tour for a watchman along which all points of P become visible?

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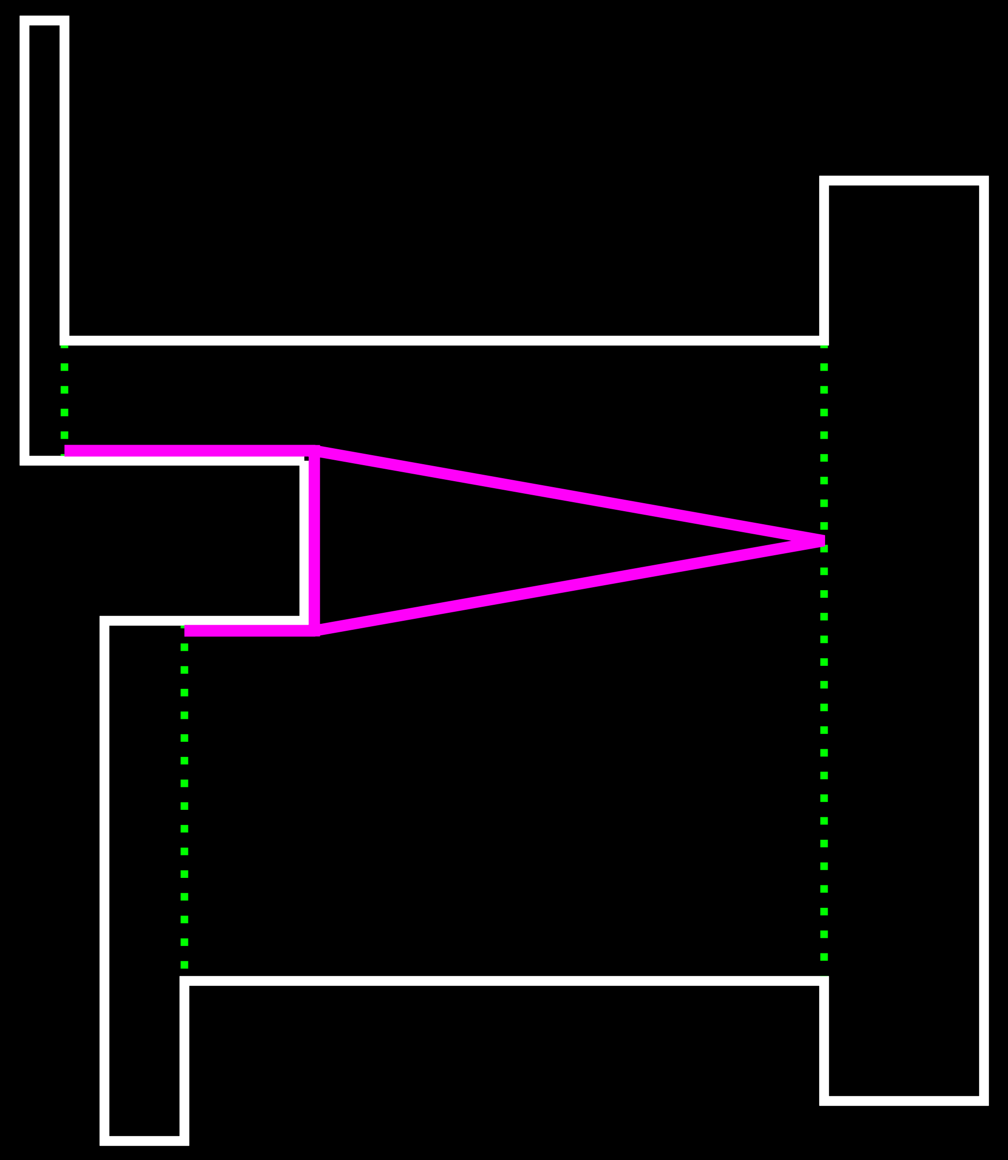

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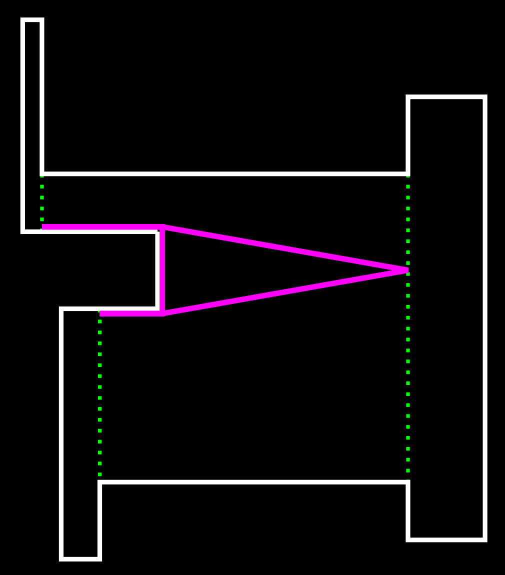

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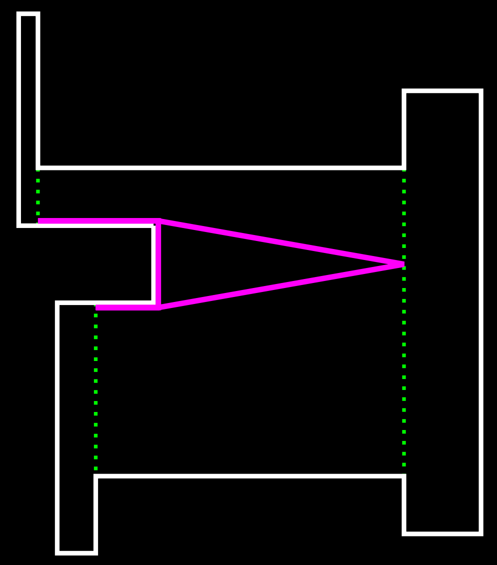

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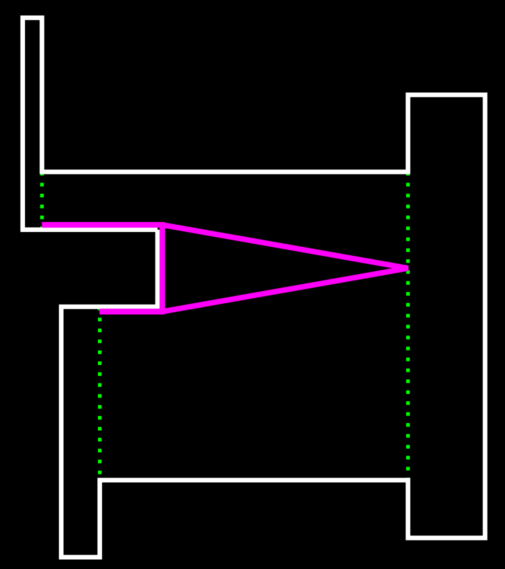

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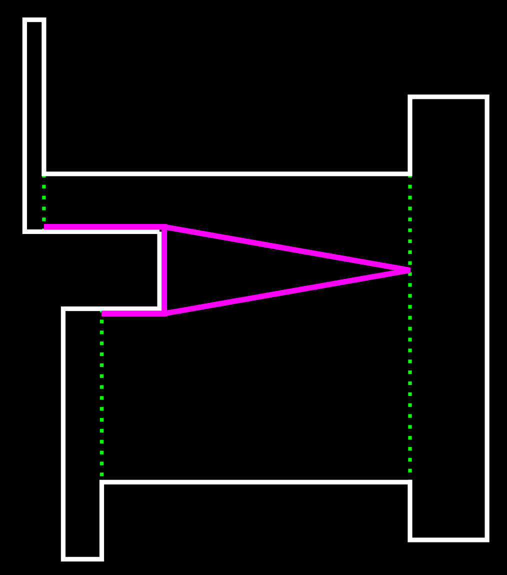

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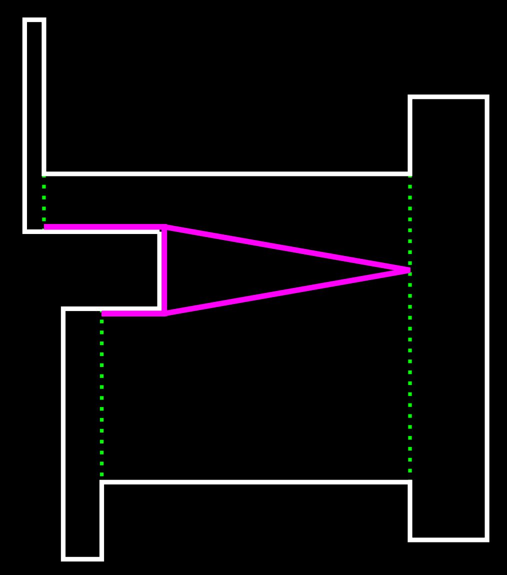

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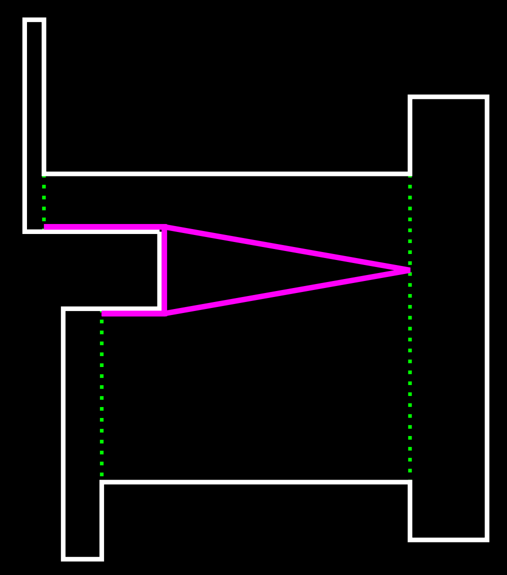

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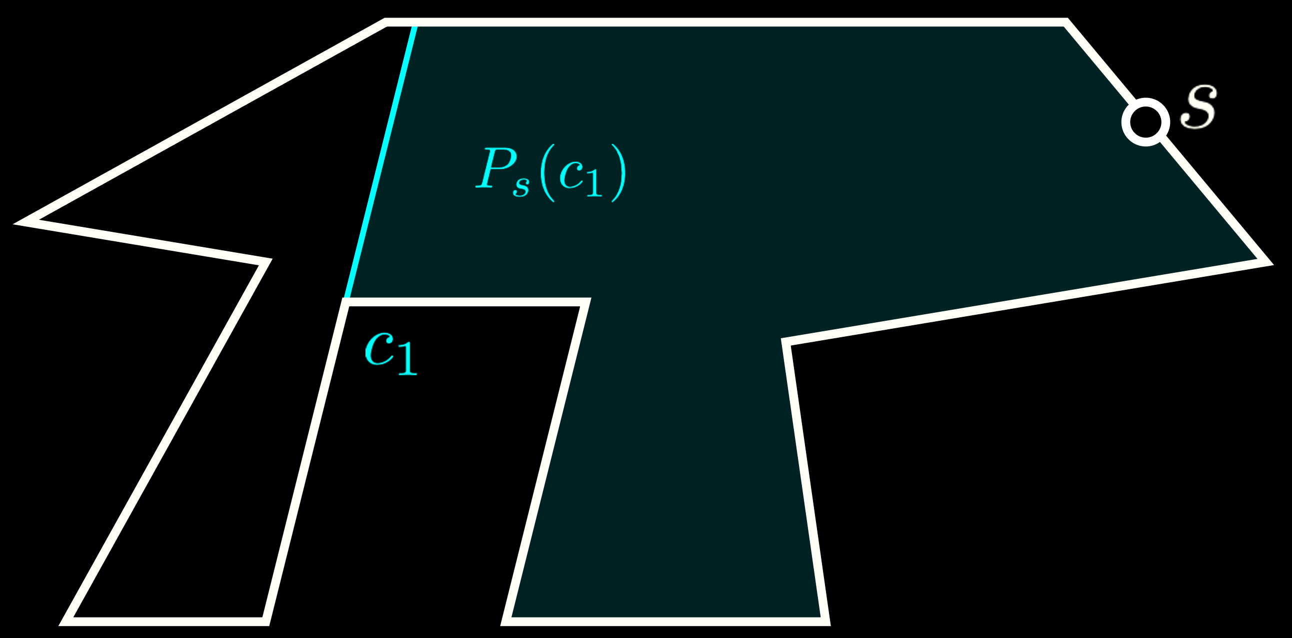

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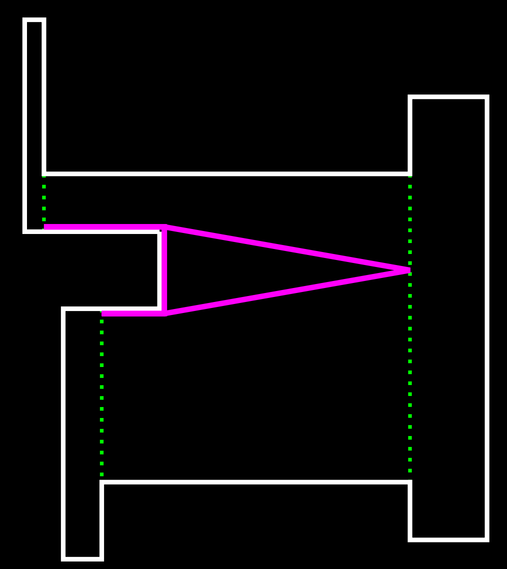

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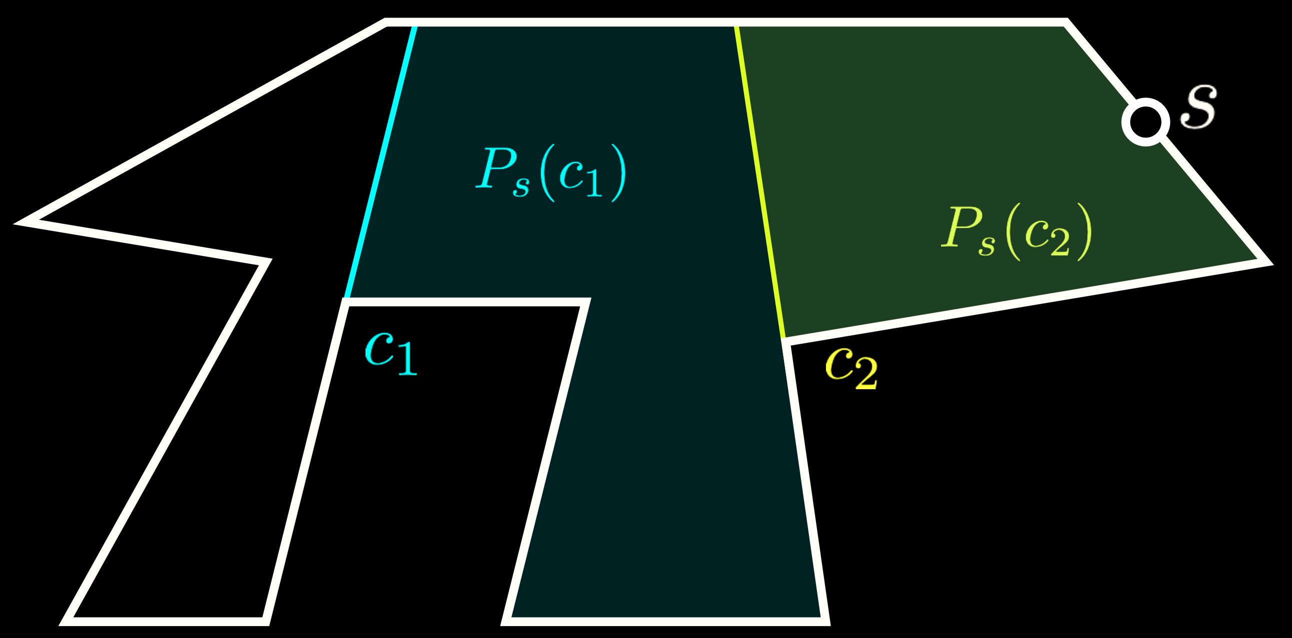

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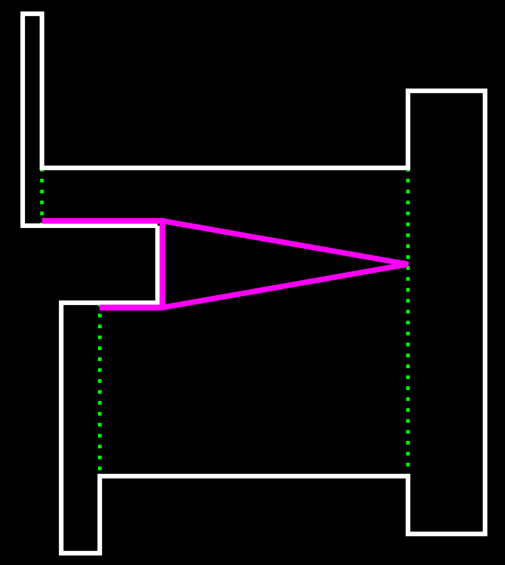

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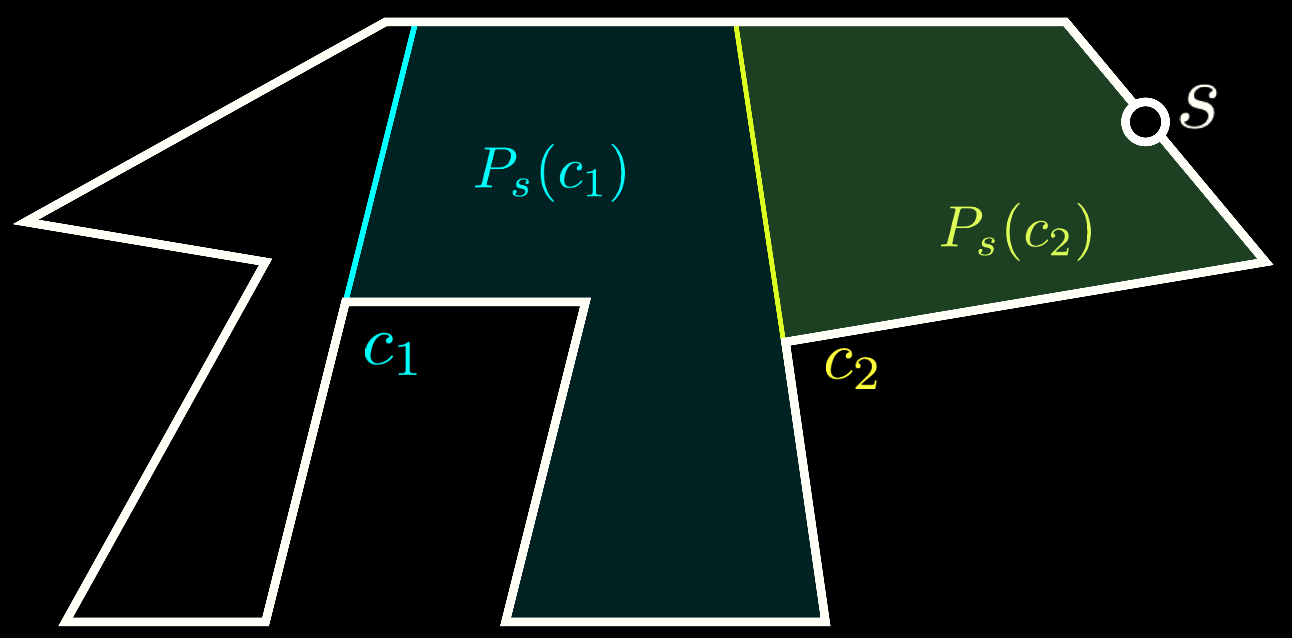

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Essential cut: not dominated by other cut

k-Transmitter Watchman Routes

[Nilsson, S., 2022]

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    k-TrWRP(*S*,*P*,*s*) or *k*-TrWRP(*S*,*P*)

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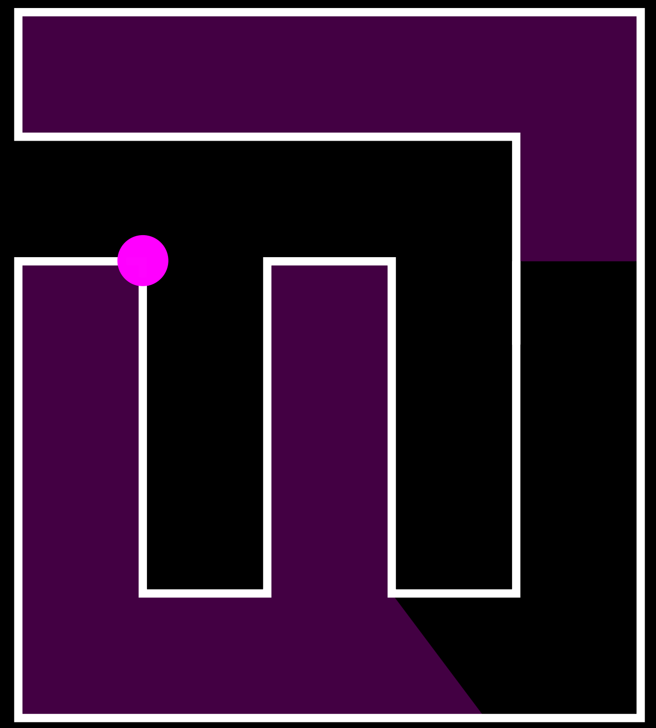
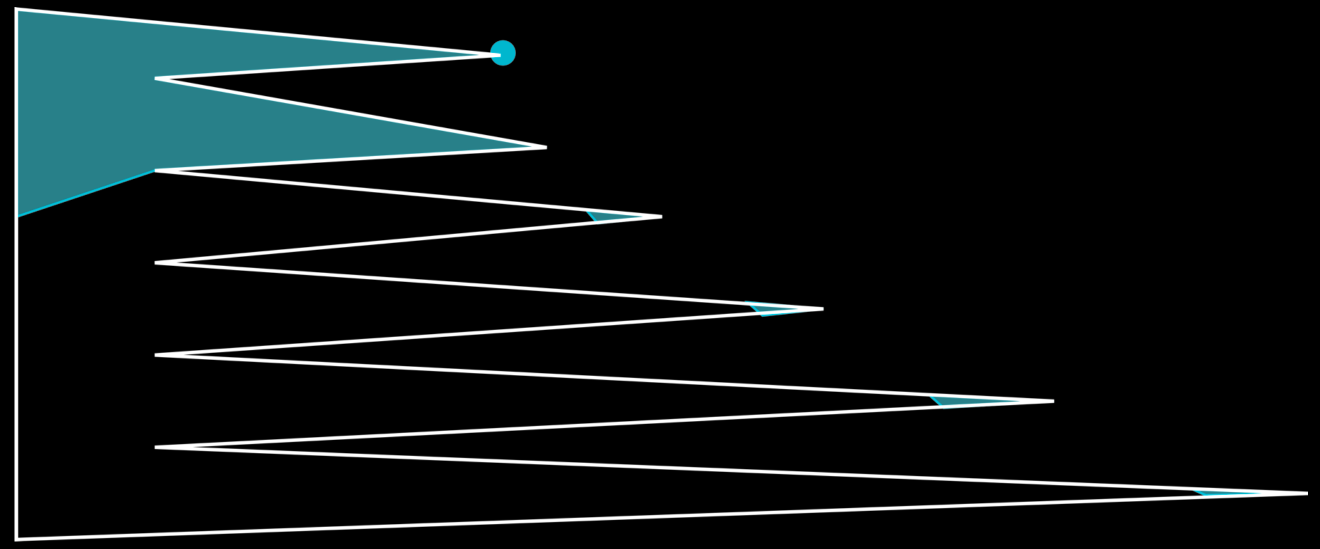
- Extensions do not translate to k-transmitters for k≥2 (no longer local!)





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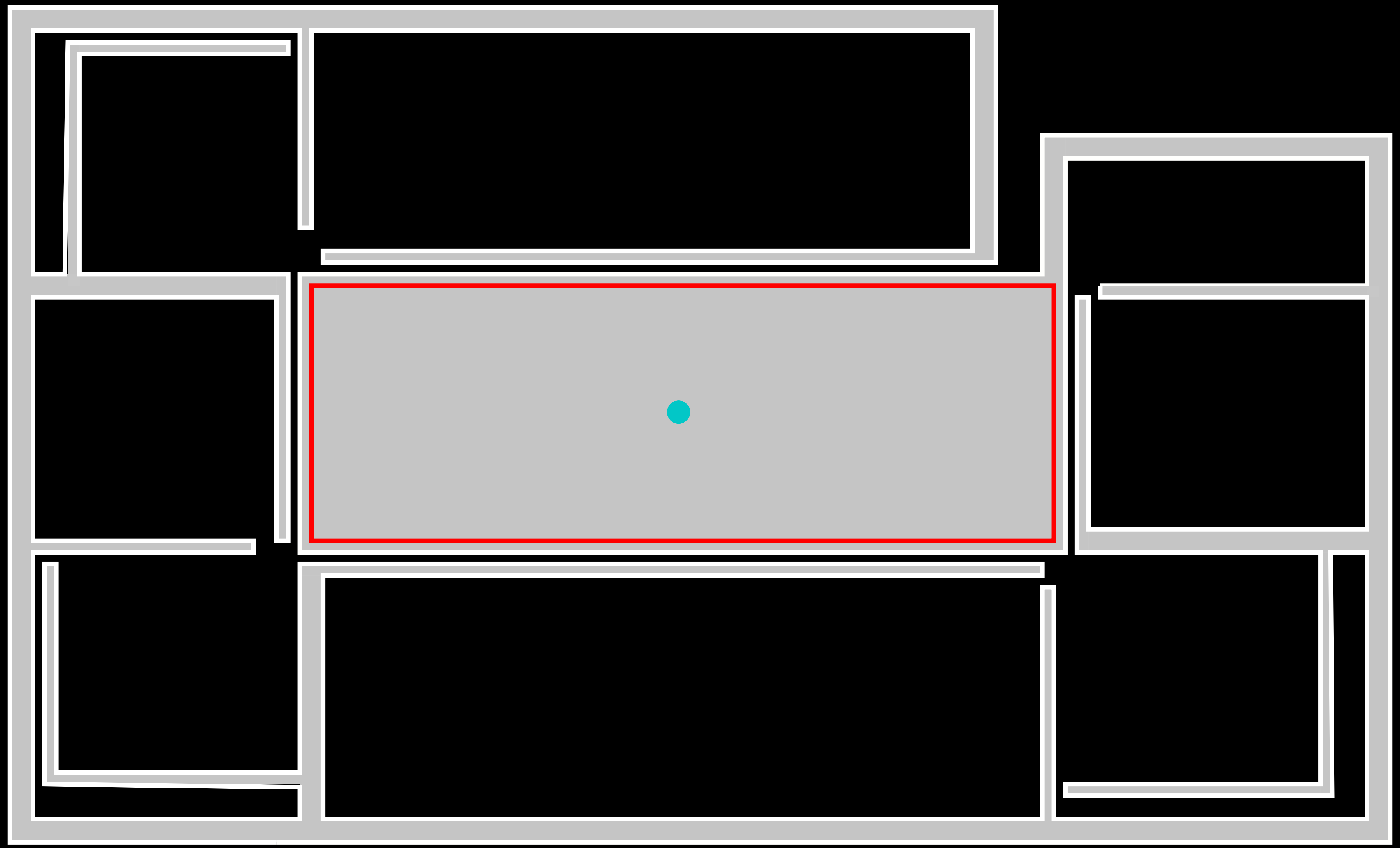
Even for a tour in a simple polygon seeing the boundary is not enough:



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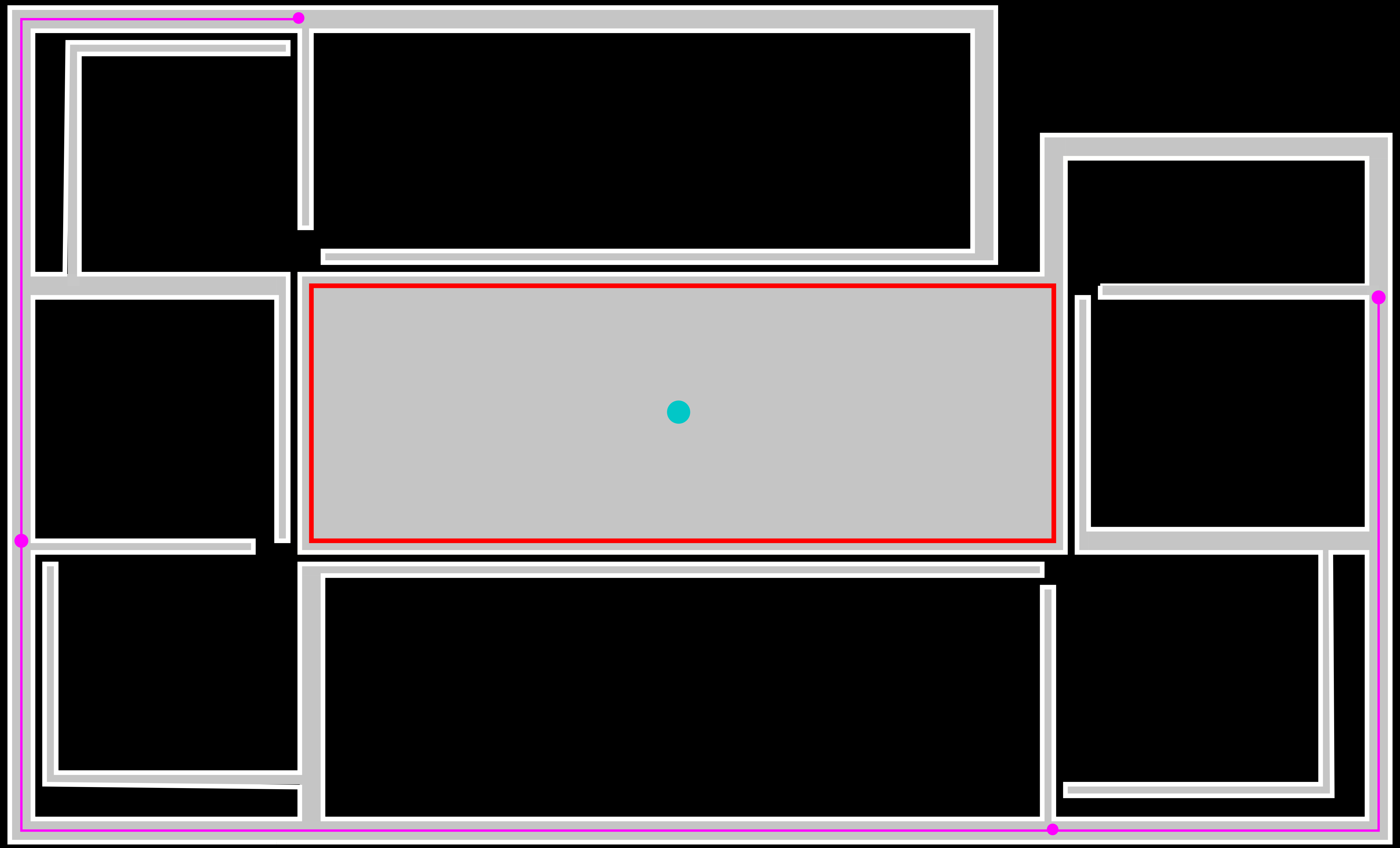

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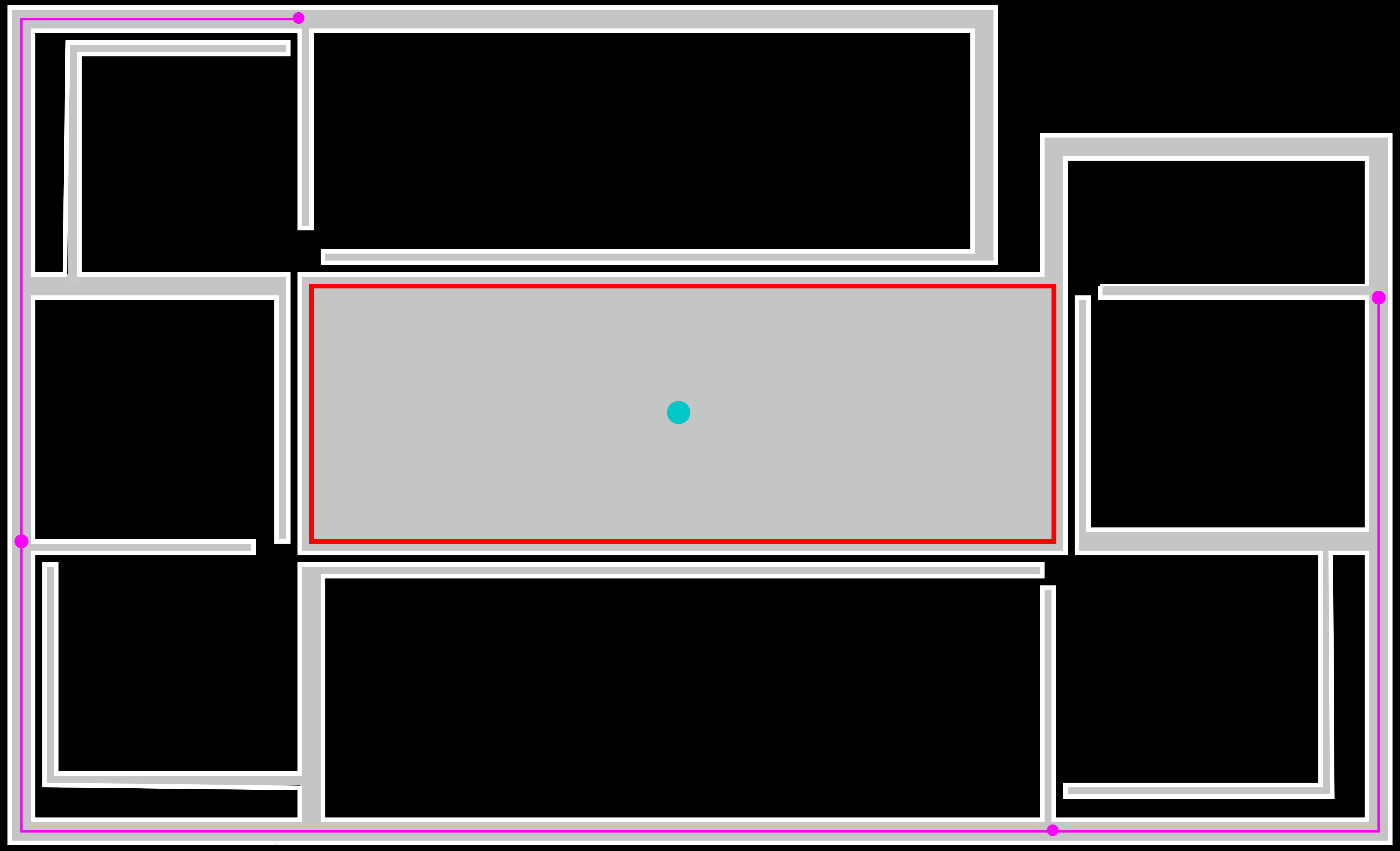

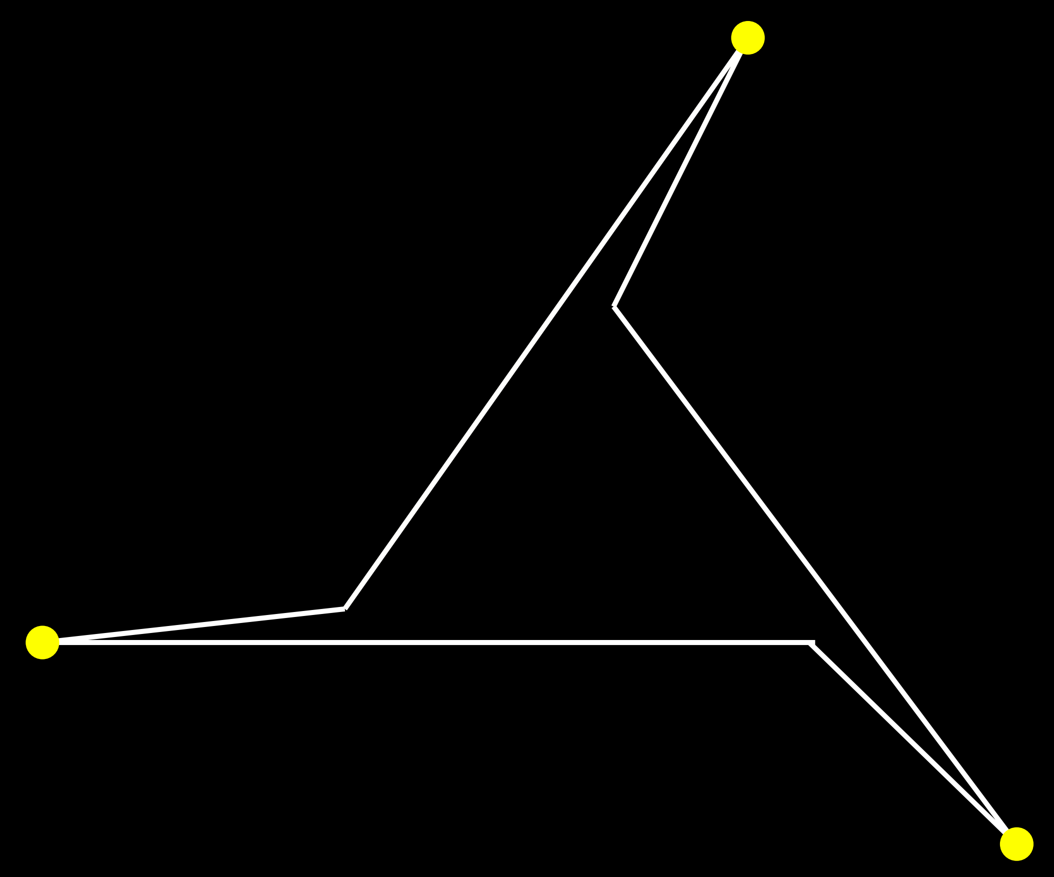

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Theorem 1: For a discrete set of points S and a simple polygon P, the k-TrWRP(S,P) does not admit a polynomial-time approximation algorithm with approximation ratio $c \ln |S|$ unless P=NP, even for k=2.

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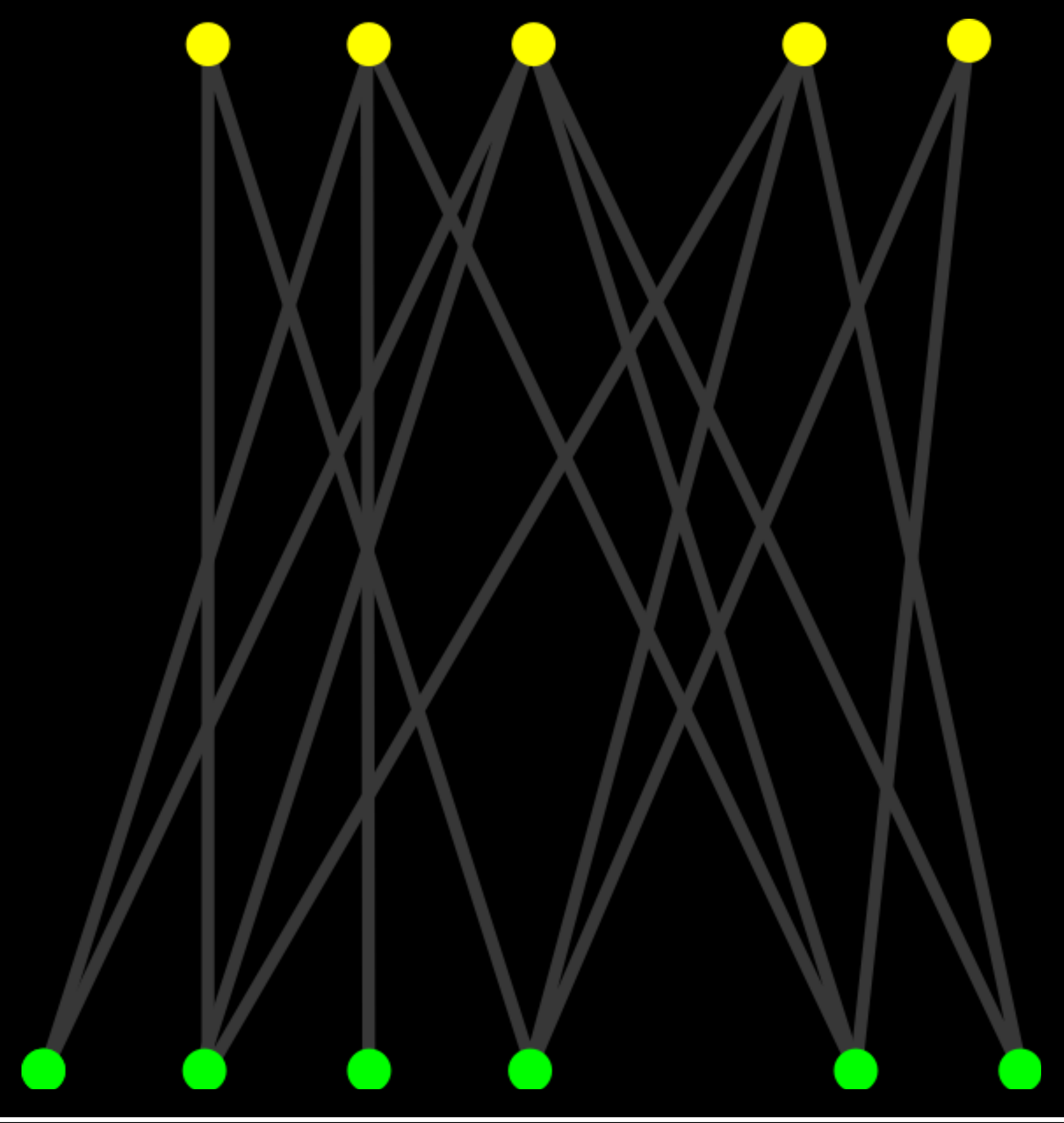

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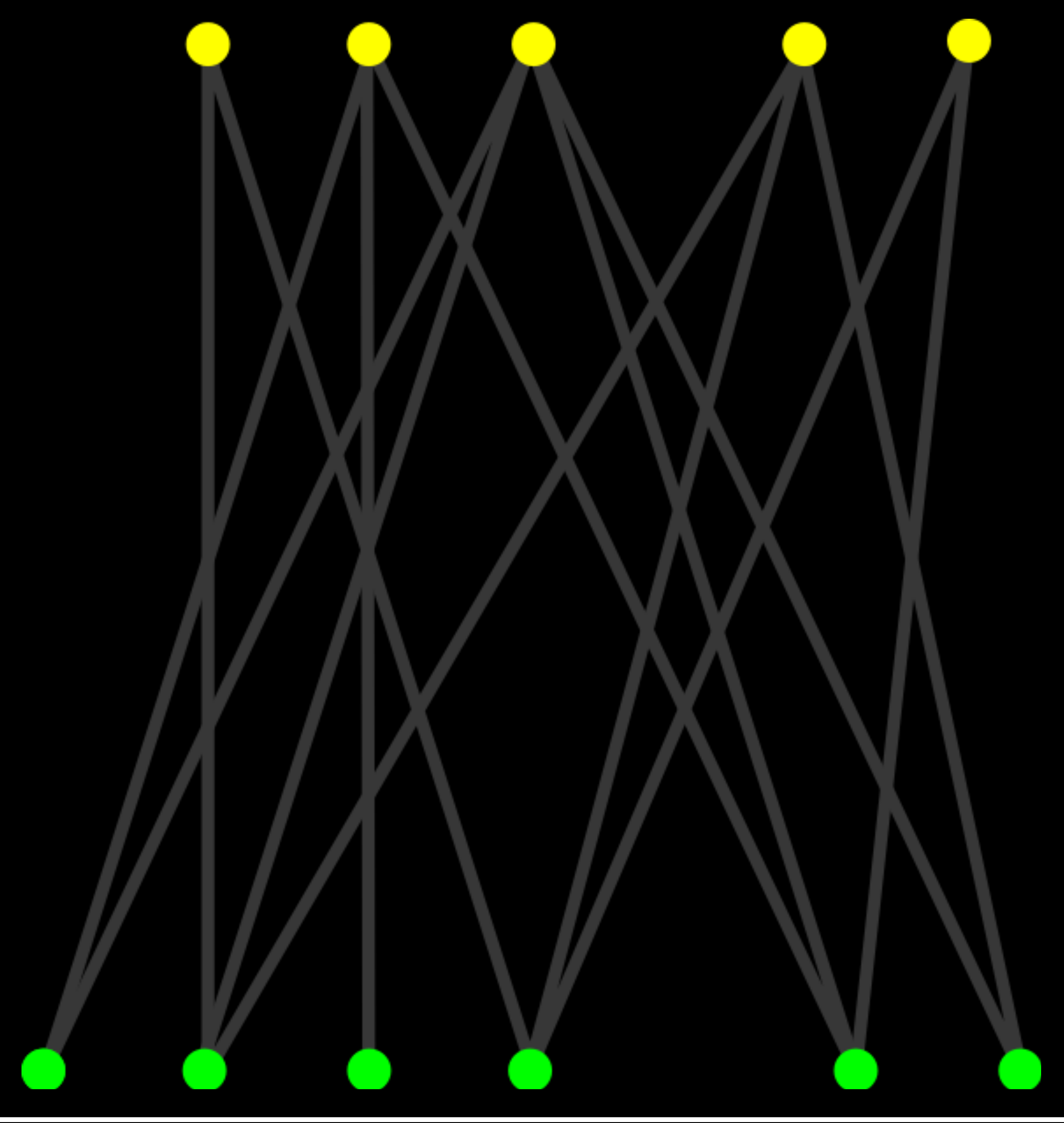

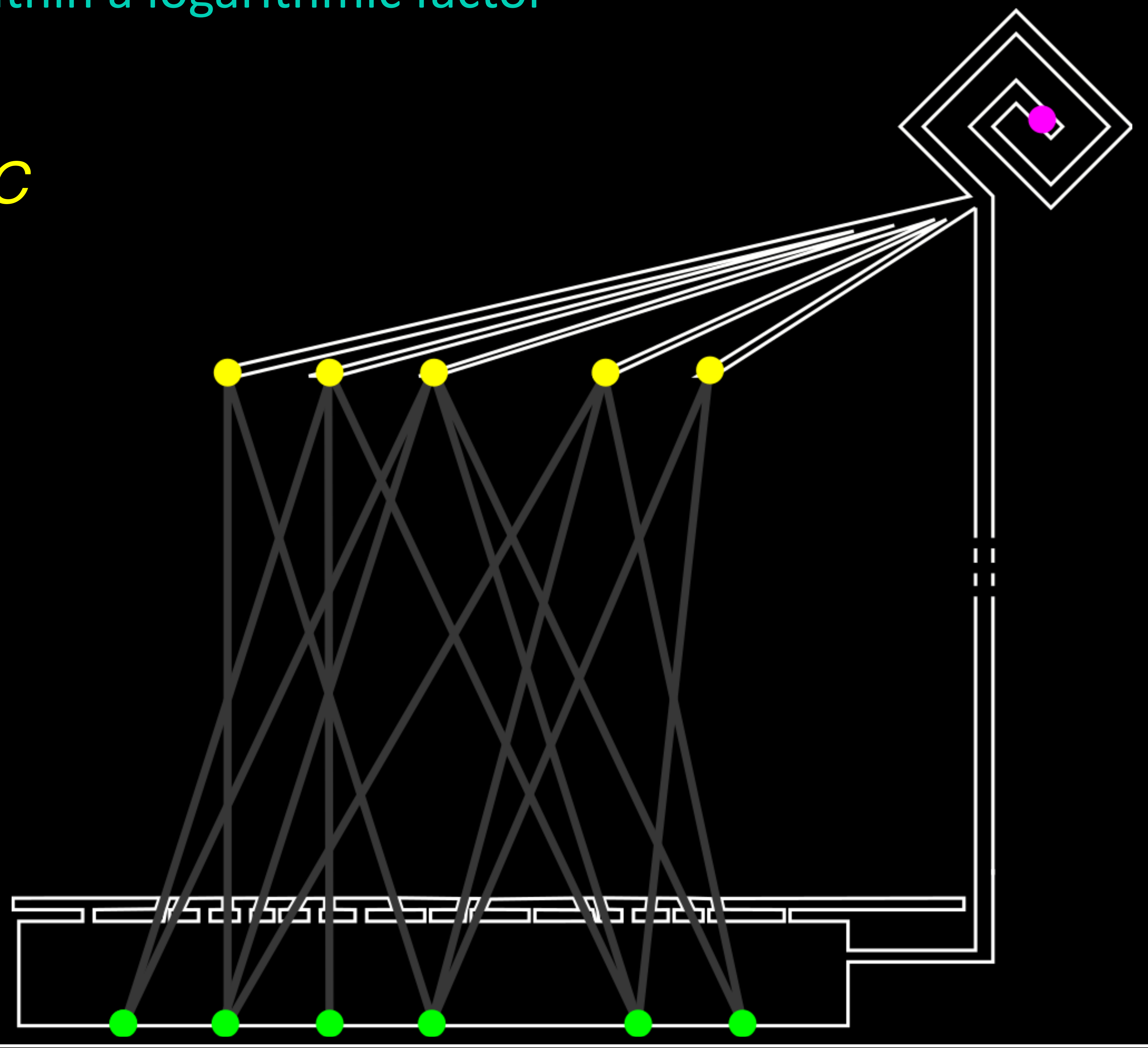

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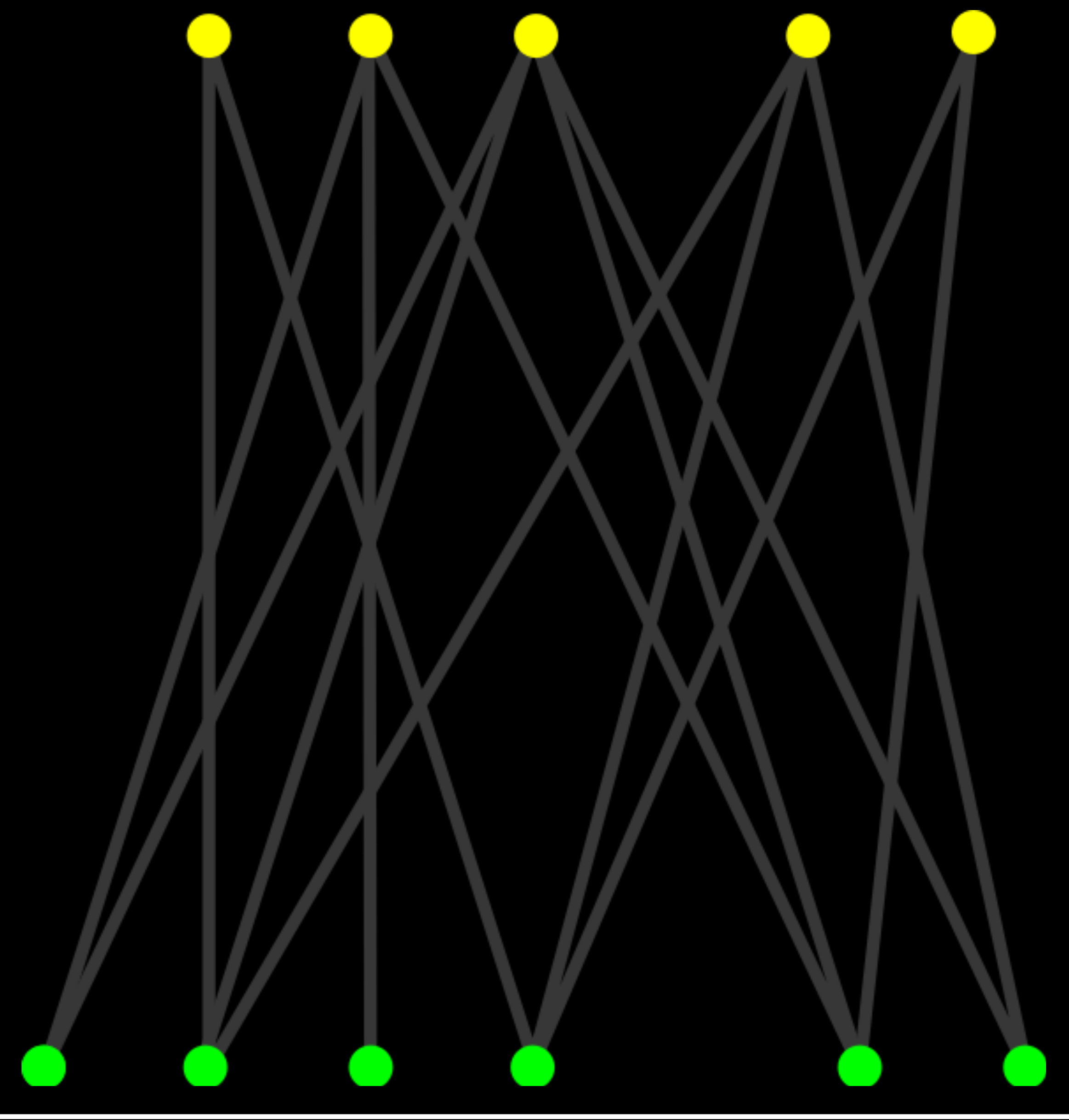

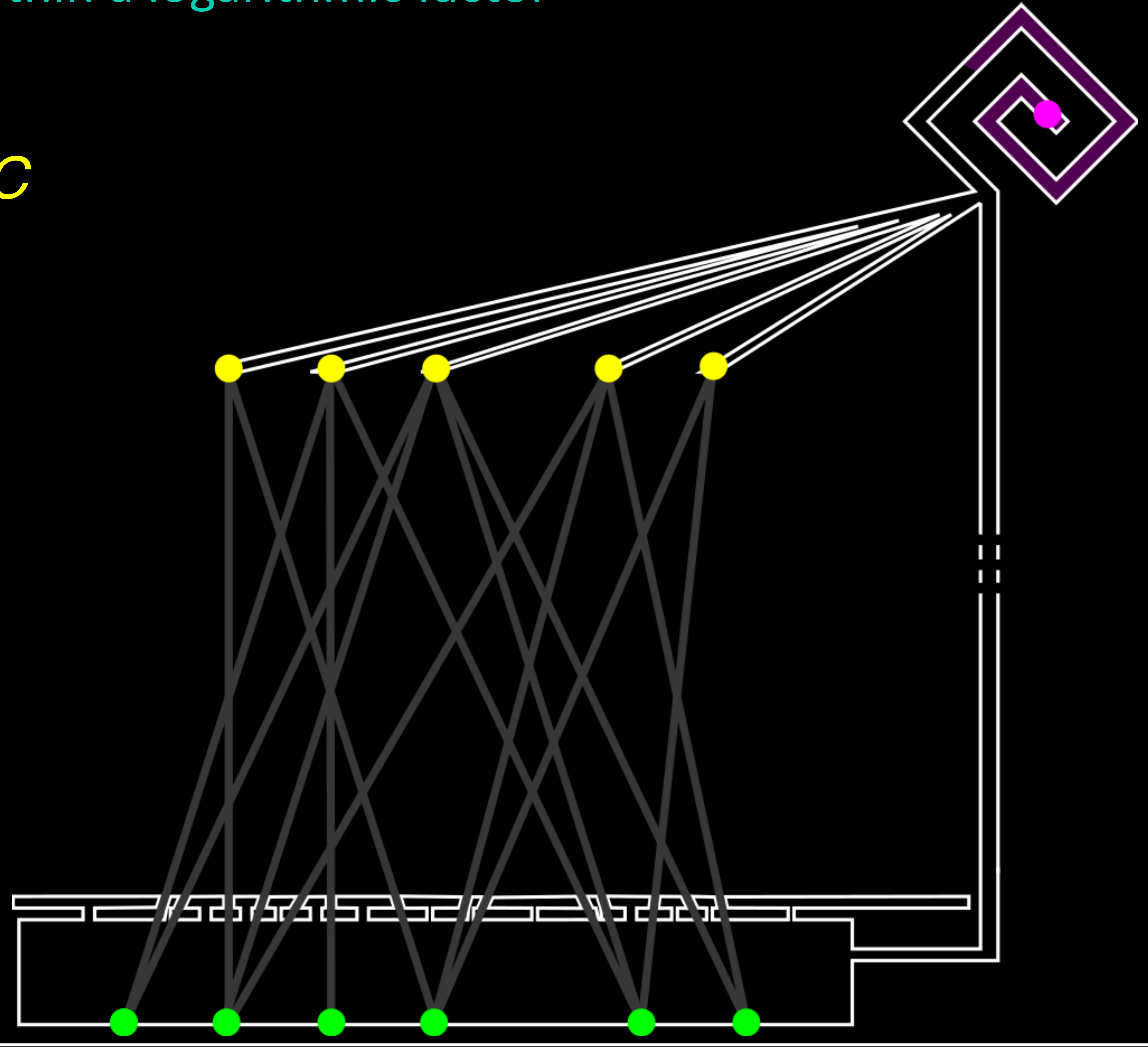

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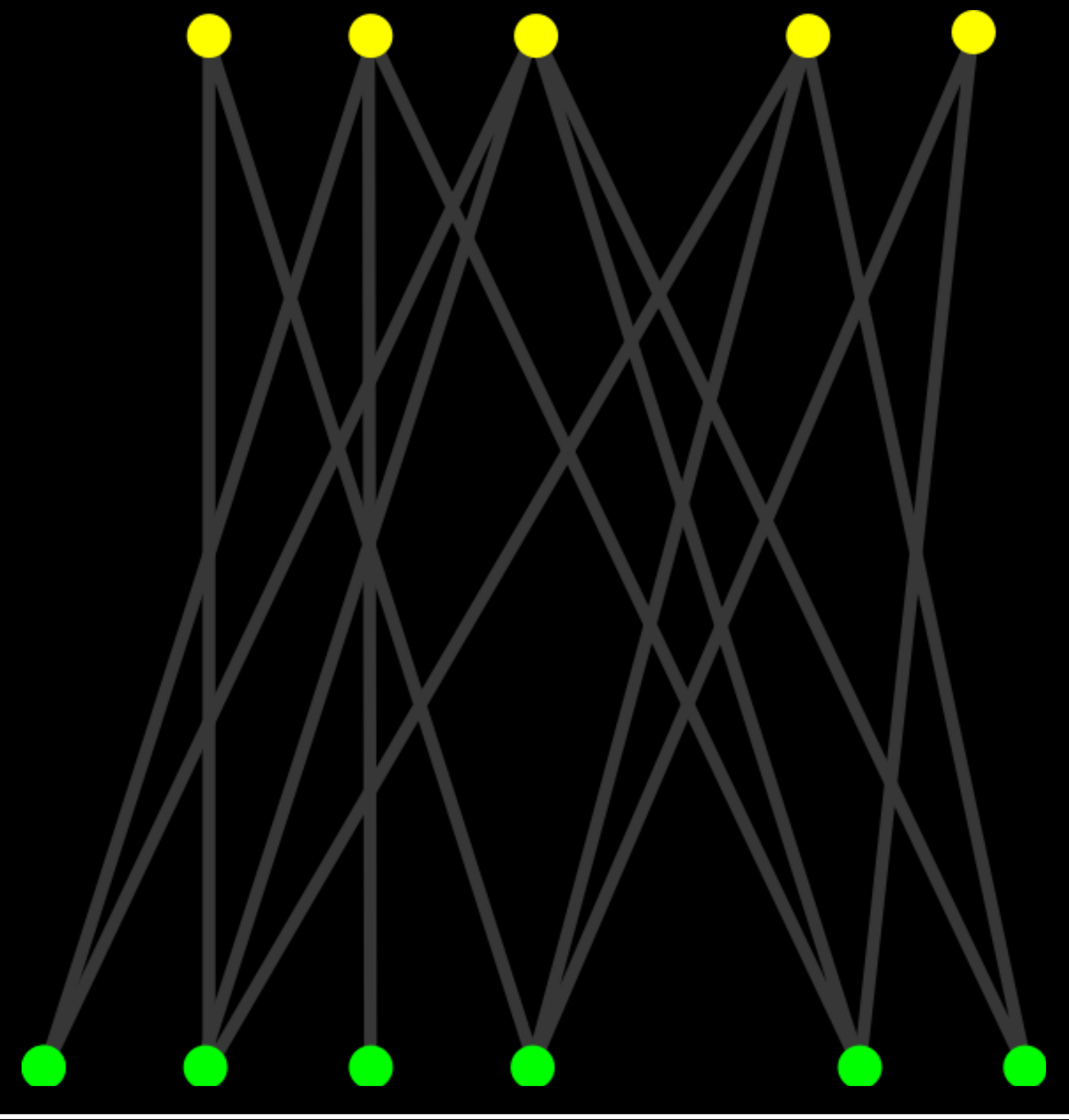

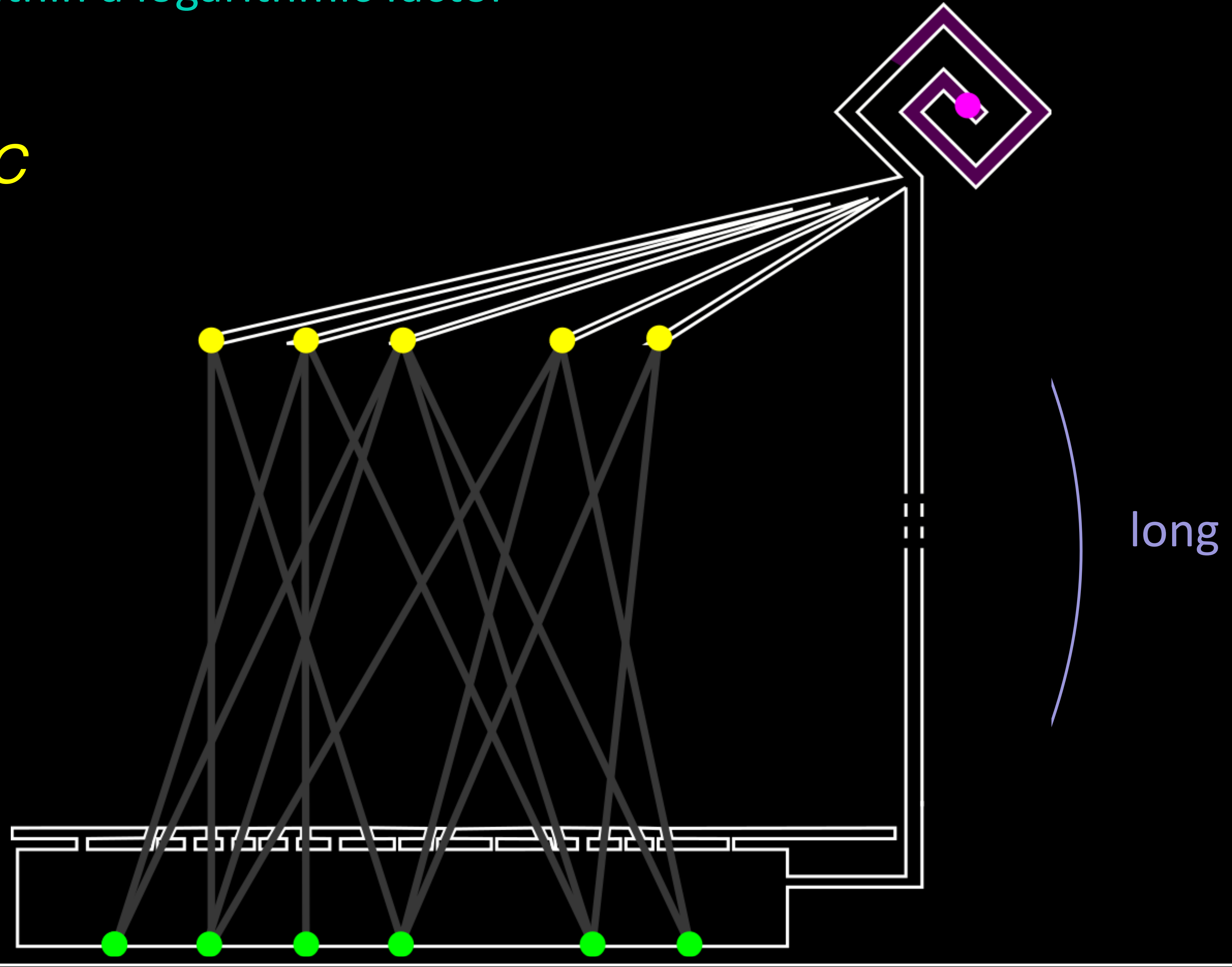

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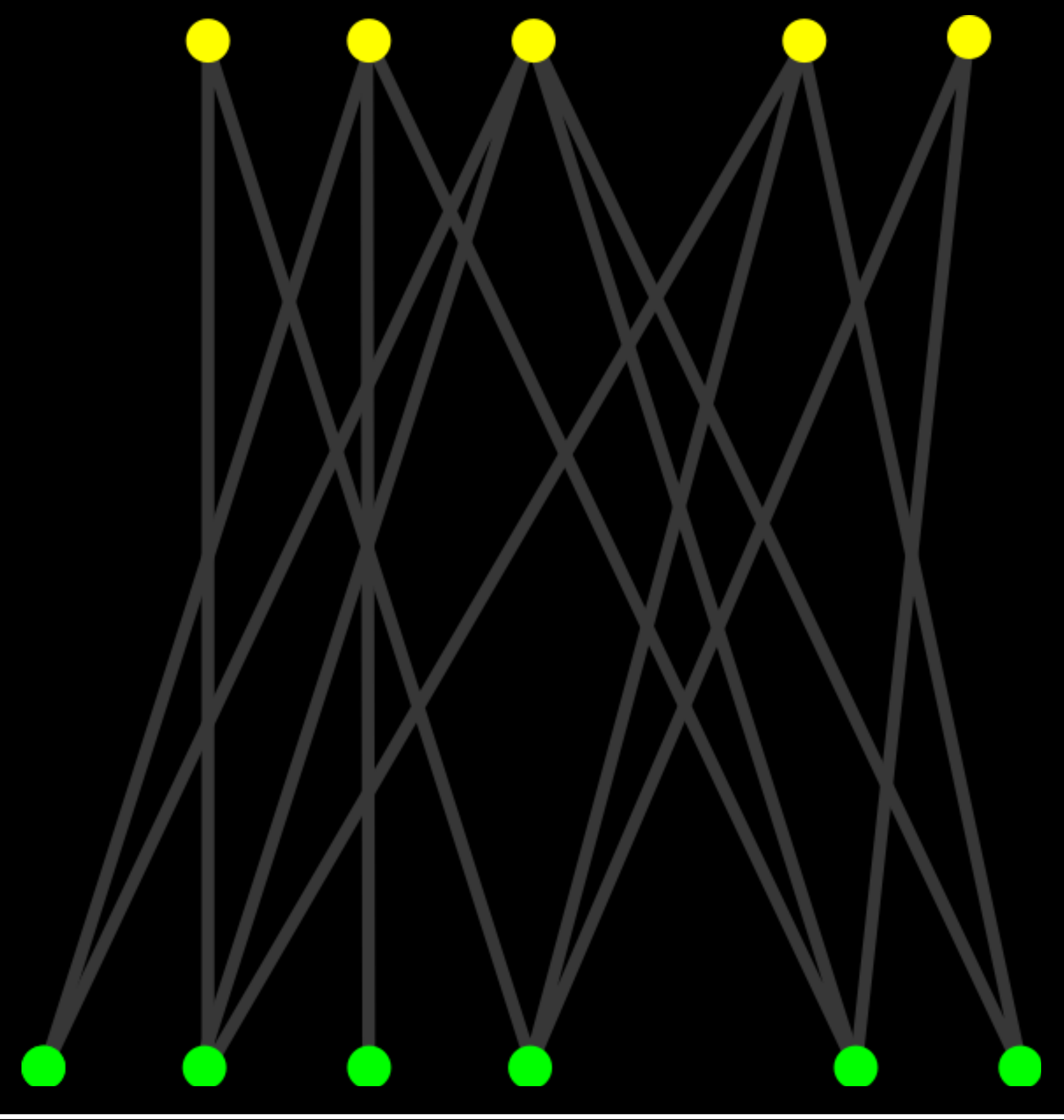

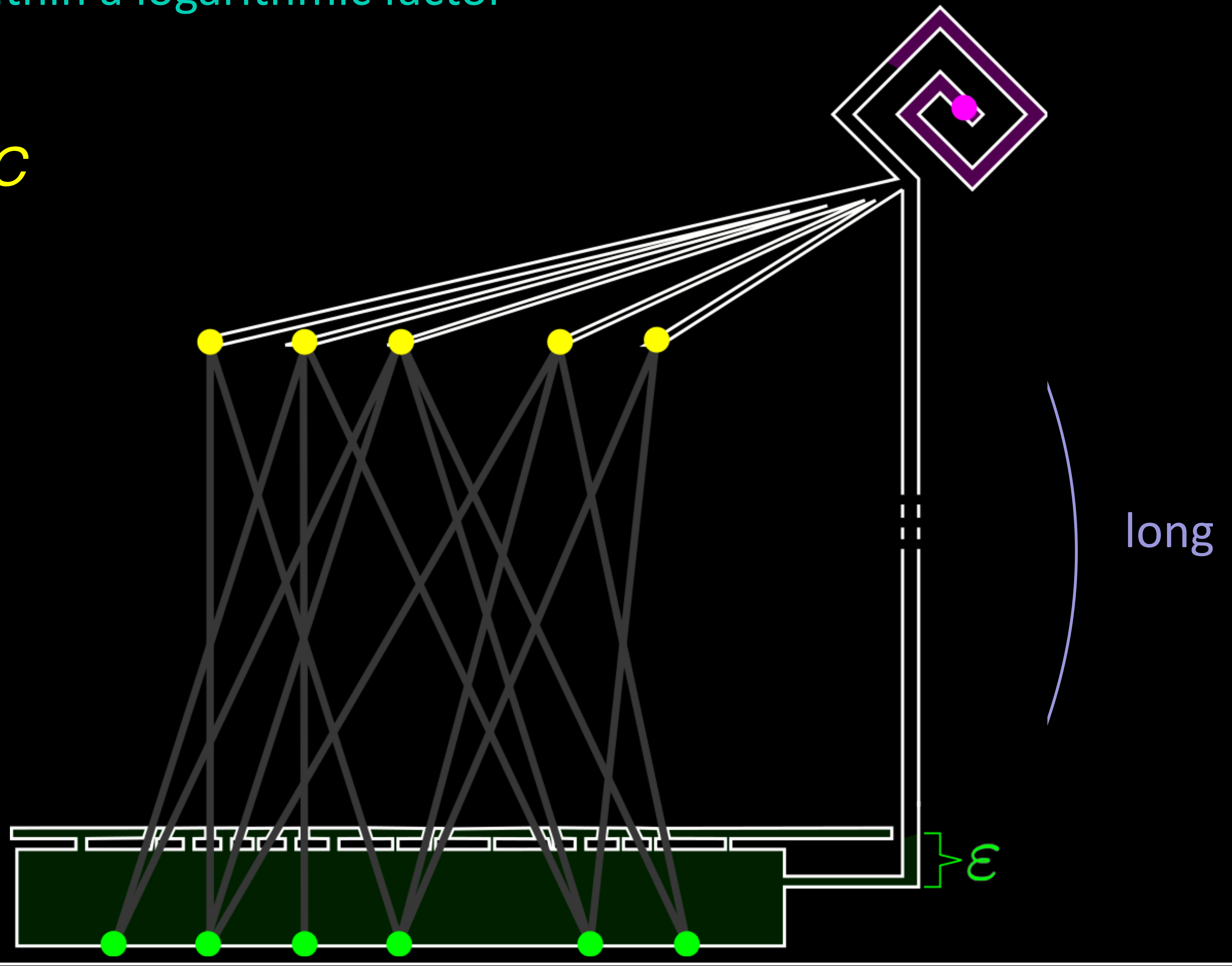

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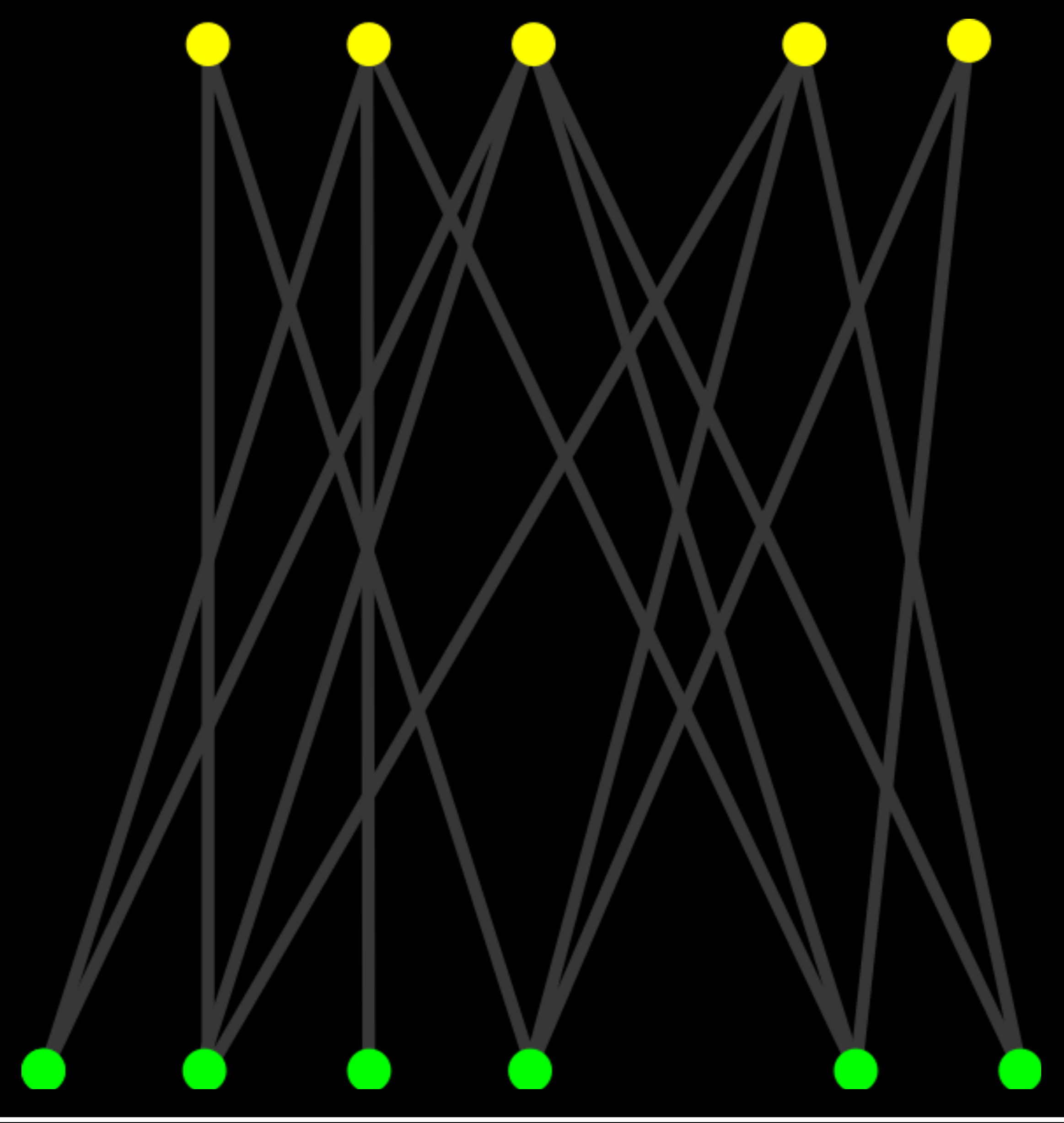

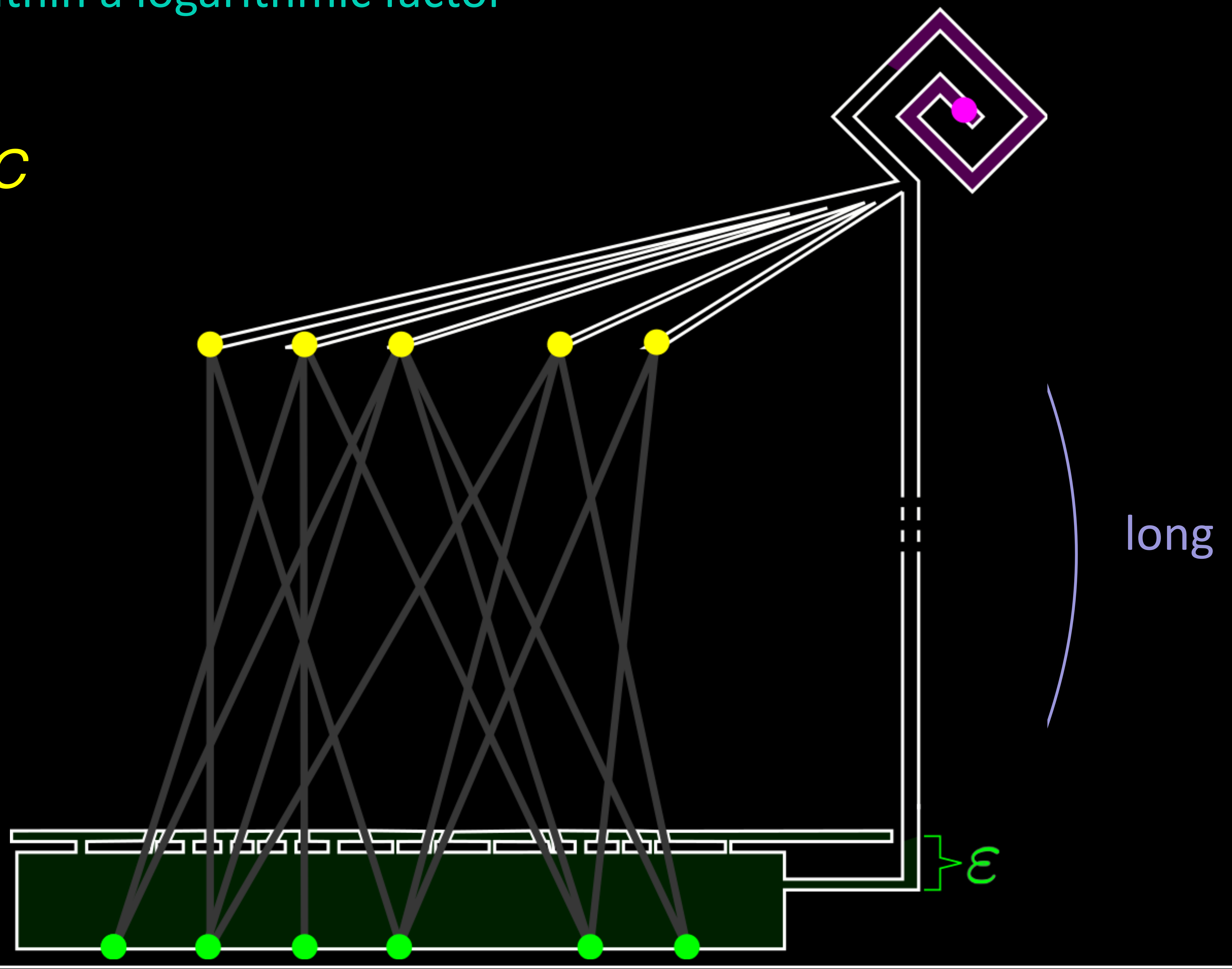

Corollary: The same holds for *k*-TrWRP(*S*,*P*,*s*).

Approximation Algorithm for *k*-TrWRP(*S*,*P*,*s*)

Approximation Algorithm for *k*-TrWRP(*S,P,s*)

Theorem 2: Let P be a simple polygon with $n=|P|$. Let OPT(S,P,s) be the optimal solution for the k-TrWRP(S,P,s) and let R be the solution by our algorithm ALG(S,P,s). Then R yields an approximation ratio of $O(\log^2(|S|\, n) \log\log(|S|\, n) \log|S|)$.

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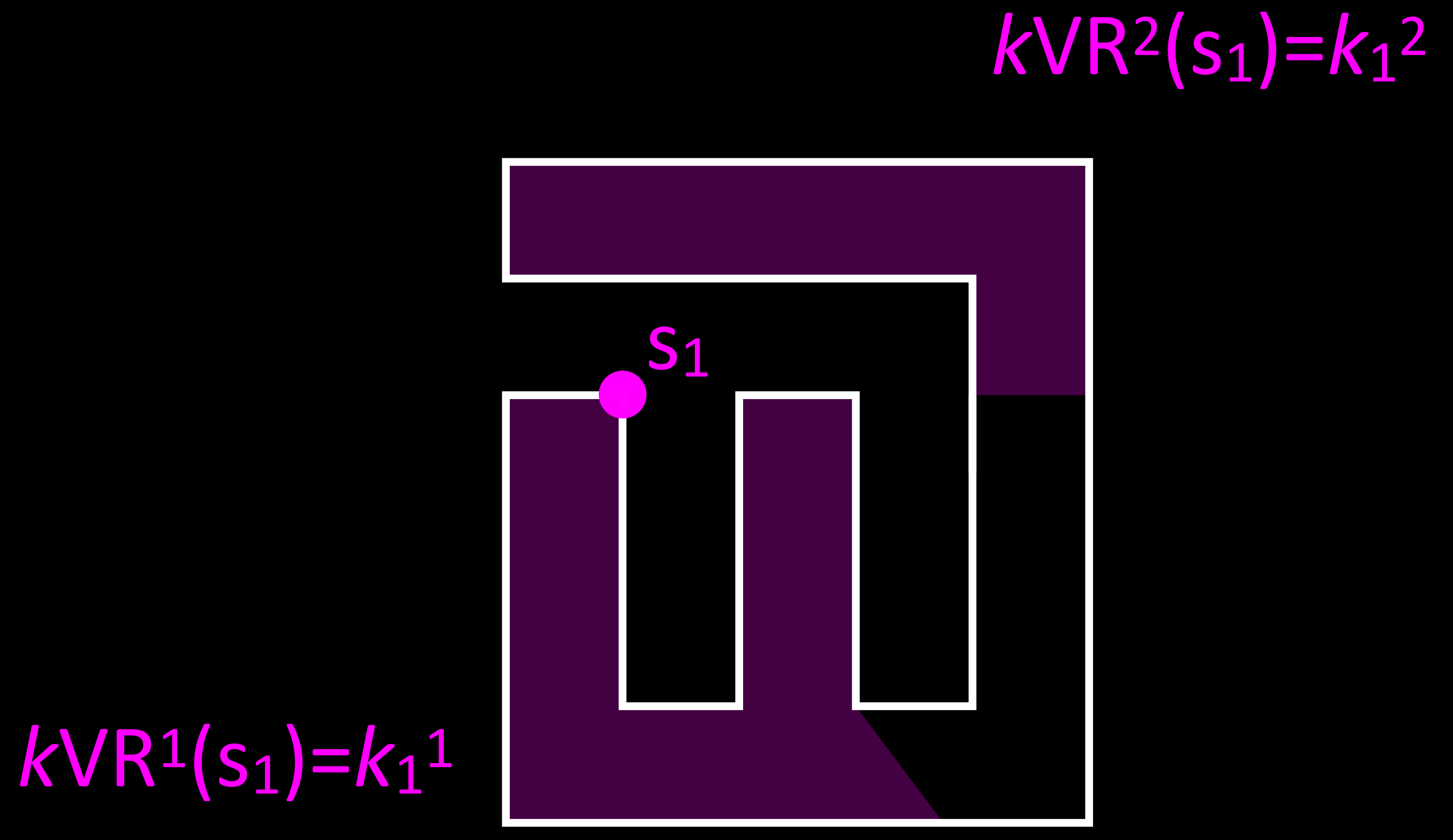

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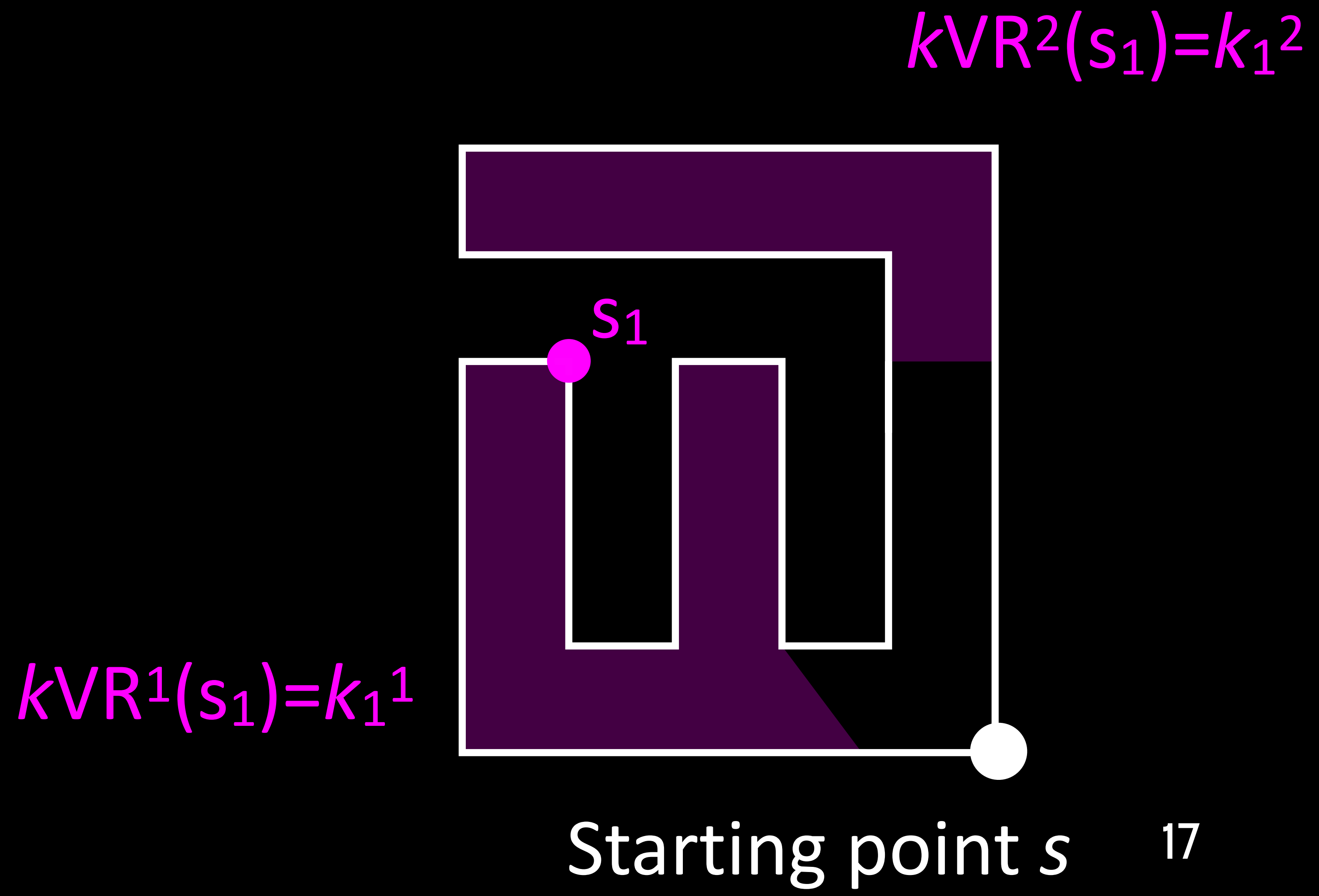

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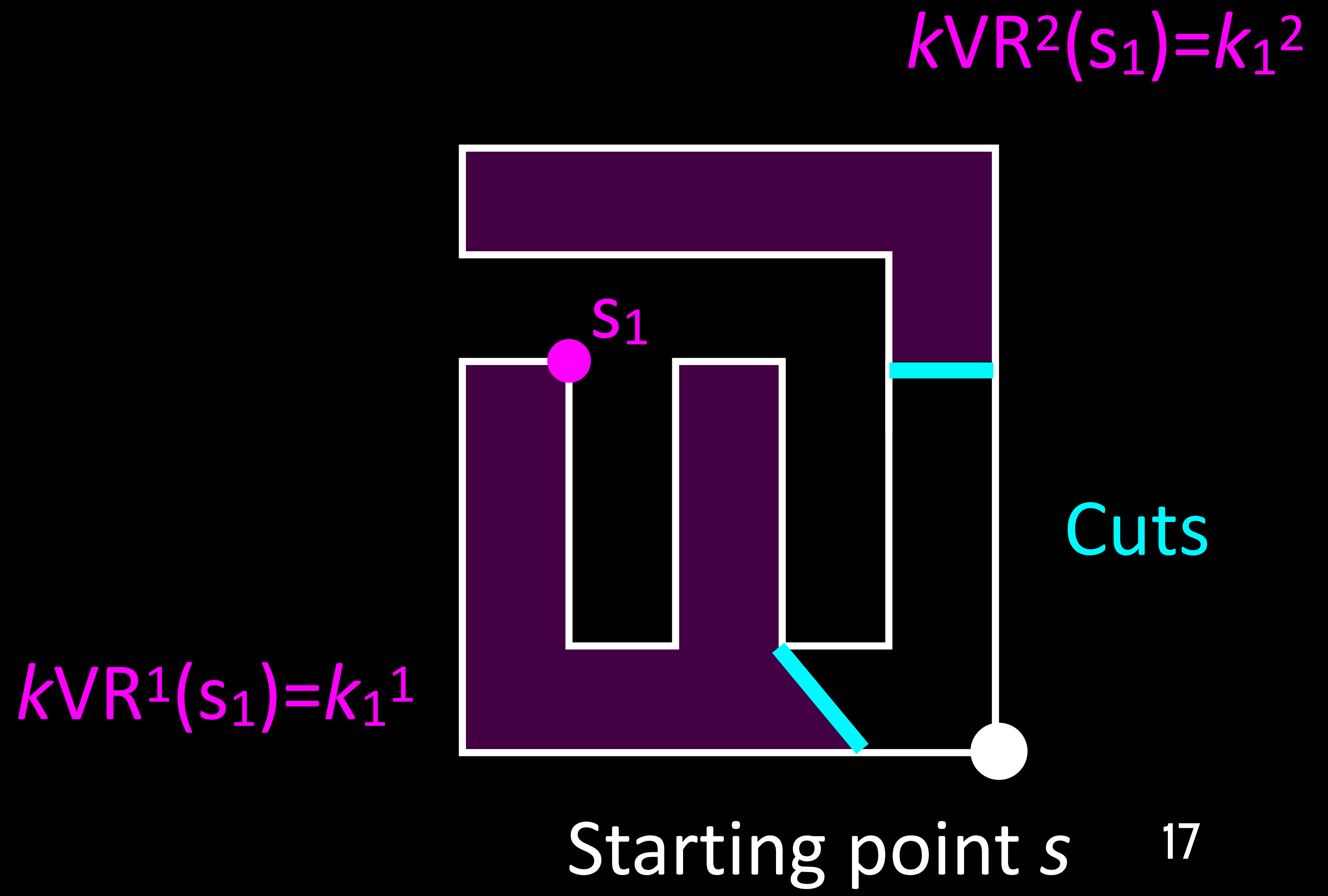

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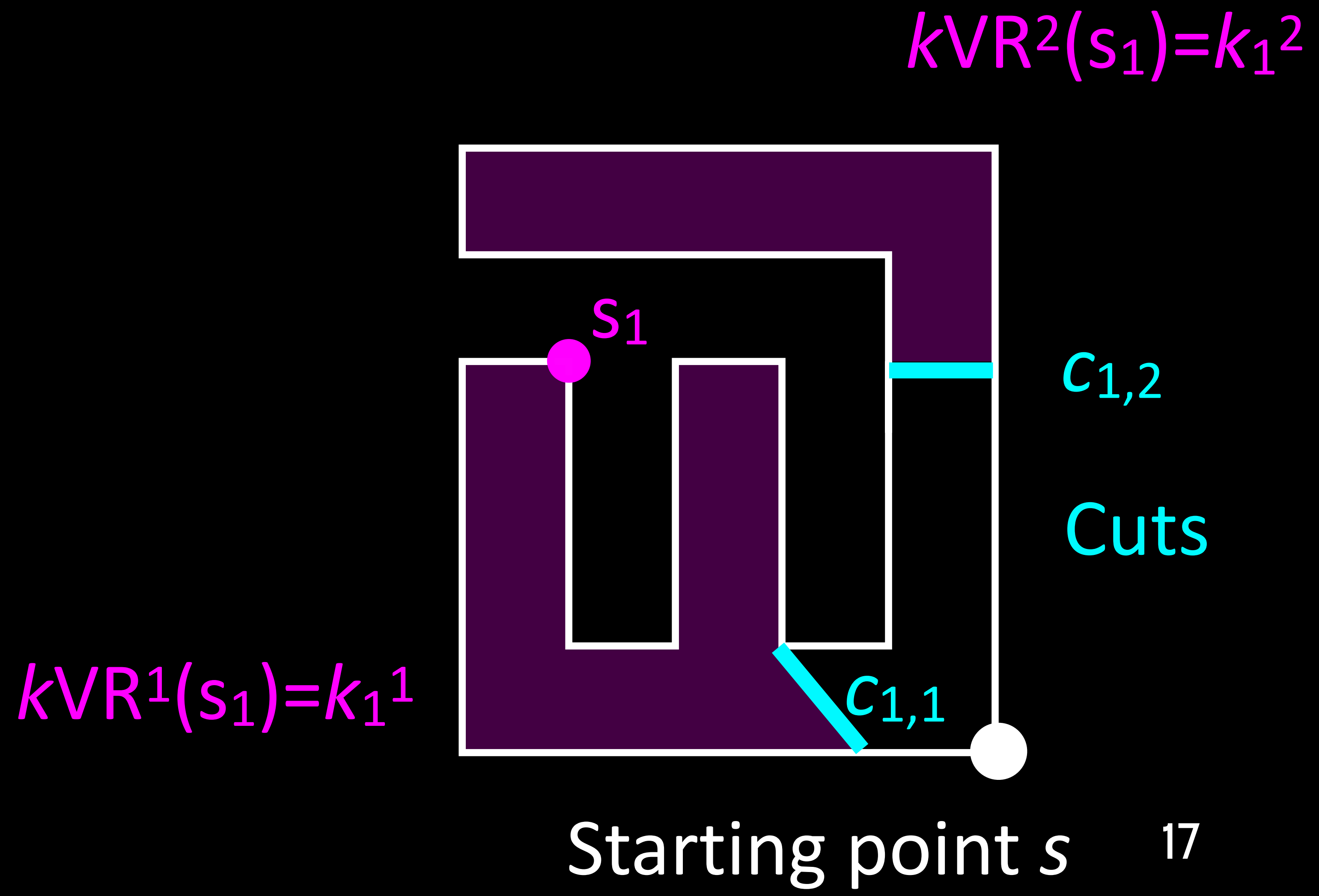

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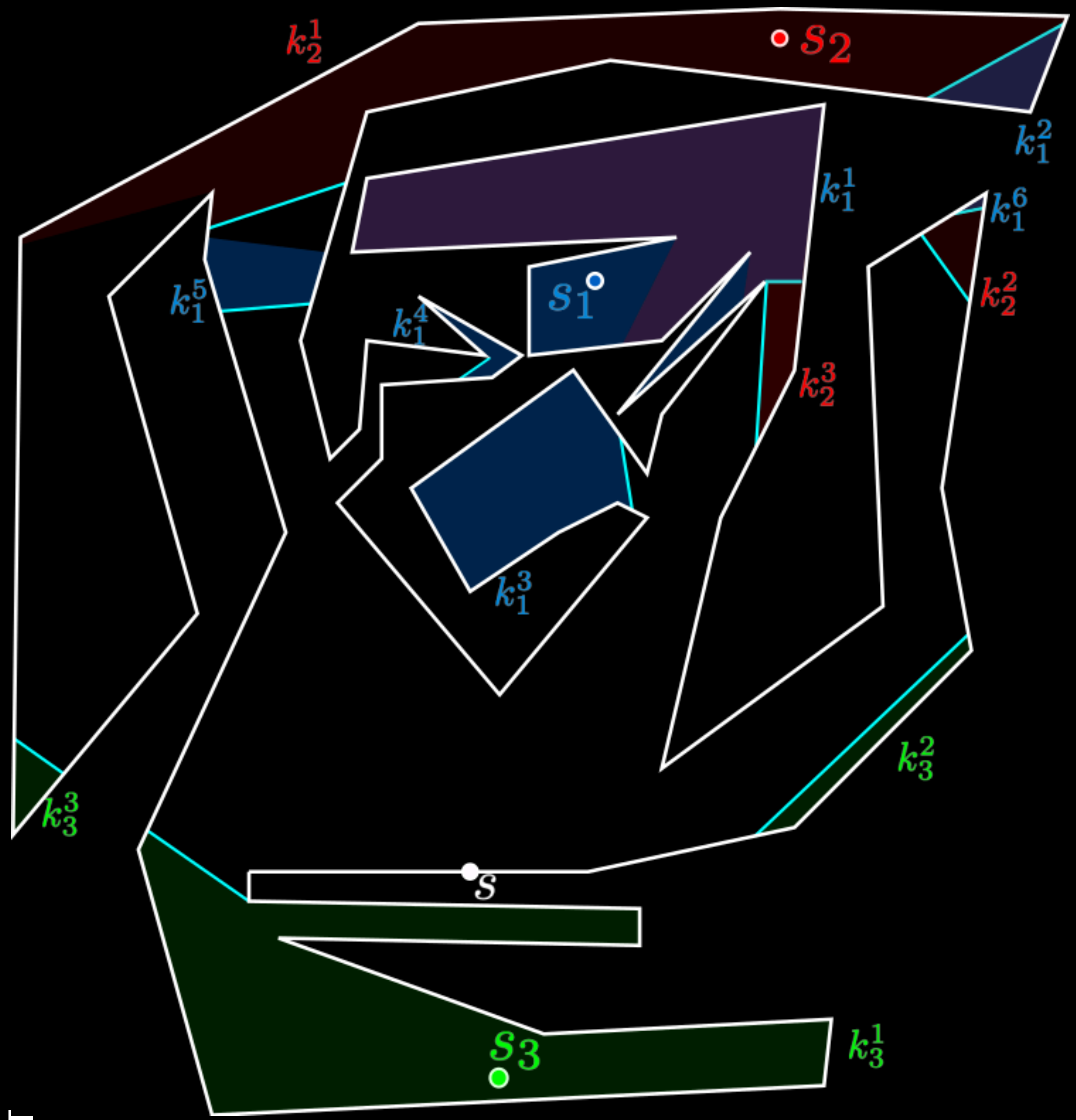

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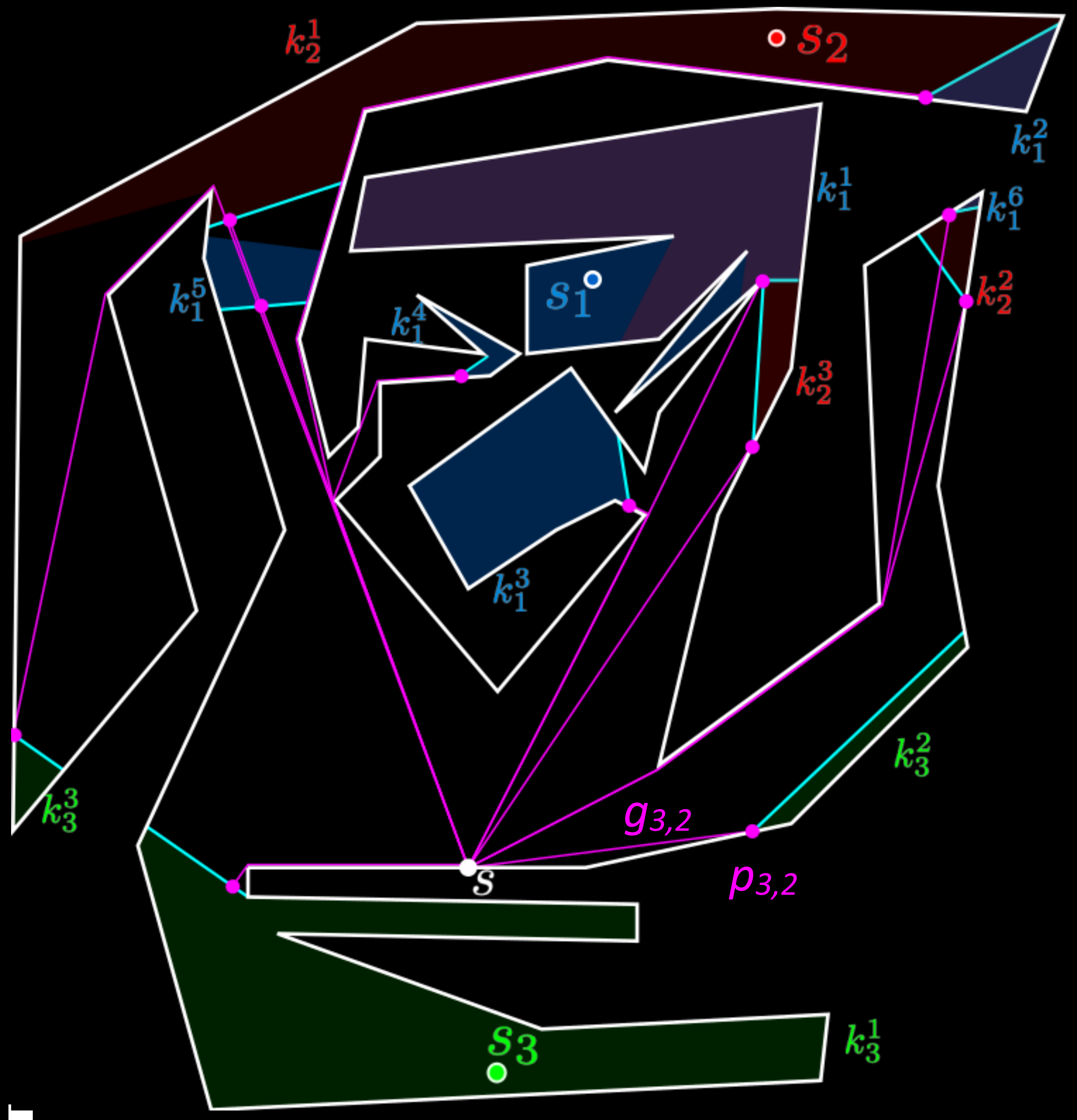

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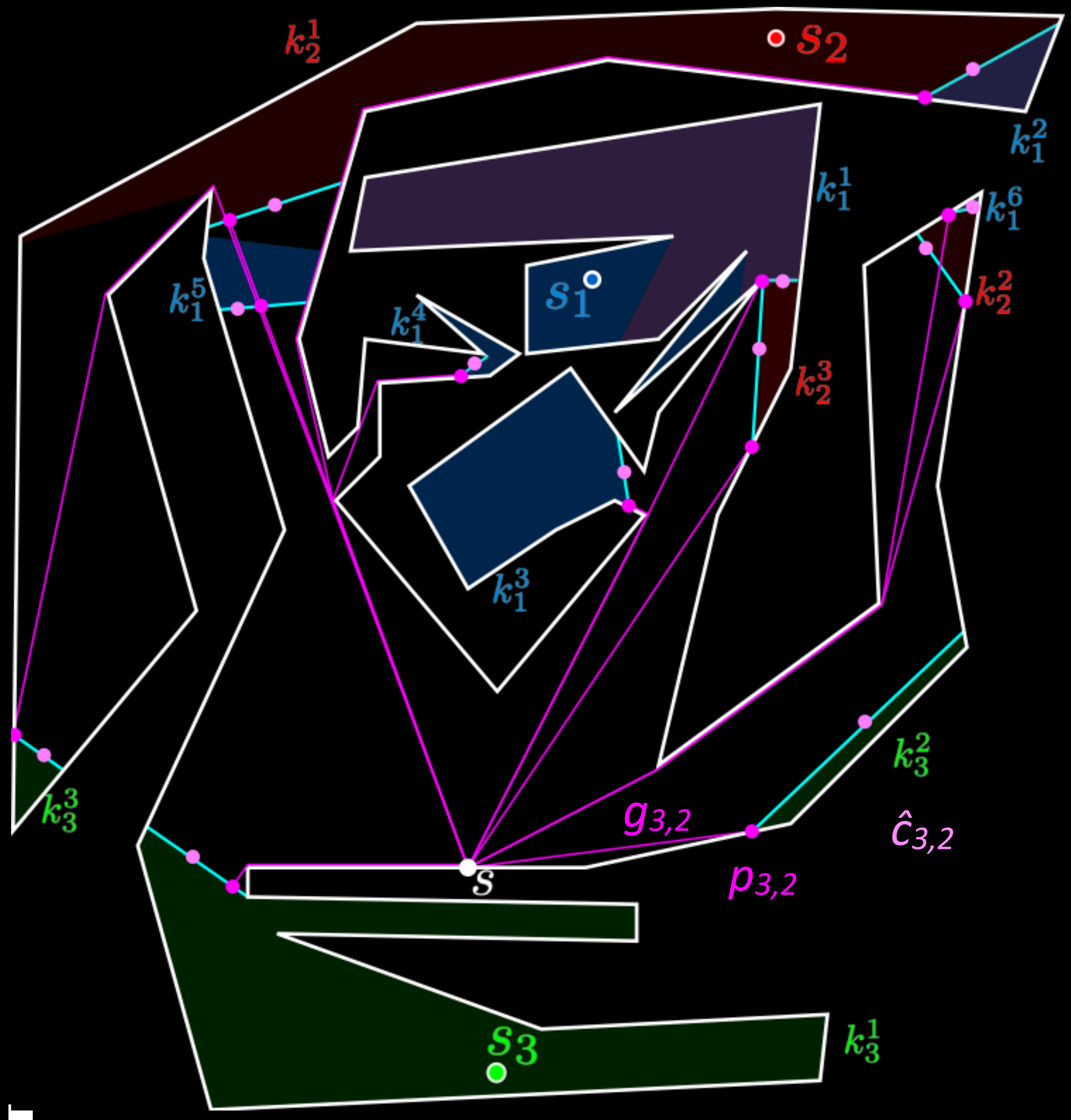

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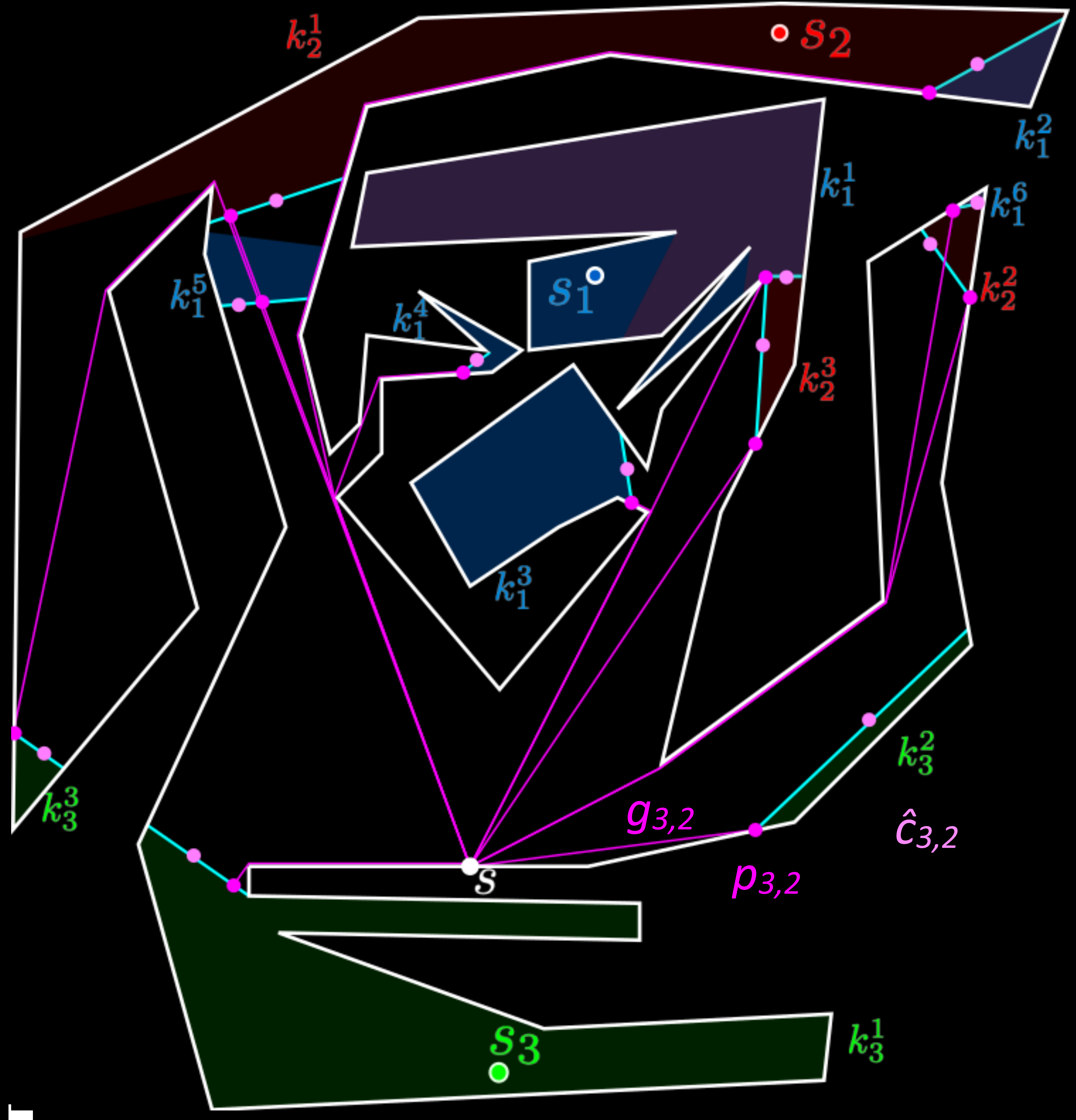

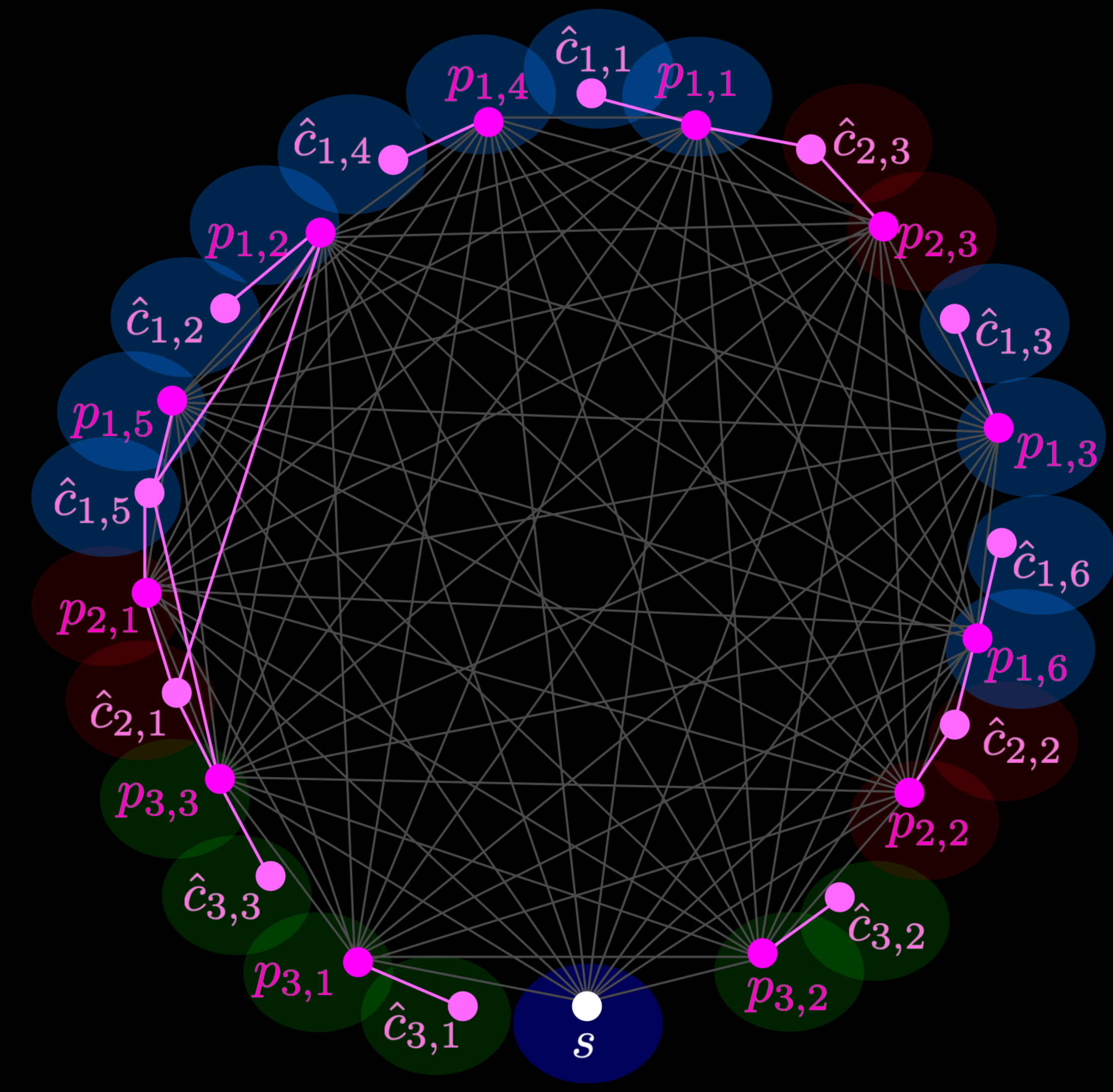

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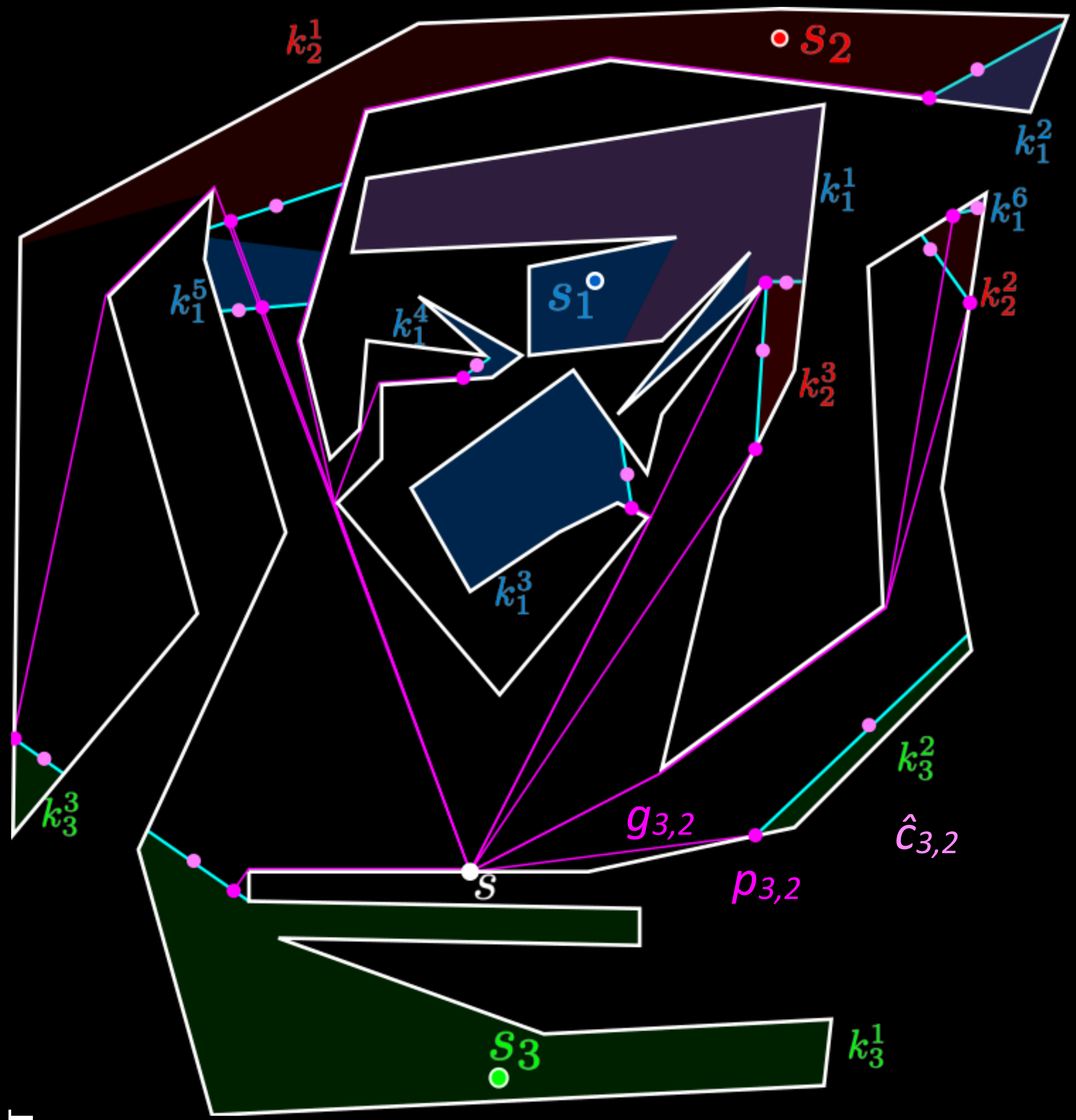

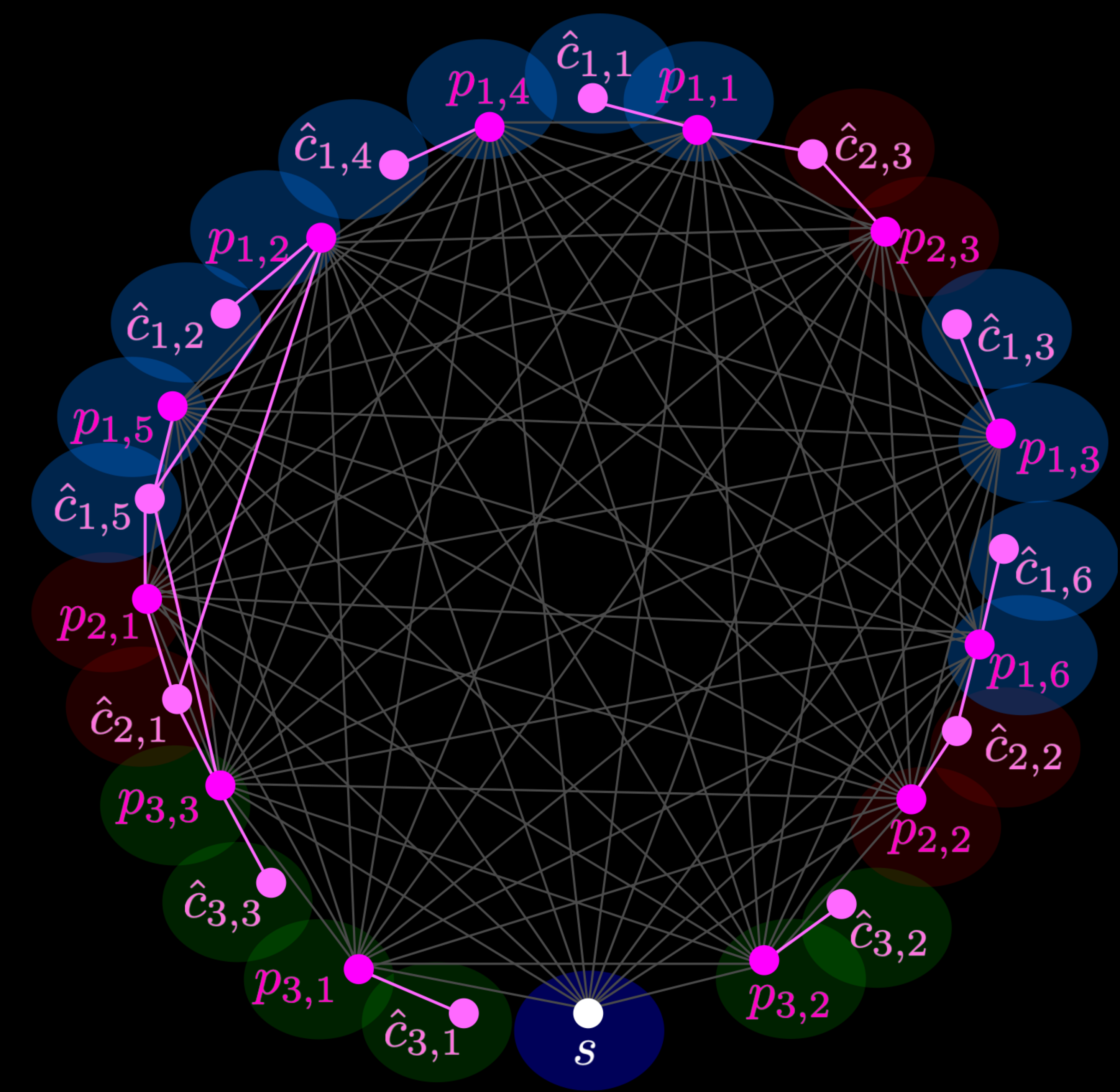

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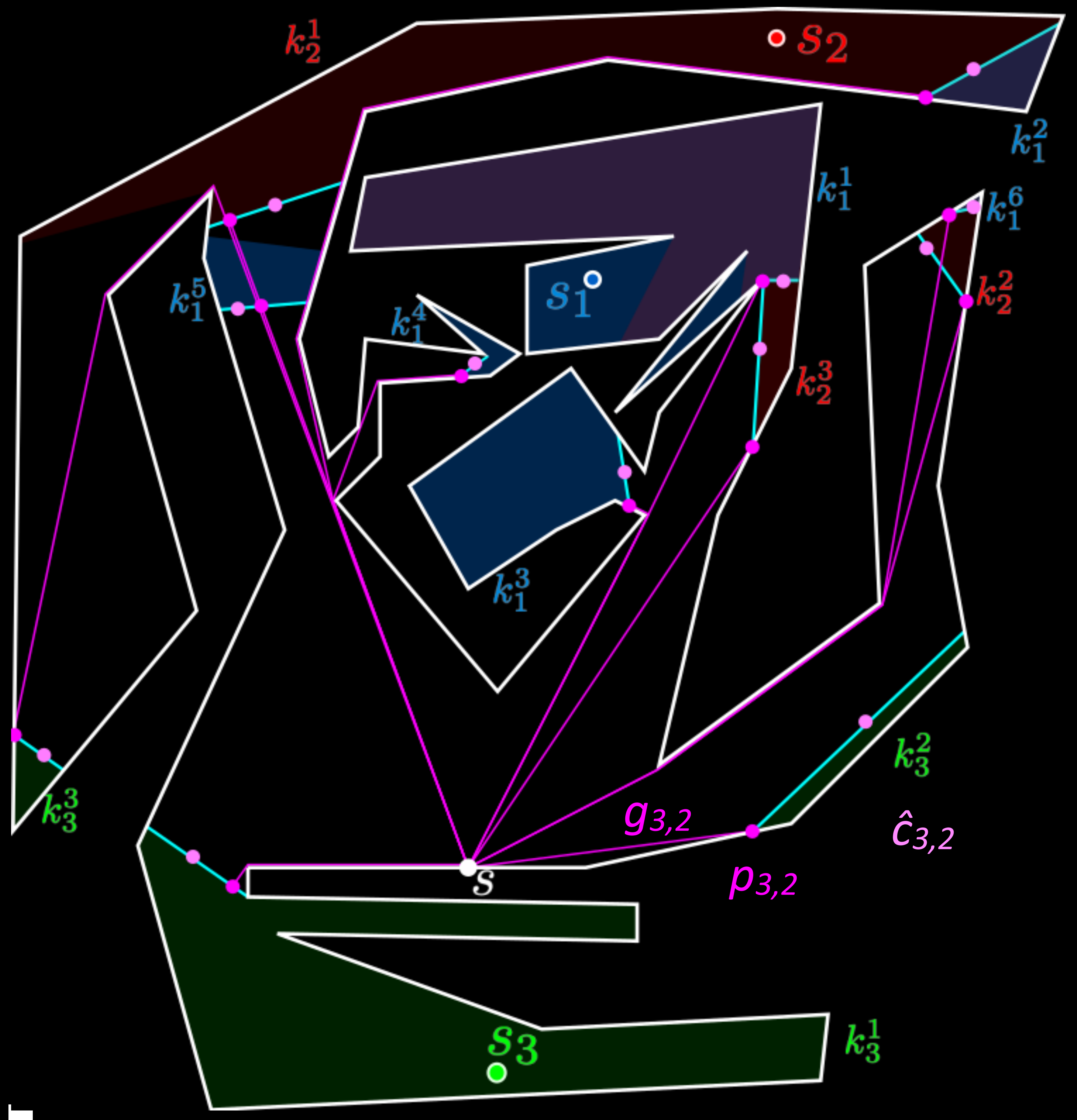

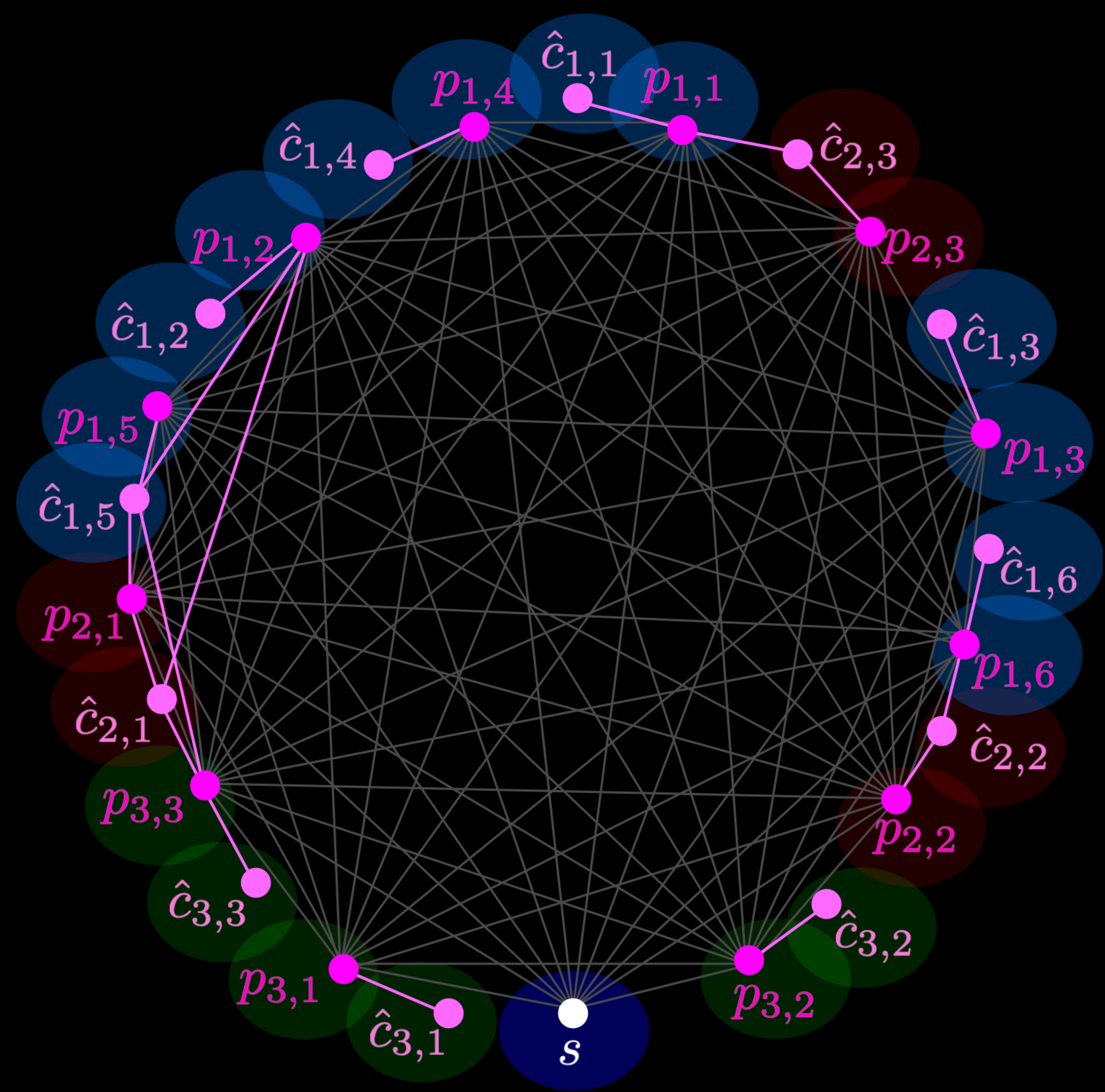

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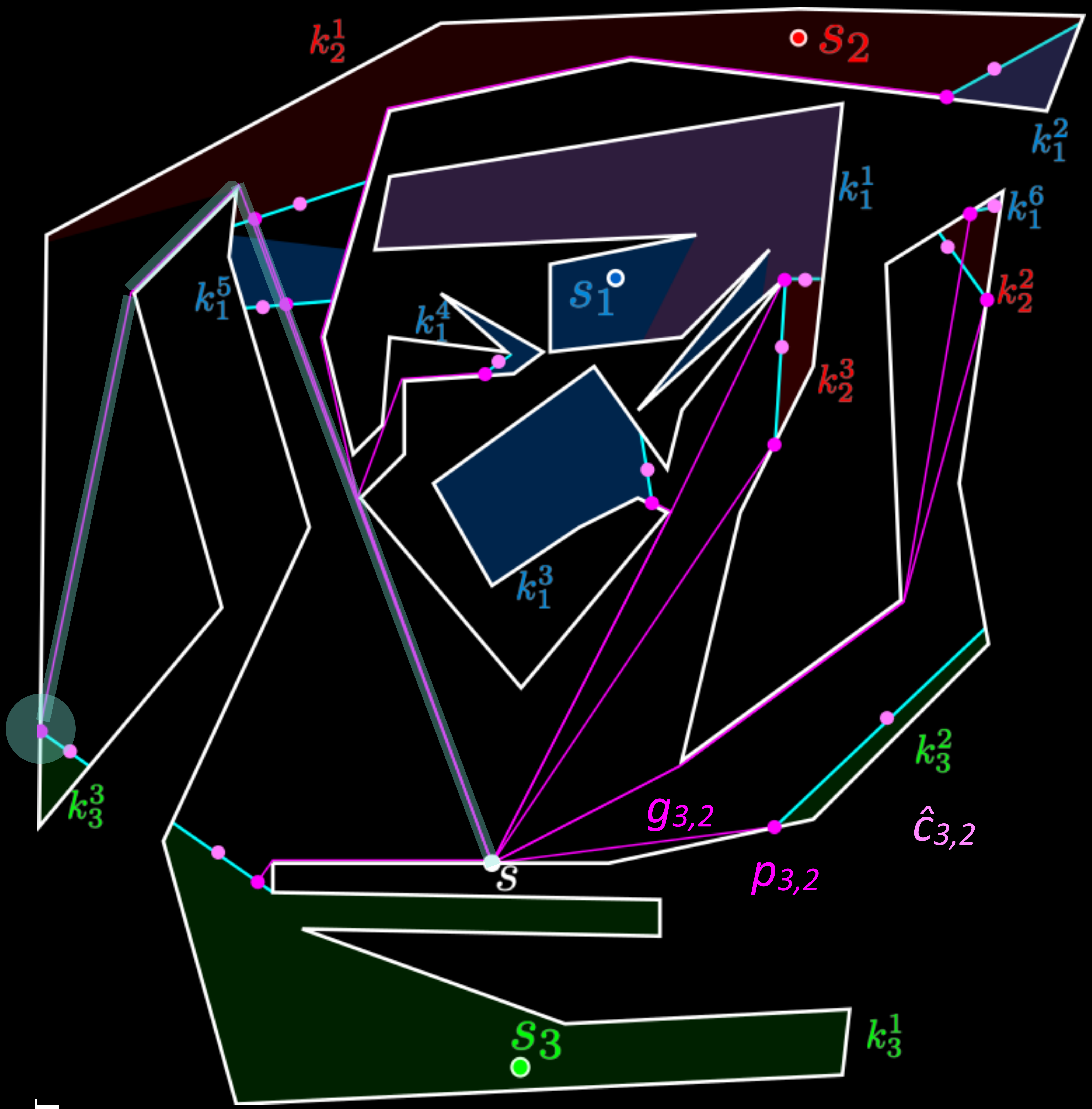
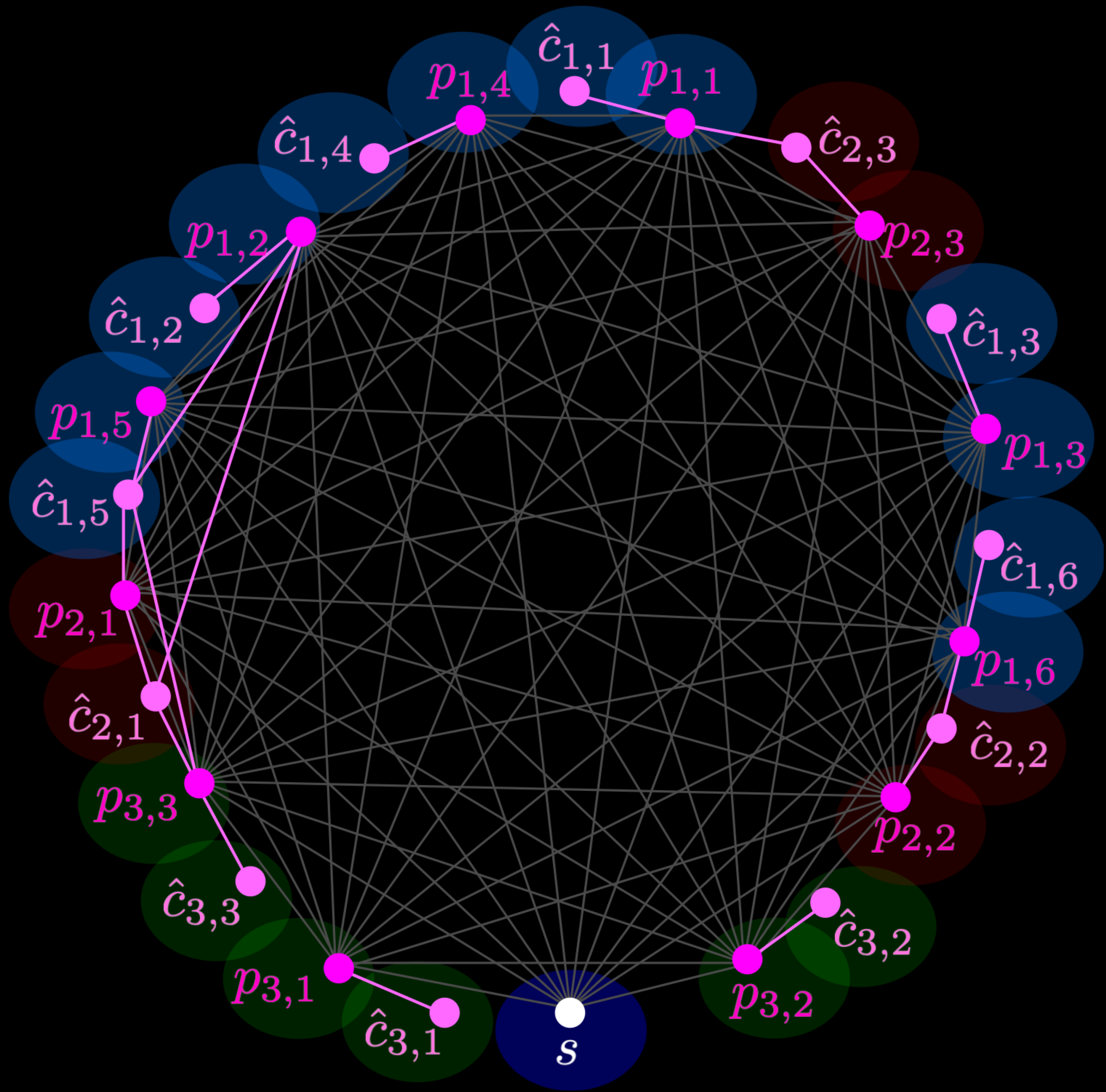
Example: When we visit k_3^3 (in point p_3^3), we also visit the cuts of k_3^3, k_2^1 and k_1^5.
Thus, we have edges from p_3^3 to $\hat{c}_3^3$, $\hat{c}_2^1$, and $\hat{c}_1^5$.





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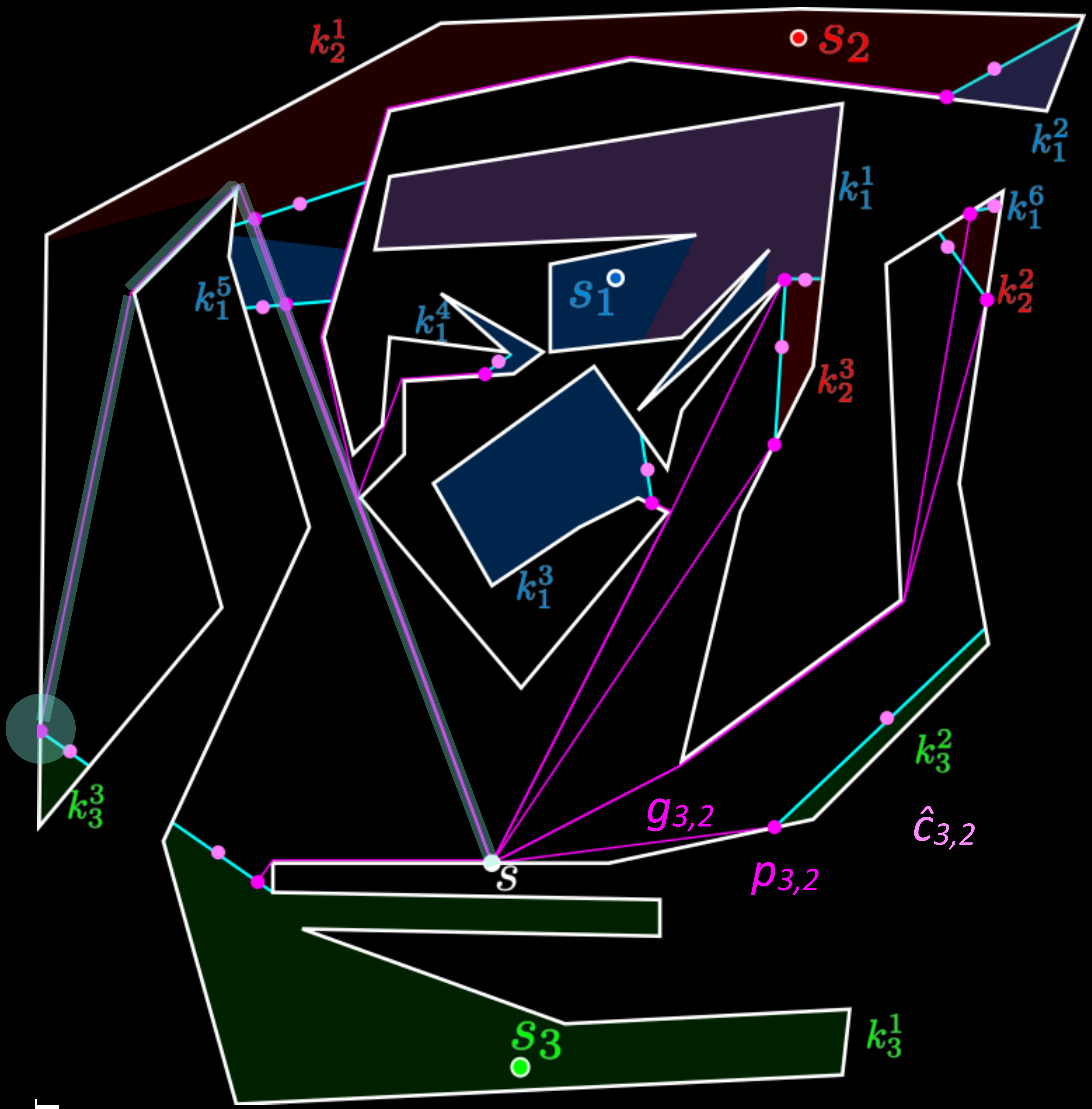

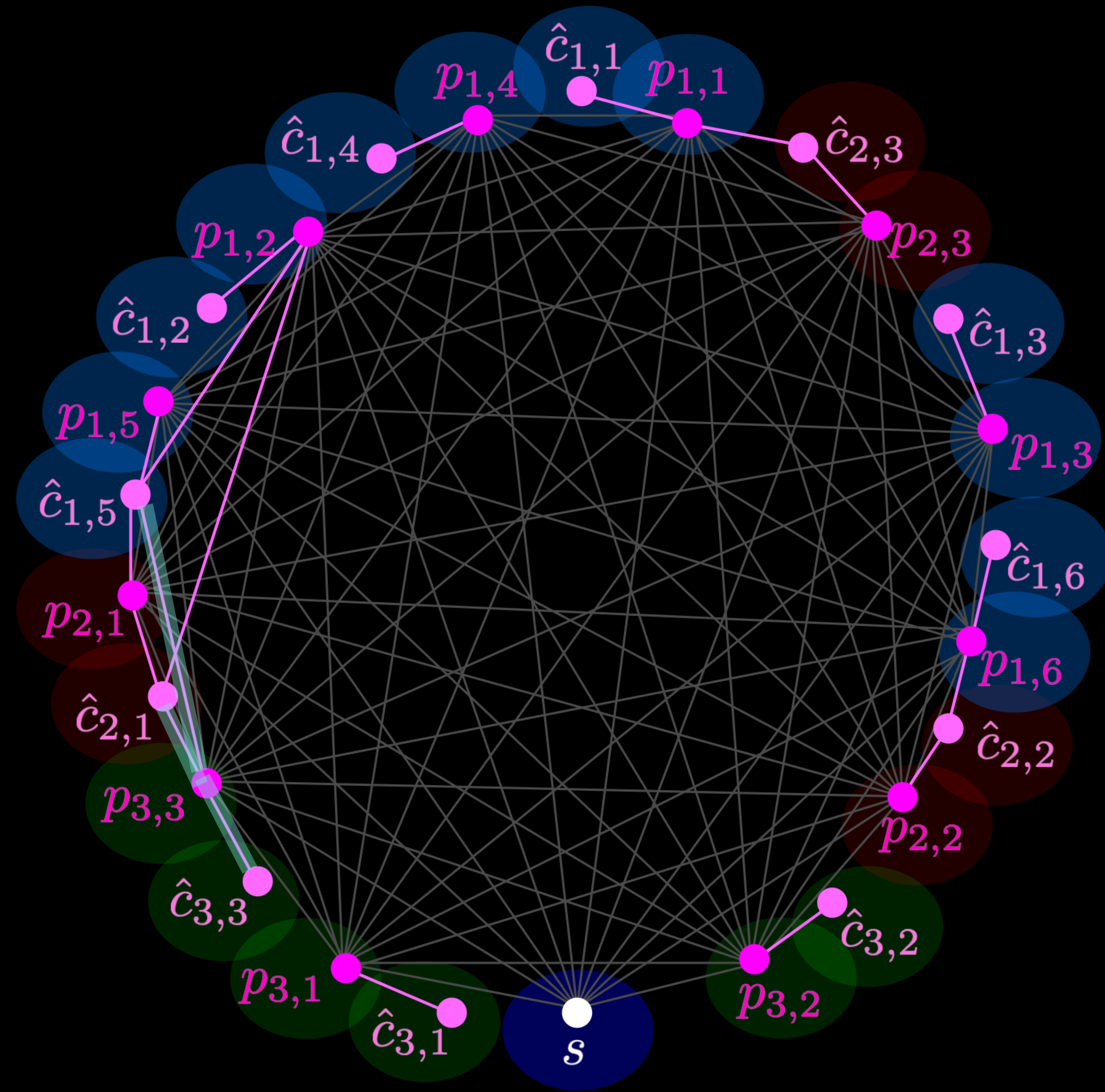

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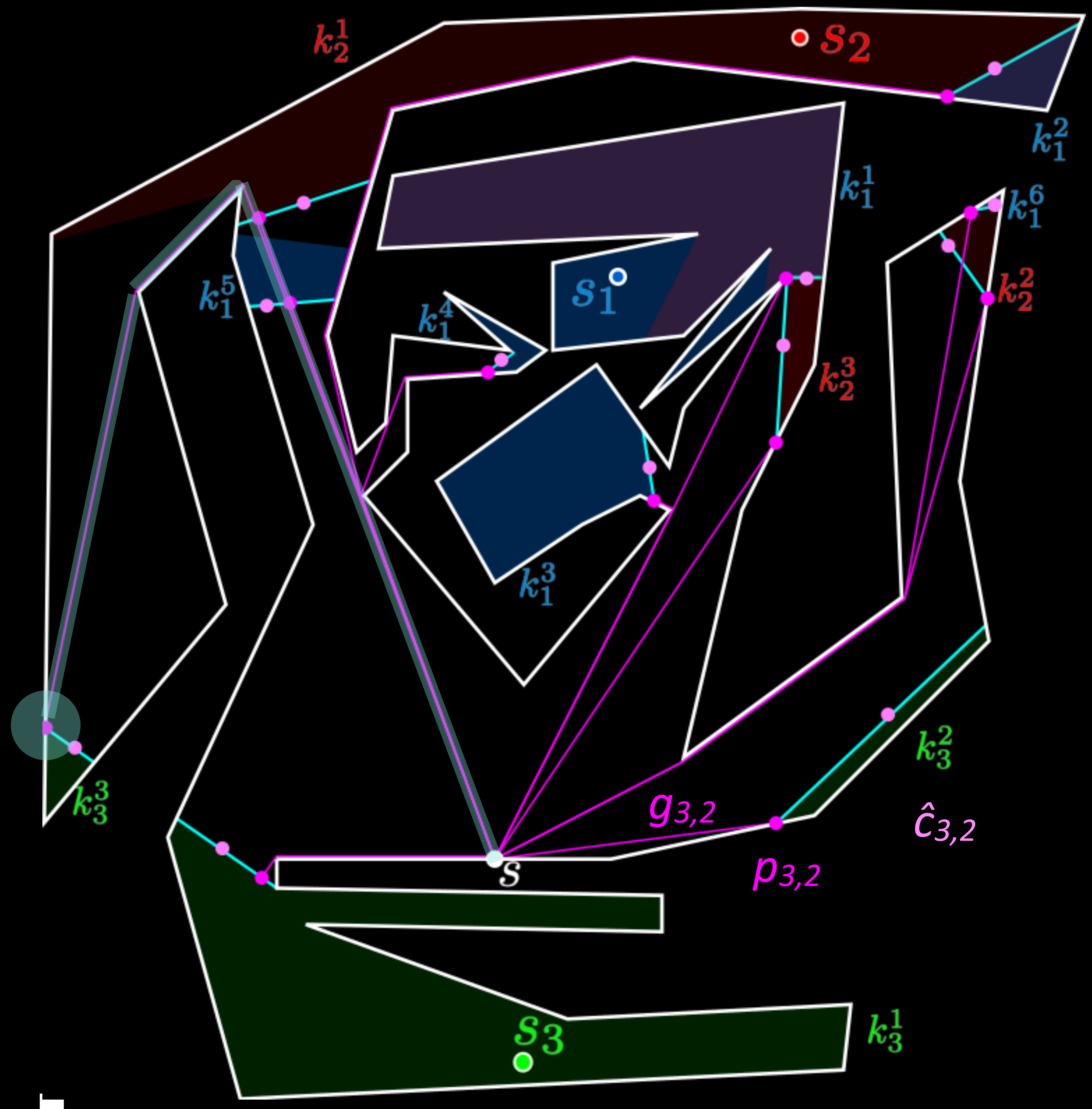

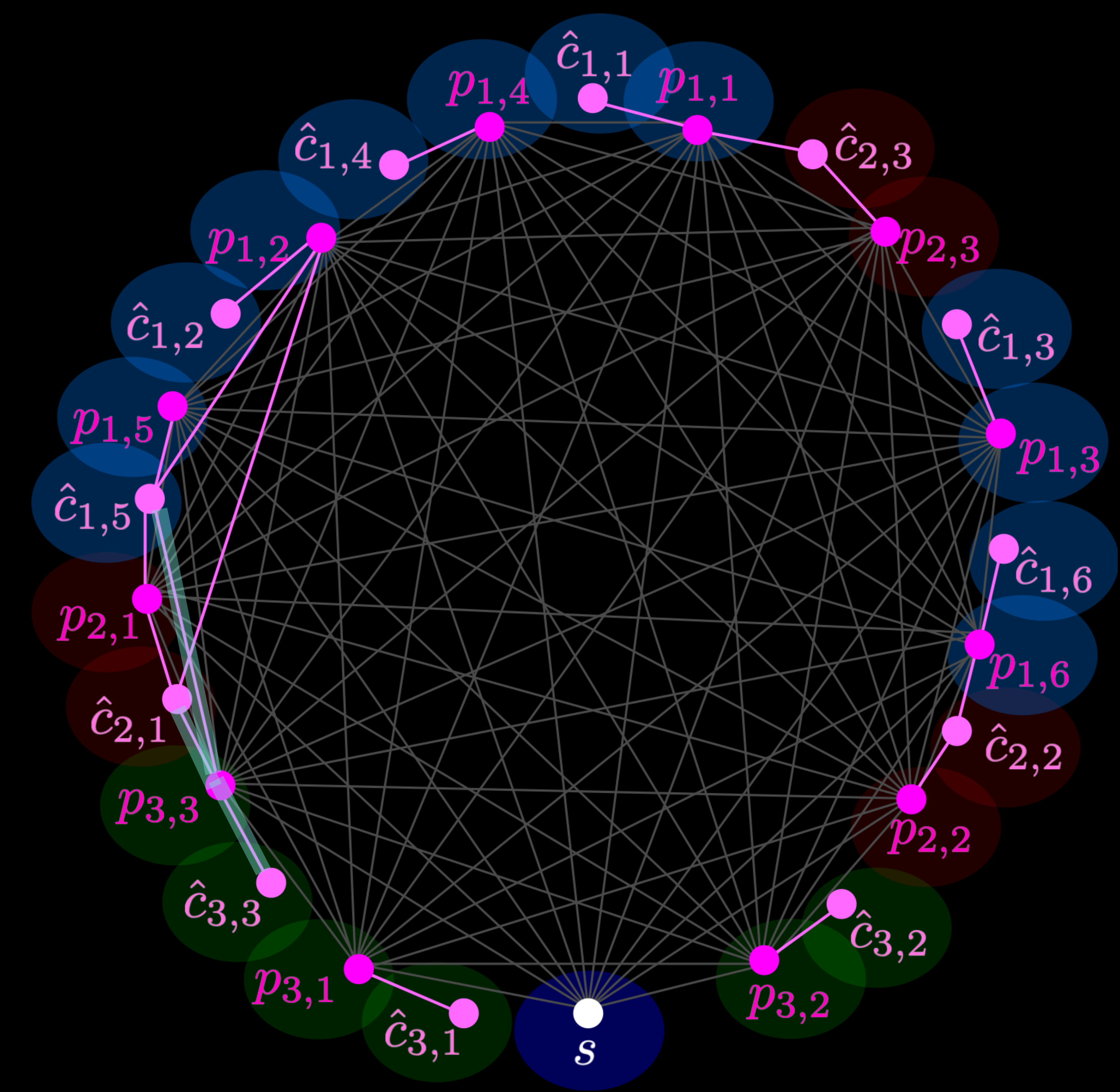

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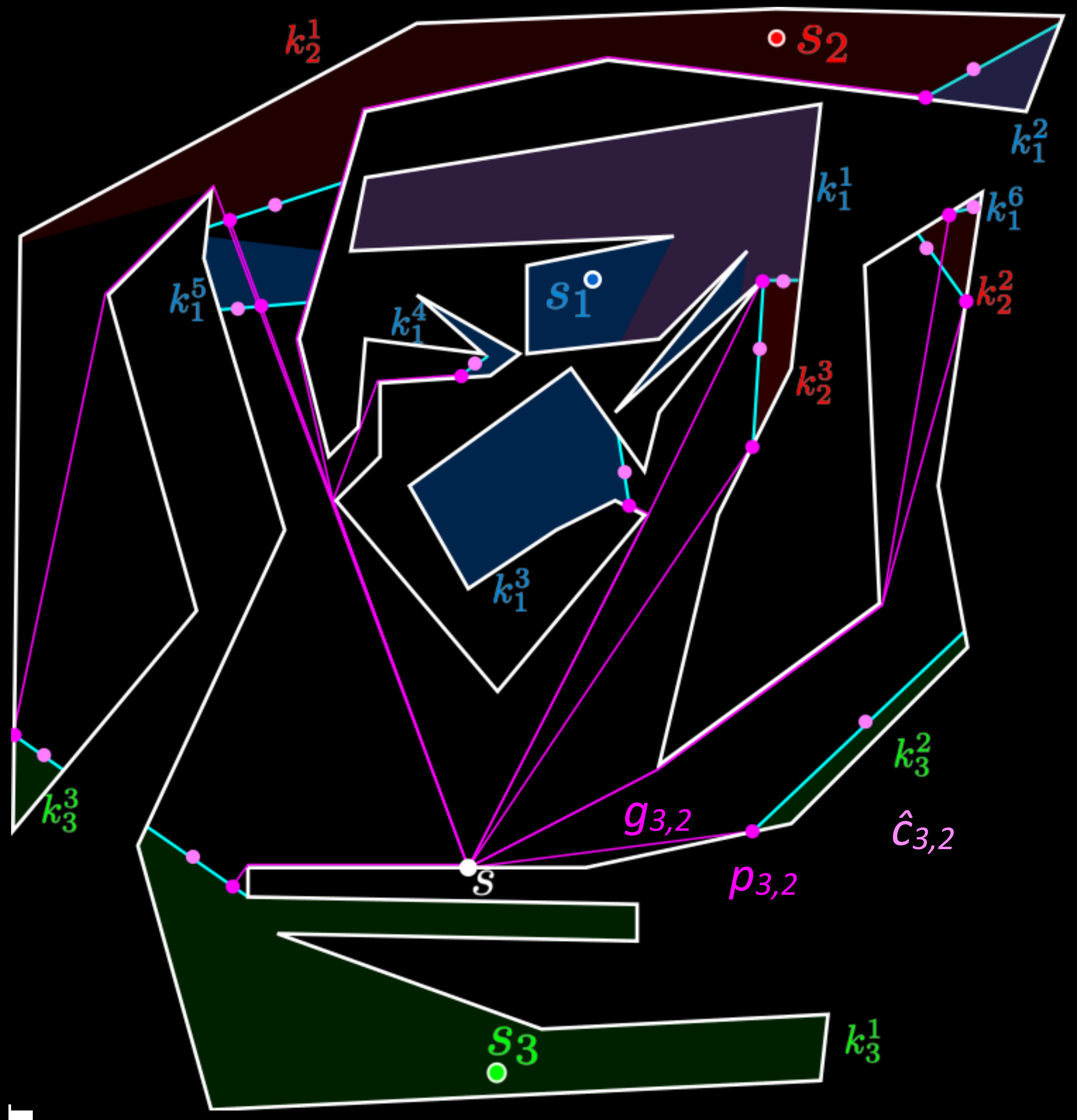

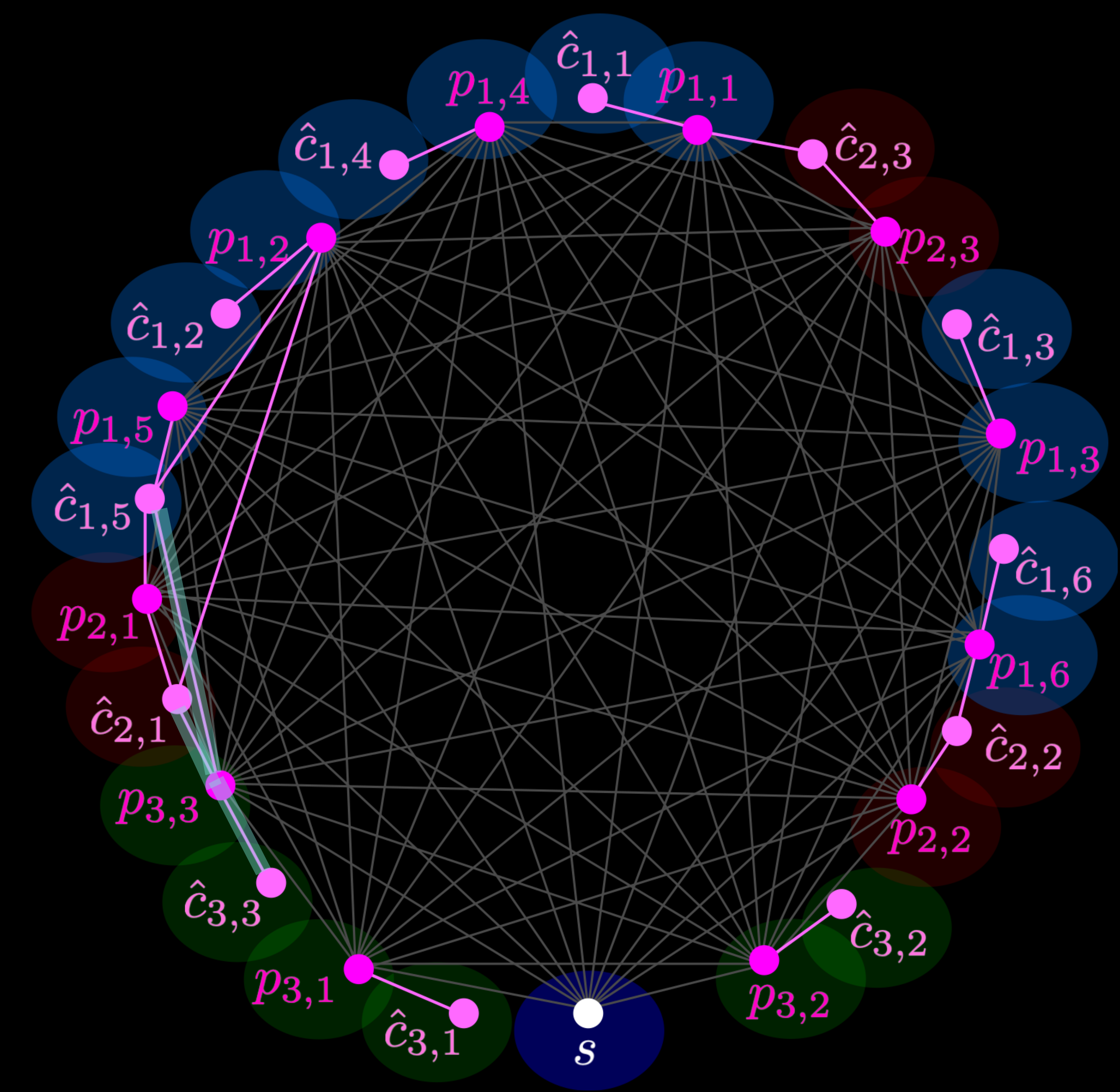

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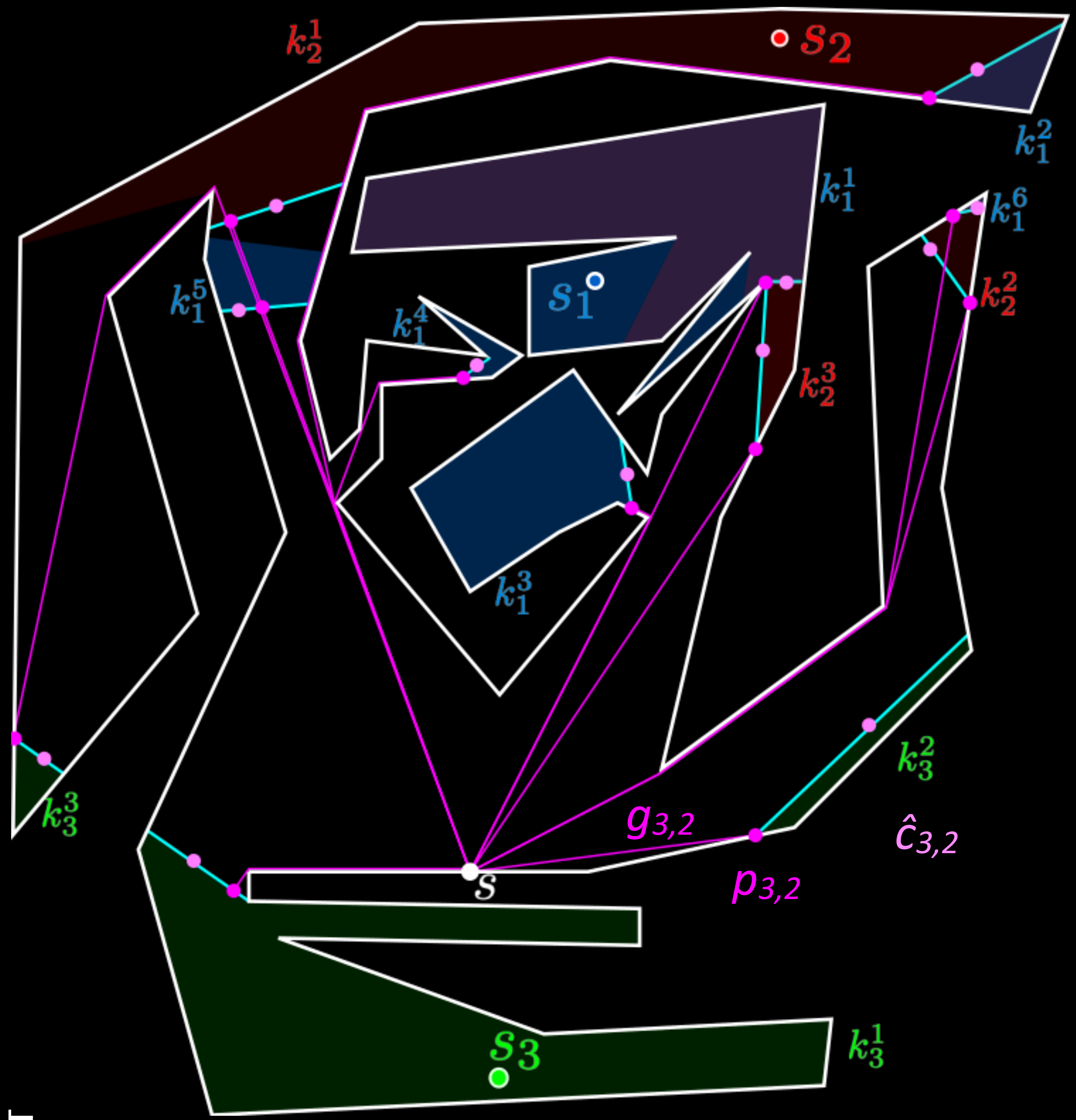
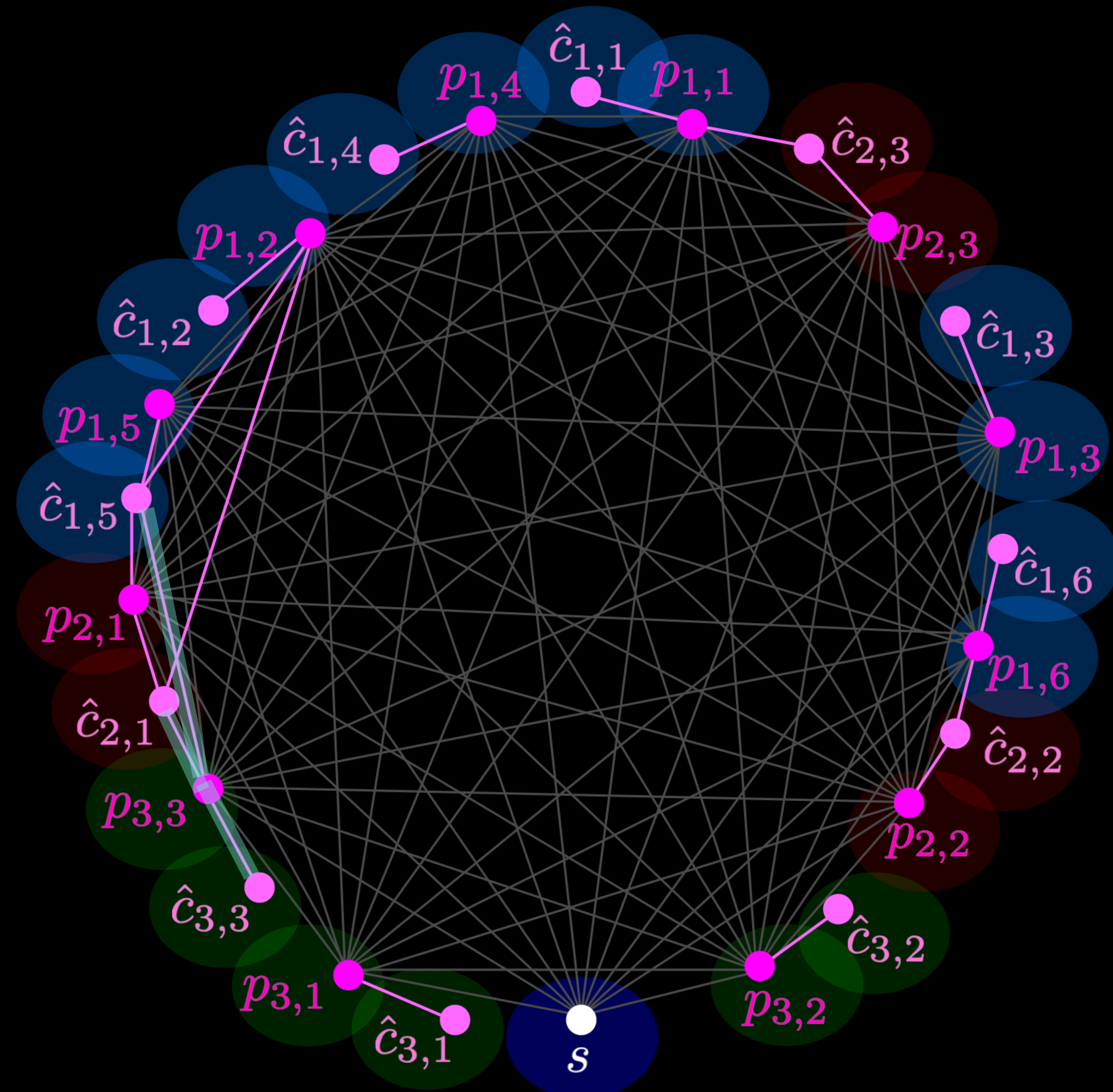
- Group all candidate points that belong to the same point in *S*: $\gamma_i = \bigcup_{j=1}^{J_i} p_{i,j} \cup \bigcup_{j=1}^{J_i} \hat{c}_{i,j}$





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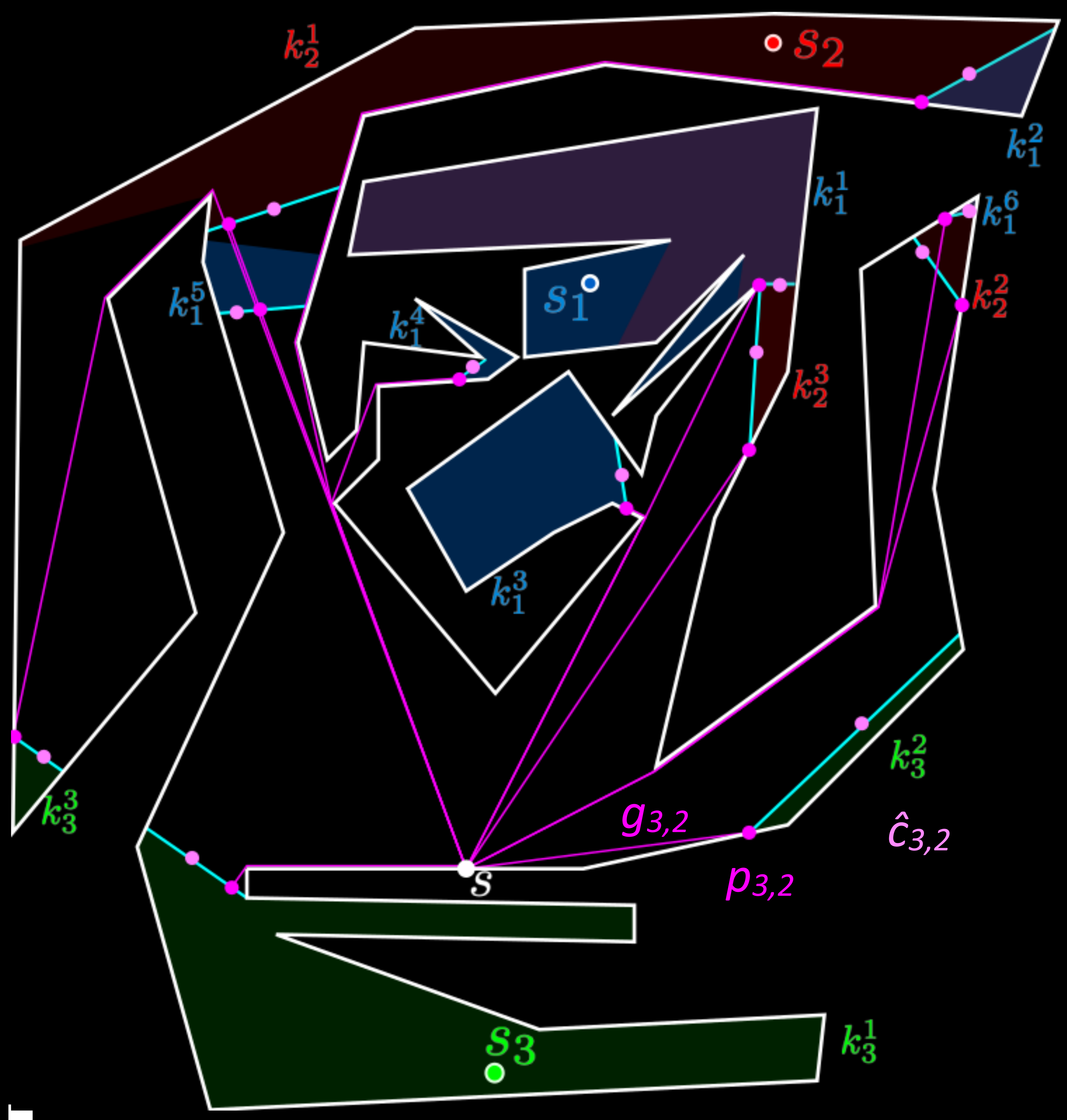
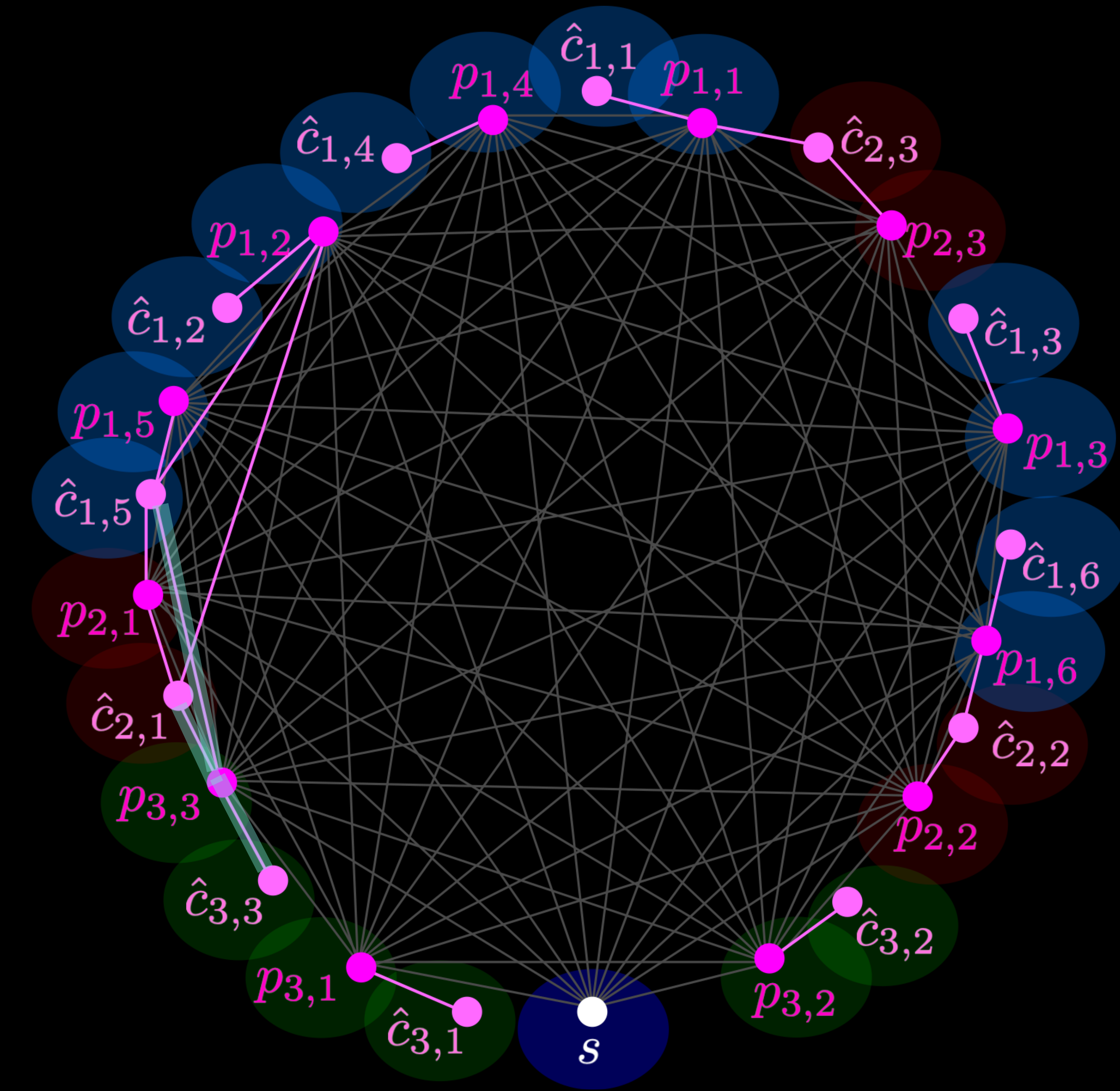


- Create a candidate point for each connected component of the *k*-visibility region of each point in *S*.
- Candidate points: intersection of geodesics from starting point *s* to cuts (C^{all} set of all cuts)
- Build complete graph G on candidate points $p_{i,j}$:
  - Gray edges: length of geodesic
  - Add pink edges: edge cost 0 (any path/tour visiting $p_{i,j}$ must visit $\hat{c}_{i,j}$)
  - $|V(G)|=O(n\,|S|)$
- Group all candidate points that belong to the same point in *S*: $\gamma_i = \bigcup_{j=1}^{J_i} p_{i,j} \cup \bigcup_{j=1}^{J_i} \hat{c}_{i,j}$
- Add $\gamma_0 = s$



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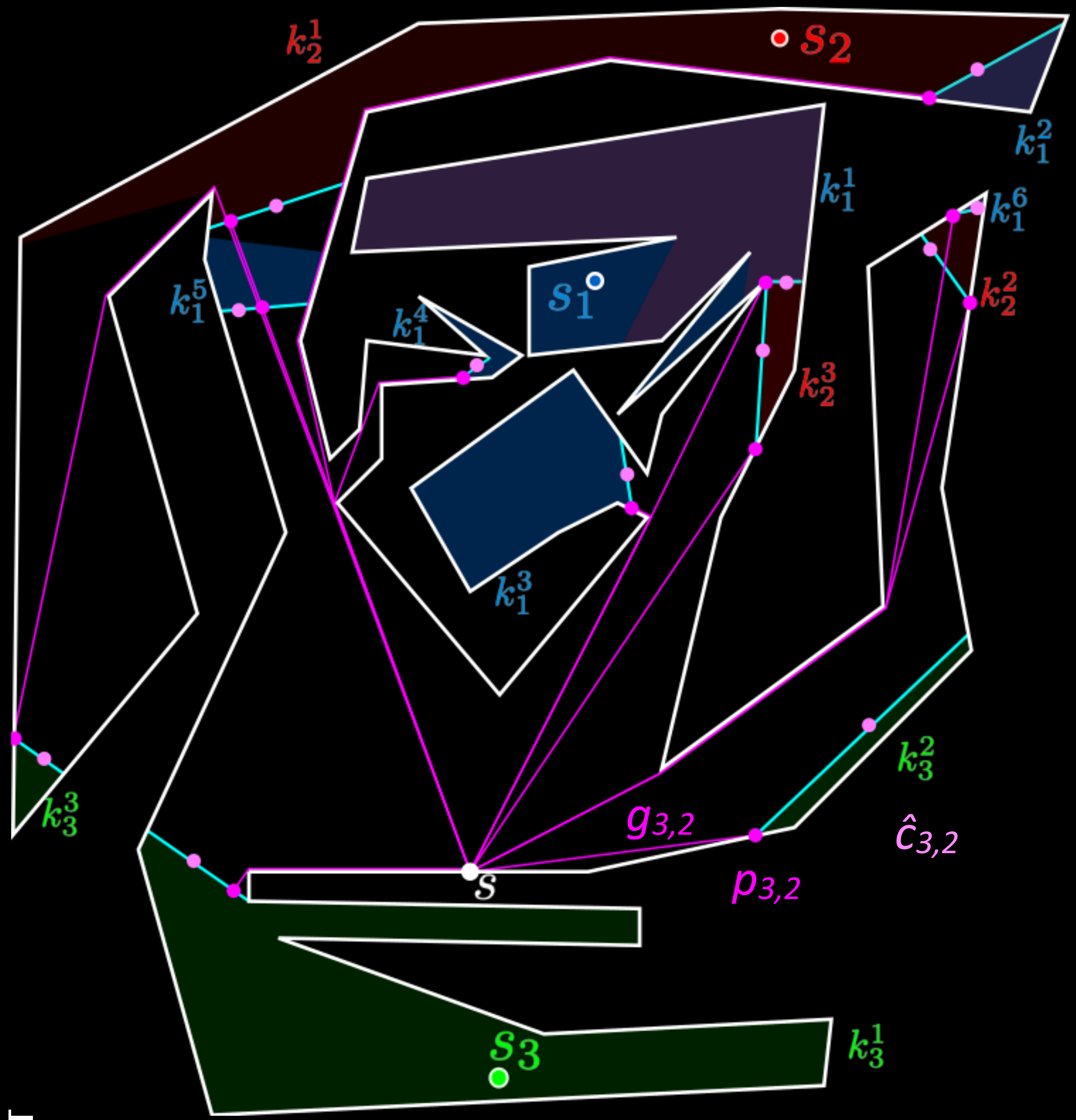

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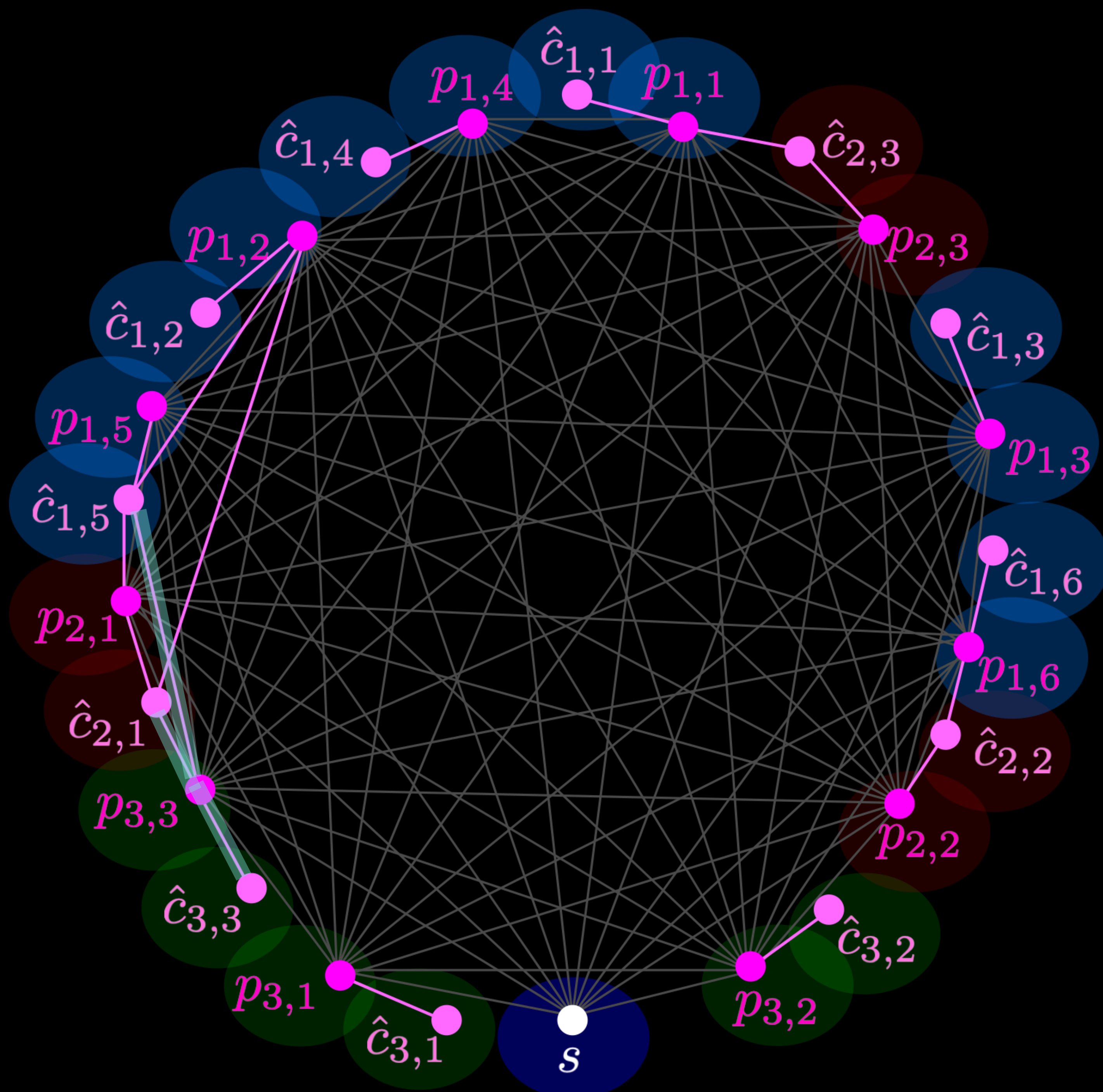
Here:

γ_1 candidate points that belong to s_1,

γ_2 candidate points that belong to s_2,

γ_3 candidate points that belong to s_3,

$\gamma_0=s$



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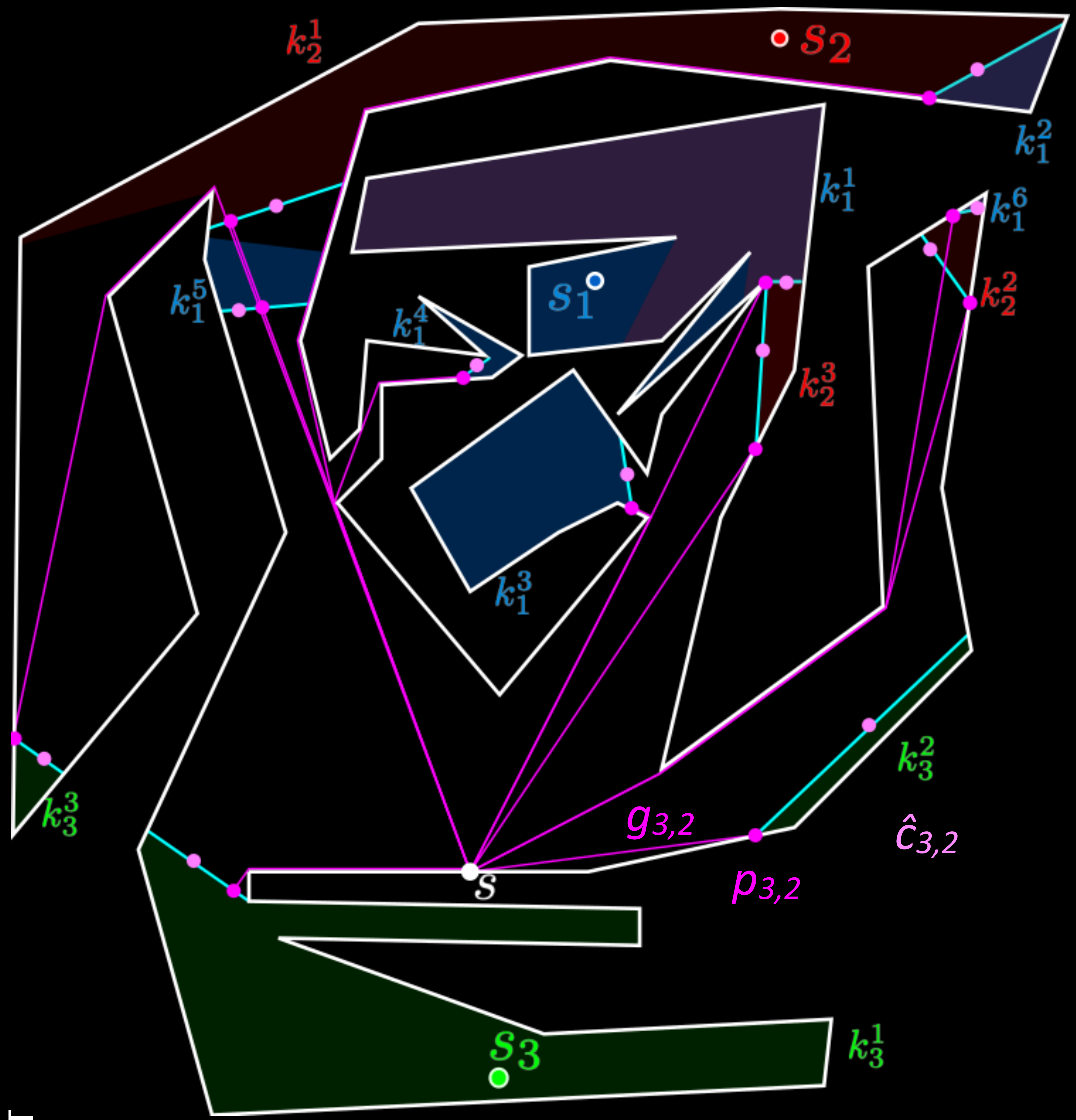

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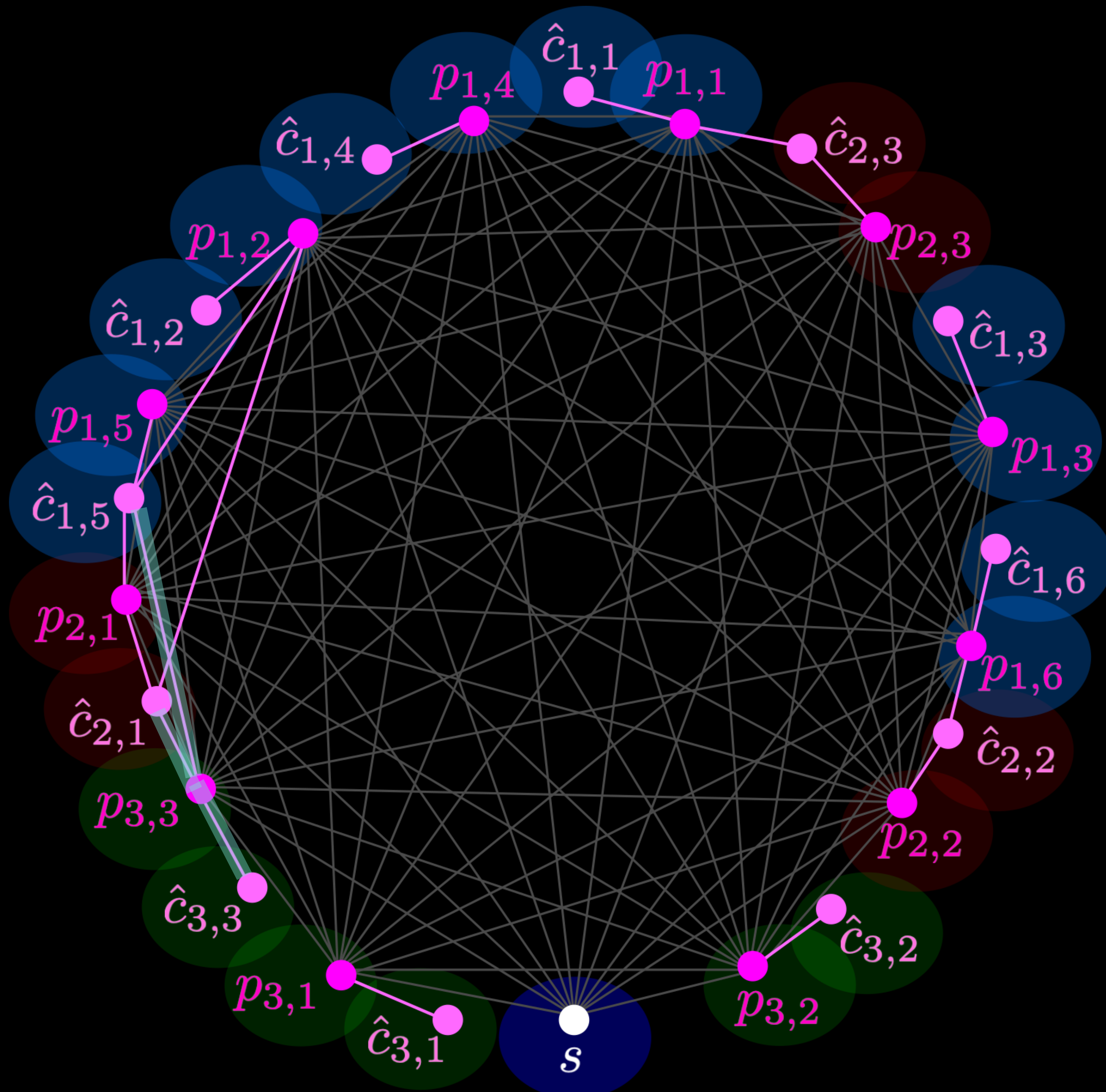

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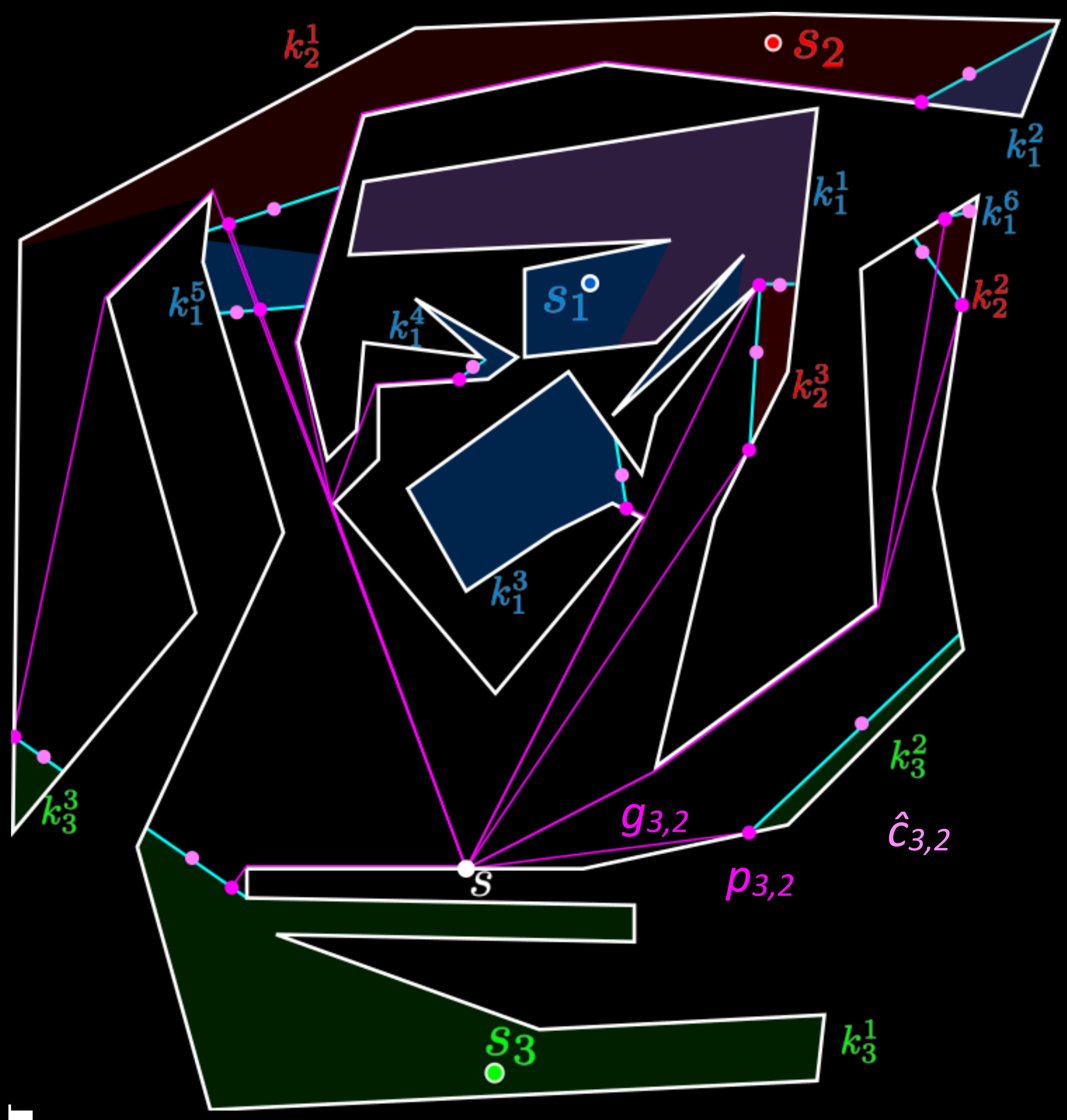

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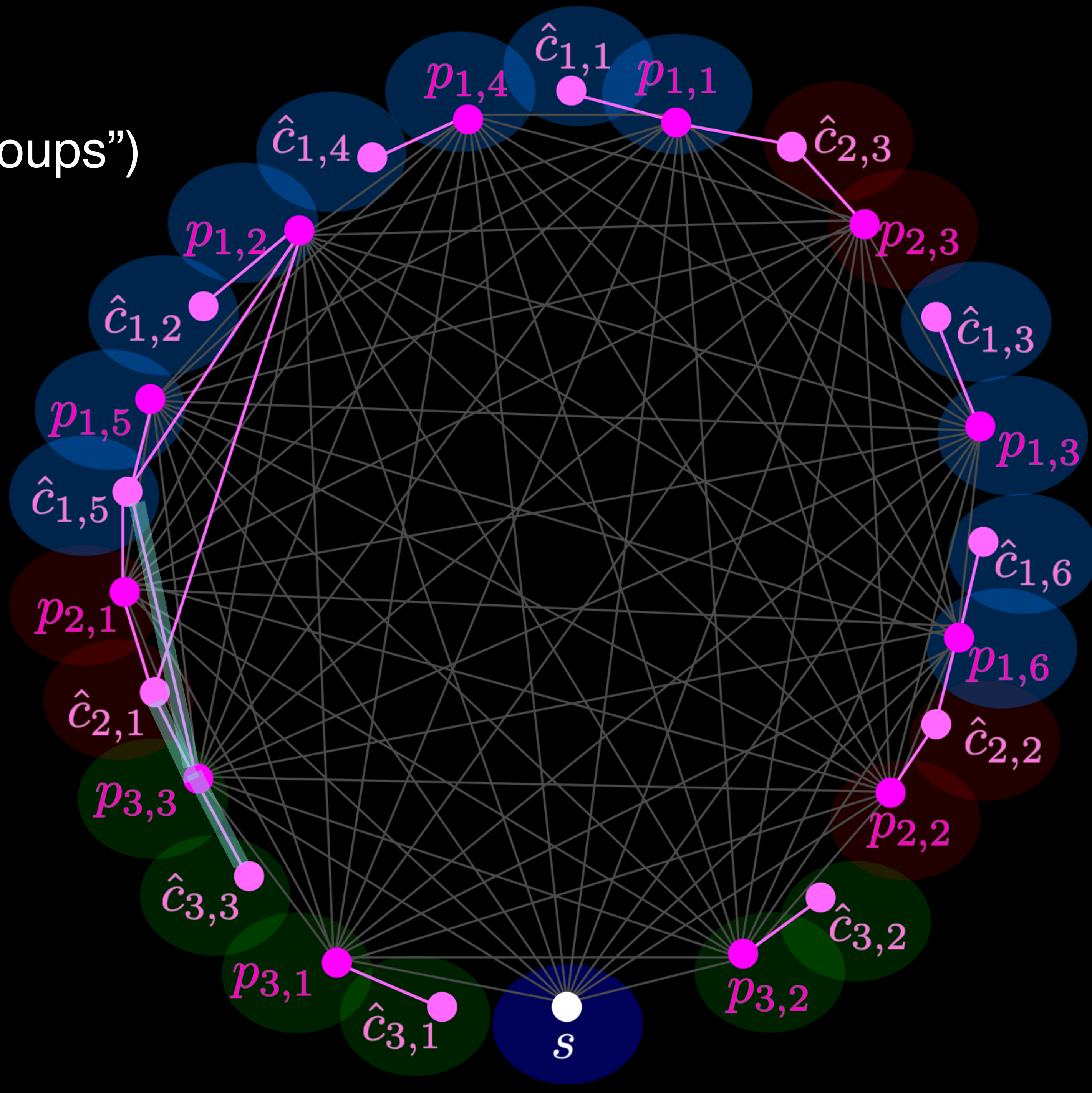

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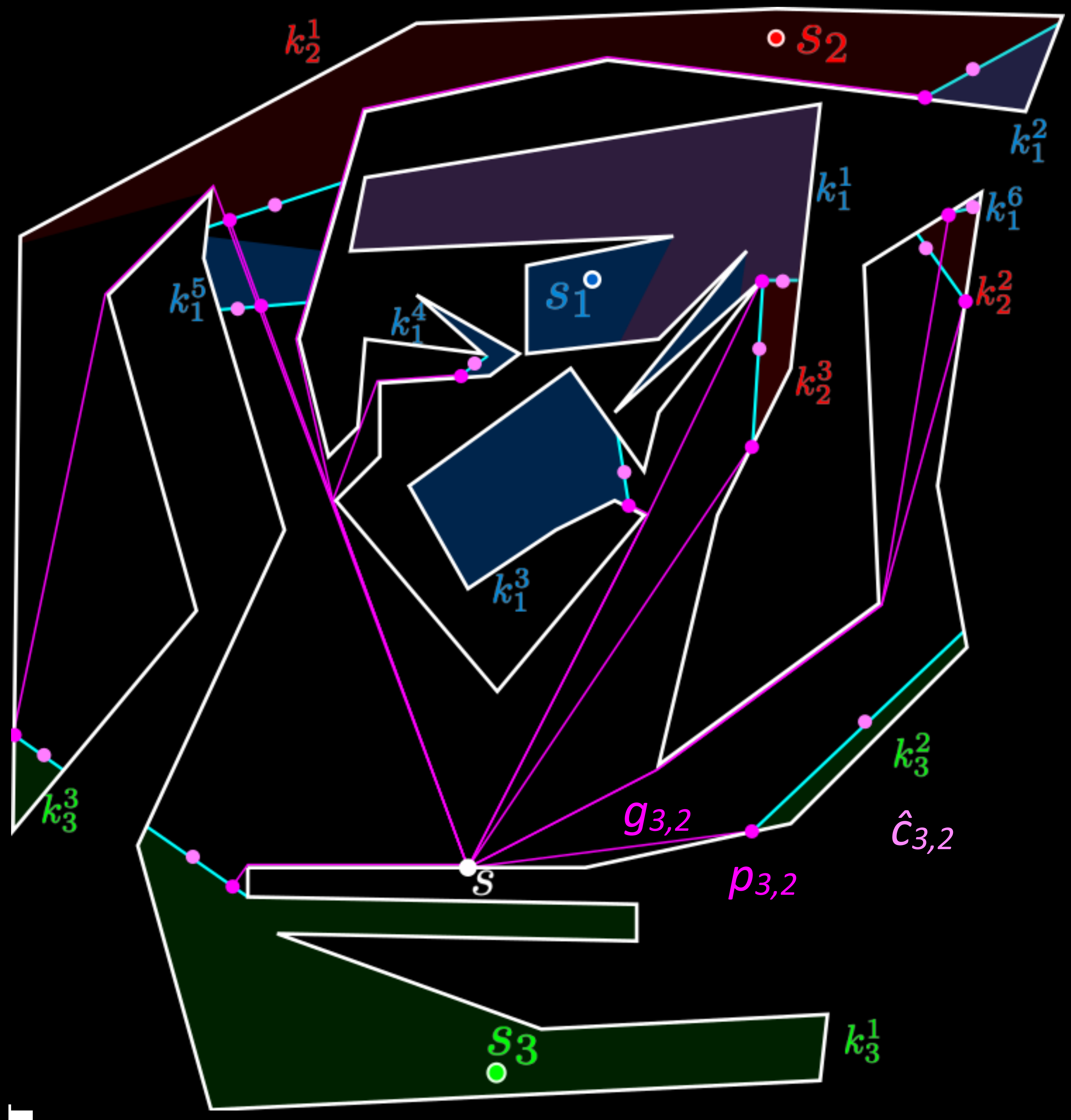

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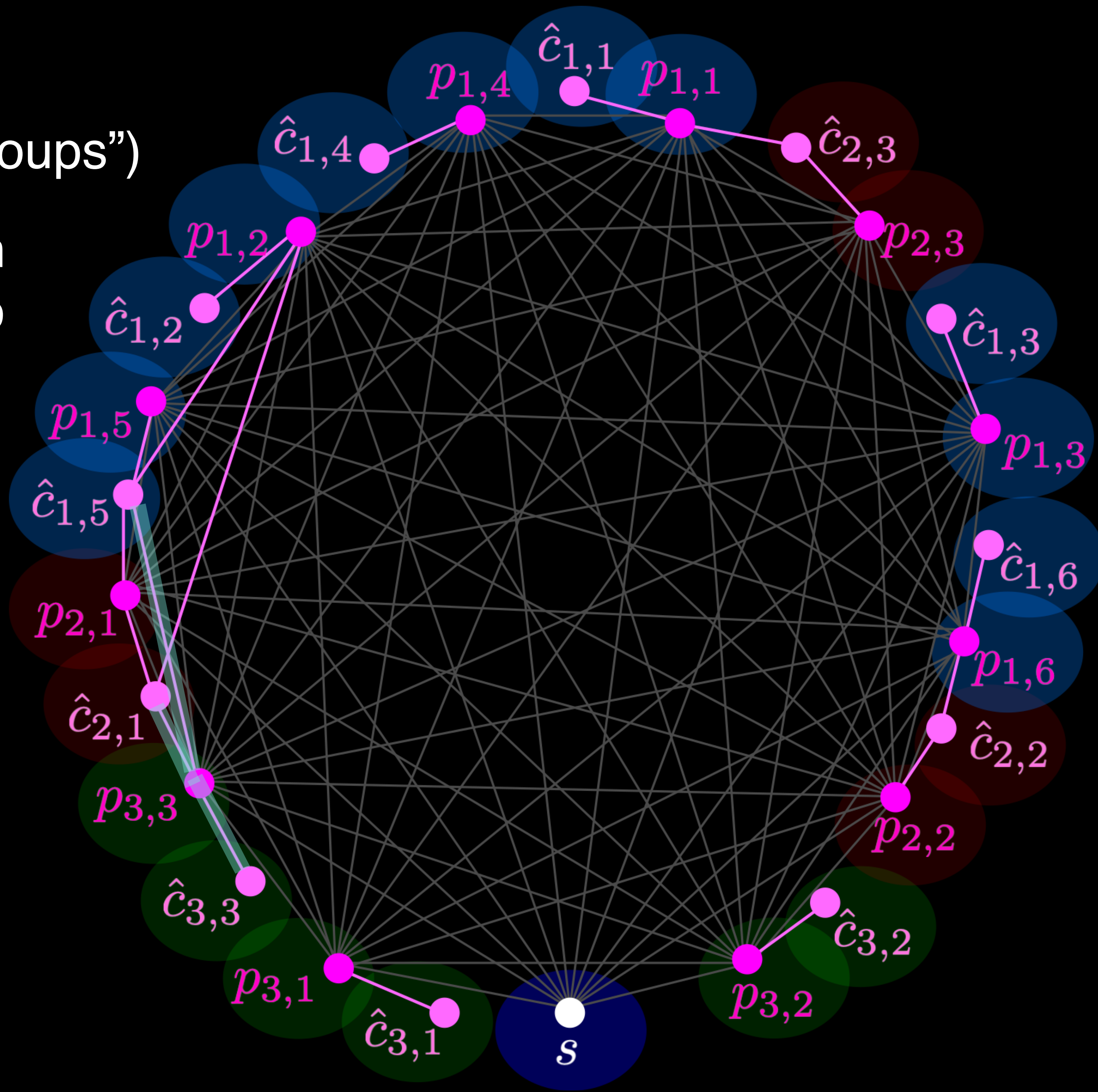

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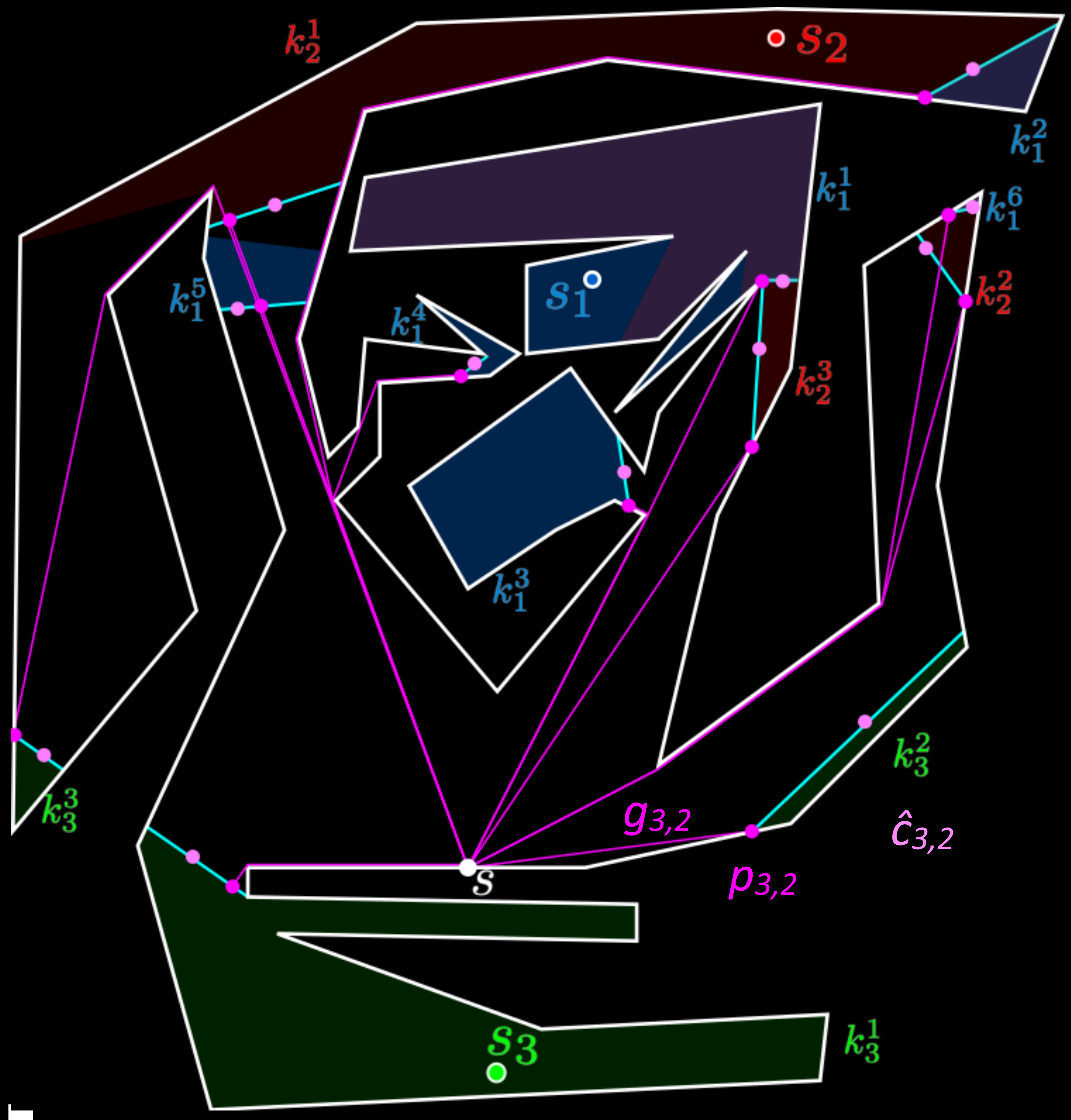

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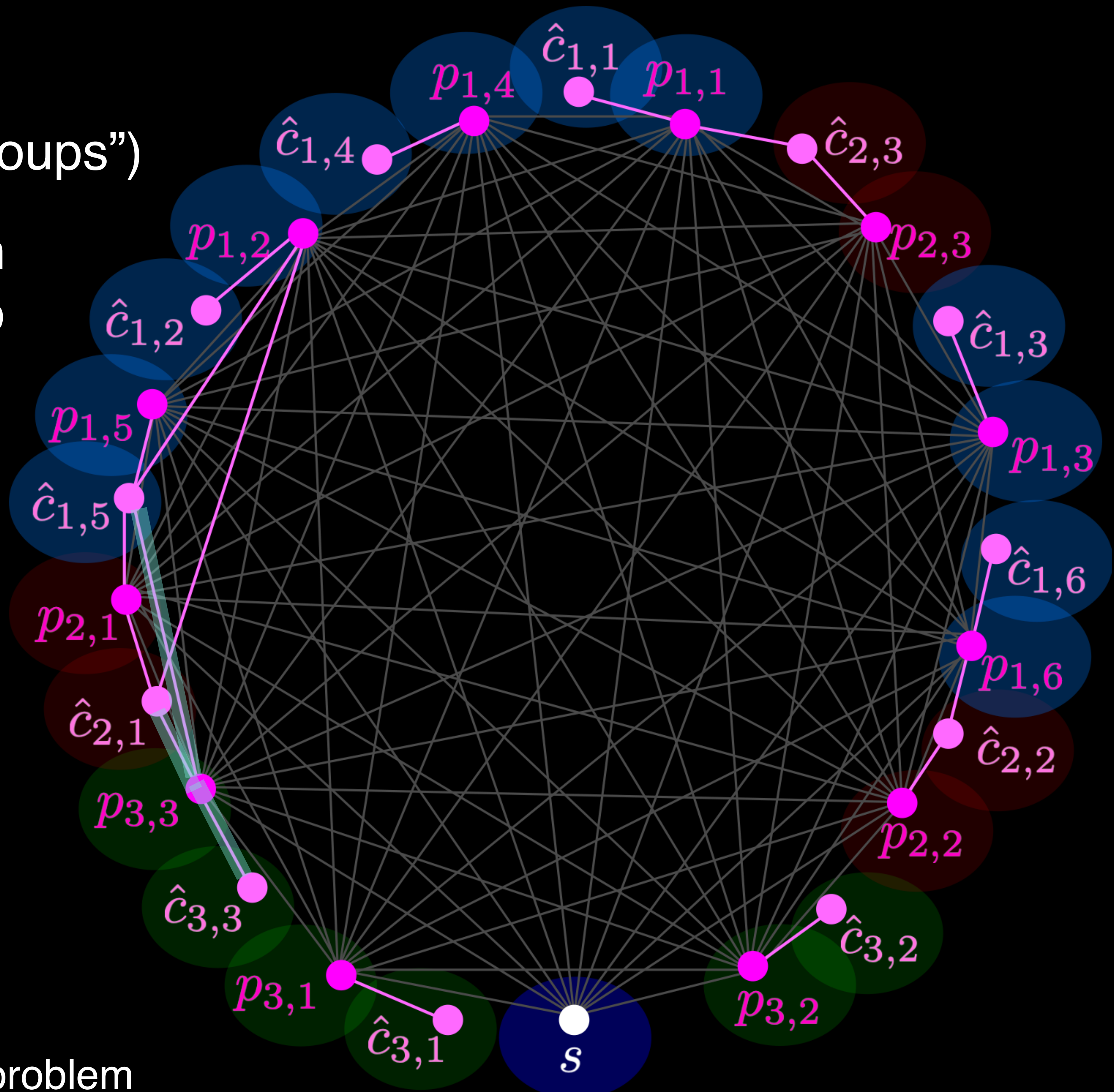

GKR00: Naveen Garg, Goran Konjevod, and R. Ravi. A polylogarithmic approximation algorithm for the group Steiner tree problem

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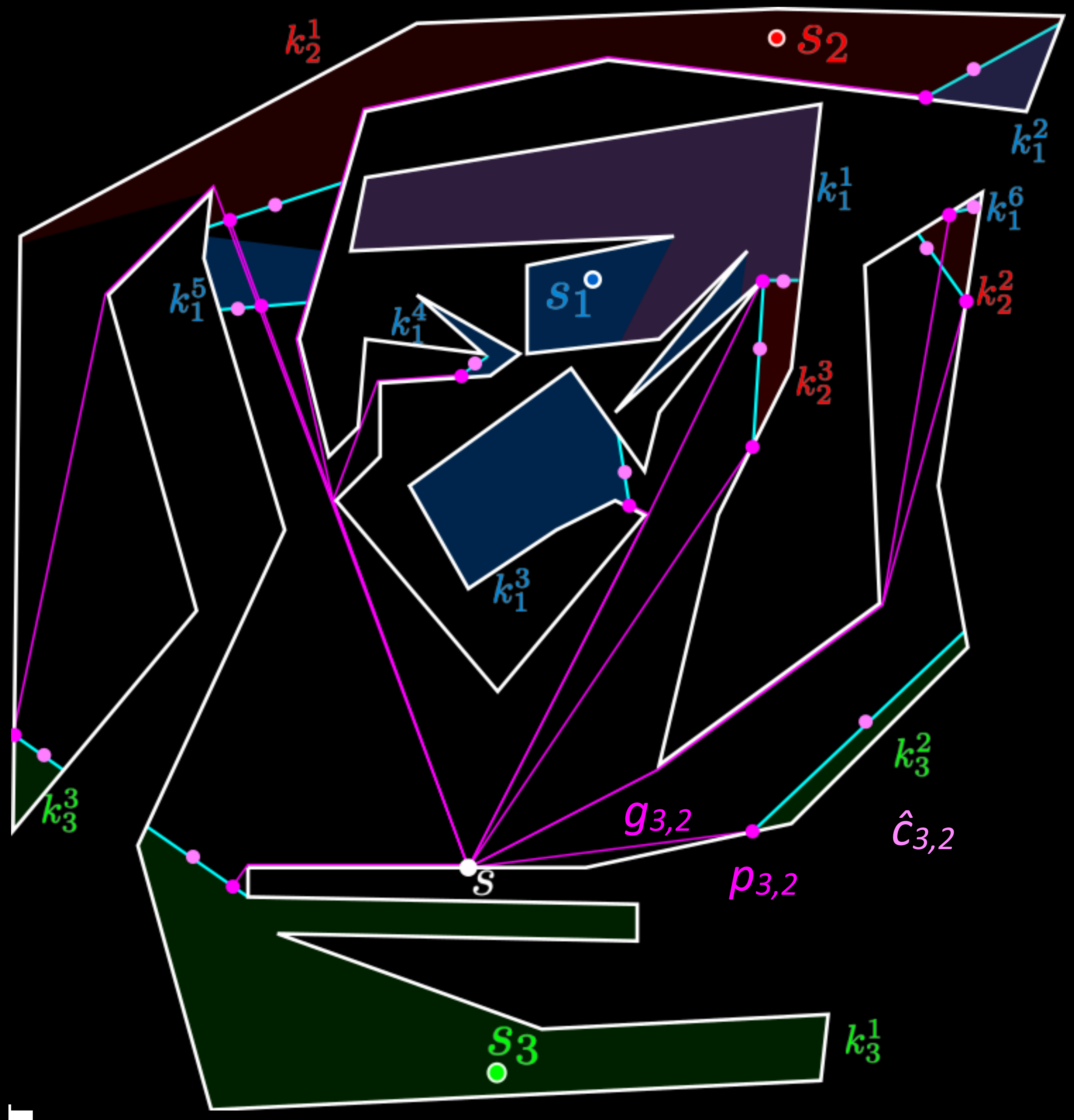

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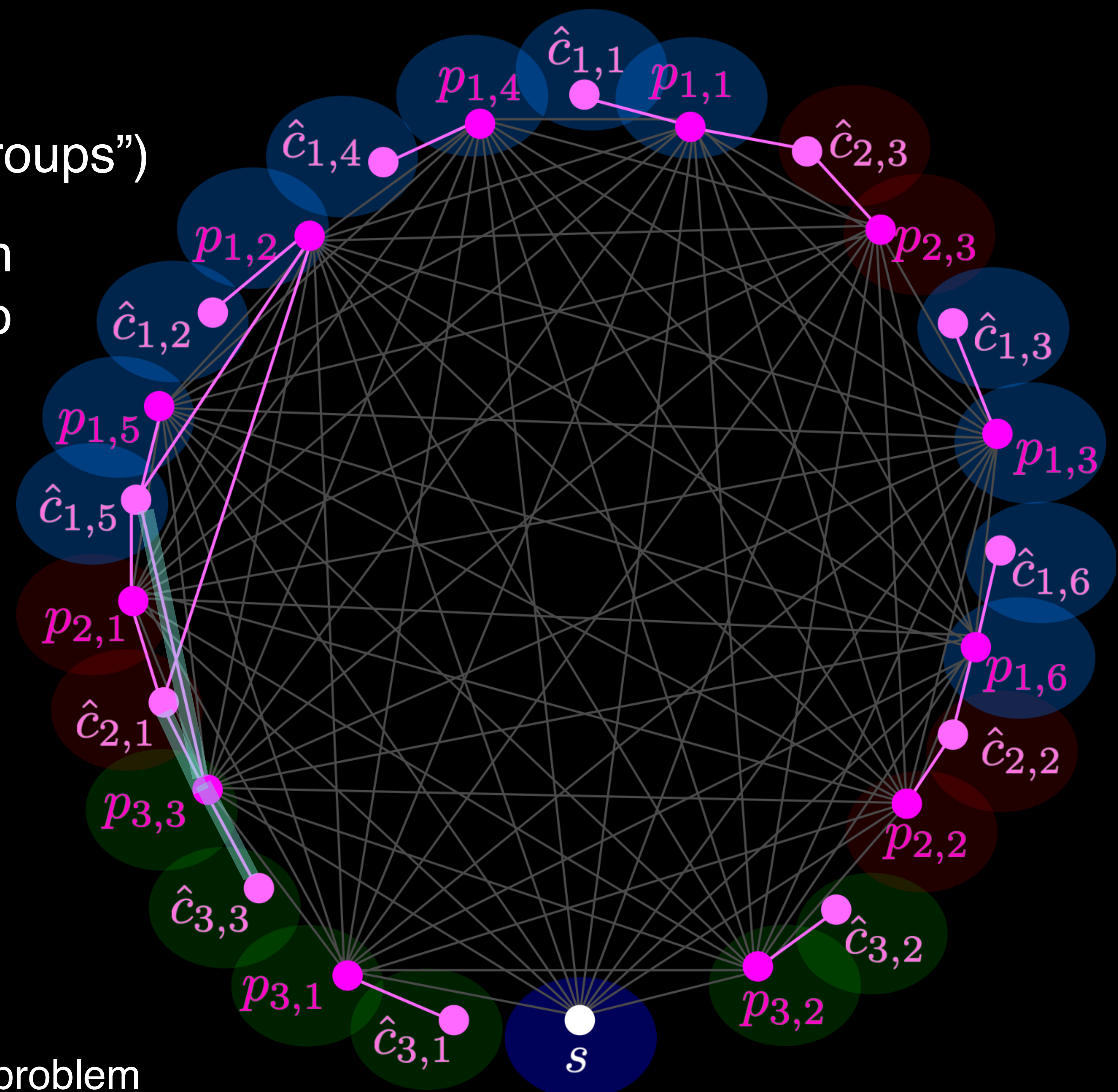

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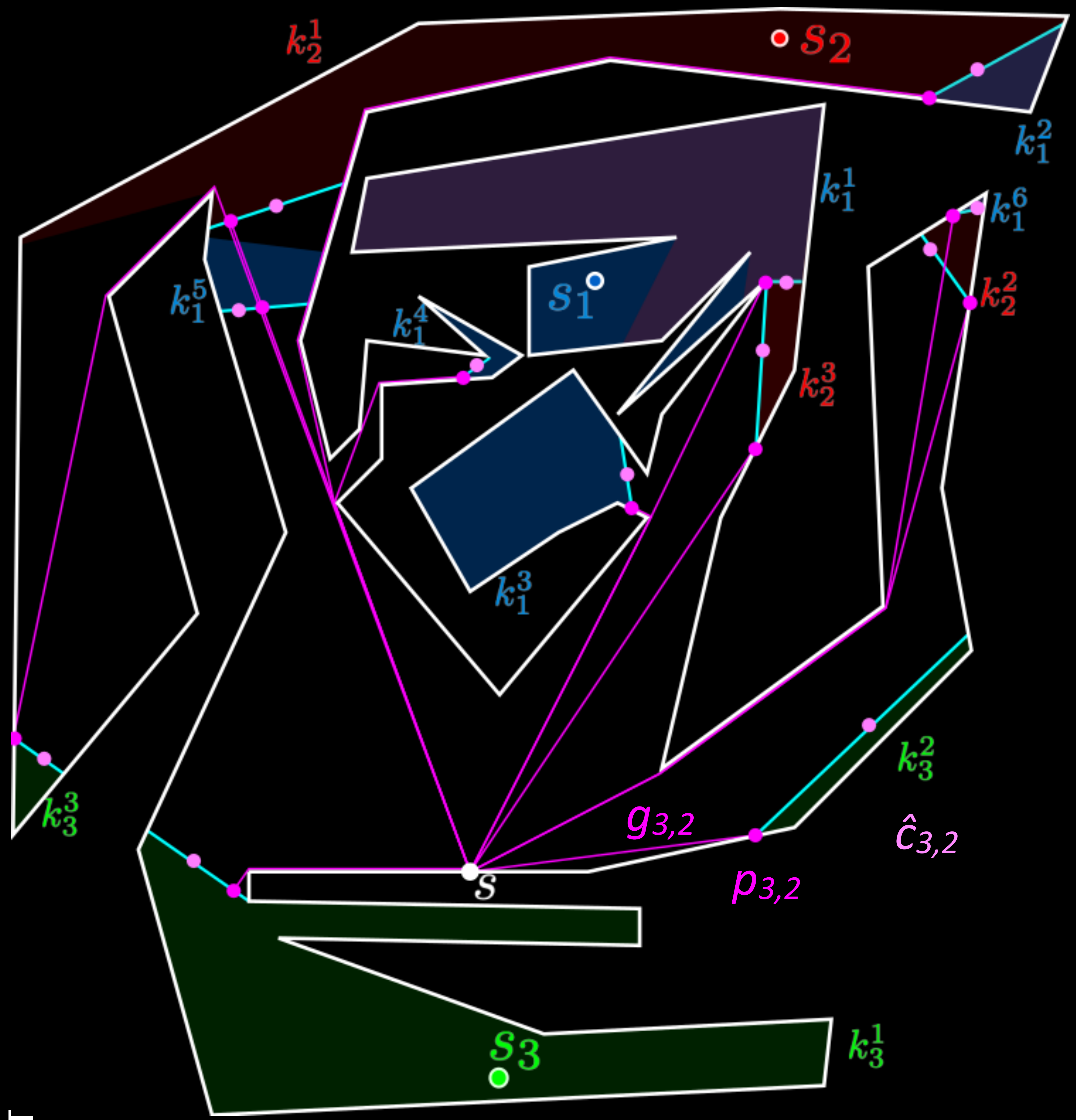

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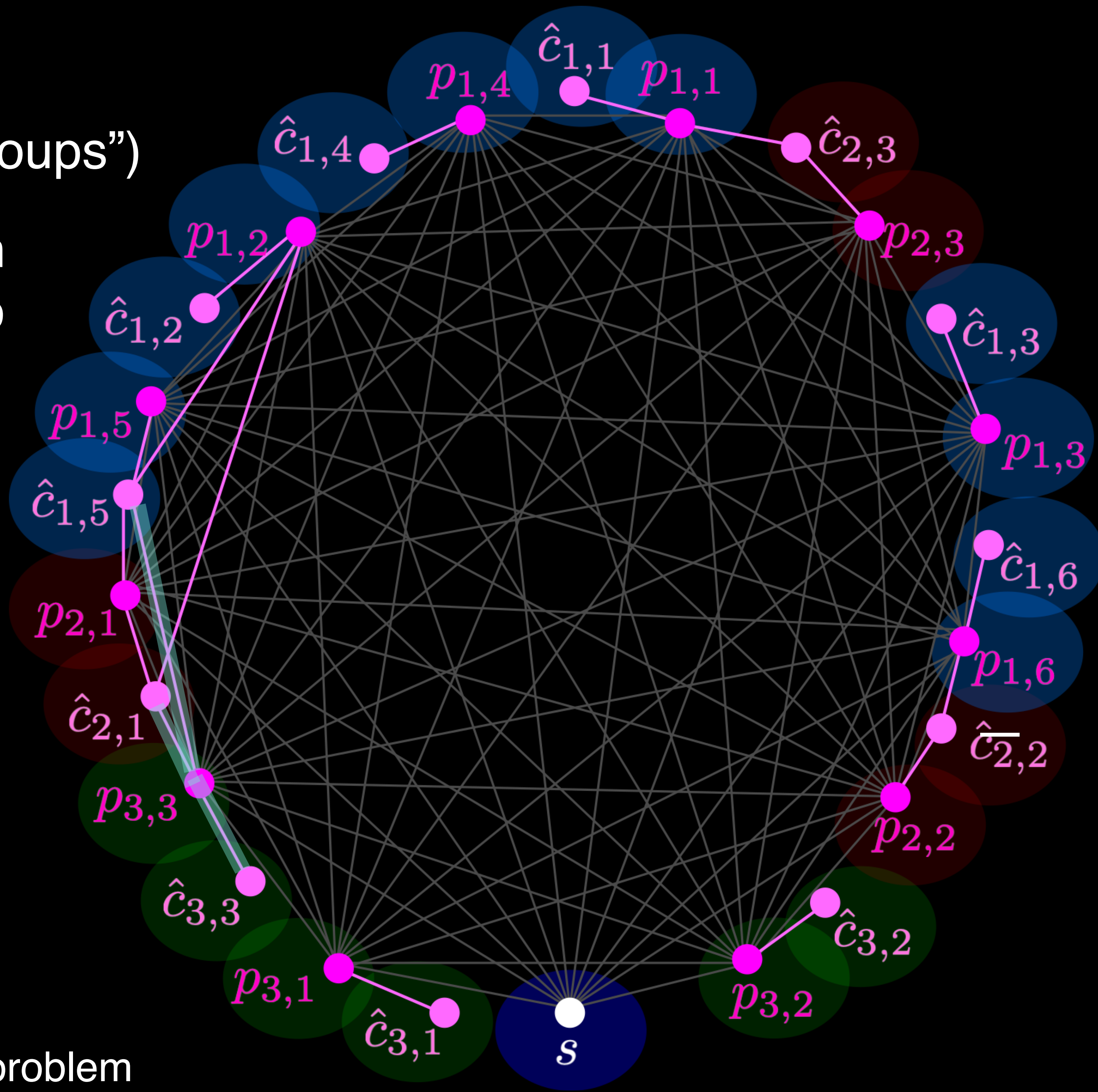

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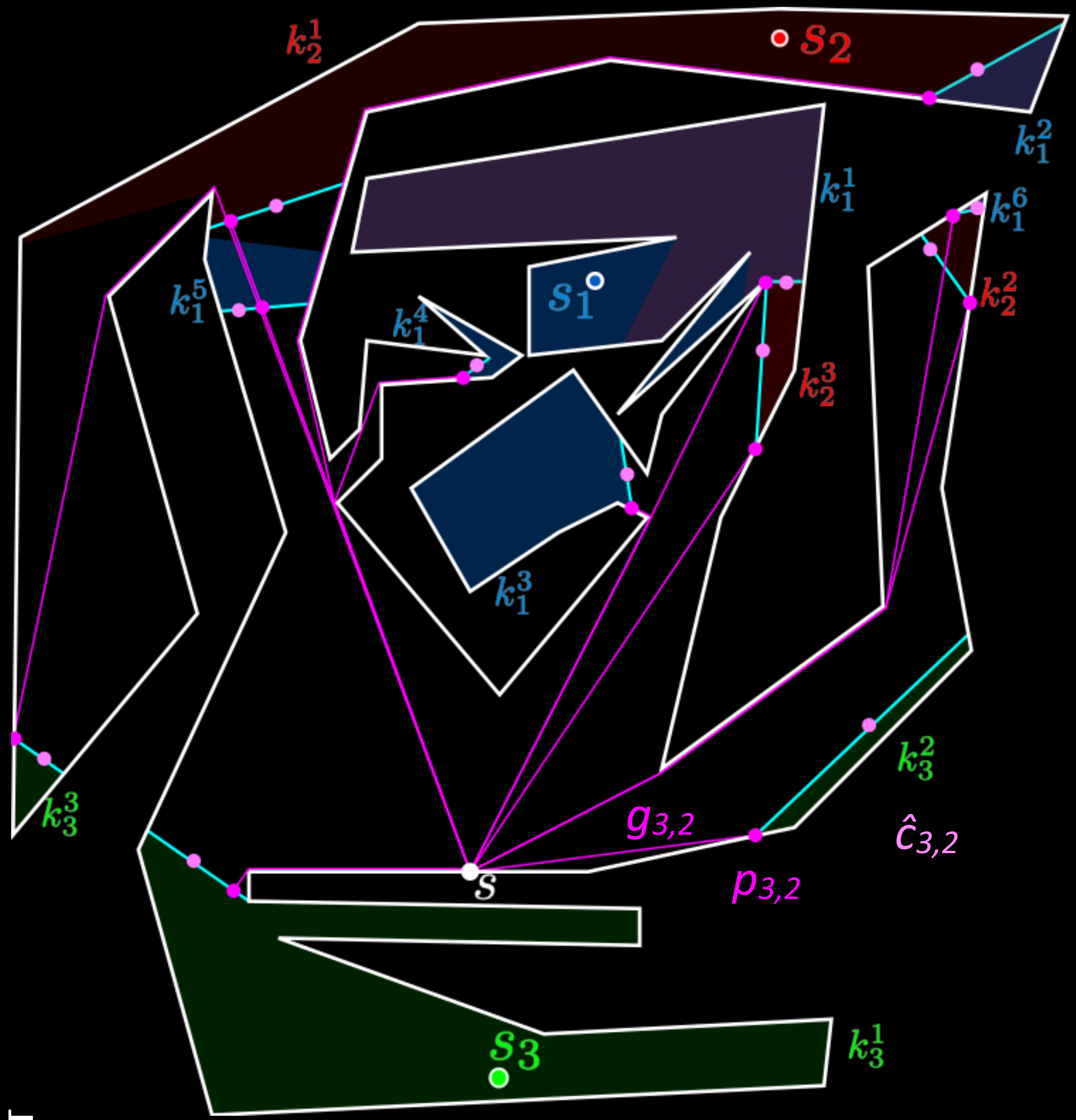
Approximation Algorithm for k-TrWRP(S,P,s)



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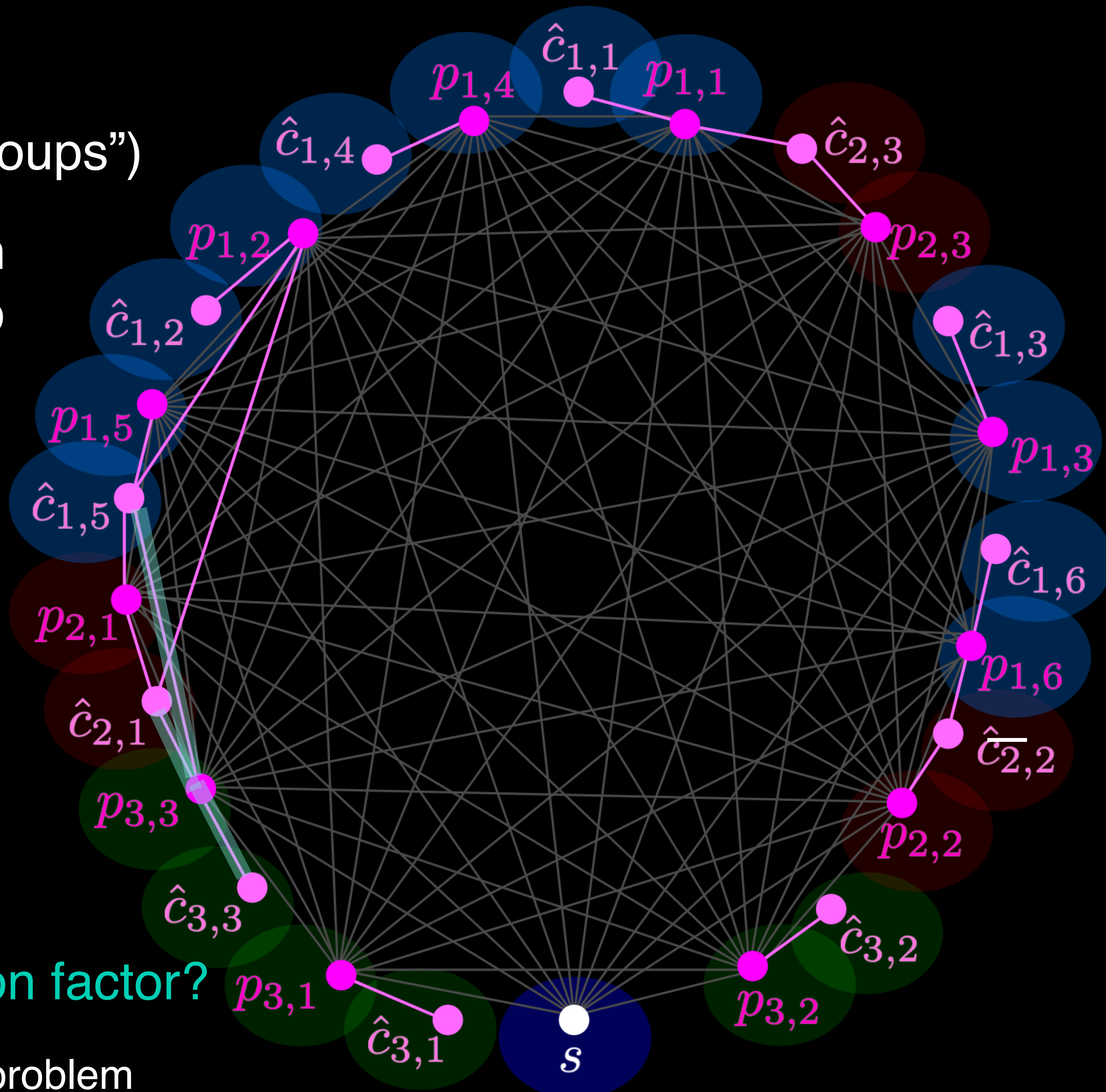

To do: why do we achieve the claimed approximation factor?

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Proof idea: alter(unknown) optimal route OPT(S,P,s) to pass through points from $V(G)$, and new tour has length at most constant$\cdot$ OPT(S,P,s)

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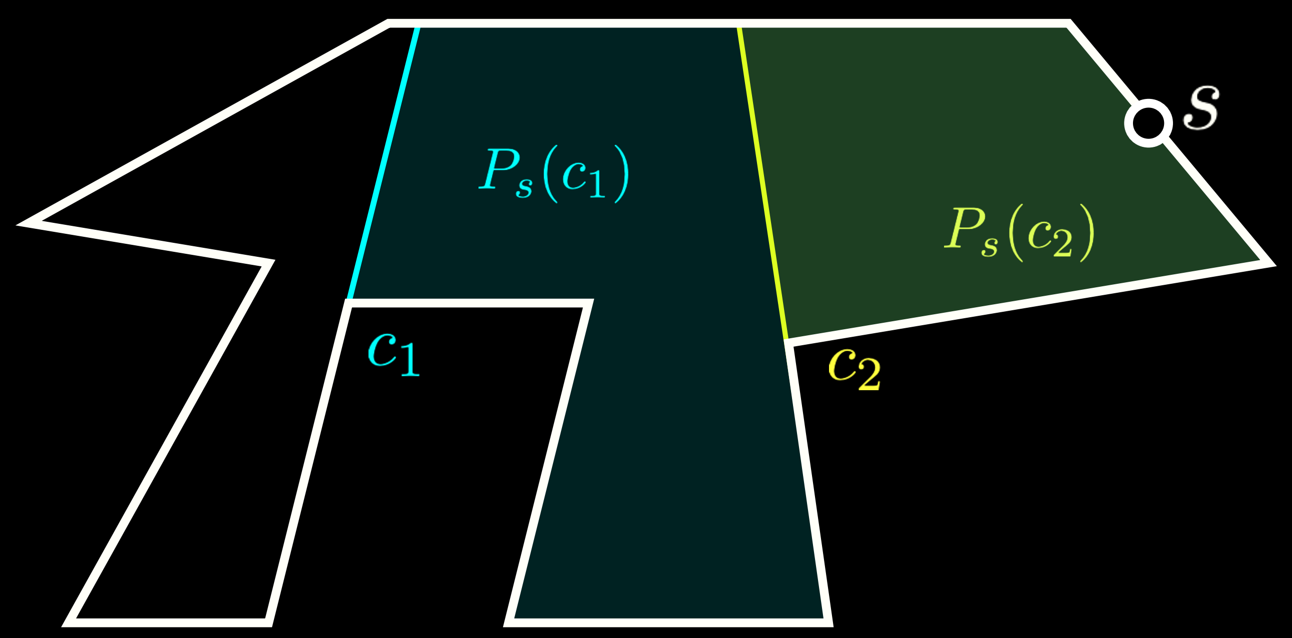
A cut c partitions polygon into two subpolygons:
$P_s(c)$—subpolygon that contains starting point s
A cut c_1 *dominates* c_2 if $P_s(c_2) \subseteq P_s(c_1)$
Essential cut: not dominated by other cut



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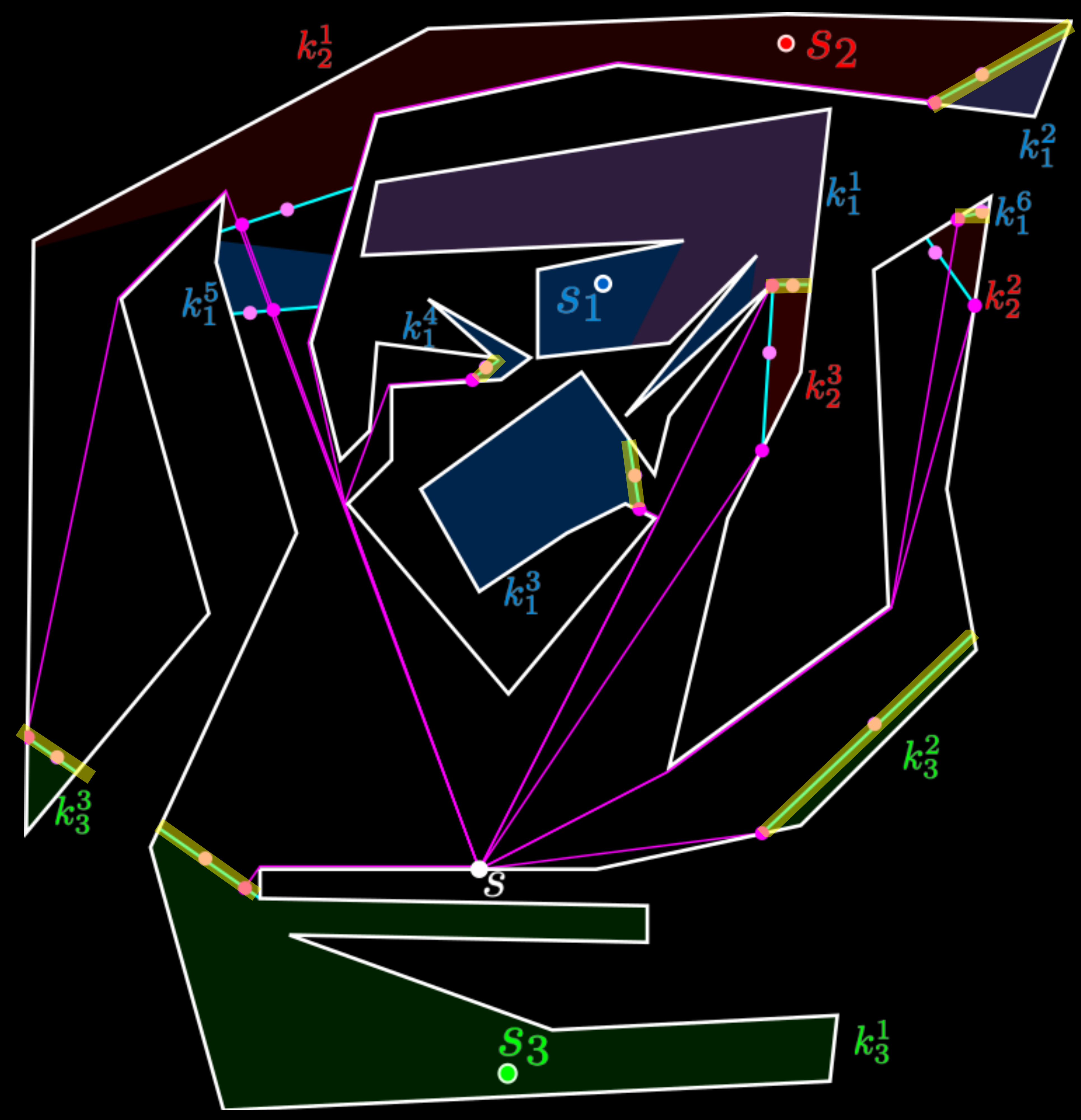

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    - $C'' \leftarrow C'' \setminus C_t$
- $\mathcal{G}_{C''}$ set of geodesics that end at cuts in C''
- Claim 1: The geodesics in $\mathcal{G}_{C''}$ are a set of *independent* geodesics, i.e., no essential cut is visited by two of these geodesics.
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- Build relative convex hull of all $o_{i,j}$ and all points in $\mathcal{P}_{C''}$ (relative w.r.t. polygon P): CH_P(OPT, $\mathcal{P}_{C''}$)

Proof idea: alter(unknown) optimal route OPT(S,P,s) to pass through points from $V(G)$, and new tour has length at most constant· OPT(S,P,s)

- Identify all cuts of the kVR(s_i) that OPT(S,P,s) visits—set C ($C \subseteq C^{all}$)
- Let $o_{i,j}$ denote the point where OPT(S,P,s) visits $c_{i,j}$ (first time)
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- Claim 3: No geodesic can intersect CH_P(OPT, $\mathcal{P}_{C''}$) between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on CH_P(OPT, $\mathcal{P}_{C''}$), we have at most two points of $\mathcal{P}_{C''}$. CH_P(OPT, $\mathcal{P}_{C''}$) has length at most 3·||OPT(S,P,s)||.

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- Claim 4: $CH_P(\mathcal{P}_{C''})$ is not longer than CH_P(OPT, $\mathcal{P}_{C''}$) and $CH_P(\mathcal{P}_{C''})$ visits one point per γ_i (except for γ_0).

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- To connect s (which may lie in the interior of $CH_P(\mathcal{P}_{C''})$, we need to connect s, which costs at most $\|OPT(S,P,s)\|$.

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- To connect s (which may lie in the interior of $\mathrm{CH}_P(\mathcal{P}_{C''})$, we need to connect s, which costs at most $\|\mathrm{OPT}(S,P,s)\|$.

$$\|R\| \leq \alpha_1 \cdot f(|V(G)|,|S|)\|\mathrm{OPT}_G(S,P,s)\| \leq \alpha_2 \cdot f(n|S|,|S|)\|\mathrm{CH}_P(\mathcal{P}_{\mathcal{C}''})\| \leq \alpha_3 \cdot f(n|S|,|S|)\|\mathrm{CH}_P(\mathrm{OPT},\mathcal{P}_{\mathcal{C}''})\|$$
$$\leq \alpha_4 \cdot f(n|S|,|S|)\|\mathrm{OPT}(S,P,s)\|$$

with $f(N,M) = \log^2 N \log\log N \log M$

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➜ After last iteration, no two remaining geodesics visit the same cut in *C'*

Claim 2: Each essential cut visited by OPT(*S*,*P*,*s*) (each cut in *C'*) is touched by exactly one of the geodesics.

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➜ All cuts in *C'* are visited

Claim 3: No geodesic can intersect $CH_P(OPT, \mathcal{P}_{C''})$ between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on $CH_P(OPT, \mathcal{P}_{C''})$, we have at most two points of $\mathcal{P}_{C''}$. $CH_P(OPT, \mathcal{P}_{C''})$ has length at most $3 \cdot \|OPT(S,P,s)\|$.

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Lemma 1: Consider a cut $c \in C''$, from CC j of a k-visibility region for $s_i \in S$, $kVR^j(s_i)$, for which both the point $o_{i,j}$ and the point $p_{i,j}$ are on $CH_P(OPT, \mathcal{P}_{C''})$. No geodesic in $\mathcal{G}_{C''}$ intersects c between $o_{i,j}$ and $p_{i,j}$.

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⚡contradiction to g_c being geodesic to c

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  ⚡contradiction to g_c being geodesic to c
- If $\ell(g_{c'}) = \ell(g_c)$: Either $\ell(g_{c'[s,p_c]}) < \ell(g_{c'}) = \ell(g_c)$ or (if p_c on c') $p_{i,j} = p_c$ (claim holds)

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Proof:

- Let $o_{i,j}$ and $o_{i',j'}$ be the two consecutive points from OPT on $CH_P(OPT, \mathcal{P}_{C''})$

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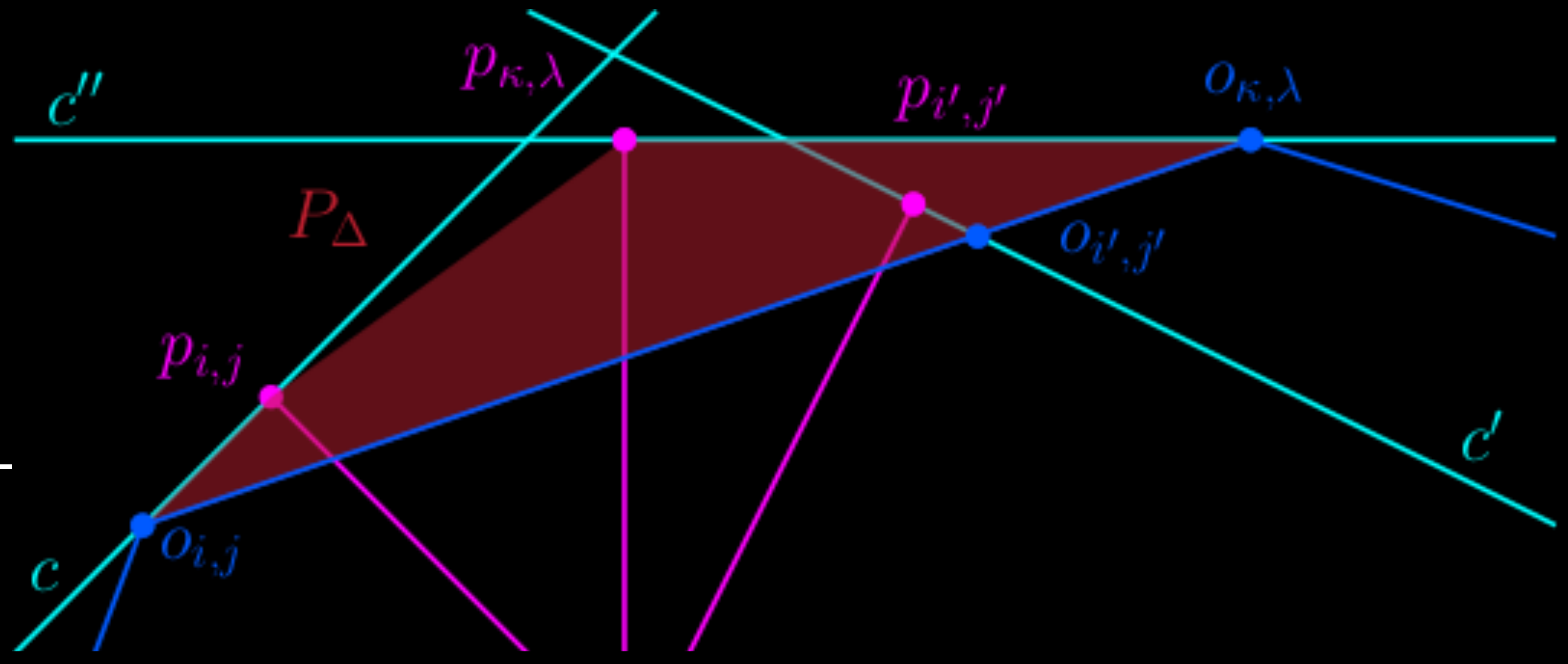

Cuts, points of the type $p_{i,j}$,
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Claim 3: No geodesic can intersect $CH_P(\text{OPT}, \mathcal{P}_{C''})$ between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on $CH_P(\text{OPT}, \mathcal{P}_{C''})$, we have at most two points of $\mathcal{P}_{C''}$. $CH_P(\text{OPT}, \mathcal{P}_{C''})$ has length at most $3 \cdot \|\text{OPT}(S,P,s)\|$.

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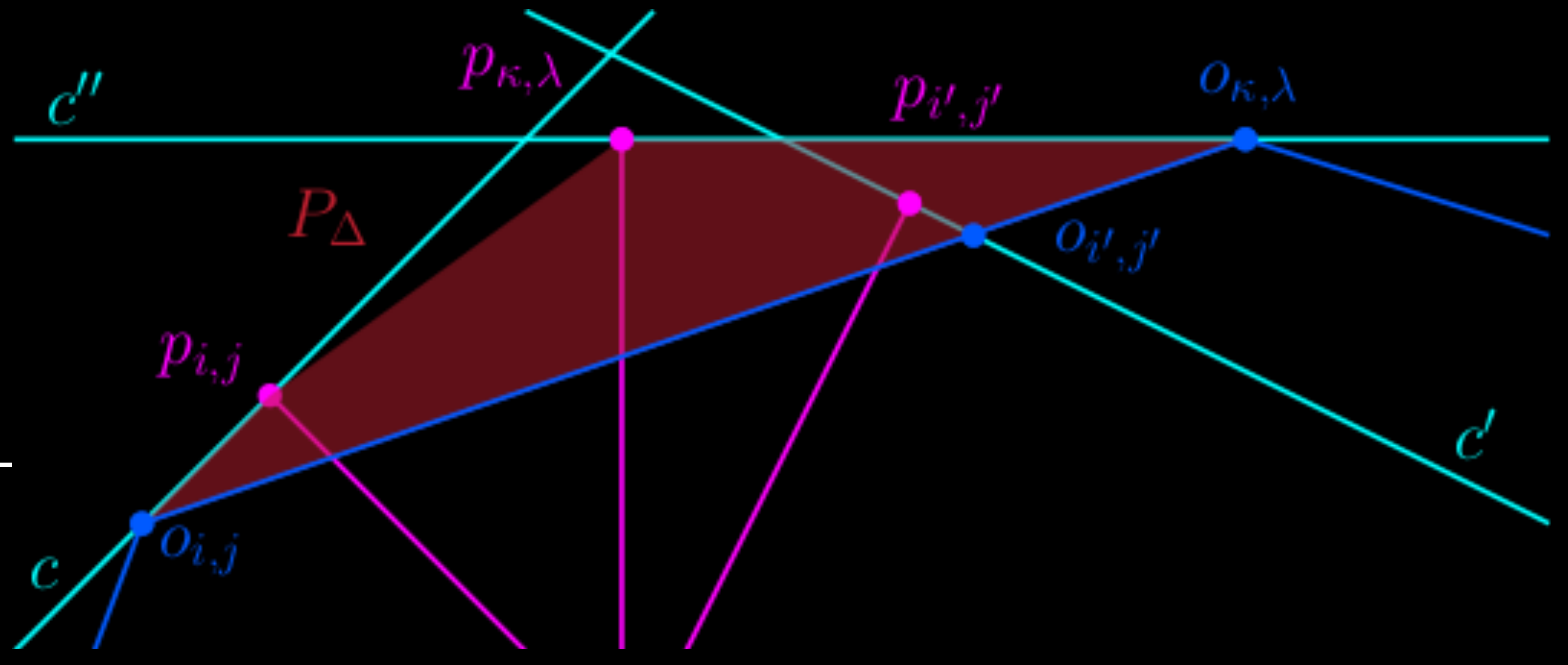
- Let $o_{i,j}$ and $o_{i',j'}$ be the two consecutive points from OPT on $CH_P(\text{OPT}, \mathcal{P}_{C''})$
- By Lemma 1, $p_{i,j}$ and $p_{i',j'}$ can lie between $o_{i,j}$ and $o_{i',j'}$
- BUT: we cannot have a point $p_{\kappa,\lambda}$ between $o_{i,j}$ and $p_{i,j}$ or between $o_{i',j'}$ and $p_{i',j'}$



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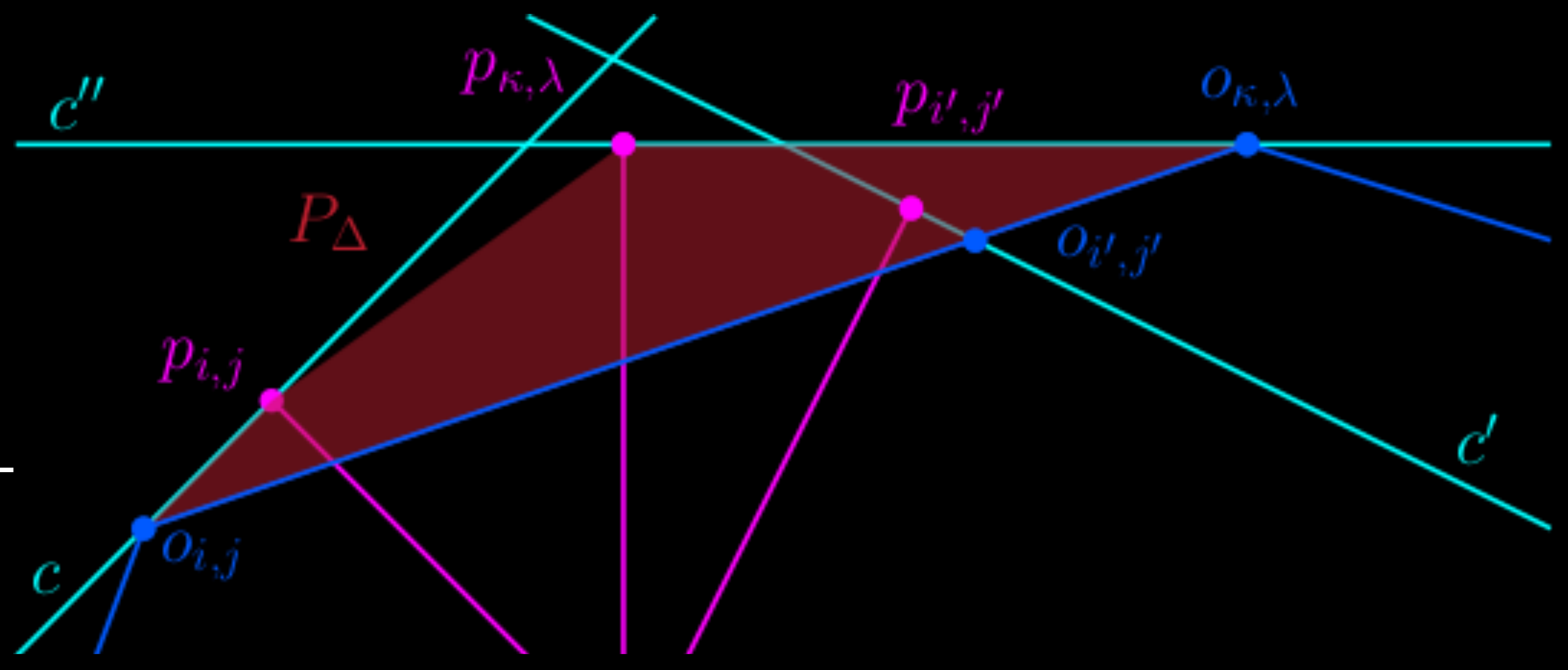
- Let $o_{i,j}$ and $o_{i',j'}$ be the two consecutive points from OPT on $CH_P(OPT, \mathcal{P}_{C''})$
- By Lemma 1, $p_{i,j}$ and $p_{i',j'}$ can lie between $o_{i,j}$ and $o_{i',j'}$
- BUT: we cannot have a point $p_{\kappa,\lambda}$ between $o_{i,j}$ and $p_{i,j}$ or between $o_{i',j'}$ and $p_{i',j'}$
- Assume there is a point $p_{\kappa,\lambda}$ between on $p_{i,j}$ and $p_{i',j'}$ on $CH_P(OPT, \mathcal{P}_{C''})$ (on cuts *c''*, *c*, *c'*, resp.)



Cuts, points of the type $p_{i,j}$,
optimal route and points of the type $o_{i,j}$

Claim 3: No geodesic can intersect $CH_P(OPT, \mathcal{P}_{C''})$ between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on $CH_P(OPT, \mathcal{P}_{C''})$, we have at most two points of $\mathcal{P}_{C''}$. $CH_P(OPT, \mathcal{P}_{C''})$ has length at most $3 \cdot \|OPT(S,P,s)\|$.

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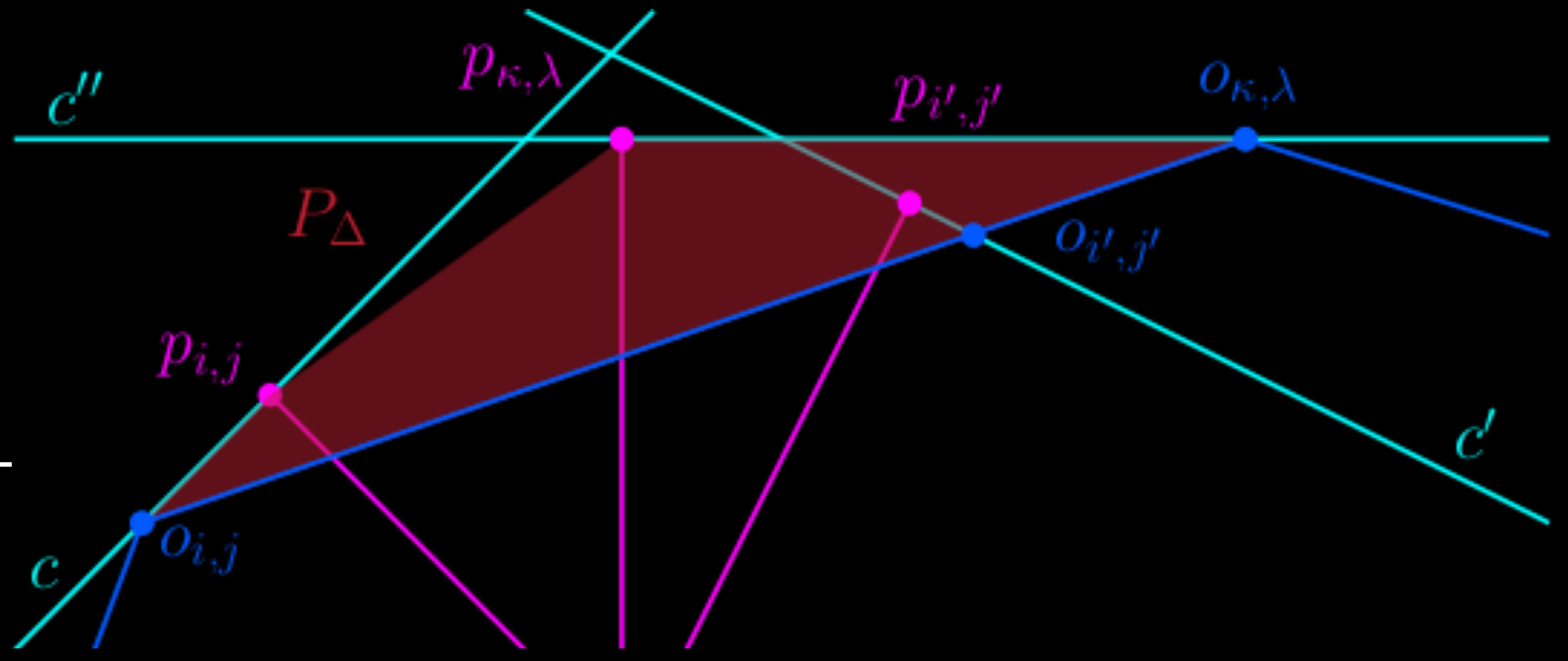
- OPT visits $o_{\kappa,\lambda}$ on c''



Cuts, points of the type $p_{i,j}$,
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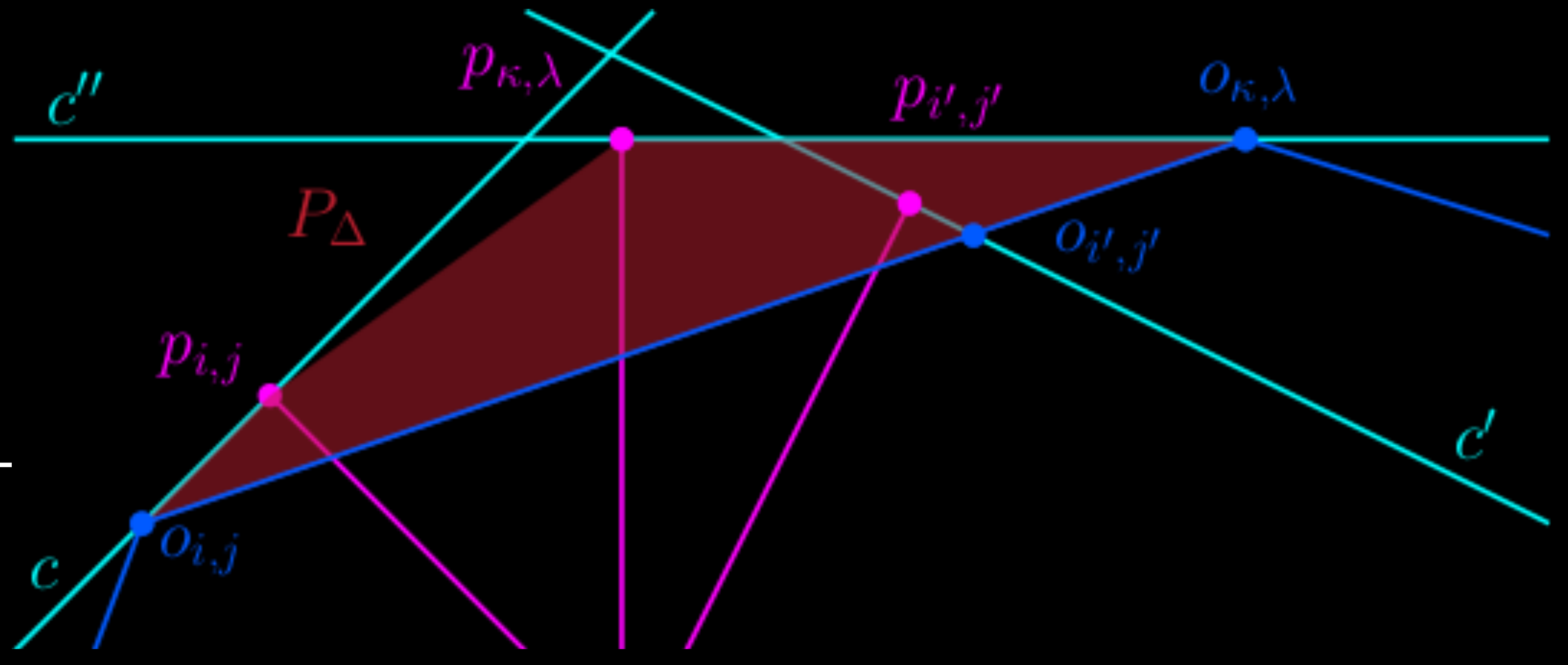
- OPT visits $o_{\kappa,\lambda}$ on c''
- $o_{i,j}$ and $o_{i',j'}$ are consecutive pts on $CH_P(OPT, \mathcal{P}_{C''})$



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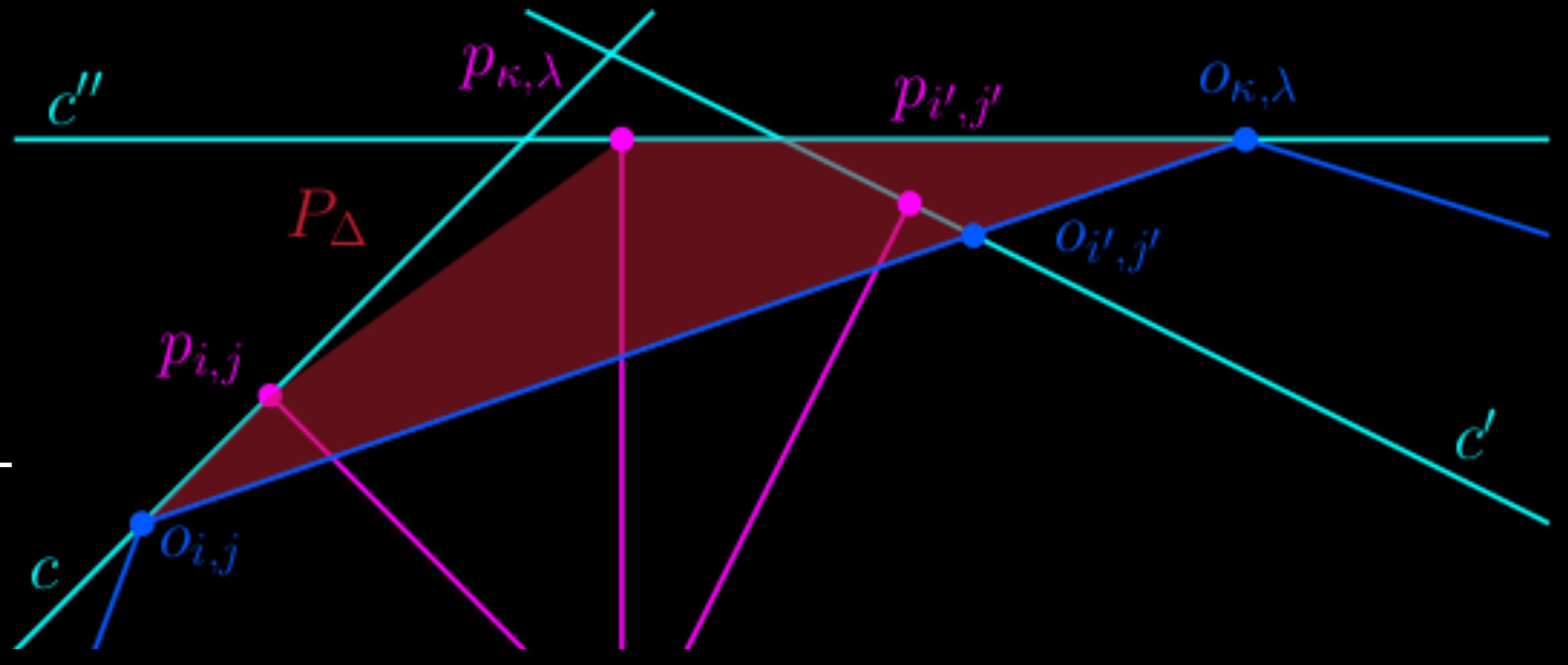
- $o_{i,j}$ and $o_{i',j'}$ are consecutive pts on $CH_P(OPT, \mathcal{P}_{C''})$

➡ Order of OPT $o_{i,j}$ $o_{i',j'}$, $o_{\kappa,\lambda}$ or $o_{\kappa,\lambda}$, $o_{i,j}$ $o_{i',j'}$, w.l.o.g. $o_{i,j}$ $o_{i',j'}$, $o_{\kappa,\lambda}$



Cuts, points of the type $p_{i,j}$,
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- $o_{i,j}$ and $o_{i',j'}$ are consecutive pts on $CH_P(OPT, \mathcal{P}_{C''})$

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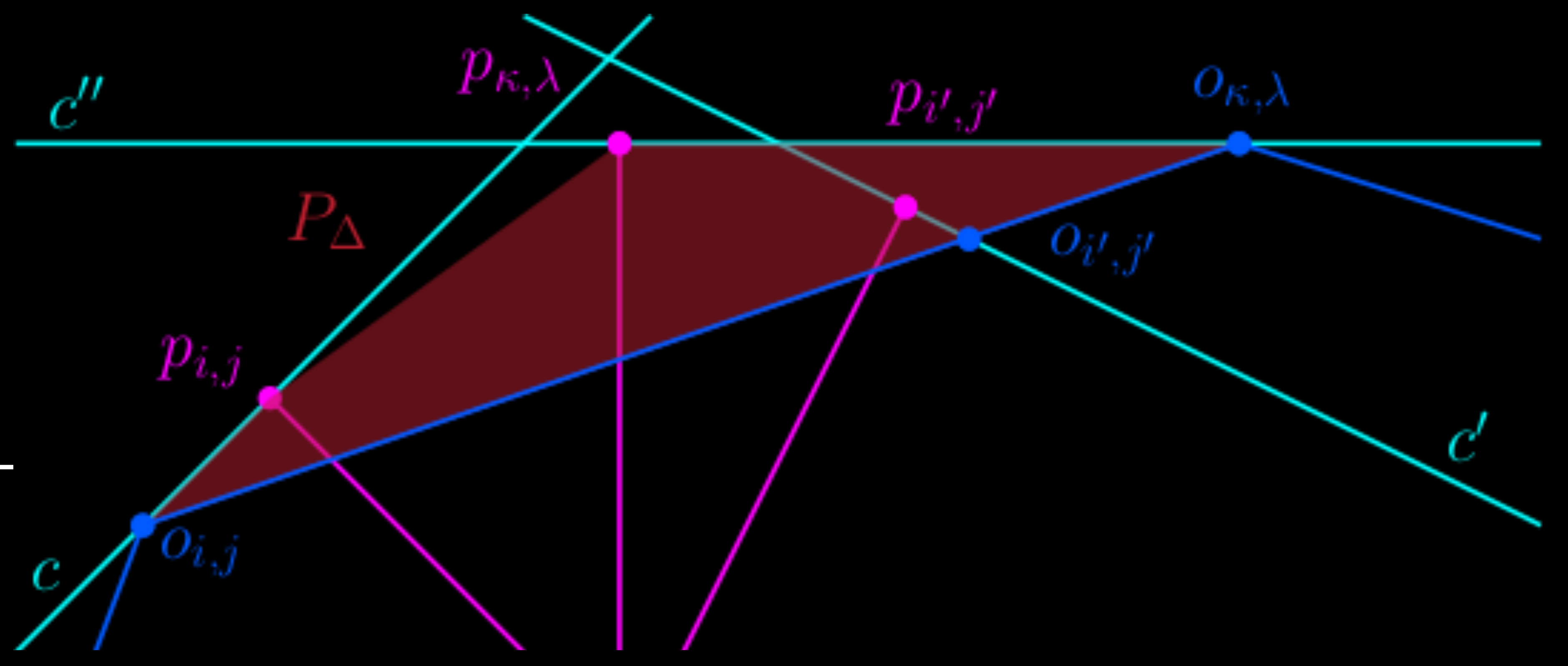
- Cut c'' is a line segment



Cuts, points of the type $p_{i,j}$,
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Claim 3: No geodesic can intersect $CH_P(OPT, \mathcal{P}_{C''})$ between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on $CH_P(OPT, \mathcal{P}_{C''})$, we have at most two points of $\mathcal{P}_{C''}$. $CH_P(OPT, \mathcal{P}_{C''})$ has length at most $3 \cdot \|OPT(S,P,s)\|$.

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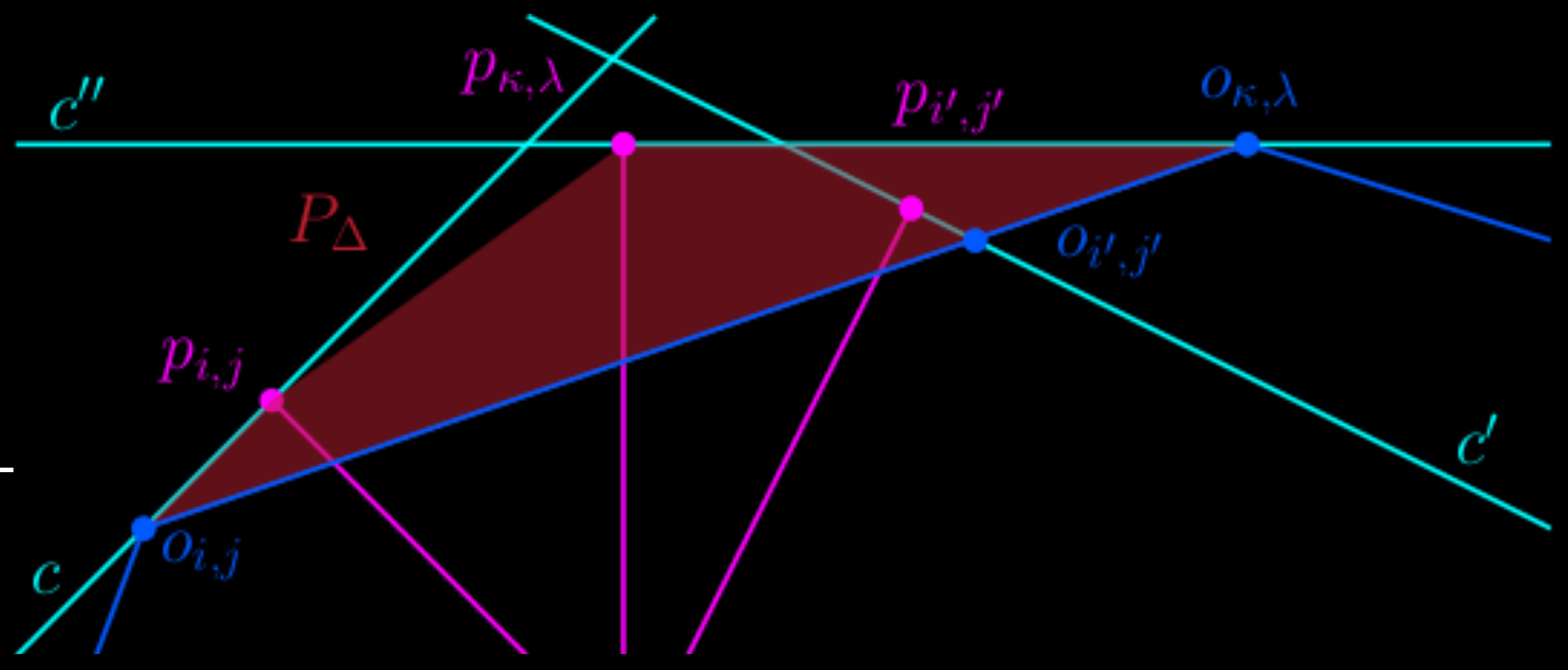
- $o_{i,j}$ and $o_{i',j'}$ are consecutive pts on $CH_P(OPT, \mathcal{P}_{C''})$

➡ Order of OPT $o_{i,j}$ $o_{i',j'}$, $o_{\kappa,\lambda}$ or $o_{\kappa,\lambda}$, $o_{i,j}$ $o_{i',j'}$, w.l.o.g. $o_{i,j}$ $o_{i',j'}$, $o_{\kappa,\lambda}$

- Cut c'' is a line segment
- Consider polgyon P_Δ with vertices $o_{i,j}$ $p_{i,j}$, $p_{\kappa,\lambda}$, $o_{\kappa,\lambda}$, $o_{i',j'}$, $o_{i,j}$



Cuts, points of the type $p_{i,j}$,
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Claim 3: No geodesic can intersect $CH_P(OPT, \mathcal{P}_{C''})$ between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on $CH_P(OPT, \mathcal{P}_{C''})$, we have at most two points of $\mathcal{P}_{C''}$. $CH_P(OPT, \mathcal{P}_{C''})$ has length at most $3 \cdot \|OPT(S,P,s)\|$.

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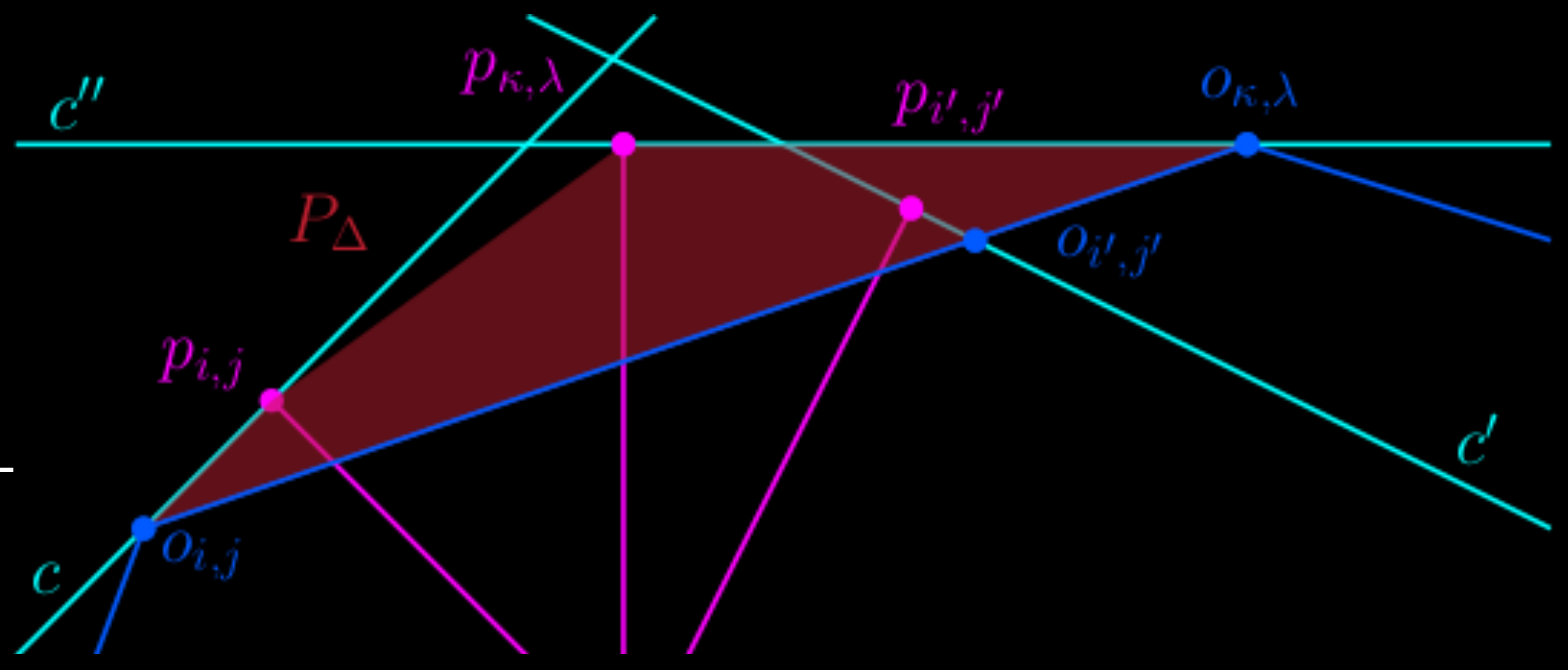
- OPT visits $o_{\kappa,\lambda}$ on c''
- $o_{i,j}$ and $o_{i',j'}$ are consecutive pts on $CH_P(OPT, \mathcal{P}_{C''})$

➡ Order of OPT $o_{i,j}$ $o_{i',j'}$, $o_{\kappa,\lambda}$ or $o_{\kappa,\lambda}$, $o_{i,j}$ $o_{i',j'}$, w.l.o.g. $o_{i,j}$ $o_{i',j'}$, $o_{\kappa,\lambda}$

- Cut c'' is a line segment
- Consider polgyon P_Δ with vertices $o_{i,j}$ $p_{i,j}$, $p_{\kappa,\lambda}$, $o_{\kappa,\lambda}$, $o_{i',j'}$, $o_{i,j}$
- Point $p_{i',j'}$ must lie in P_Δ's interior + $o_{i',j'}$ cannot lie on $CH_P(OPT, \mathcal{P}_{C''})$ ⚡



Cuts, points of the type $p_{i,j}$,
optimal route and points of the type $o_{i,j}$

Claim 3: No geodesic can intersect $CH_P(OPT, \mathcal{P}_{C''})$ between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on $CH_P(OPT, \mathcal{P}_{C''})$, we have at most two points of $\mathcal{P}_{C''}$. $CH_P(OPT, \mathcal{P}_{C''})$ has length at most $3 \cdot \|OPT(S,P,s)\|$.

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Lemma 3: $\|CH_P(OPT, \mathcal{P}_{C''})\| \leq 3 \cdot \|OPT(S,P,s)\|$.

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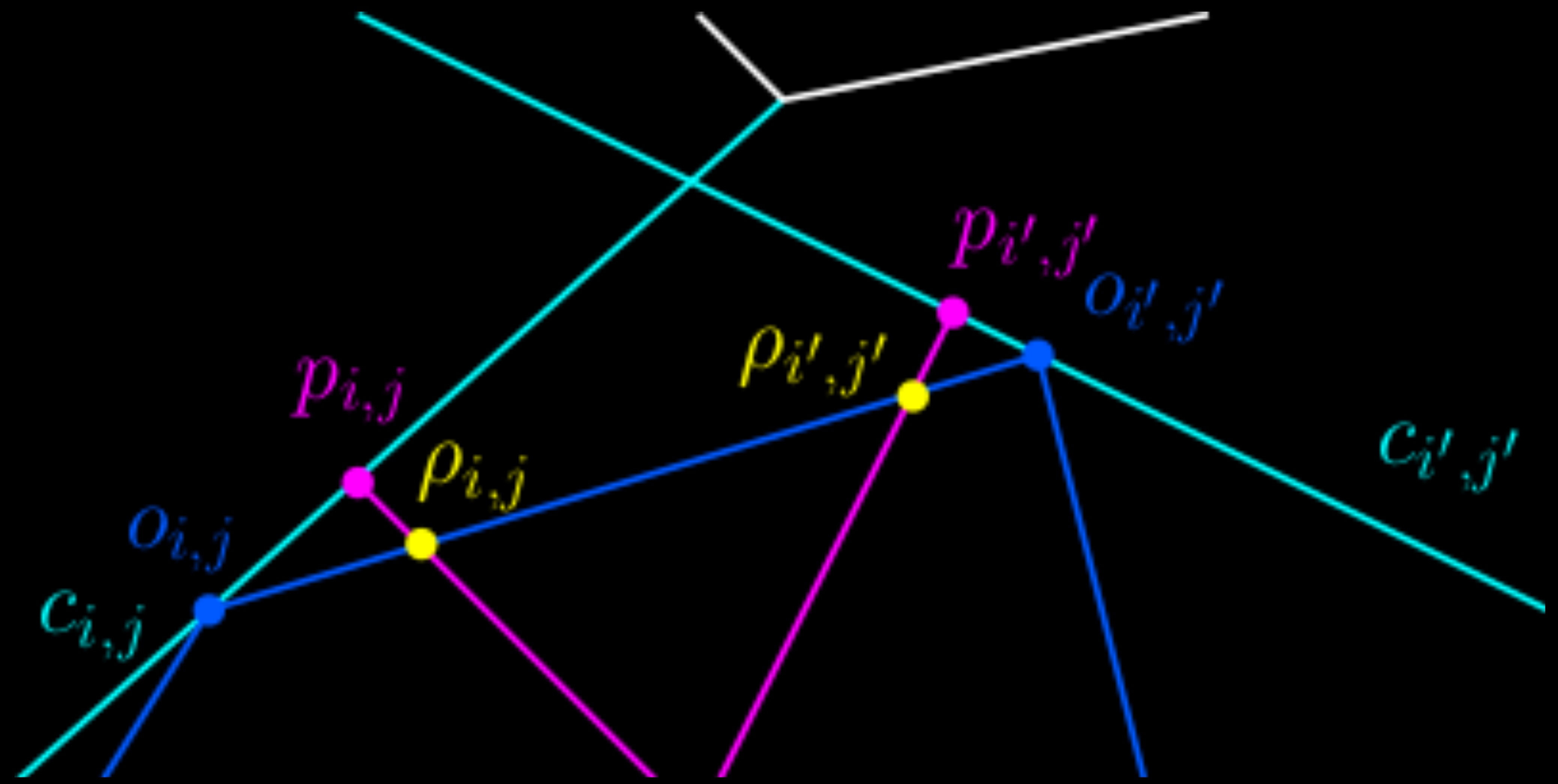
Lemma 3: $\|CH_P(OPT, \mathcal{P}_{C''})\| \leq 3 \cdot \|OPT(S,P,s)\|$.

Proof:

- Lemmas 1,2 ➜ Between two consecutive points of $OPT(S,P,s)$ on $CH_P(OPT, \mathcal{P}_{C''})$, $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$



Cuts, points of the type $p_{i,j}$, polygon boundary, optimal route and points of the type $o_{i,j}$

Claim 3: No geodesic can intersect $CH_P(\text{OPT}, \mathcal{P}_{C''})$ between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on $CH_P(\text{OPT}, \mathcal{P}_{C''})$, we have at most two points of $\mathcal{P}_{C''}$. $CH_P(\text{OPT}, \mathcal{P}_{C''})$ has length at most $3 \cdot ||\text{OPT}(S,P,s)||$.

Lemma 3: $||CH_P(\text{OPT}, \mathcal{P}_{C''})|| \leq 3 \cdot ||\text{OPT}(S,P,s)||$.

Proof:

- Lemmas 1,2 ➜ Between two consecutive points of $\text{OPT}(S,P,s)$ on $CH_P(\text{OPT}, \mathcal{P}_{C''})$, $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
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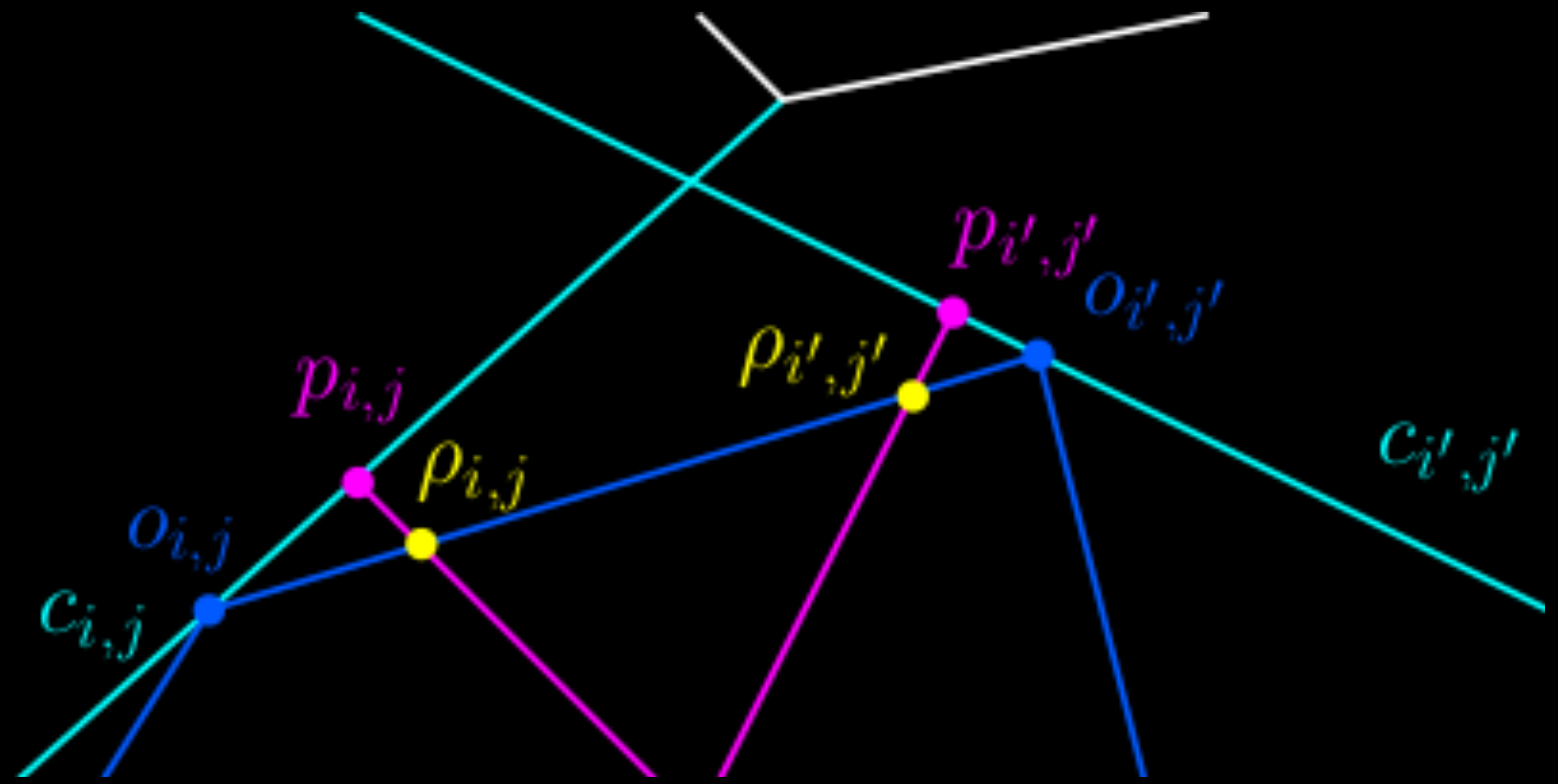

Cuts, points of the type $p_{i,j}$, polygon boundary, optimal route and points of the type $o_{i,j}$

Claim 3: No geodesic can intersect CH_P(OPT, $\mathcal{P}_{C''}$) between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on CH_P(OPT, $\mathcal{P}_{C''}$), we have at most two points of $\mathcal{P}_{C''}$. CH_P(OPT, $\mathcal{P}_{C''}$) has length at most 3·||OPT(S,P,s)||.

Lemma 3: ||CH_P(OPT, $\mathcal{P}_{C''}$)|| ≤ 3·||OPT(S,P,s)||.

Proof:

- Lemmas 1,2 ➜ Between two consecutive points of OPT(S,P,s) on CH_P(OPT, $\mathcal{P}_{C''}$), $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
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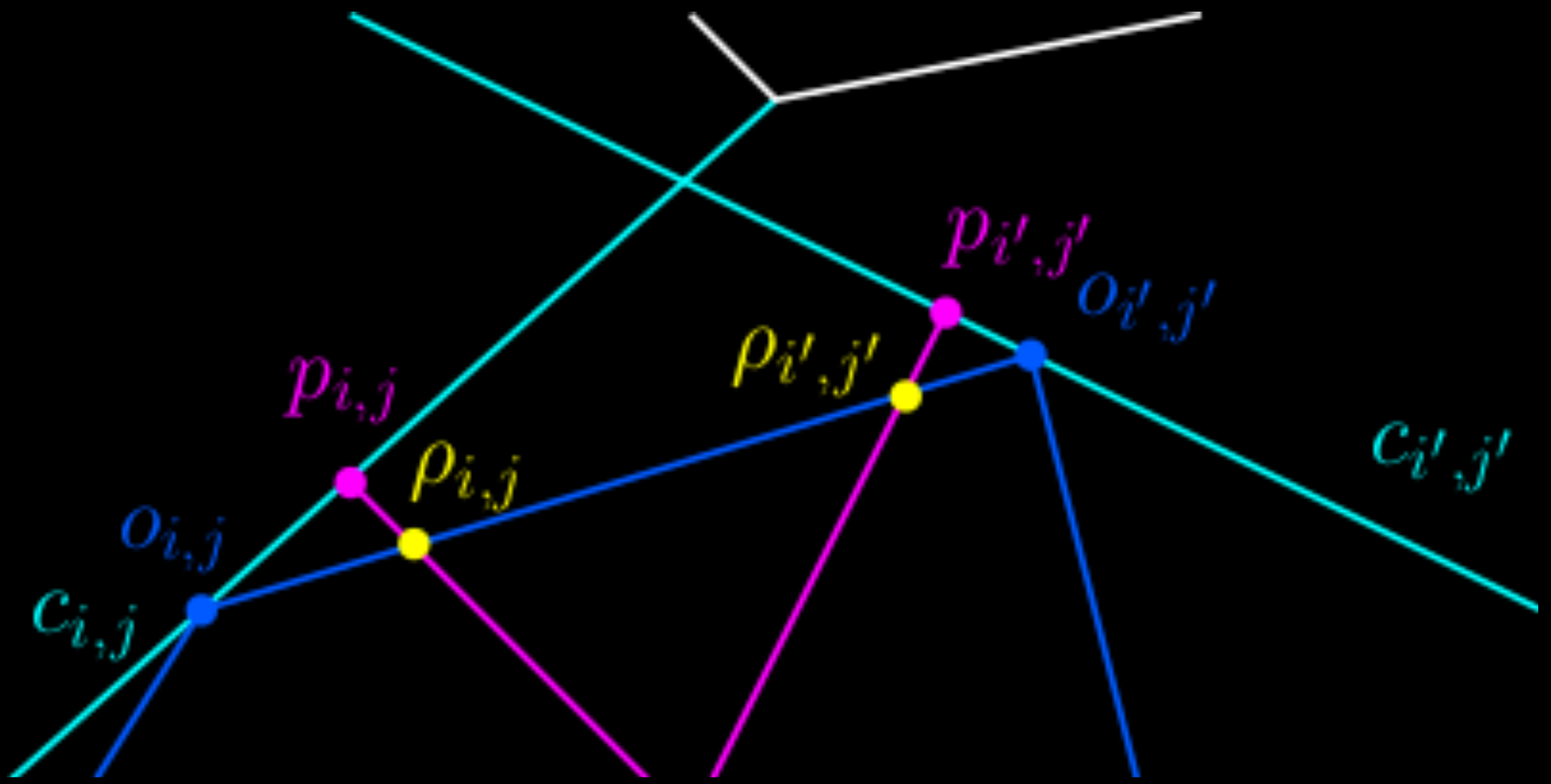
➡ $g_{i,j}$ intersects OPT(S,P,s) between $o_{i,j}$ and $o_{i',j'}$—in point: $\varrho_{i,j}$



Cuts, points of the type $p_{i,j}$, polygon boundary, optimal route and points of the type $o_{i,j}$

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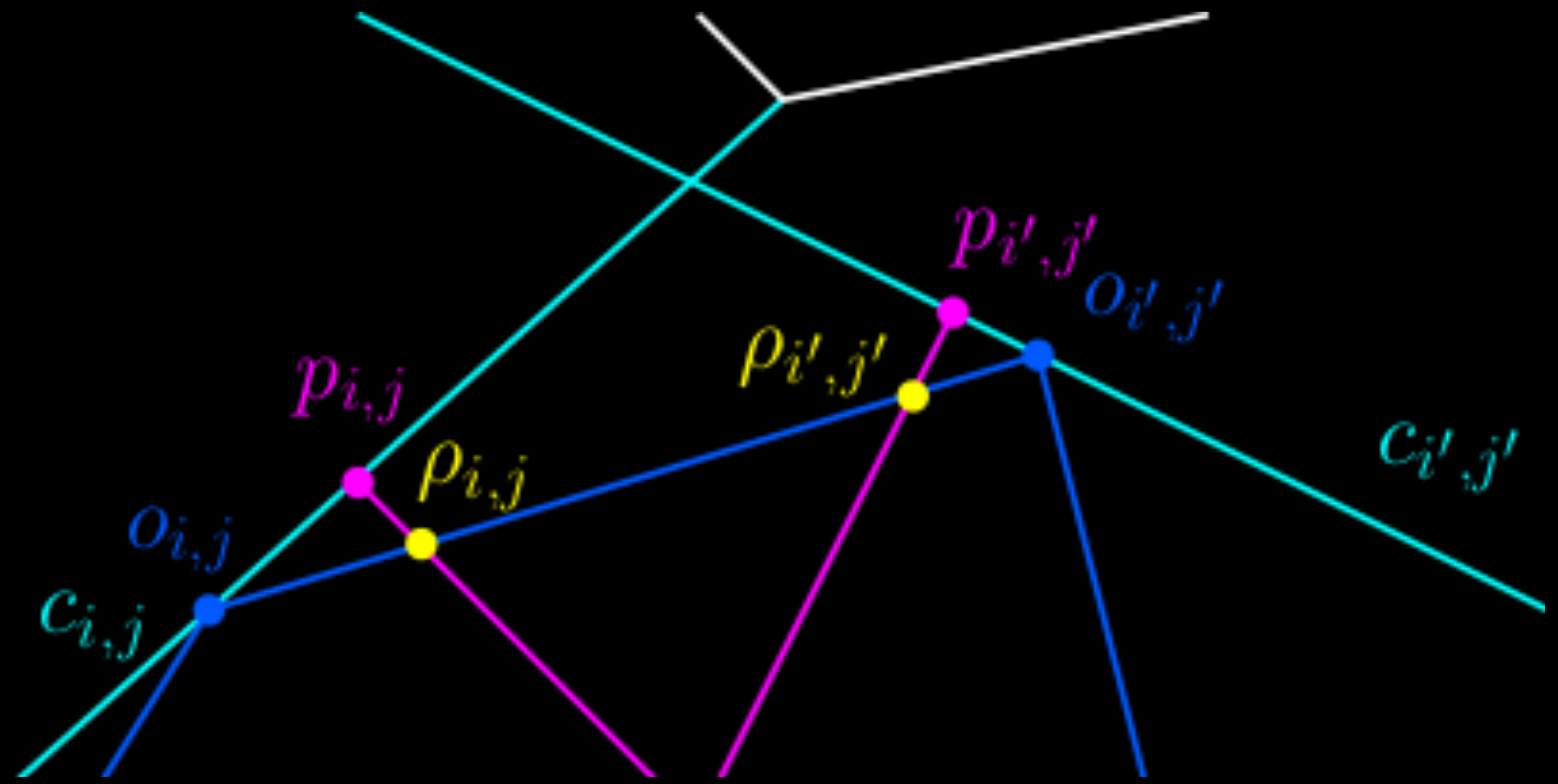
Lemma 3: $||CH_P(OPT, \mathcal{P}_{C''})|| \leq 3 \cdot ||OPT(S,P,s)||$.

Proof:

- Lemmas 1,2 ➜ Between two consecutive points of $OPT(S,P,s)$ on $CH_P(OPT, \mathcal{P}_{C''})$, $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
- Points $o_{i,j}$ and $p_{i,j}$ both on $c_{i,j}$ / points $o_{i',j'}$ and $p_{i',j'}$ both on $c_{i',j'}$

➡ $g_{i,j}$ intersects $OPT(S,P,s)$ between $o_{i,j}$ and $o_{i',j'}$—in point: $\varrho_{i,j}$

- $g_{i,j}$ is geodesic



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Lemma 3: $\|CH_P(OPT, \mathcal{P}_{C''})\| \leq 3 \cdot \|OPT(S,P,s)\|$.

Proof:

- Lemmas 1,2 ➜ Between two consecutive points of $OPT(S,P,s)$ on $CH_P(OPT, \mathcal{P}_{C''})$, $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
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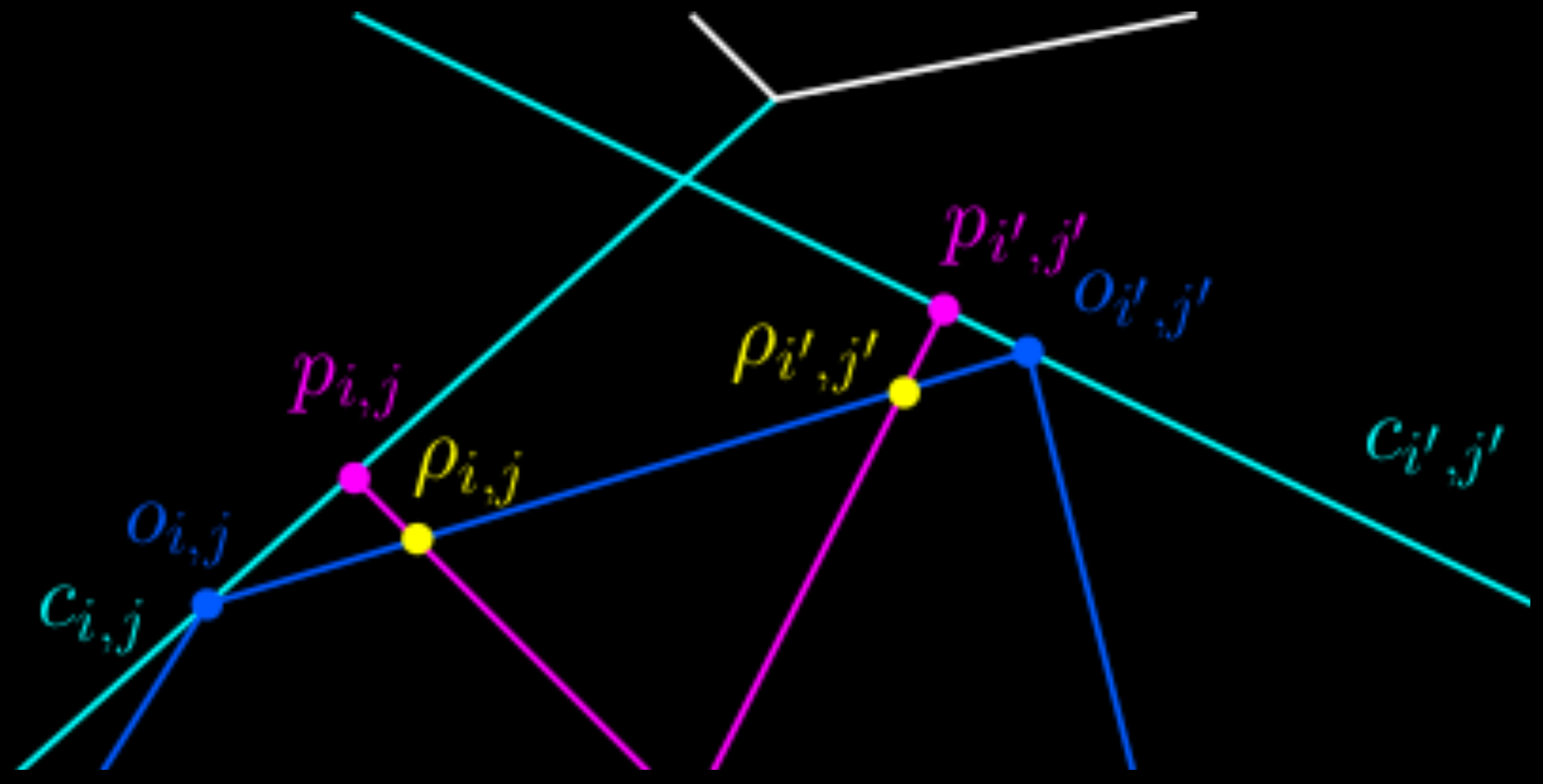
➡ $g_{i,j}$ intersects $OPT(S,P,s)$ between $o_{i,j}$ and $o_{i',j'}$—in point: $\varrho_{i,j}$

- $g_{i,j}$ is geodesic

➡ $\ell(\varrho_{i,j}, p_{i,j}) \leq \ell(\varrho_{i,j}, o_{i,j})$ (and $\ell(\varrho_{i',j'}, p_{i',j'}) \leq \ell(\varrho_{i',j'}, o_{i',j'})$)



Cuts, points of the type $p_{i,j}$, polygon boundary, optimal route and points of the type $o_{i,j}$

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Lemma 3: II$CH_P(OPT, \mathcal{P}_{C''})$II ≤ 3·IIOPT($S,P,s$)II.

Proof:

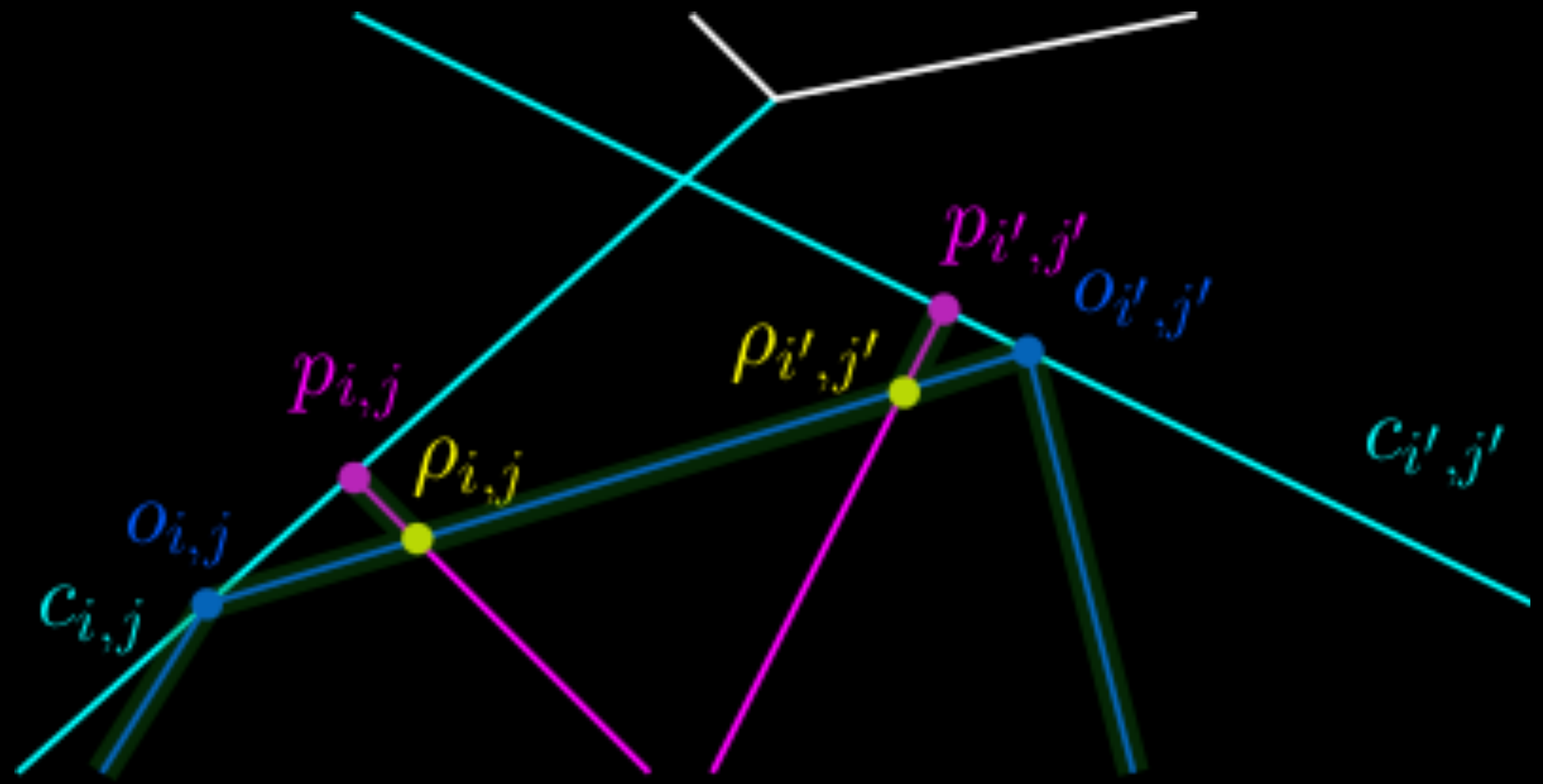
- Lemmas 1,2 ➜ Between two consecutive points of OPT(S,P,s) on $CH_P(OPT, \mathcal{P}_{C''})$, $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
- Points $o_{i,j}$ and $p_{i,j}$ both on $c_{i,j}$ / points $o_{i',j'}$ and $p_{i',j'}$ both on $c_{i',j'}$

➡ $g_{i,j}$ intersects OPT(S,P,s) between $o_{i,j}$ and $o_{i',j'}$—in point: $\varrho_{i,j}$

- $g_{i,j}$ is geodesic

➡ $\ell(\varrho_{i,j}, p_{i,j}) \leq \ell(\varrho_{i,j}, o_{i,j})$ (and $\ell(\varrho_{i',j'}, p_{i',j'}) \leq \ell(\varrho_{i',j'}, o_{i',j'})$)

- Alter OPT(S,P,s) between $o_{i,j}$ and $o_{i',j'}$: $o_{i,j}\ \varrho_{i,j}\ p_{i,j}\ \varrho_{i,j}\ \varrho_{i',j'}\ p_{i',j'}\ \varrho_{i',j'}\ o_{i',j'}$



Cuts, points of the type $p_{i,j}$, polygon boundary, optimal route and points of the type $o_{i,j}$

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Lemma 3: $\|CH_P(OPT, \mathcal{P}_{C''})\| \leq 3 \cdot \|OPT(S,P,s)\|$.

Proof:

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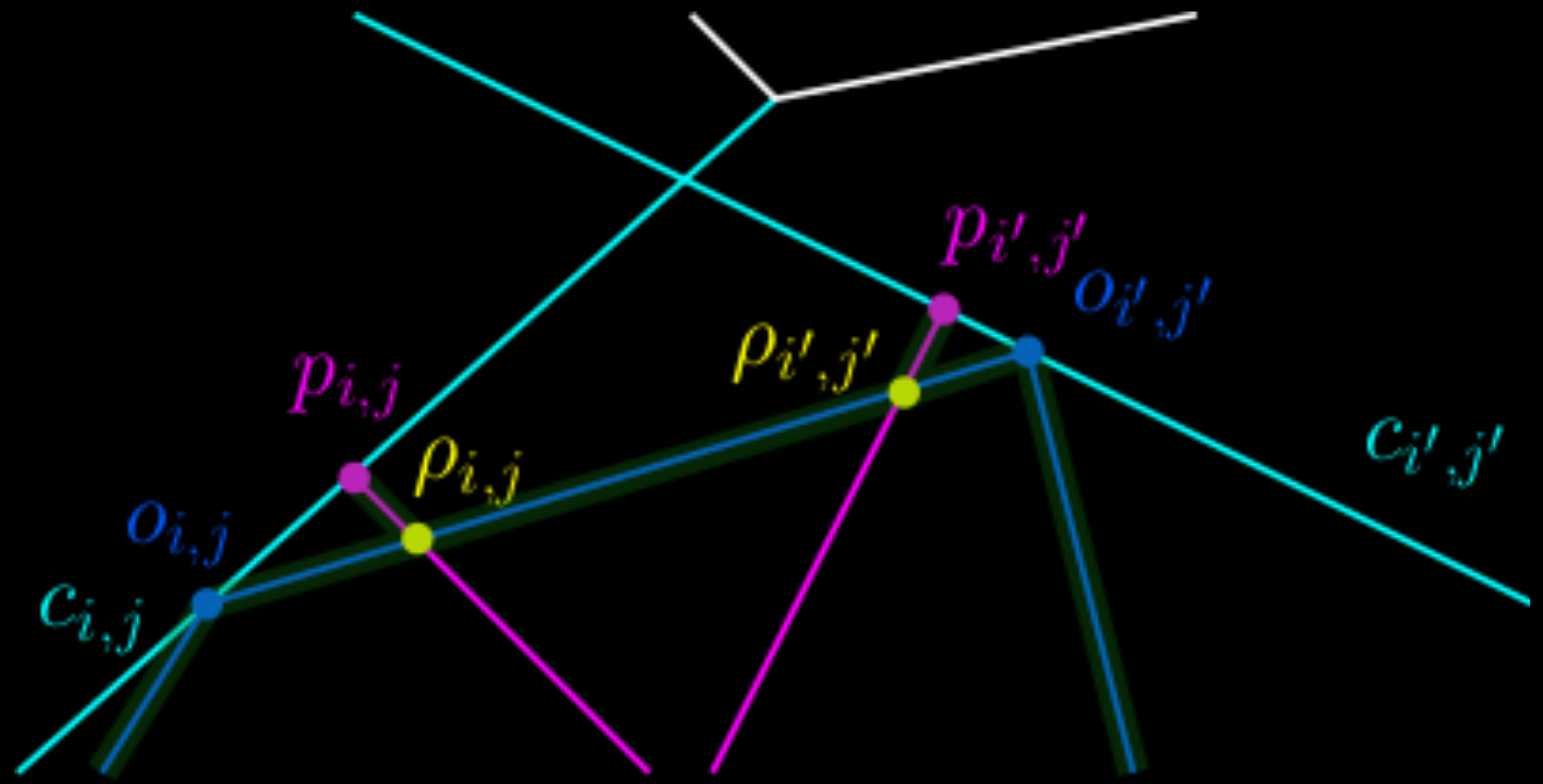
➡ $g_{i,j}$ intersects $OPT(S,P,s)$ between $o_{i,j}$ and $o_{i',j'}$—in point: $\varrho_{i,j}$

- $g_{i,j}$ is geodesic

➡ $\ell(\varrho_{i,j}, p_{i,j}) \leq \ell(\varrho_{i,j}, o_{i,j})$ (and $\ell(\varrho_{i',j'}, p_{i',j'}) \leq \ell(\varrho_{i',j'}, o_{i',j'})$)

- Alter $OPT(S,P,s)$ between $o_{i,j}$ and $o_{i',j'}$: $o_{i,j}\ \varrho_{i,j}\ p_{i,j}\ \varrho_{i,j}\ \varrho_{i',j'}\ p_{i',j'}\ \varrho_{i',j'}\ o_{i',j'}$

➡ New tour T: visits all points on $CH_P(OPT, \mathcal{P}_{C''})$



Cuts, points of the type $p_{i,j}$, polygon boundary, optimal route and points of the type $o_{i,j}$

Claim 3: No geodesic can intersect CH_P(OPT, $\mathcal{P}_{C''}$) between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on CH_P(OPT, $\mathcal{P}_{C''}$), we have at most two points of $\mathcal{P}_{C''}$. CH_P(OPT, $\mathcal{P}_{C''}$) has length at most 3·||OPT(S,P,s)||.

Lemma 3: ||CH_P(OPT, $\mathcal{P}_{C''}$)|| ≤ 3·||OPT(S,P,s)||.

Proof:

- Lemmas 1,2 ➜ Between two consecutive points of OPT(S,P,s) on CH_P(OPT, $\mathcal{P}_{C''}$), $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
- Points $o_{i,j}$ and $p_{i,j}$ both on $c_{i,j}$ / points $o_{i',j'}$ and $p_{i',j'}$ both on $c_{i',j'}$

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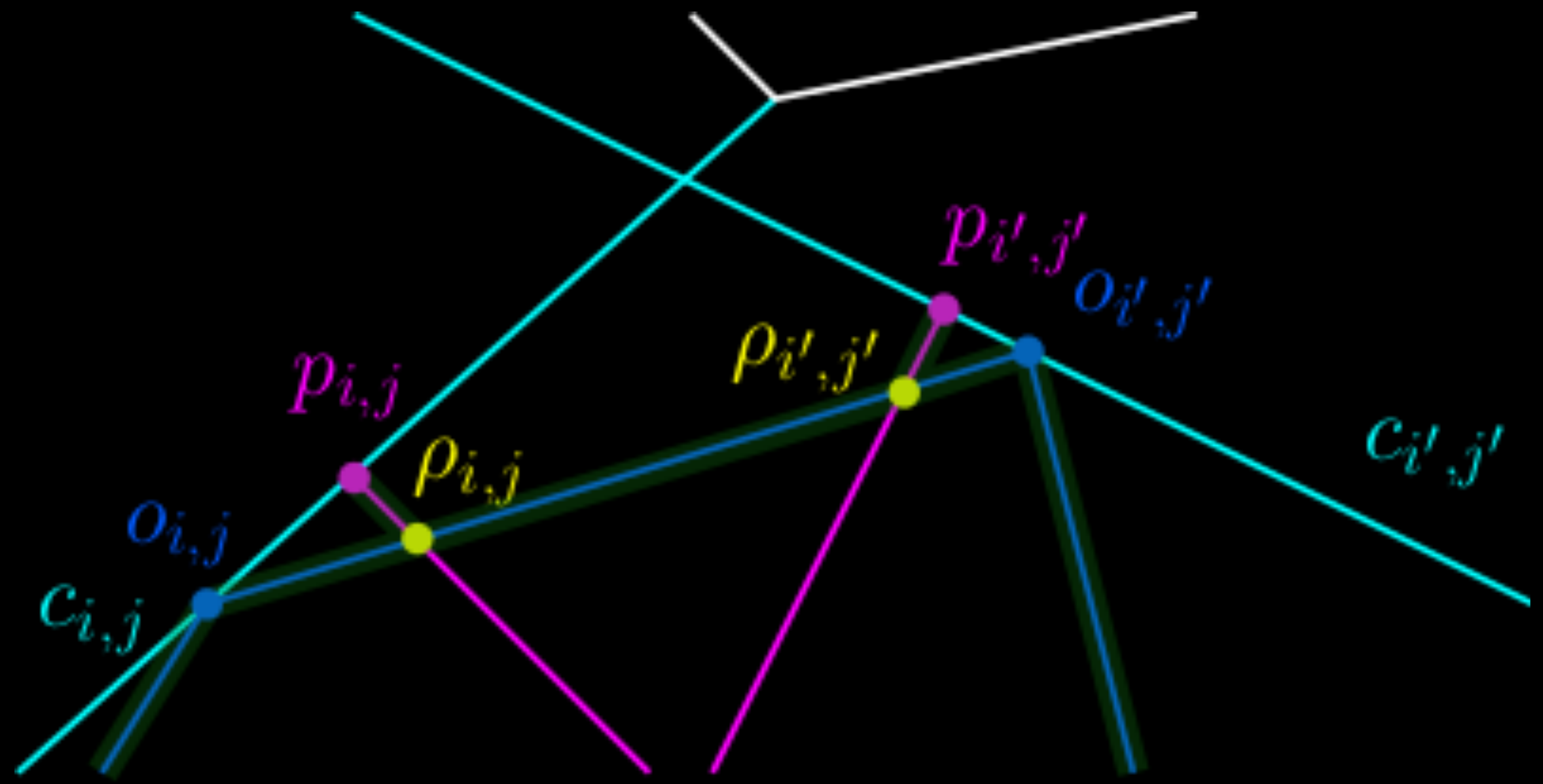

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Lemma 3: $\|CH_P(OPT, \mathcal{P}_{C''})\| \leq 3 \cdot \|OPT(S,P,s)\|$.

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- Lemmas 1,2 ➜ Between two consecutive points of $OPT(S,P,s)$ on $CH_P(OPT, \mathcal{P}_{C''})$, $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
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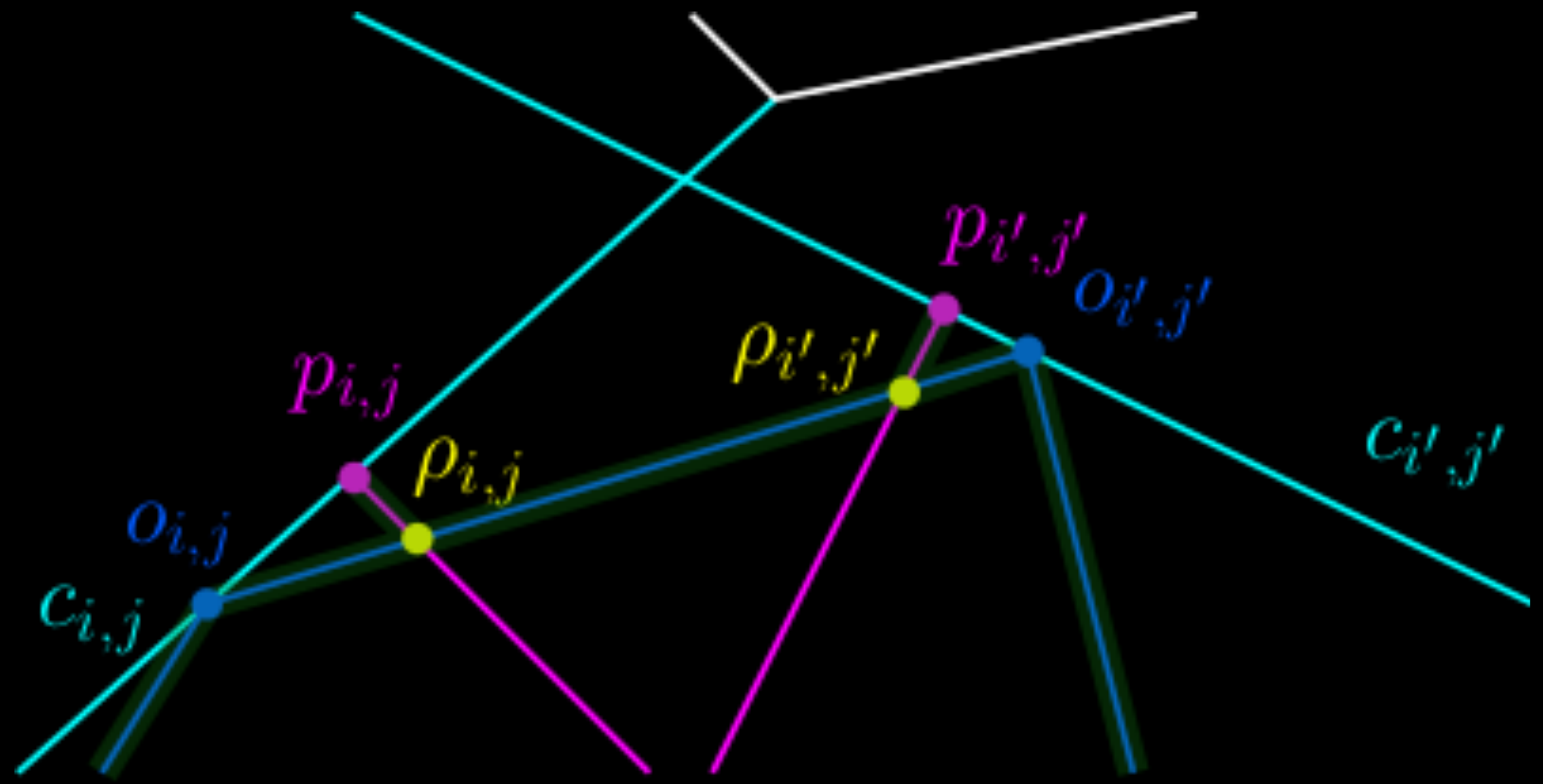
➡ New tour T: visits all points on $CH_P(OPT, \mathcal{P}_{C''})$

➡ $\|T\| \leq 3 \cdot \|OPT(S,P,s)\|$

- $CH_P(OPT, \mathcal{P}_{C''})$ is shortest tour to visit these points



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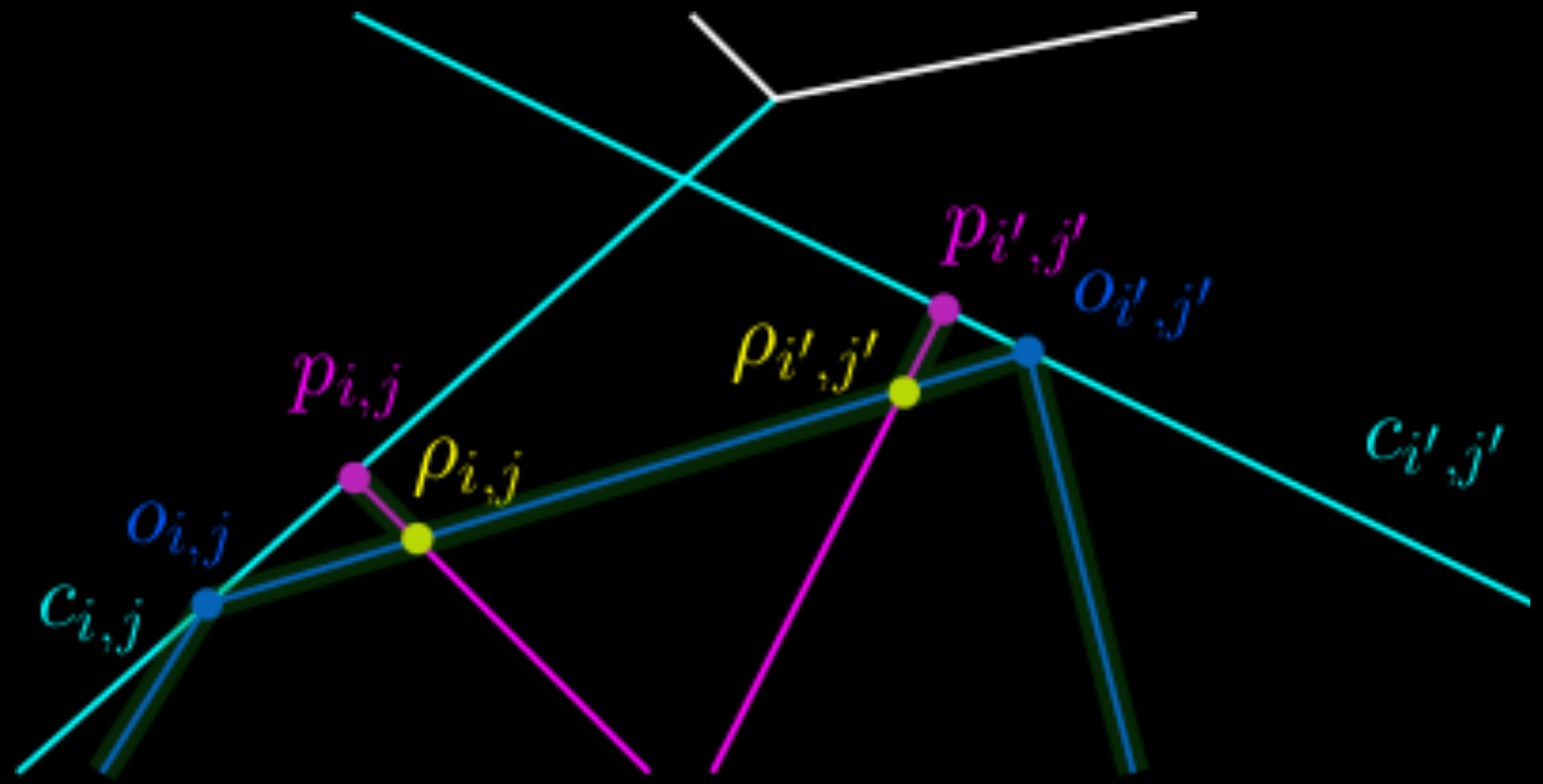

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Lemma 3: $||CH_P(OPT, \mathcal{P}_{C''})|| \leq 3 \cdot ||OPT(S,P,s)||$.

Proof:

- Lemmas 1,2 ➜ Between two consecutive points of $OPT(S,P,s)$ on $CH_P(OPT, \mathcal{P}_{C''})$, $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
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- ➡ New tour T: visits all points on $CH_P(OPT, \mathcal{P}_{C''})$
- ➡ $||T|| \leq 3 \cdot ||OPT(S,P,s)||$
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- ➡ $||CH_P(OPT, \mathcal{P}_{C''})|| \leq ||T||$
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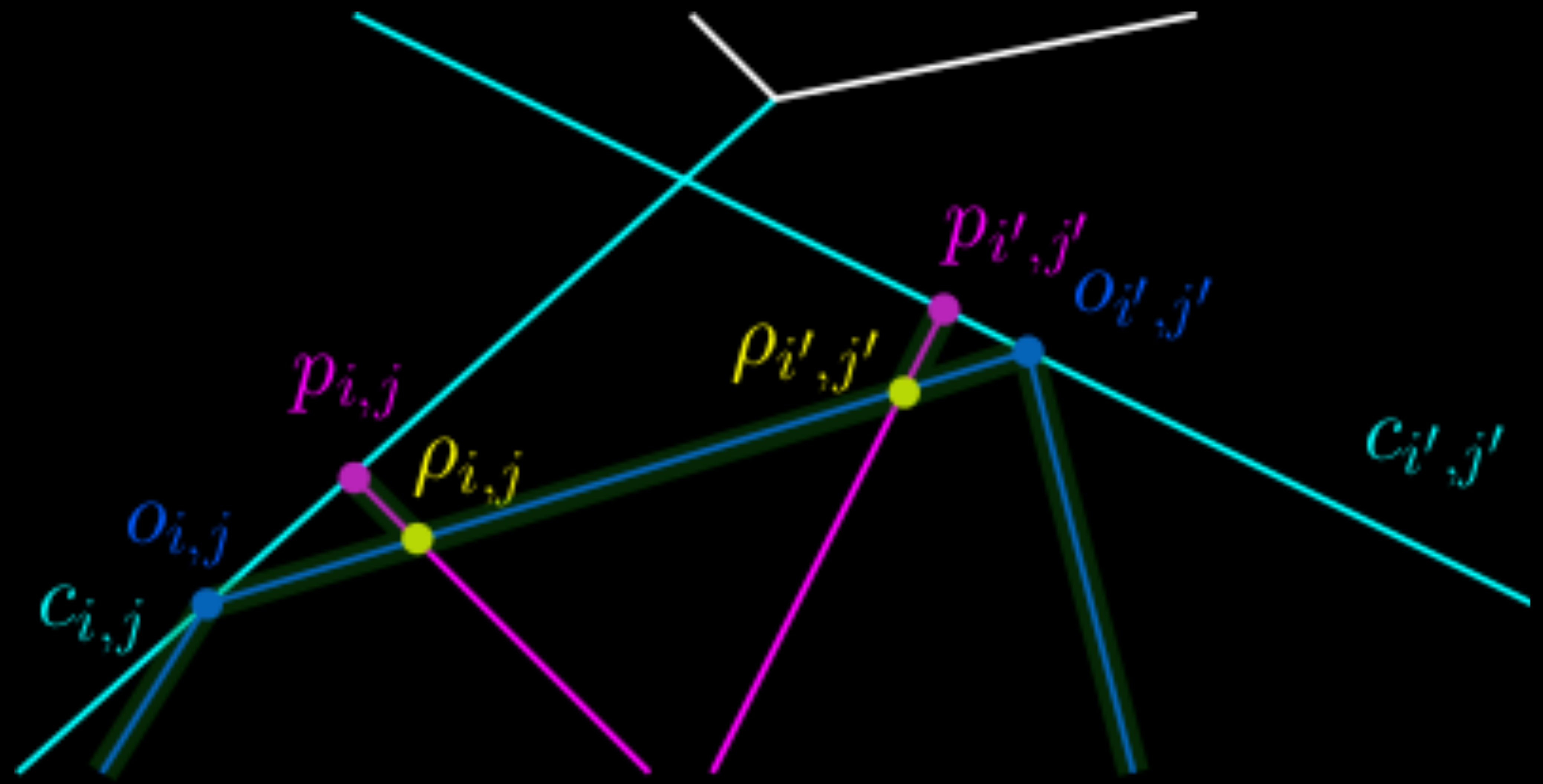

Cuts, points of the type $p_{i,j}$, polygon boundary, optimal route and points of the type $o_{i,j}$

Claim 4: $\mathrm{CH}_P(\mathcal{P}_{C''})$ is not longer than $\mathrm{CH}_P(\mathrm{OPT}, \mathcal{P}_{C''})$ and $\mathrm{CH}_P(\mathcal{P}_{C''})$ visits one point per γ_i (except for γ_0).

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Lemma 4: $\|CH_P(\mathcal{P}_{C''})\| \leq \|CH_P(OPT, \mathcal{P}_{C''})\|$.

Proof: $\mathcal{P}_{C''} \subseteq \mathcal{P}_{C''} \cup OPT$ ➜ ☑

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➡ No $g_c \in \mathcal{G}_{C''}$ intersects $c_{i,j}$

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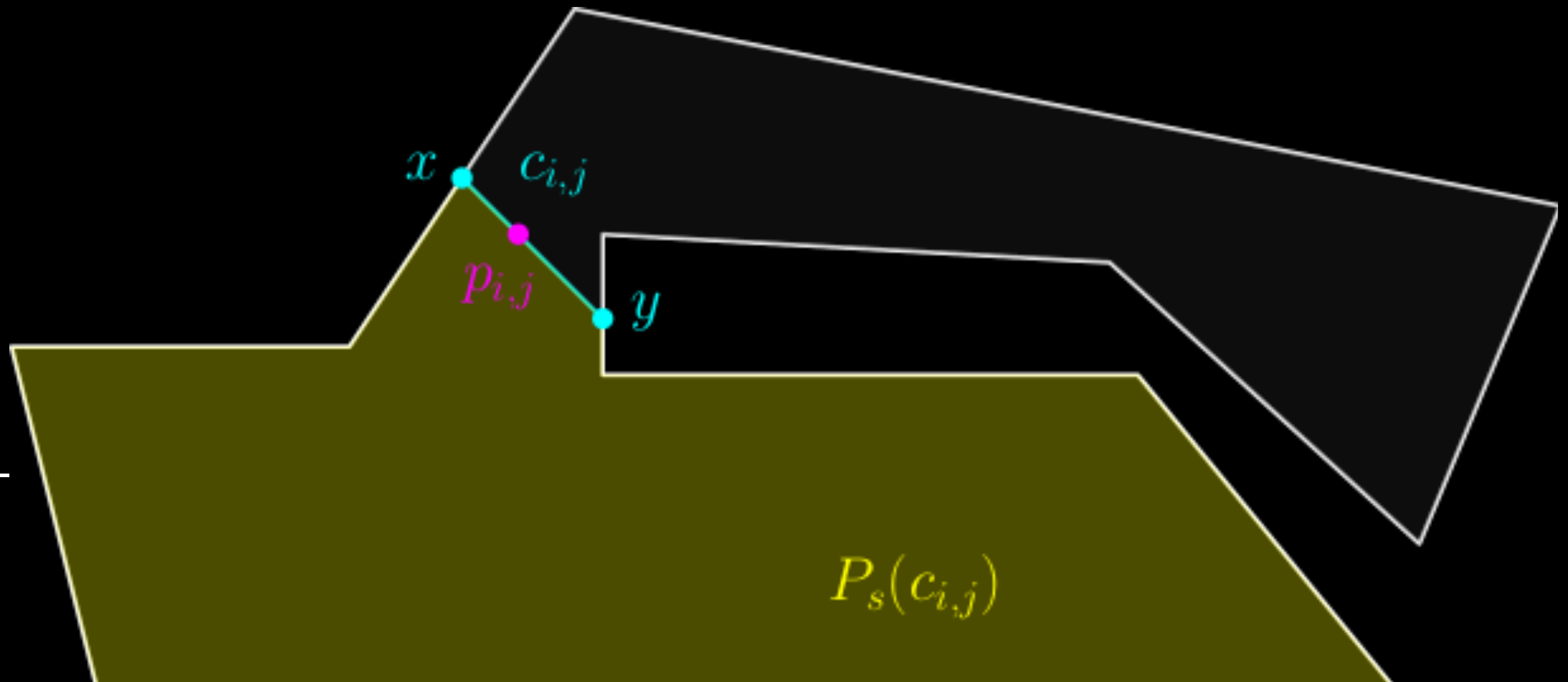

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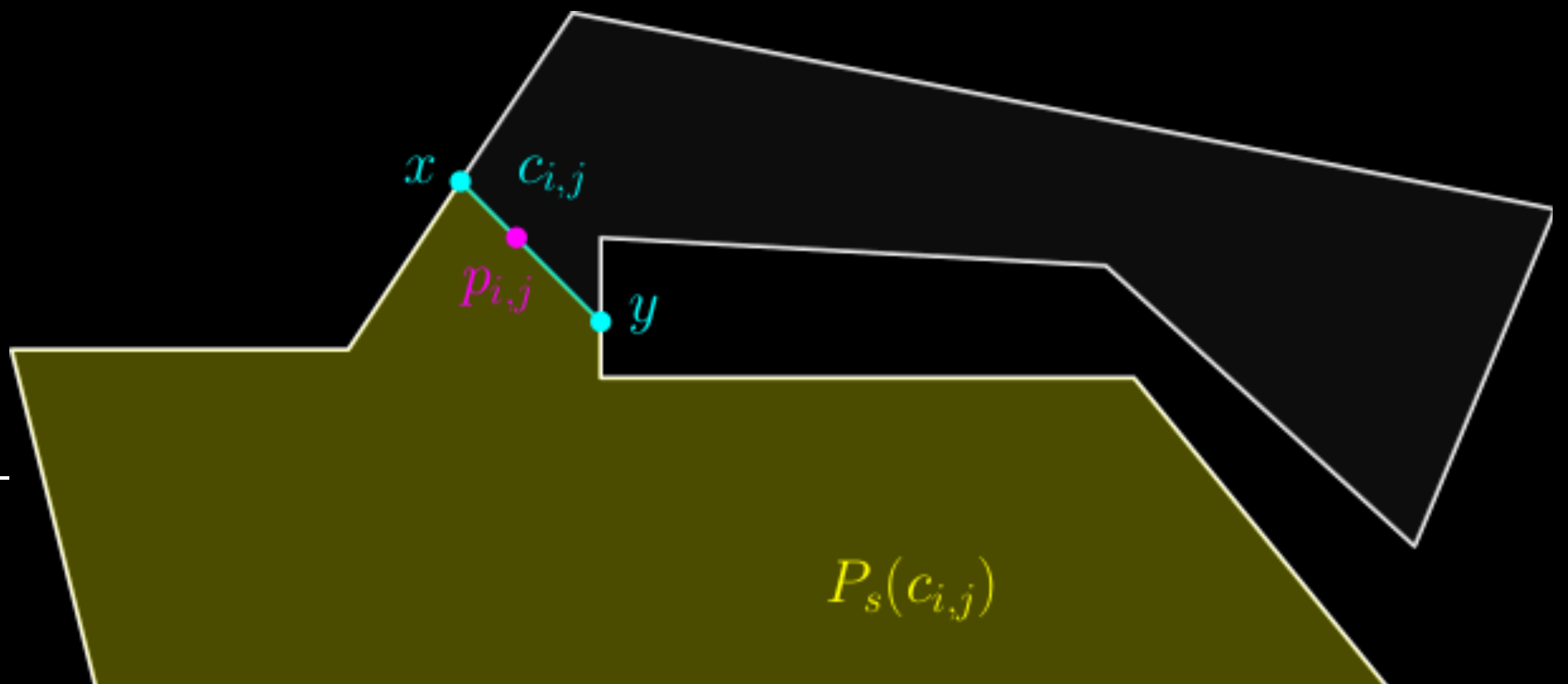

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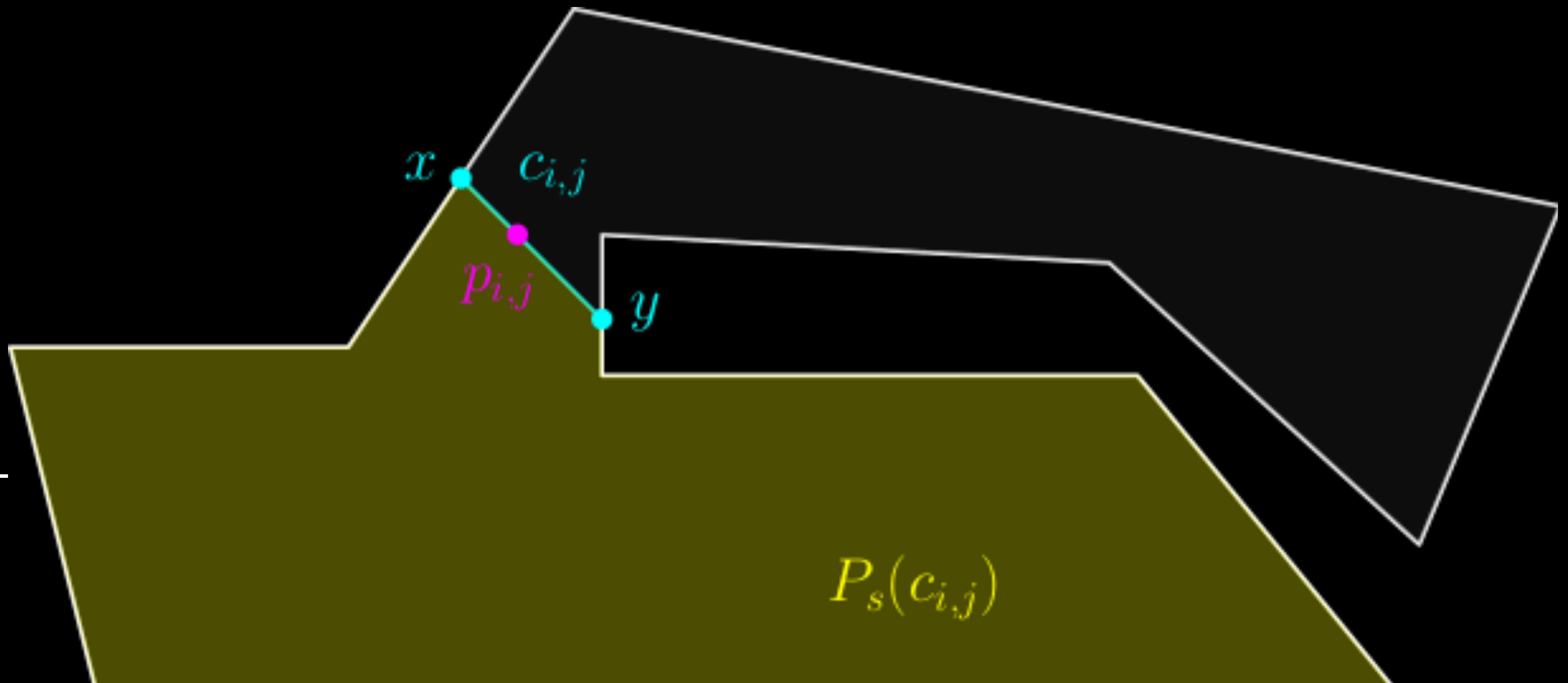
➡ $p_{i,j}$ must lie on $\mathrm{CH}_P(\mathcal{P}_{C''})$ ⚡



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Proof:

- $C = C'' \cup \{C' \setminus C''\} \cup \{C \setminus C'\}$ (Reminder: C visited by OPT, $C' \subseteq C$ essential cuts, $C'' \subseteq C'$ cuts with independent geodesics)

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- Assume $c \in \{C' \setminus C''\}$ not visited by $\text{CH}_P(\mathcal{P}_{C''})$

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- Assume $c \in \{C' \setminus C''\}$ not visited by $\mathrm{CH}_P(\mathcal{P}_{C''})$

➡ Cut c not in C'' (we filtered its geodesic g_c out)

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➡ There is a geodesic $g_{c_{i,j}} \in \mathcal{G}_{C''}$ with $\ell(g_{c_{i,j}}) \geq \ell(g_c)$ that intersects c; visits $c_{i,j}$ in the point $p_{i,j}$

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➡ Cut c not in C'' (we filtered its geodesic g_c out)

➡ There is a geodesic $g_{ci,j} \in \mathcal{G}_{C''}$ with $\ell(g_{ci,j}) \geq \ell(g_c)$ that intersects c; visits $c_{i,j}$ in the point $p_{i,j}$

➡ Any tour that visits s and $p_{i,j}$ must intersect c

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Approximation Algorithm for *k*-TrWRP(*S,P,s*)

Theorem 2: Let P be a simple polygon with $n=|P|$. Let OPT(S,P,s) be the optimal solution for the k-TrWRP(S,P,s) and let R be the solution by our algorithm ALG(S,P,s). Then R yields an approximation ratio of $O(\log^2 (|S|\, n) \log\log (|S|\, n) \log |S|)$.

Open Problem: k-Transmitter Combinatorics

k-Transmitters

- "Art Gallery Theorems"
  - AFFHUV2018: tight bounds for monotone and monotone orthogonal polygons ($\lceil \frac{n-2}{2k+3} \rceil$ k-transmitters are sometimes necessary and always sufficient to cover a monotone n-gon)
  - BBBDDDFHILMSSU2010:
    - Bounds for line segments in the plane
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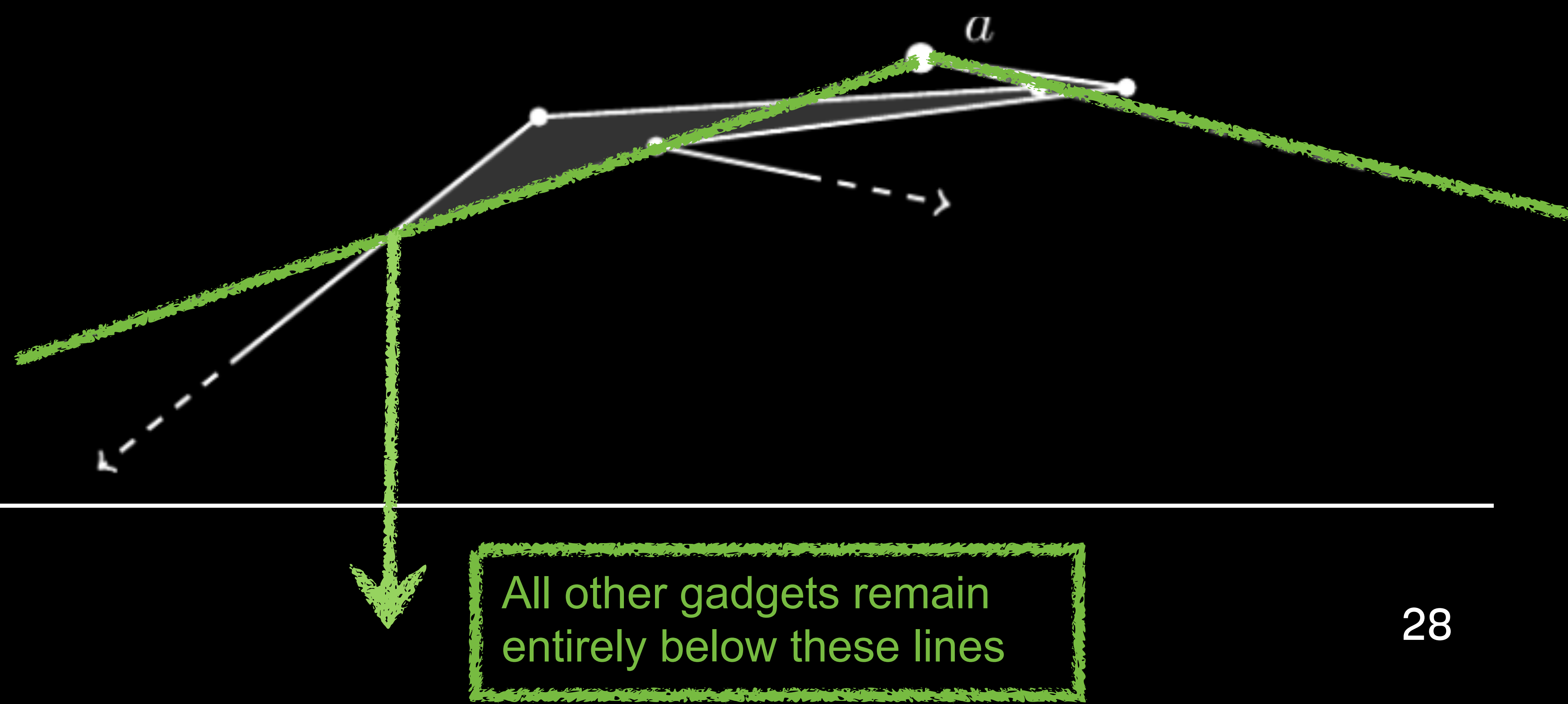

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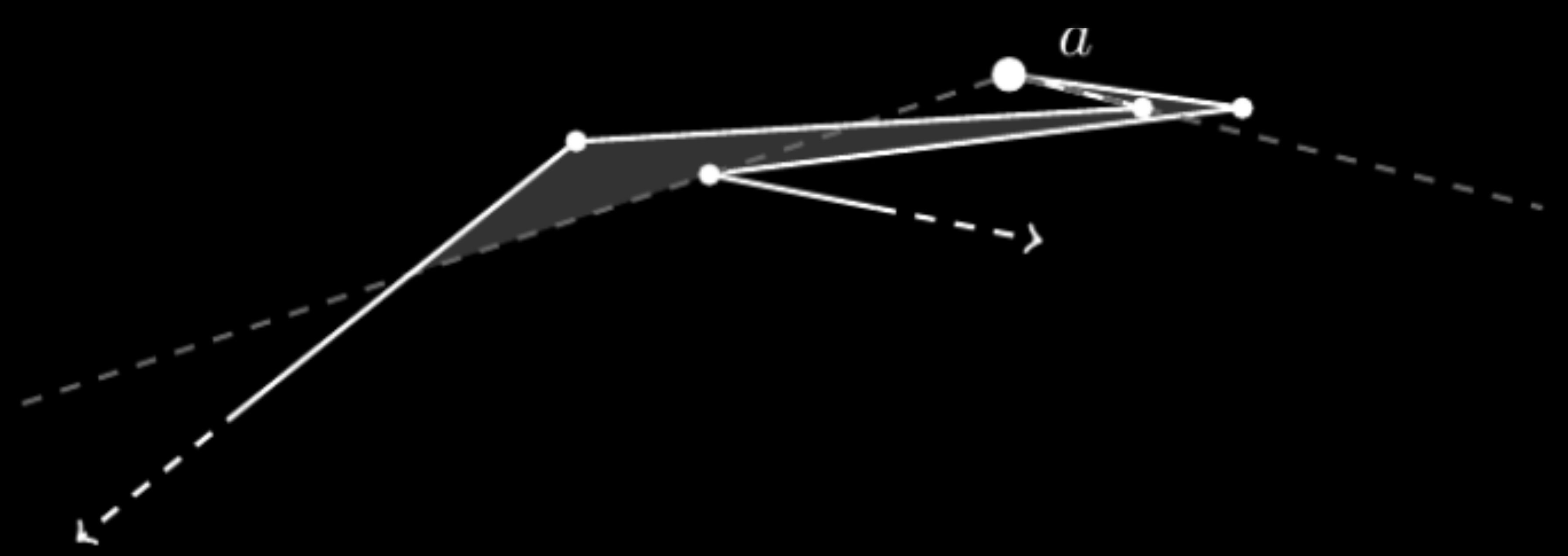

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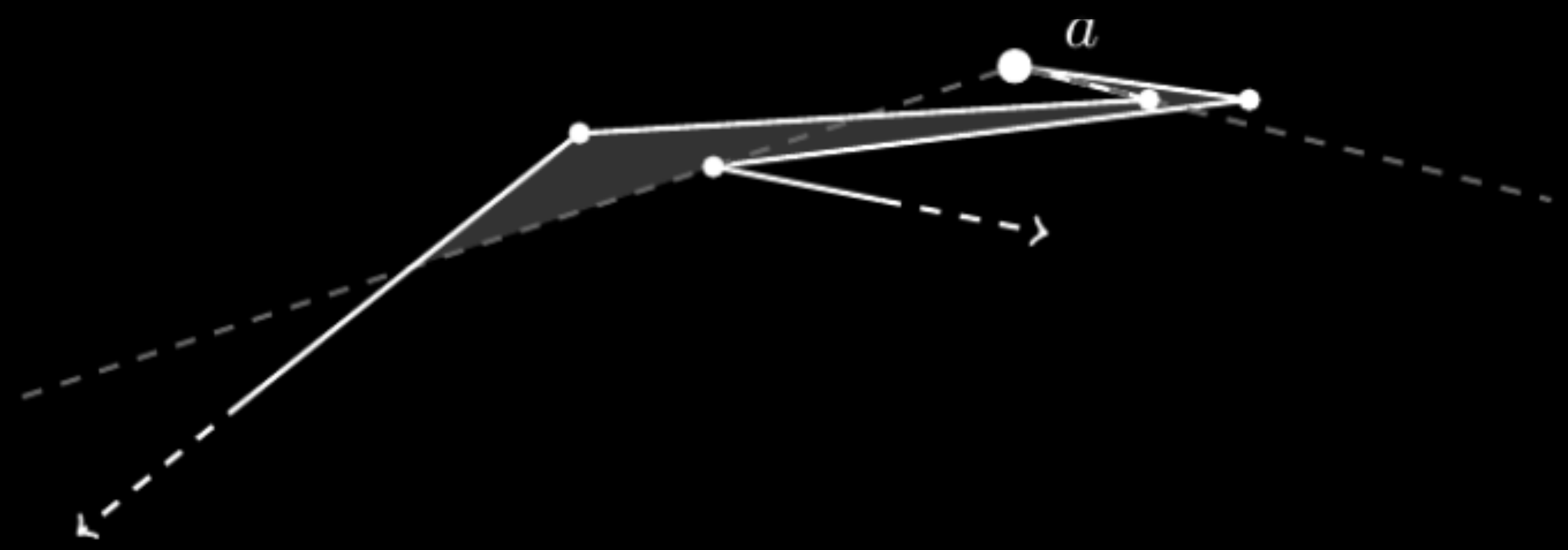

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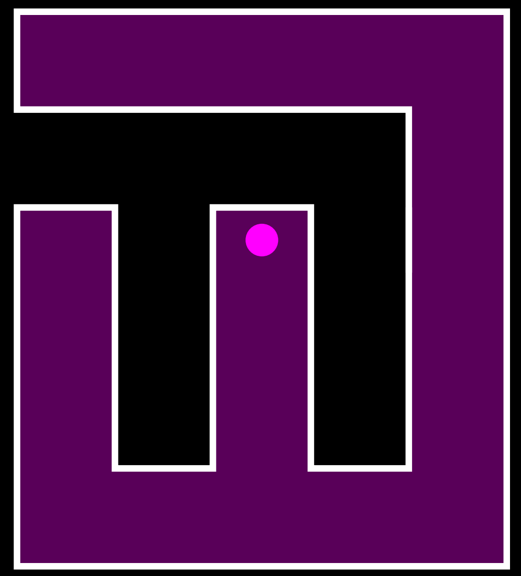

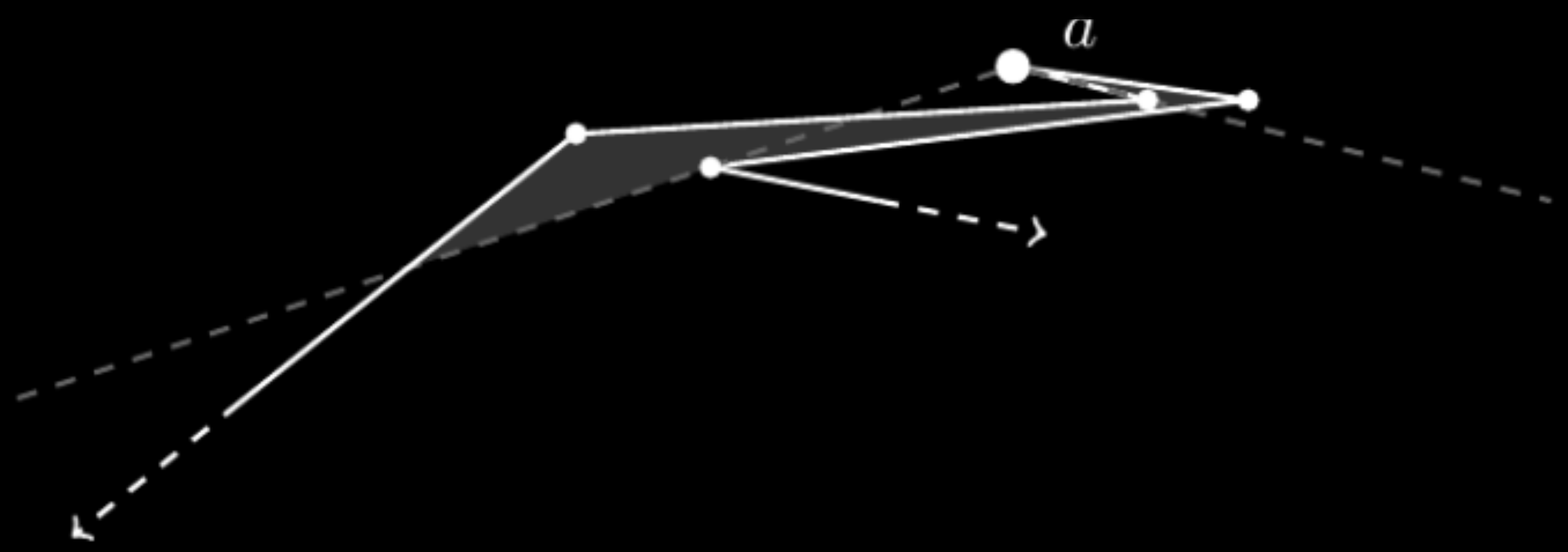

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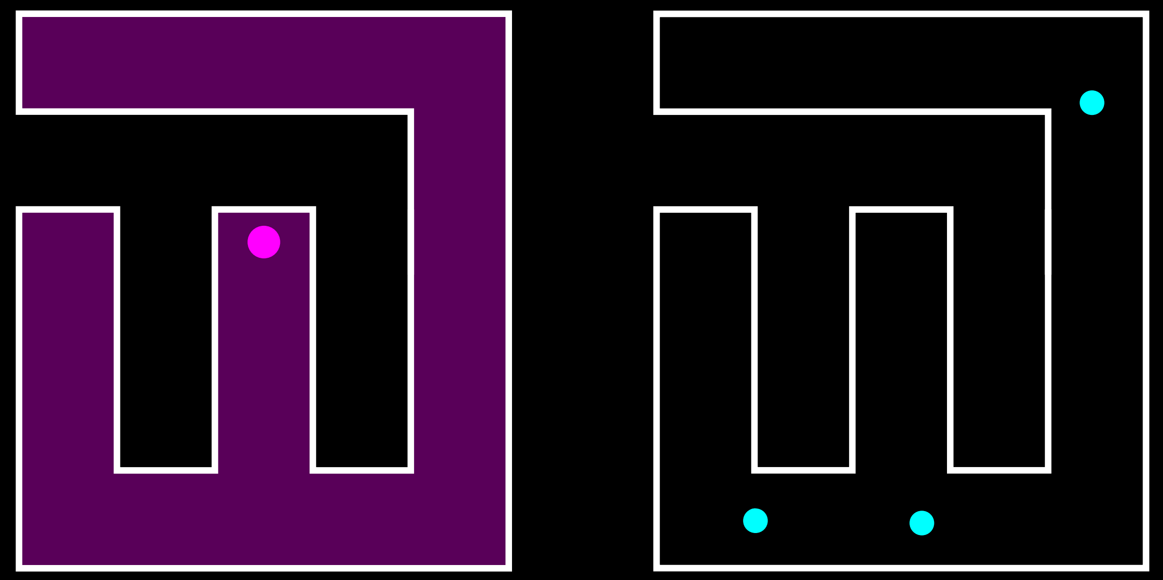

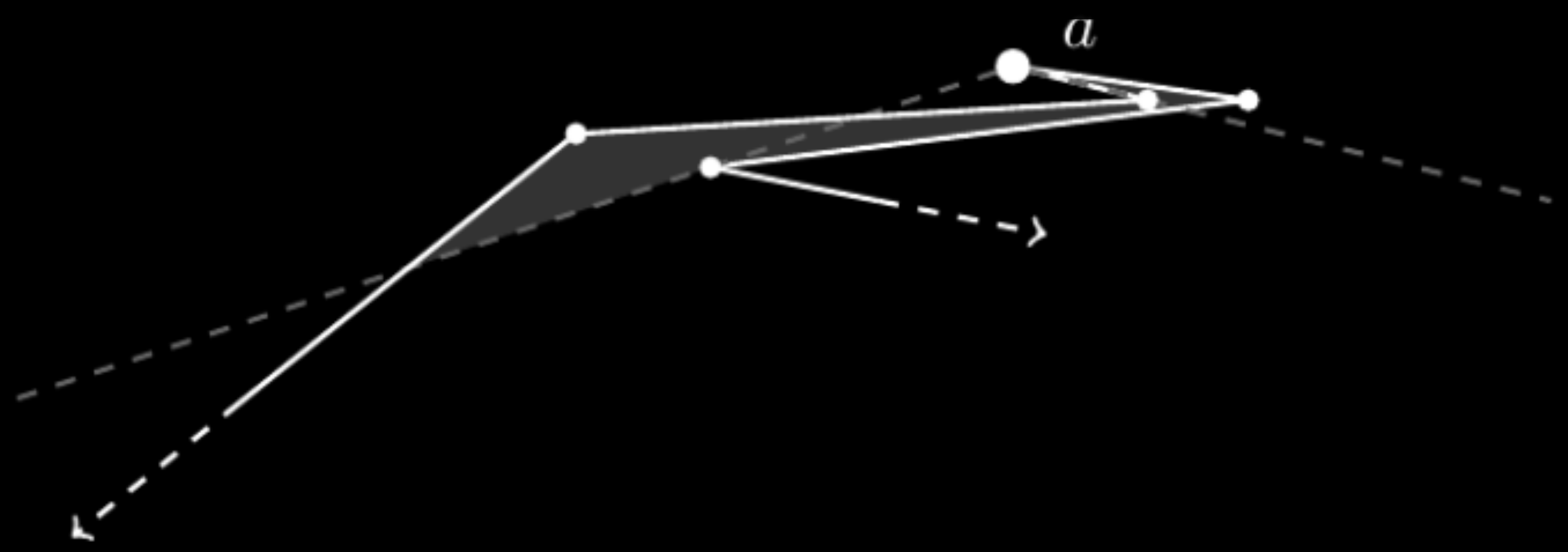

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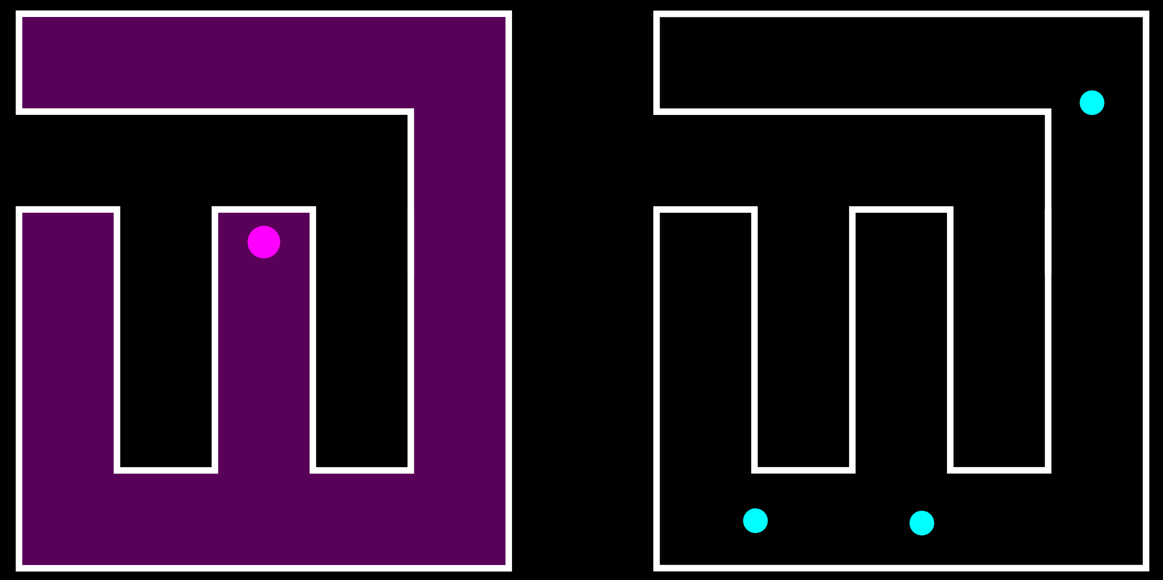

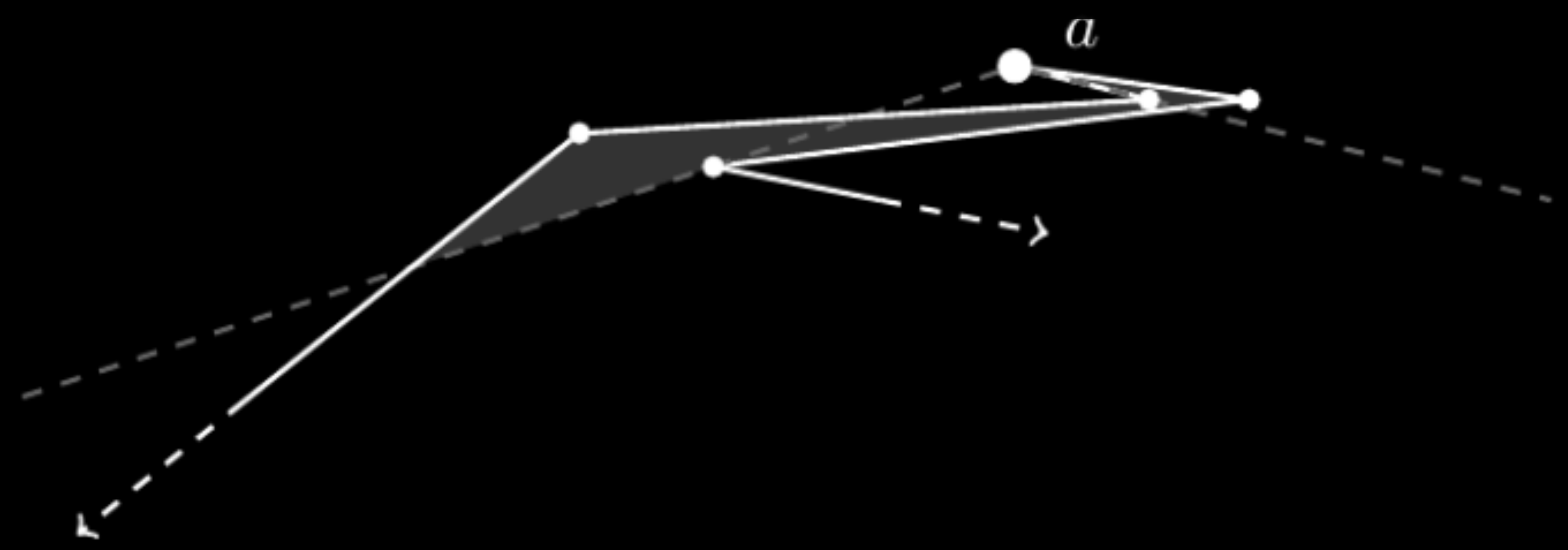

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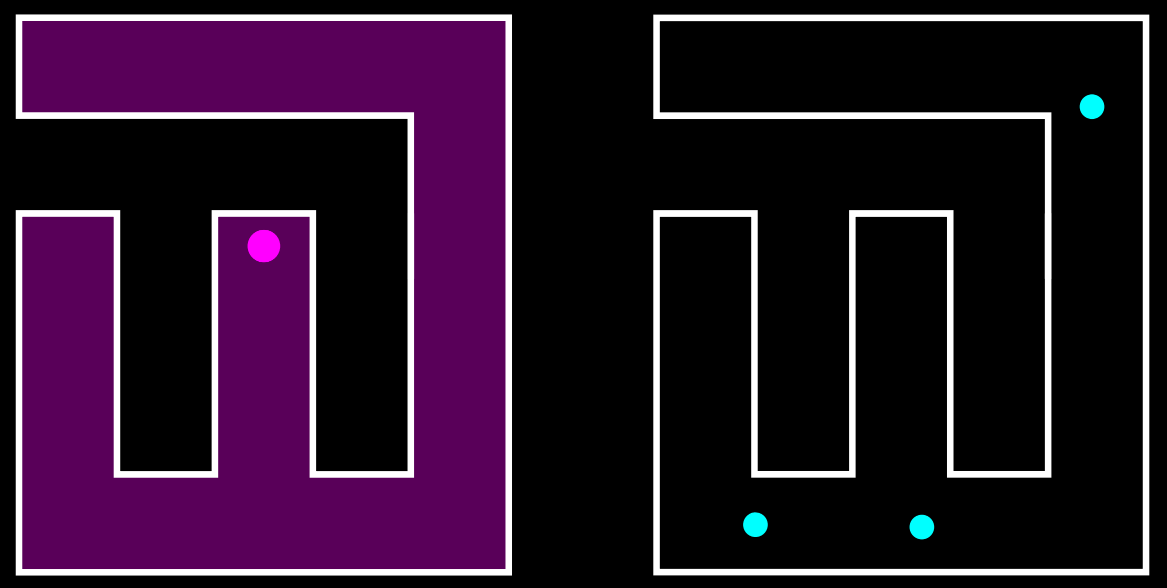

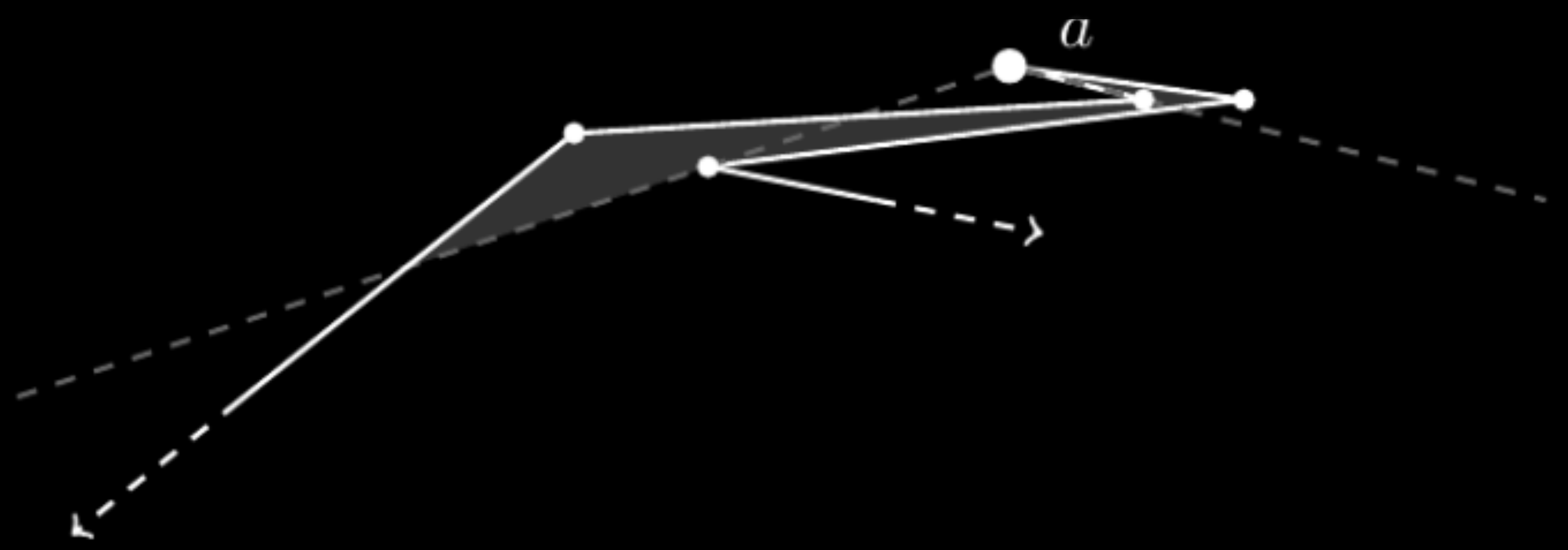

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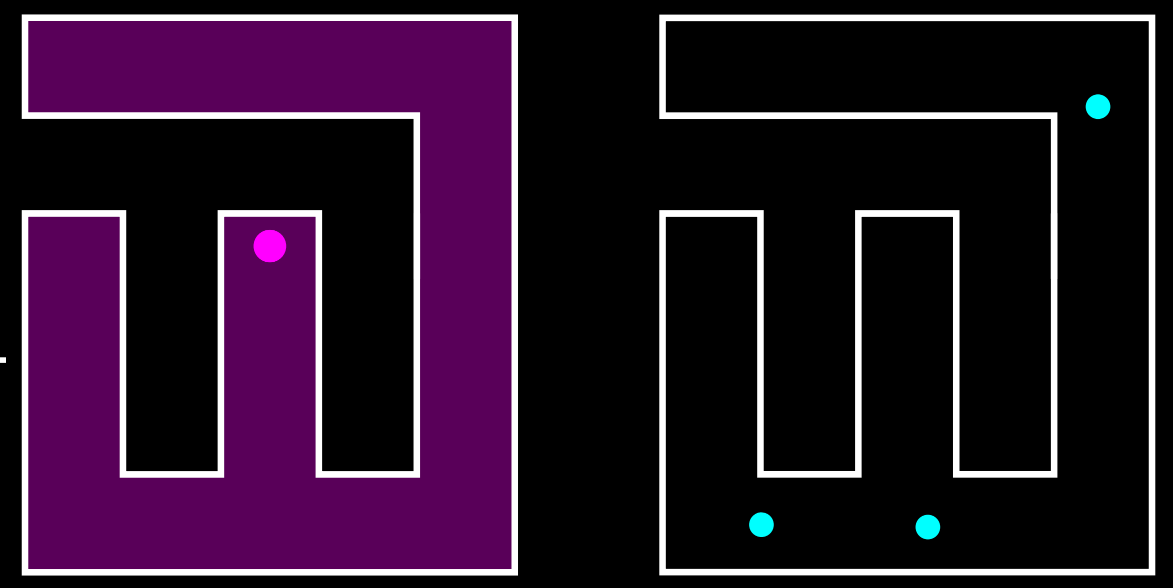

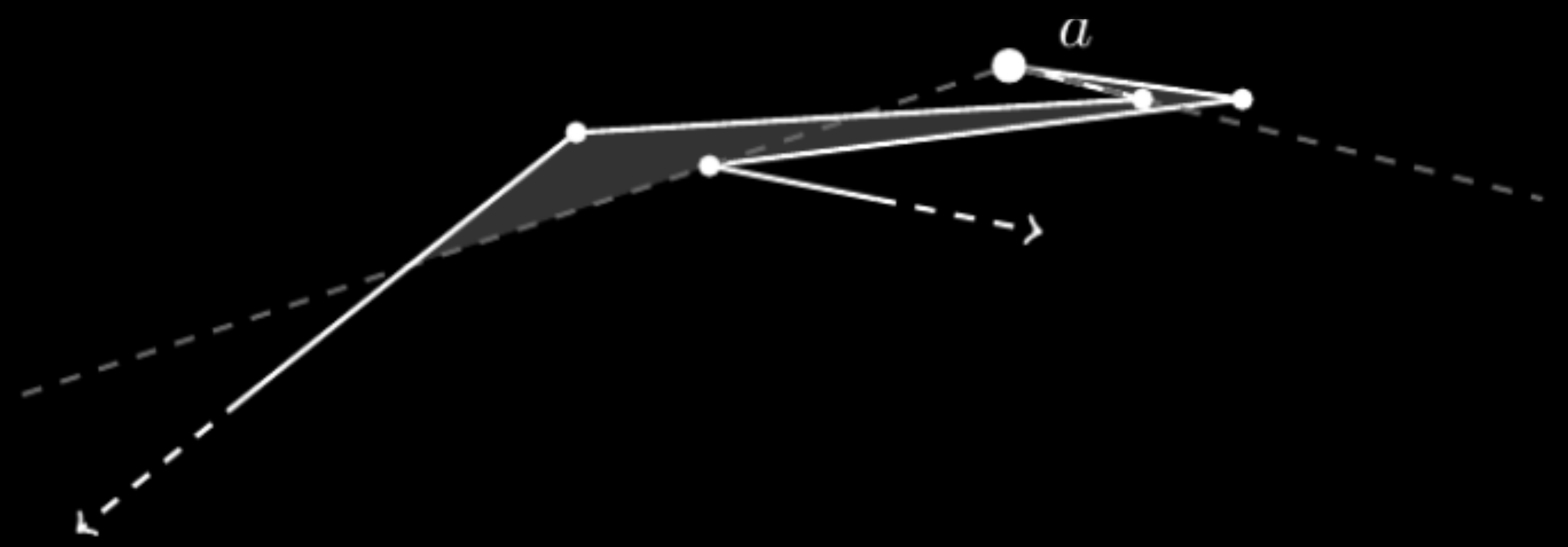

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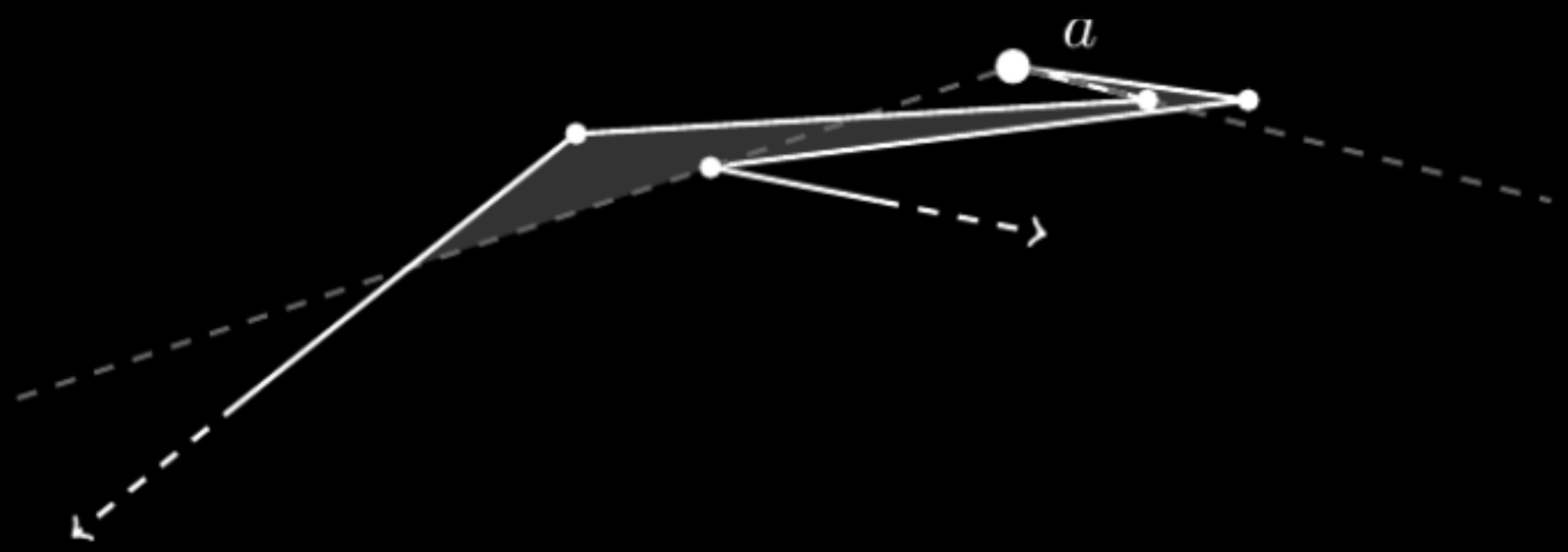

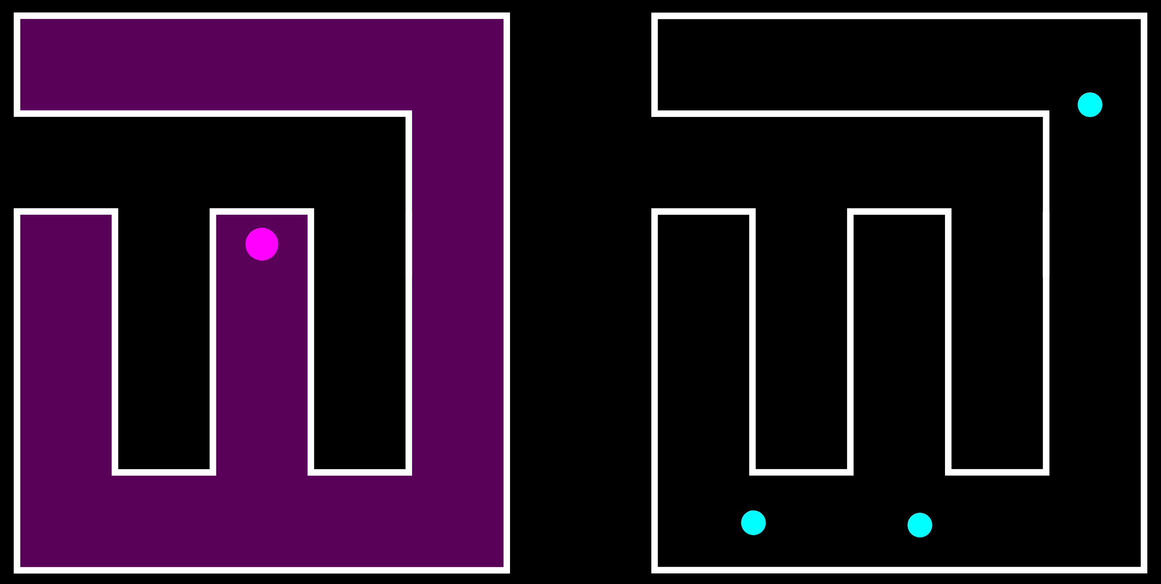

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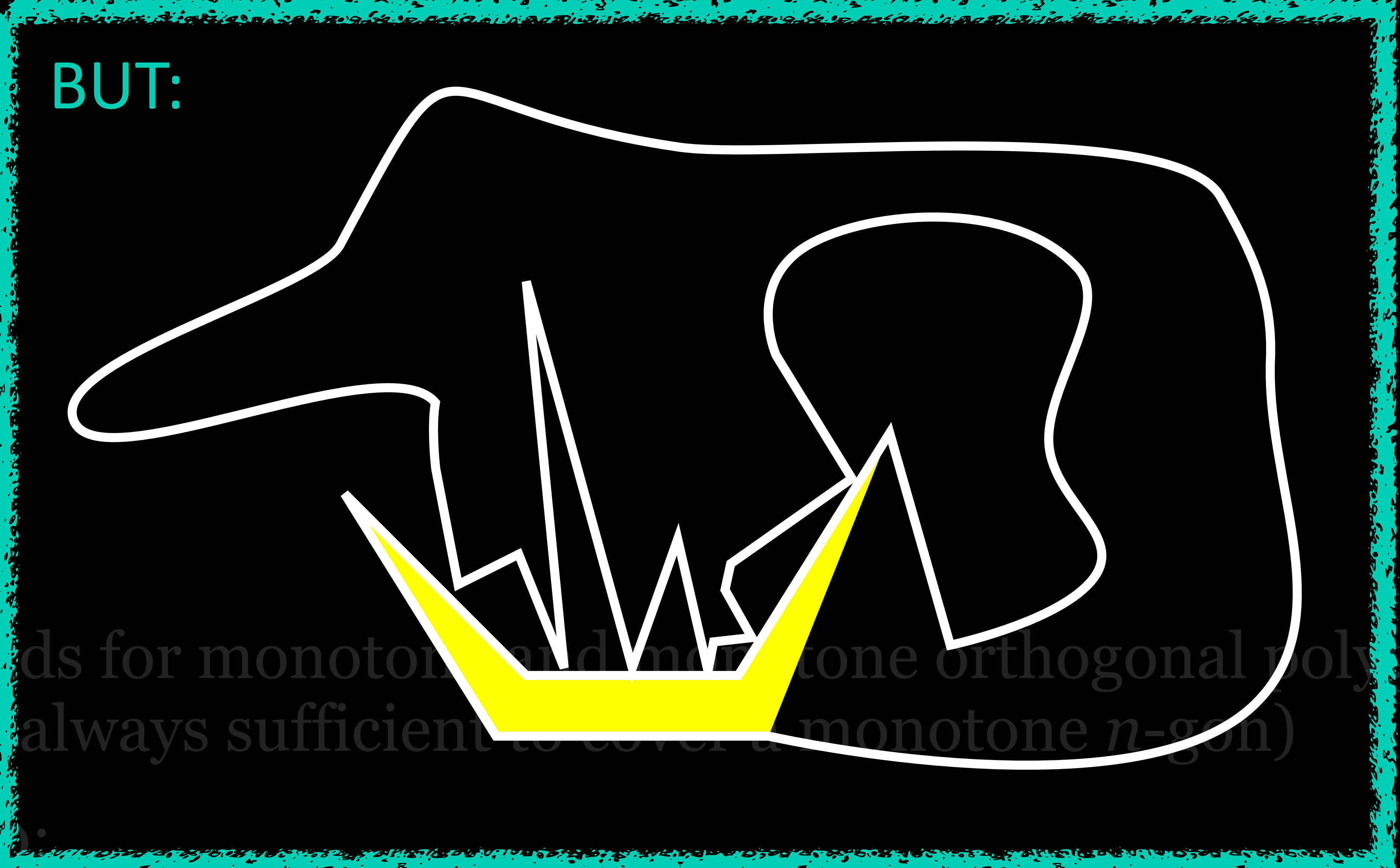

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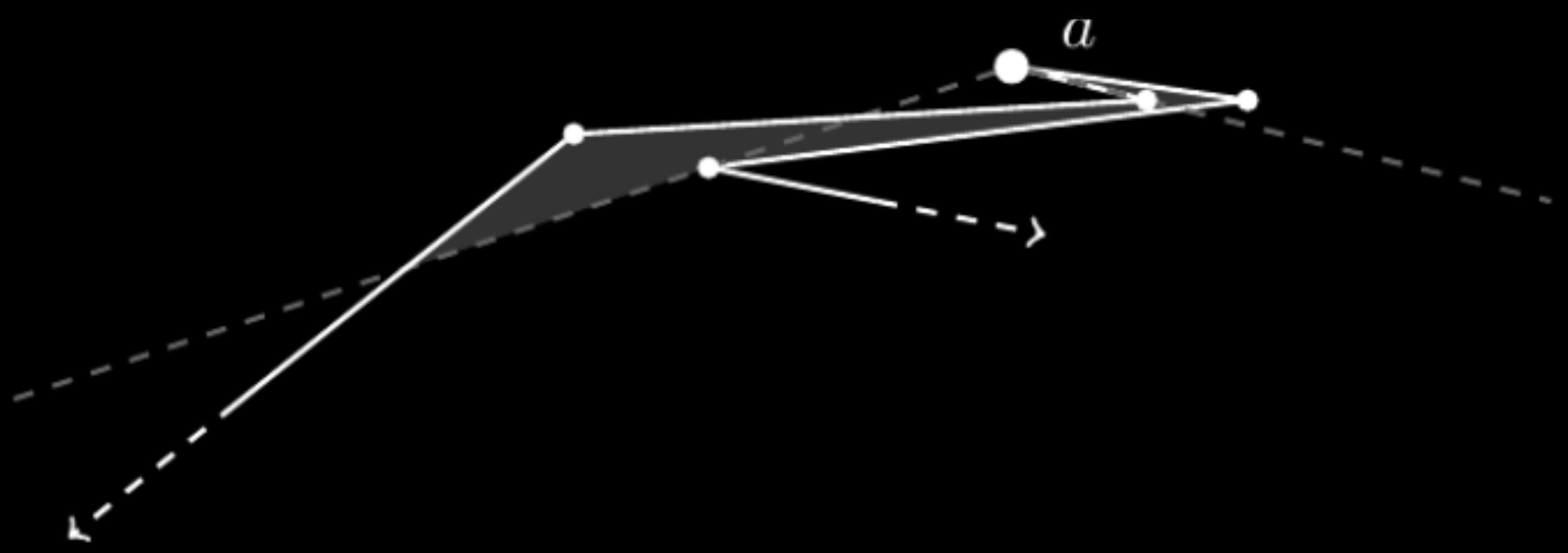

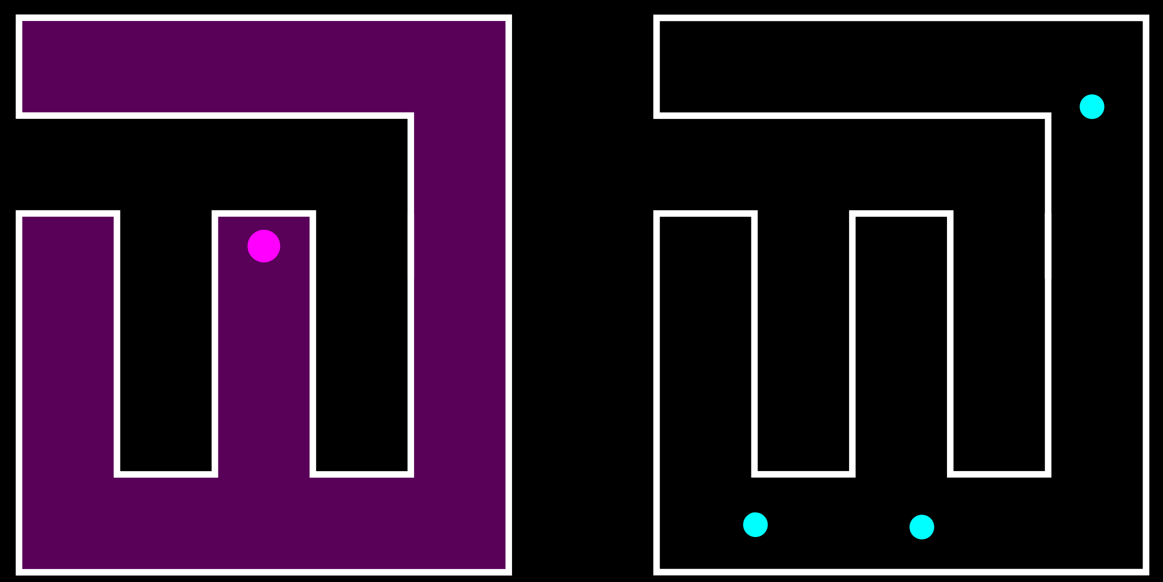

Outlook

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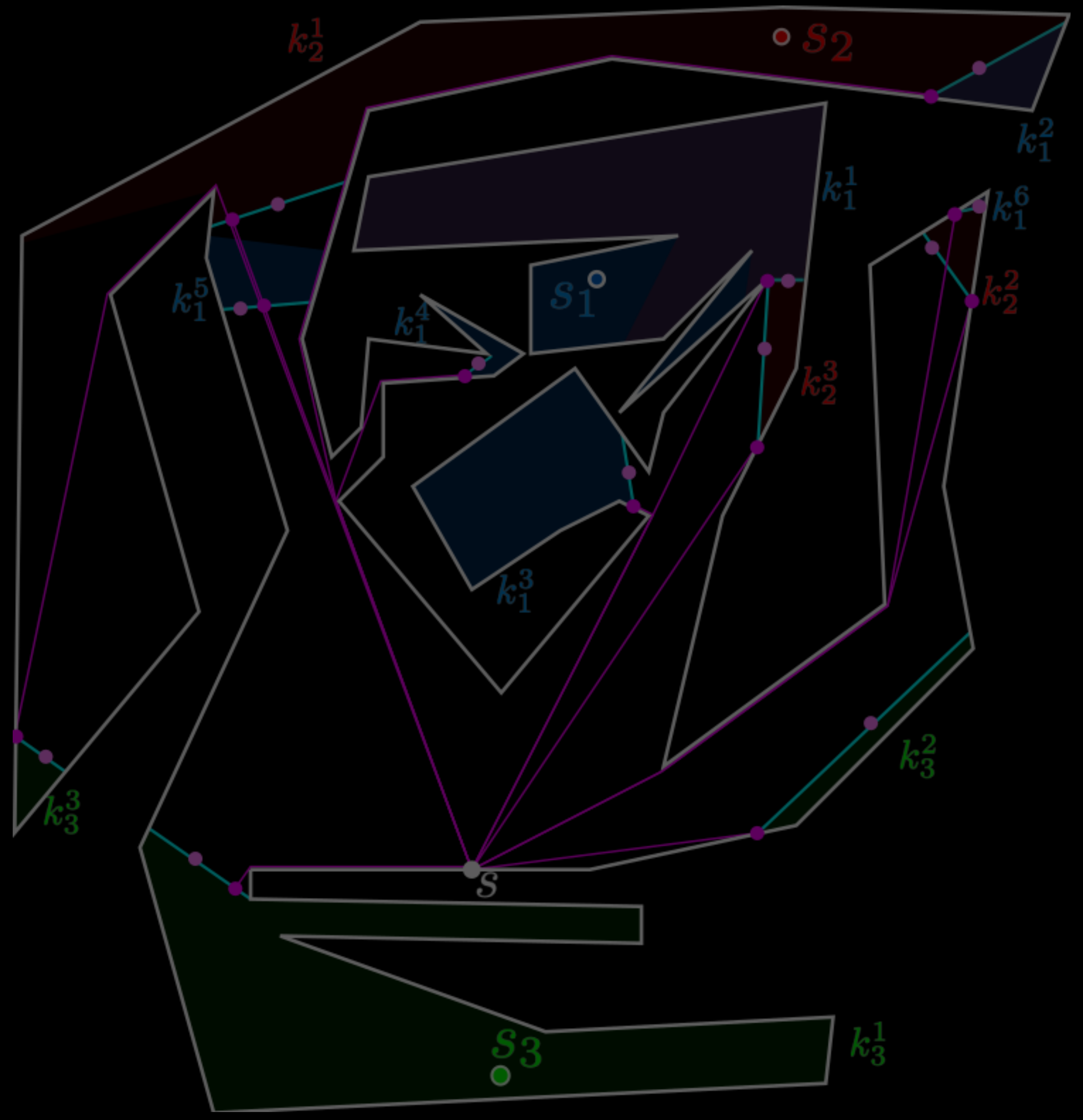
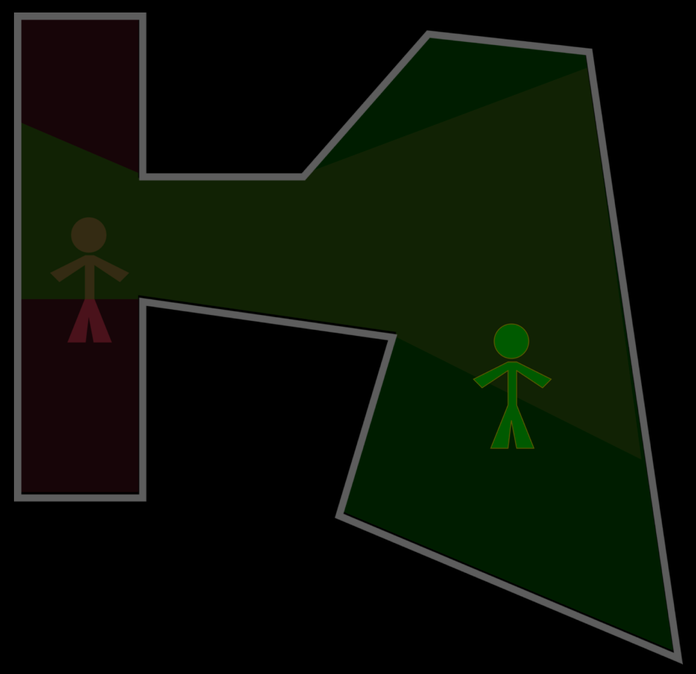
- Approximation for watchmen routes for k-transmitters without given starting point and/or when all of *P* should be monitored?
- Structural analogue for extensions for 0-transmitters?
- Improved combinatorial bounds for 2-/k-transmitter covers—in particular, better upper bounds for simple polygons than the one stemming from 0-transmitters





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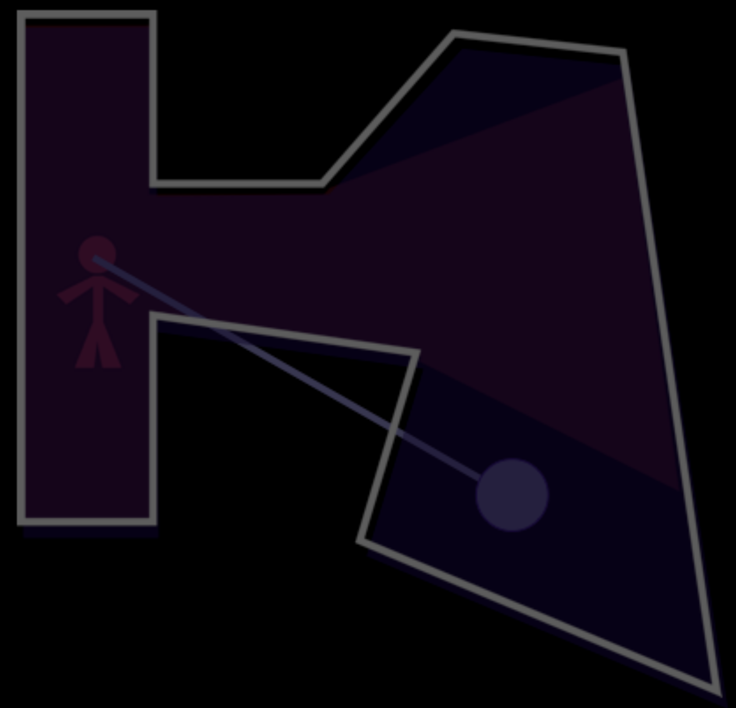

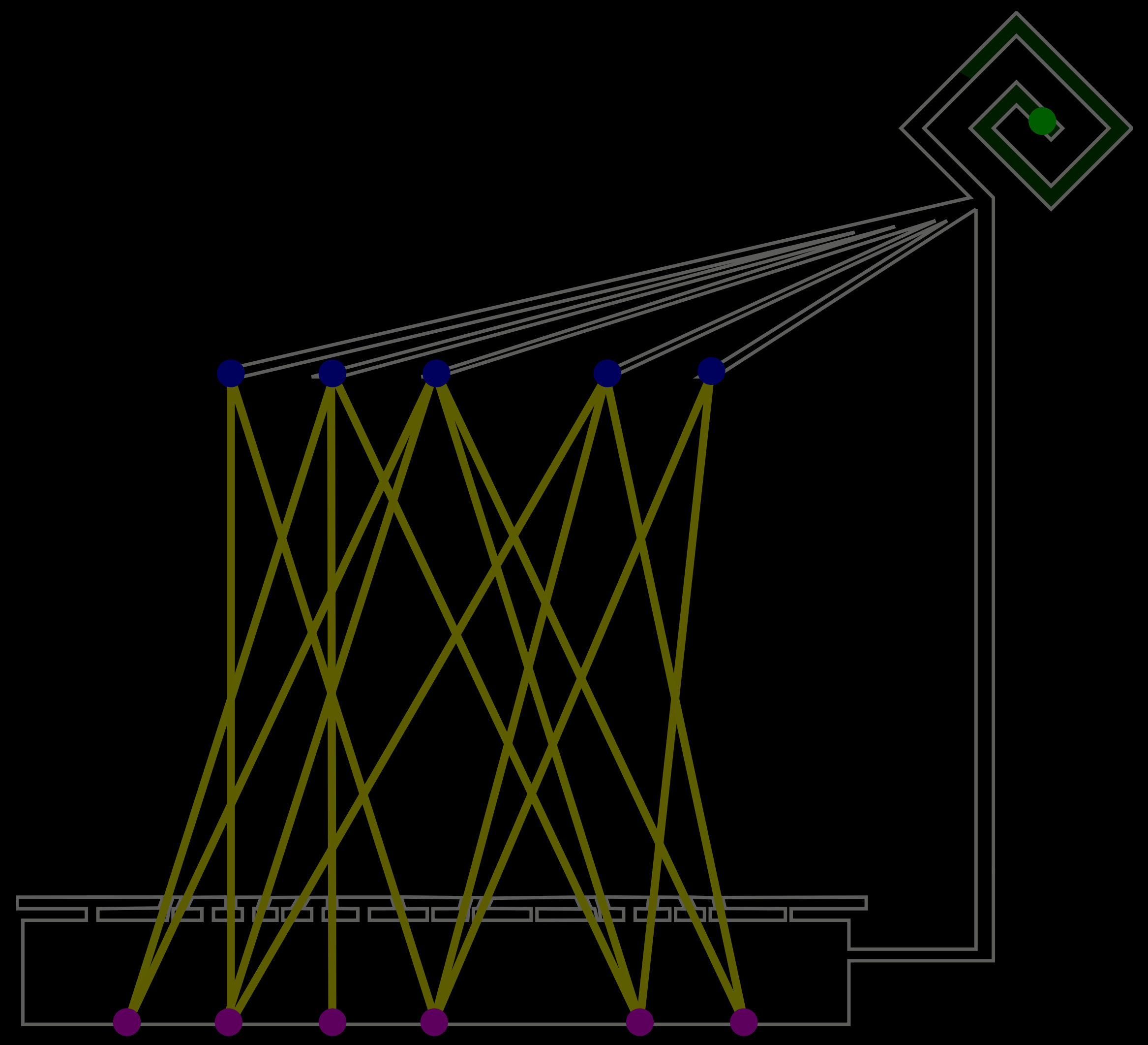

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