k-Transmitters/k-Modems

Christiane Schmidt

XX Spanish Meeting on Computational Geometry

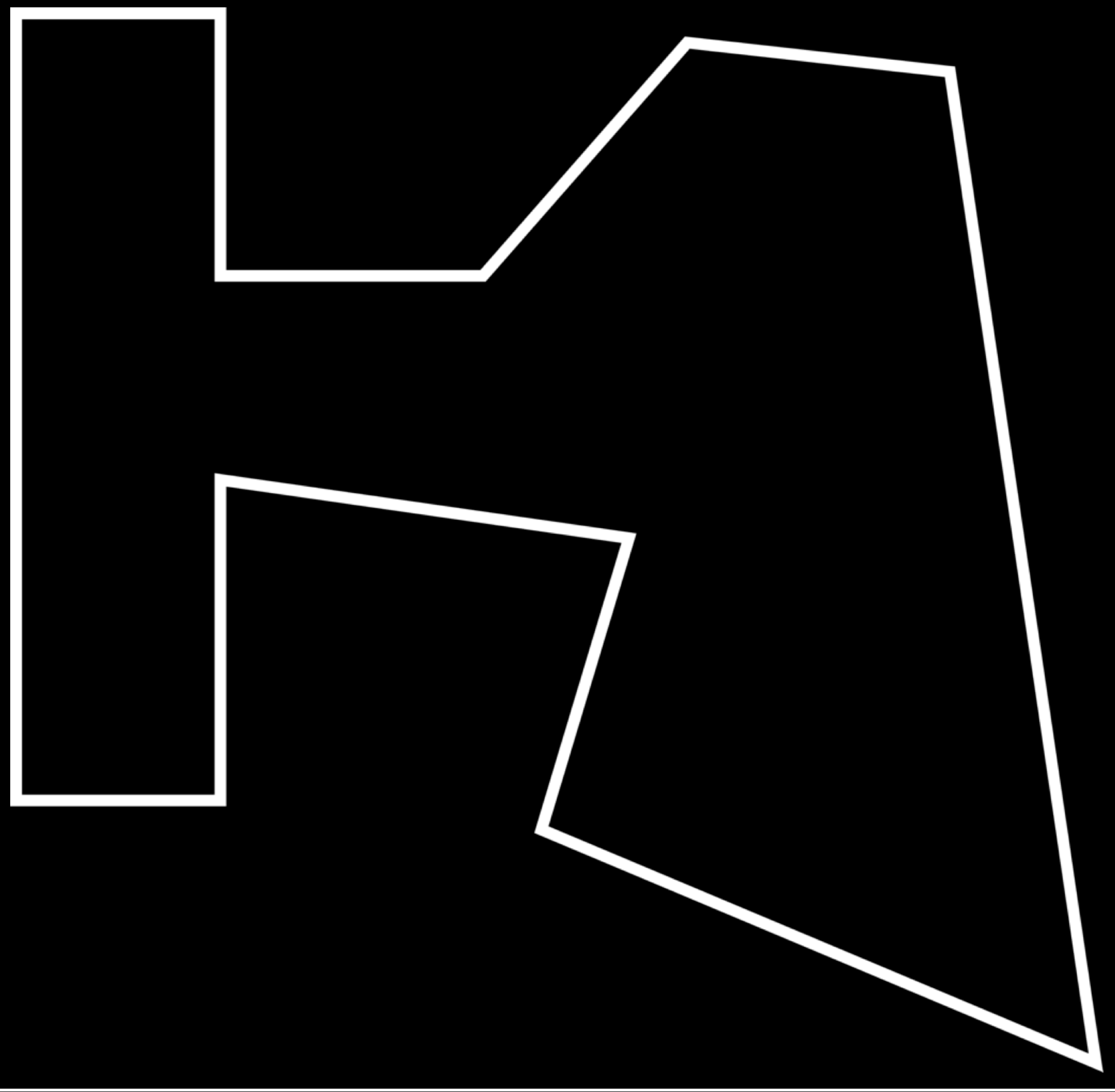
2023

LINKÖPING UNIVERSITY

Agenda

- The Art Gallery Problem
- *k*-Transmitters
- Art Gallery Theorems for *k*-Transmitters
- Computation of the *k*-Visibility Region
- Computational Complexity
- Sliding *k*-Transmitters
- The Watchman Route Problem (WRP)
- *k*-Transmitter Watchman Routes
- Outlook

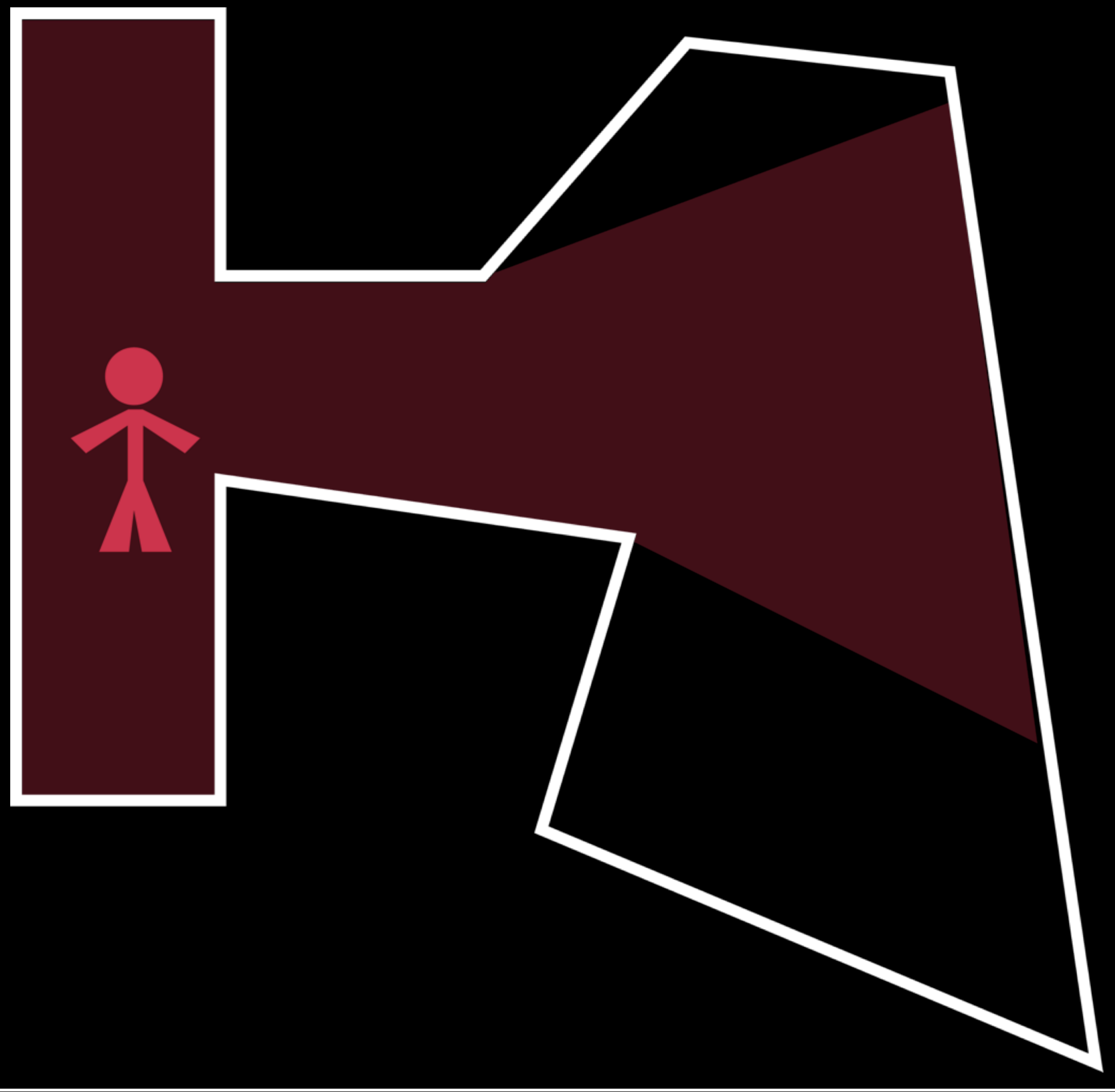
The Art Gallery Problem (AGP)



Given: Polygon P

How many guards do we need to monitor P?

The Art Gallery Problem (AGP)



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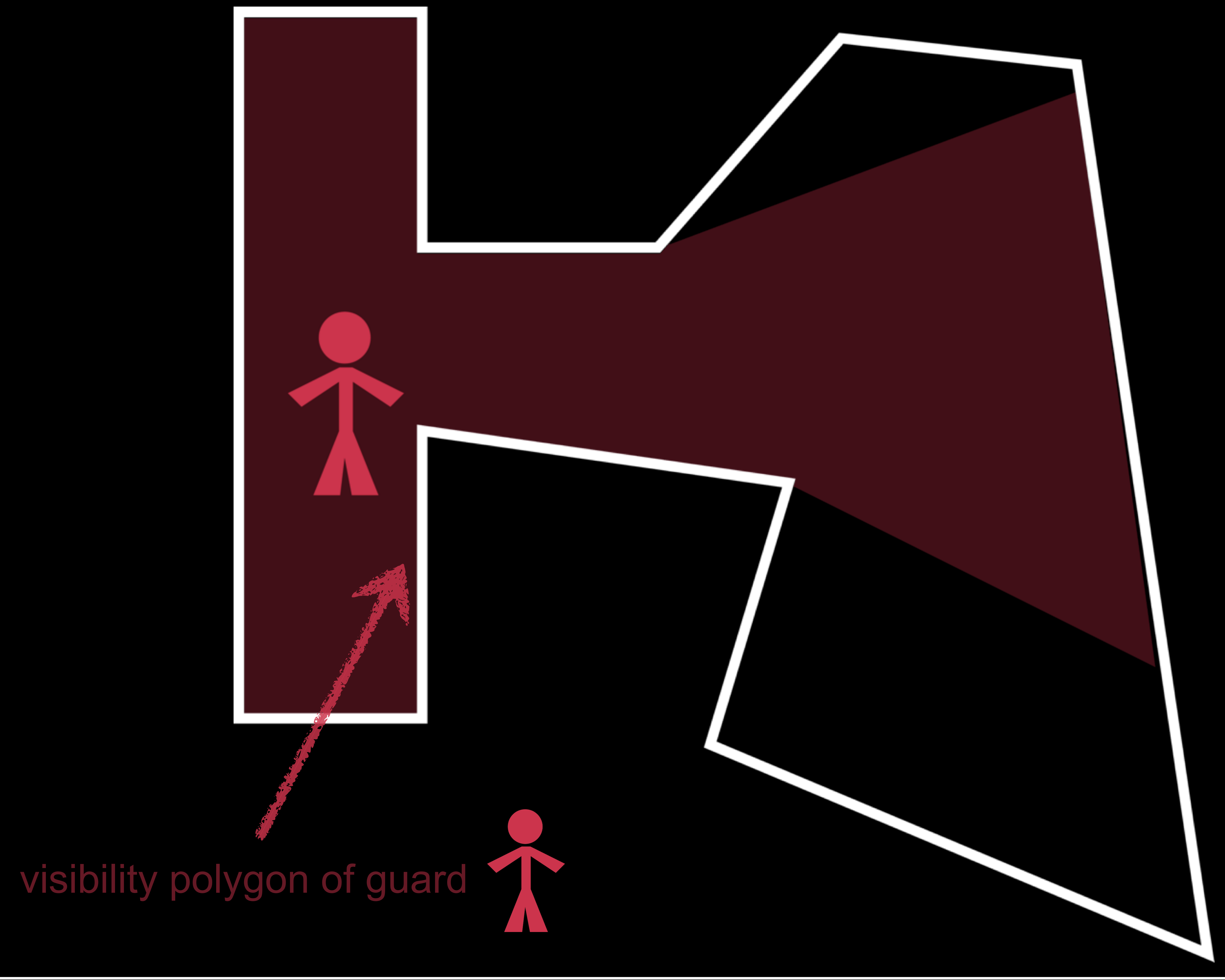

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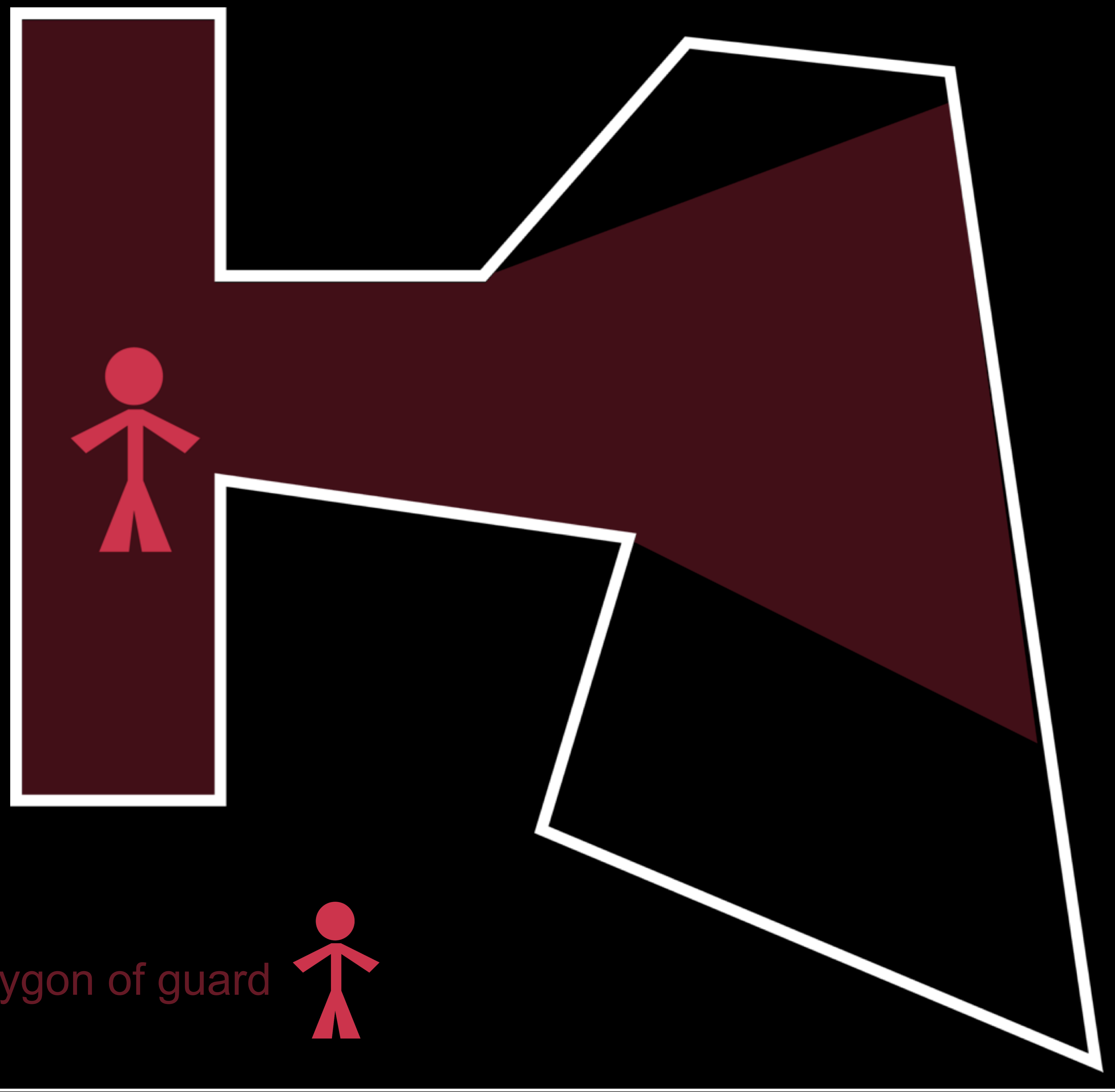

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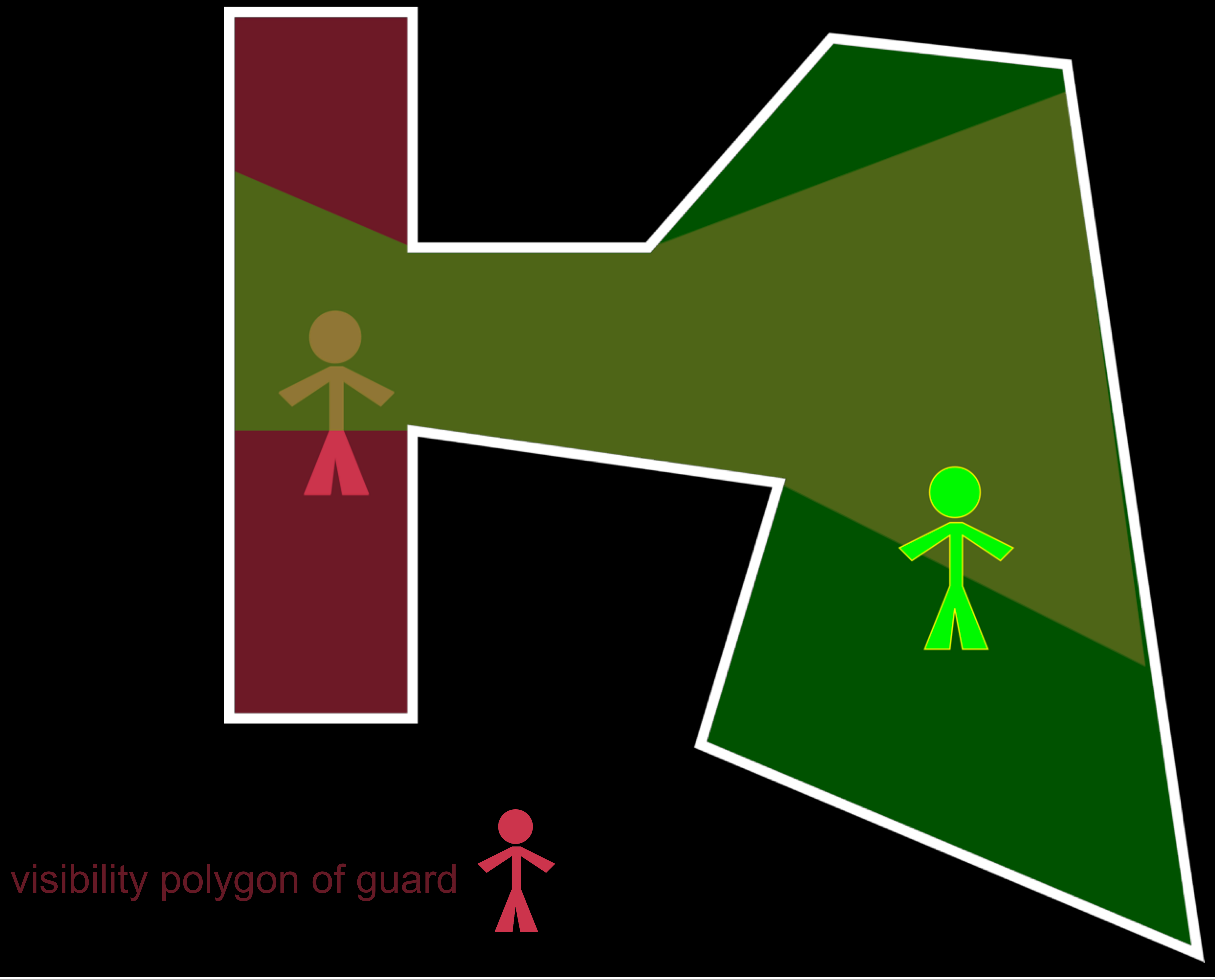
visibility polygon of guard

The Art Gallery Problem (AGP)



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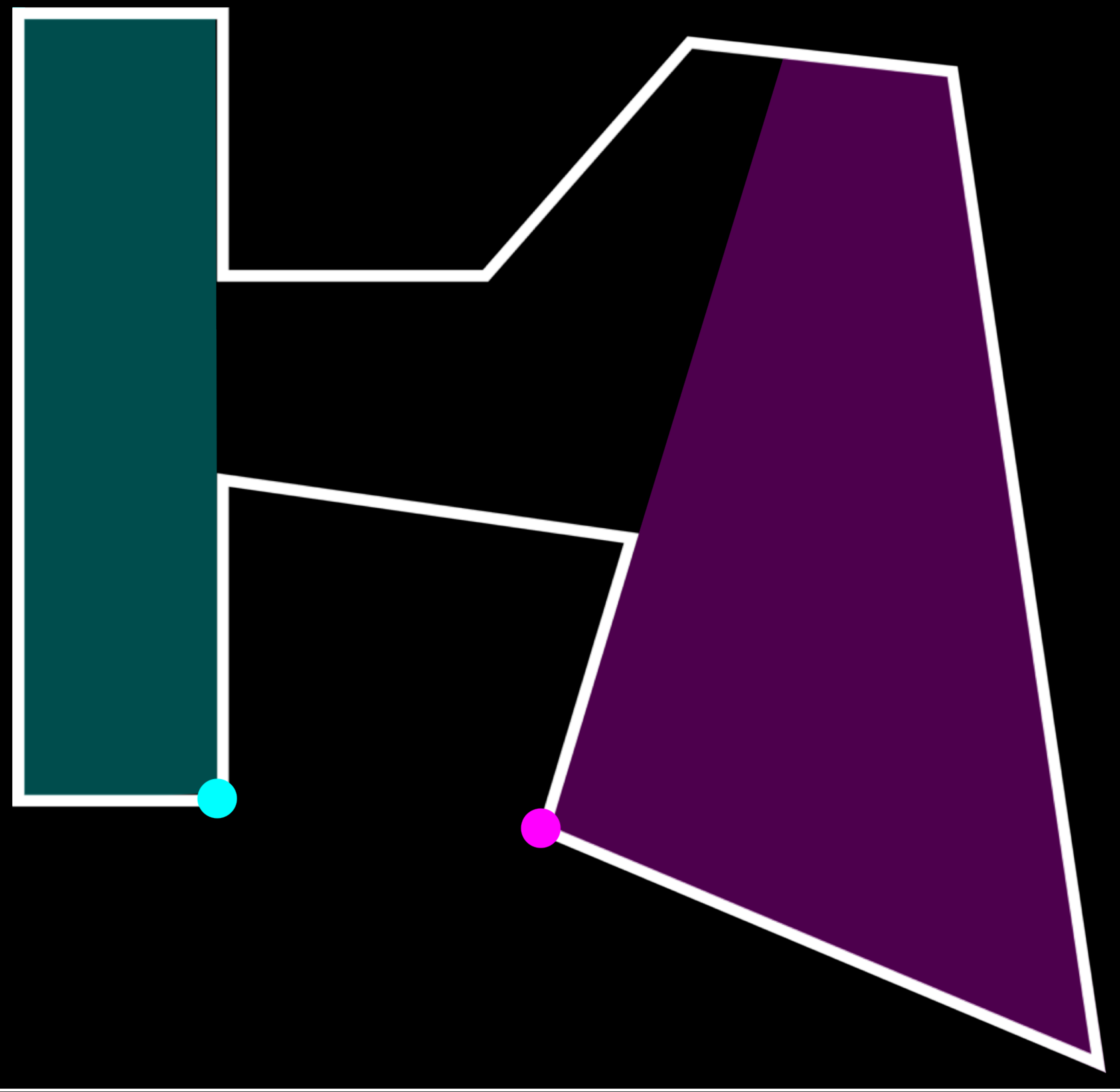

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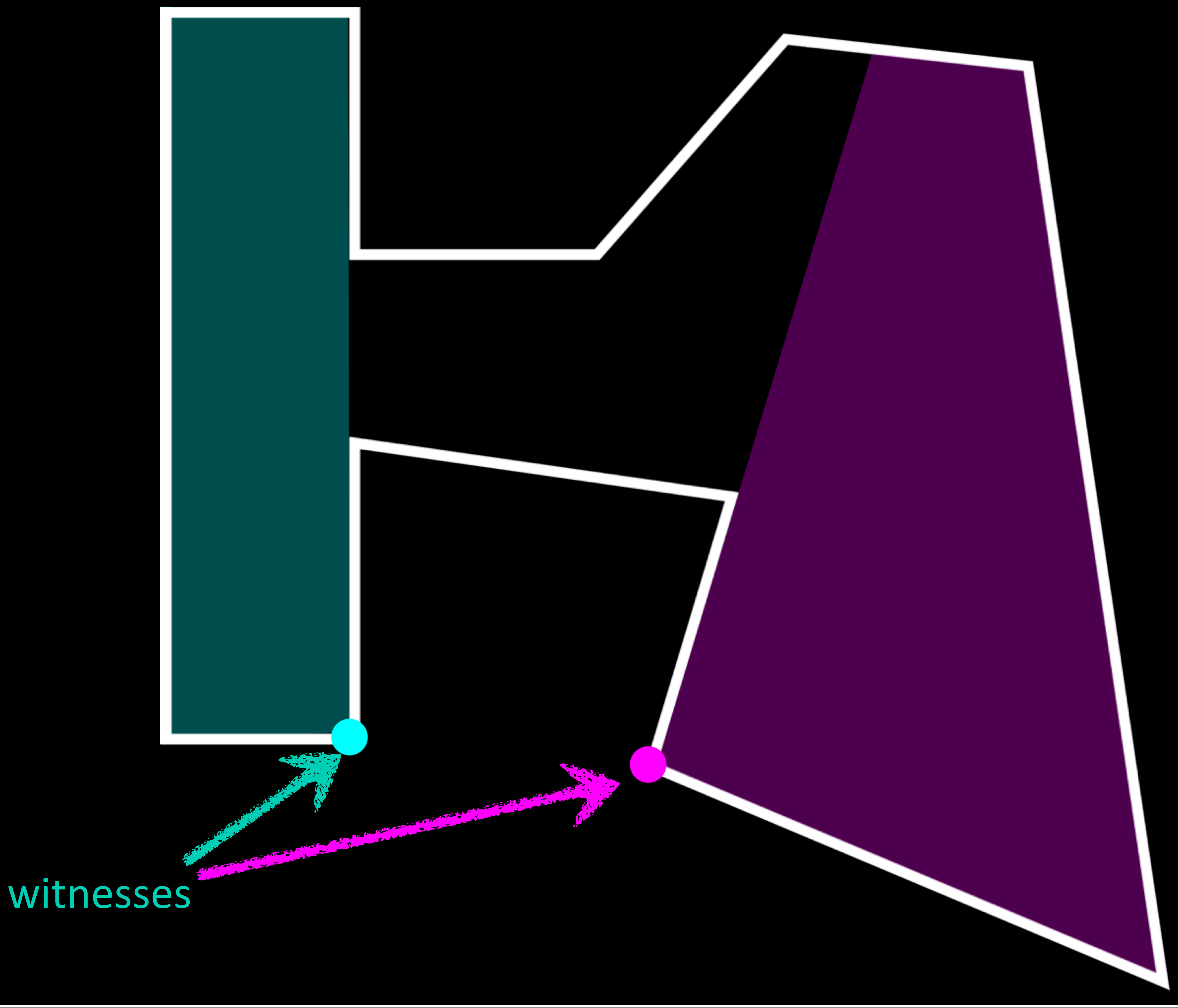
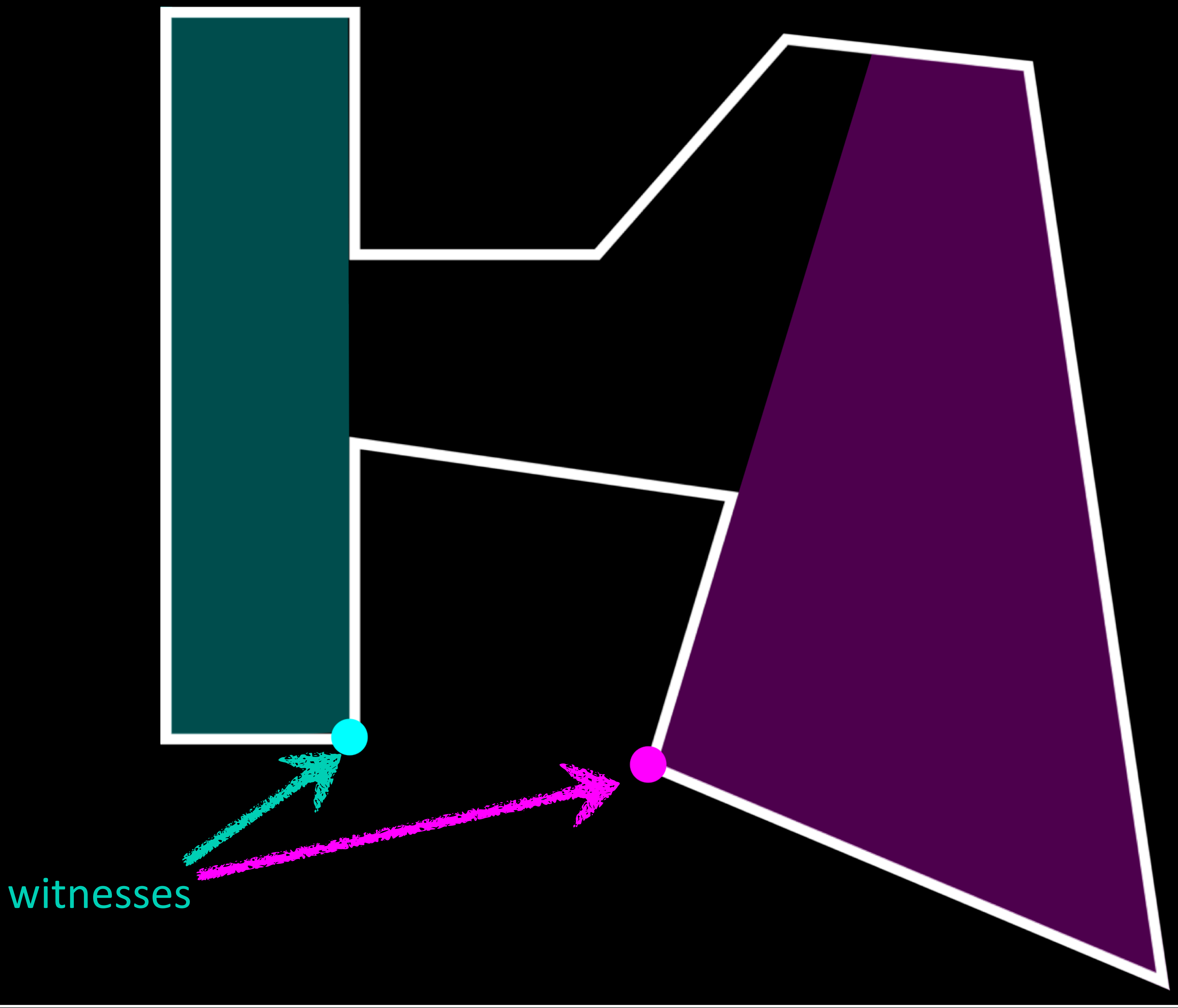


The Art Gallery Problem (AGP)

➜ Lower bound of 2
However, generally, the ratio between minimum number of guards and maximum number of witnesses can be arbitrarily bad:



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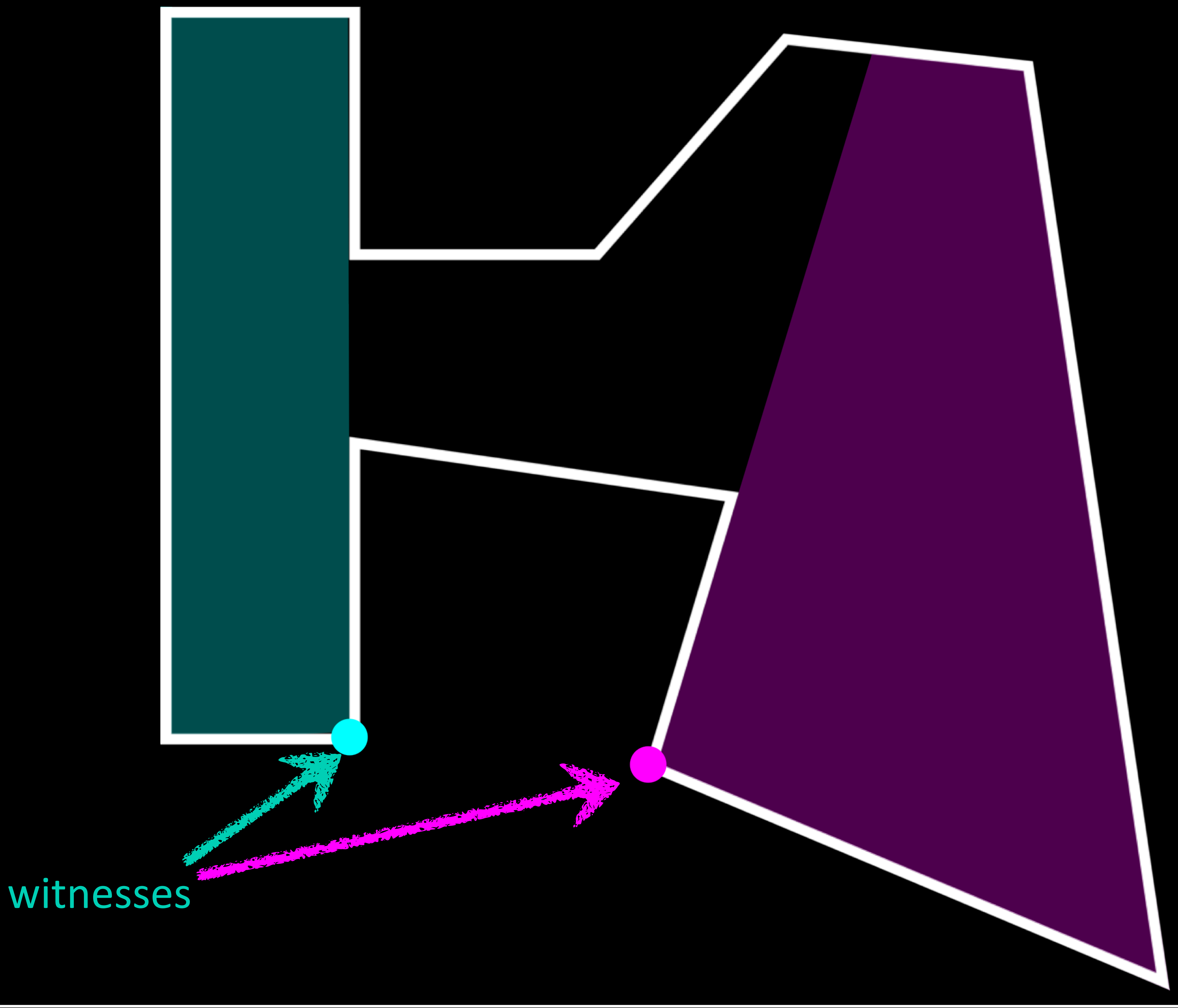

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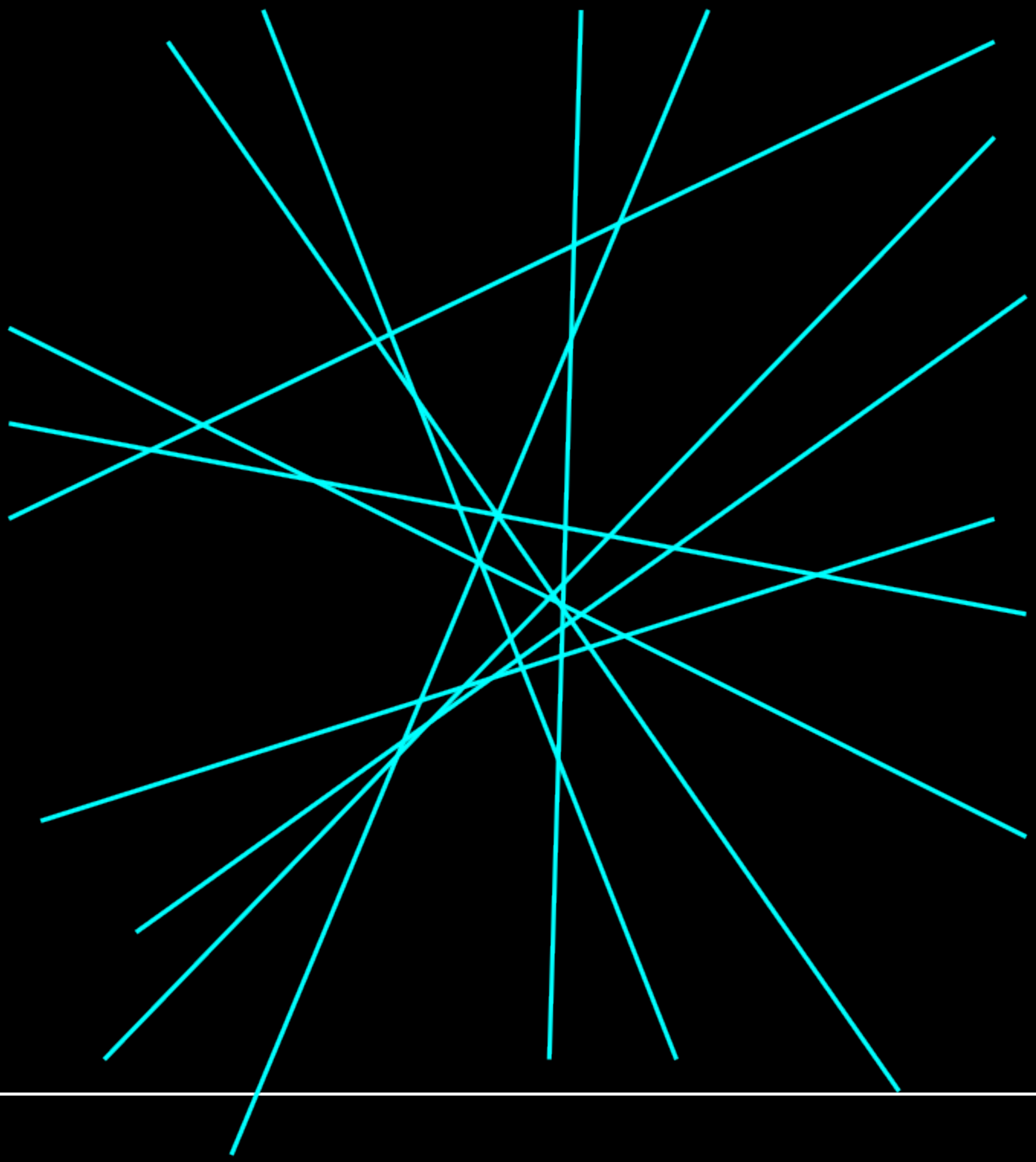

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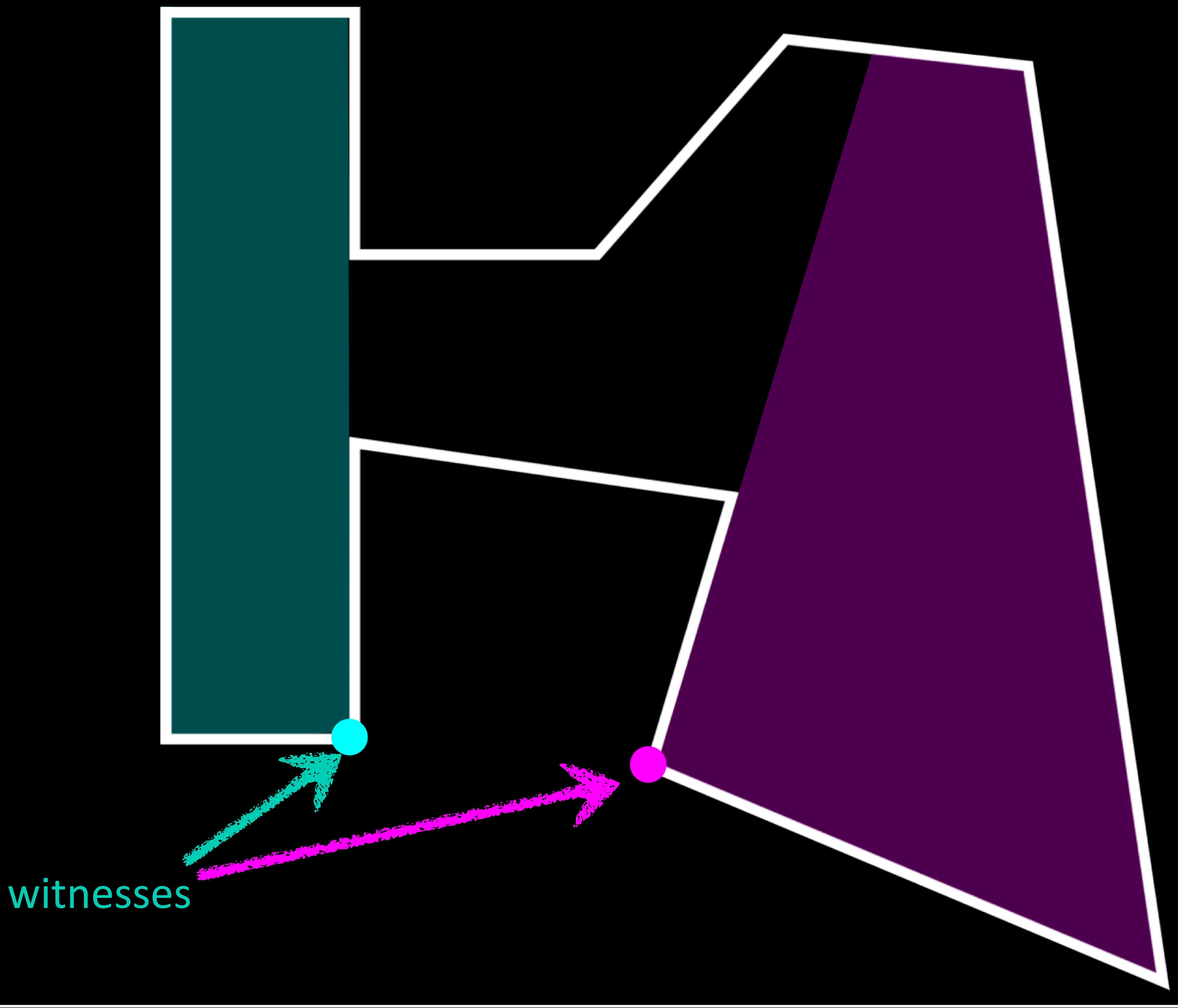

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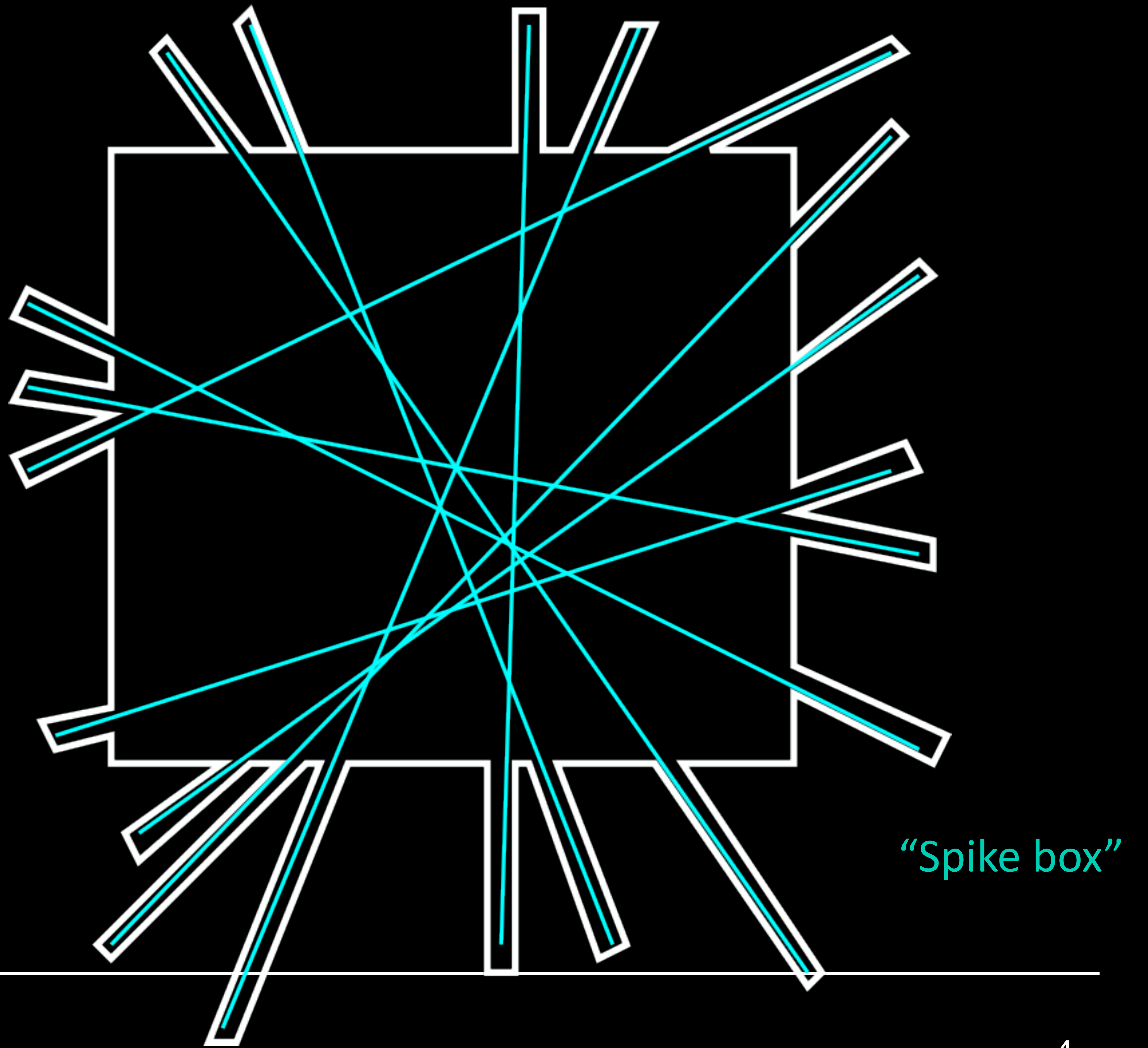

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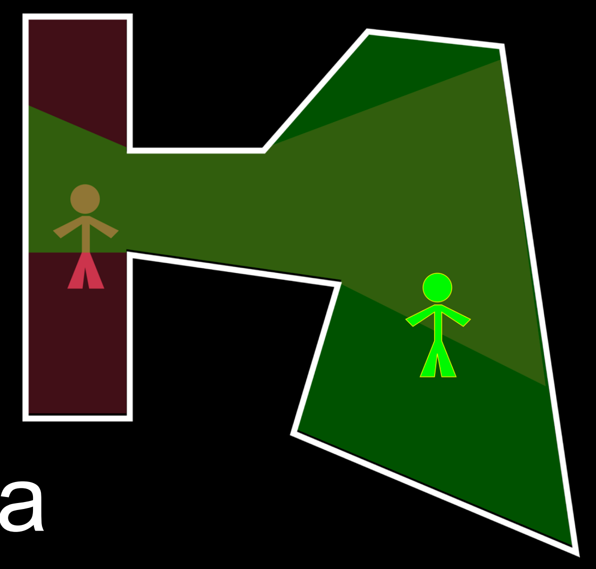
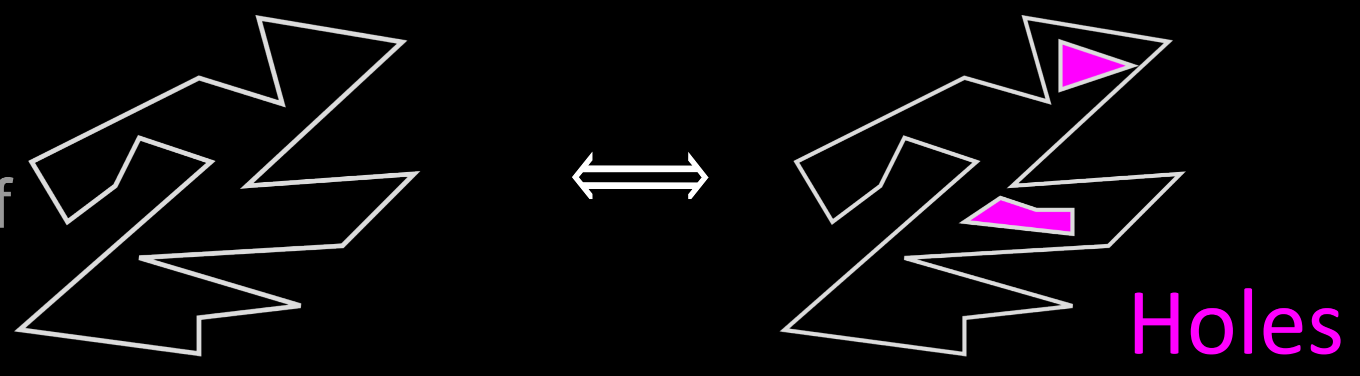


So-called “Art Gallery Theorems”: x guards are always sufficient and sometimes necessary to guard a polygon with n vertices (polygon from a specific class)

Simple polygon:

- Does not intersect itself
- No holes



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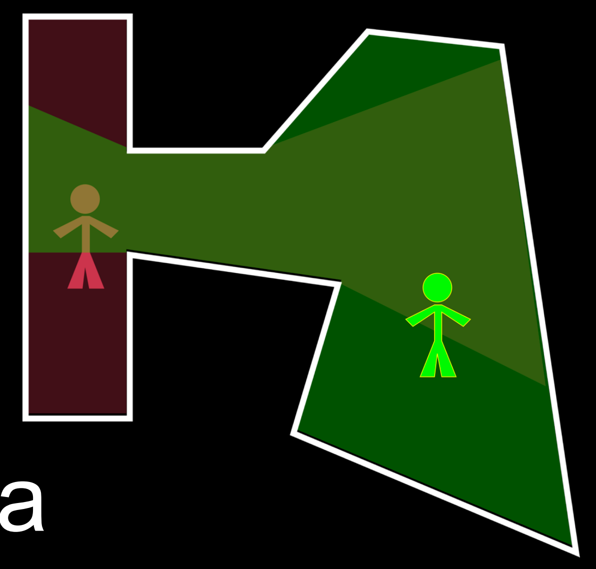

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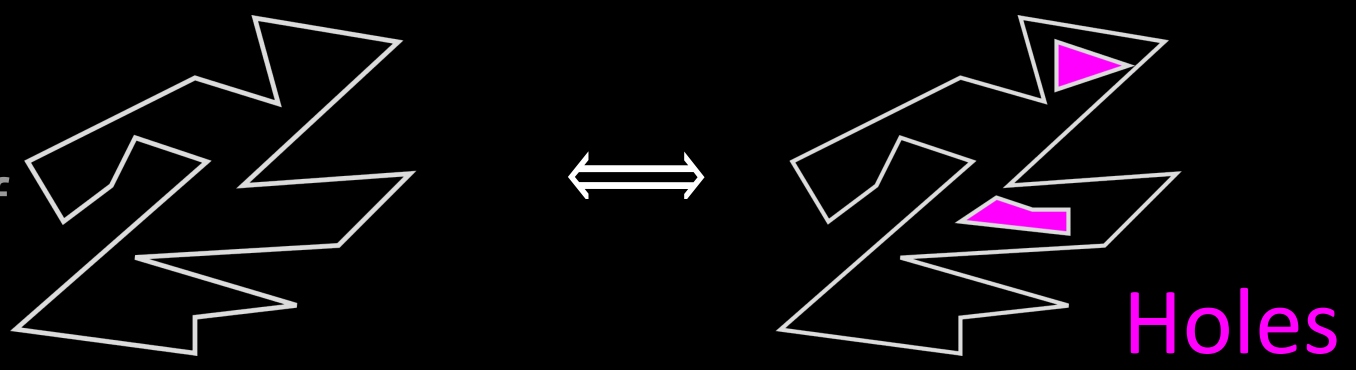

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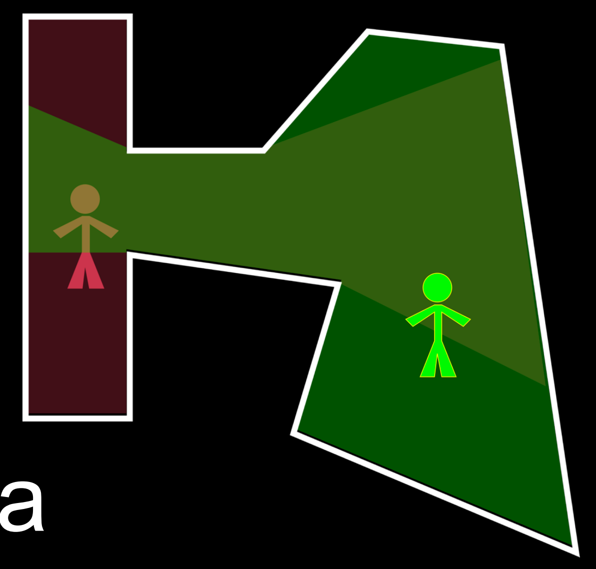

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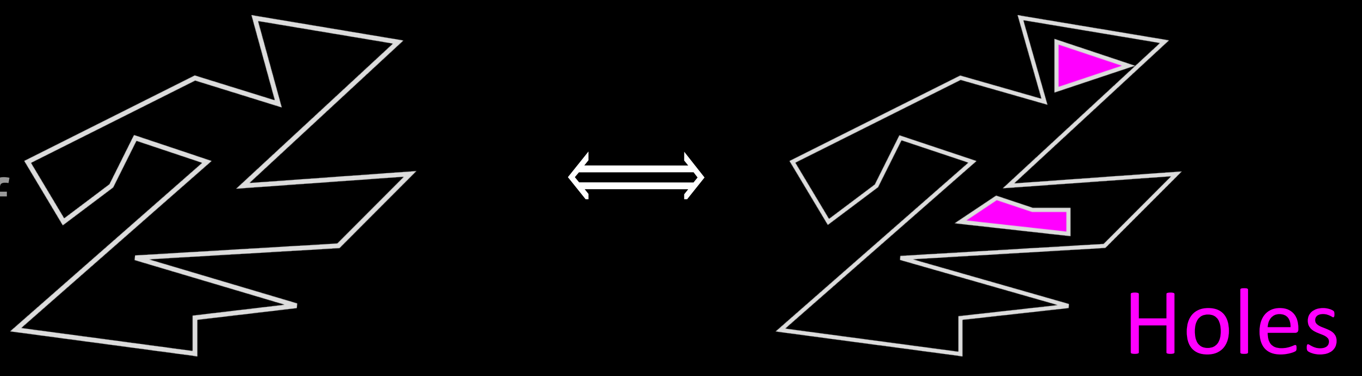

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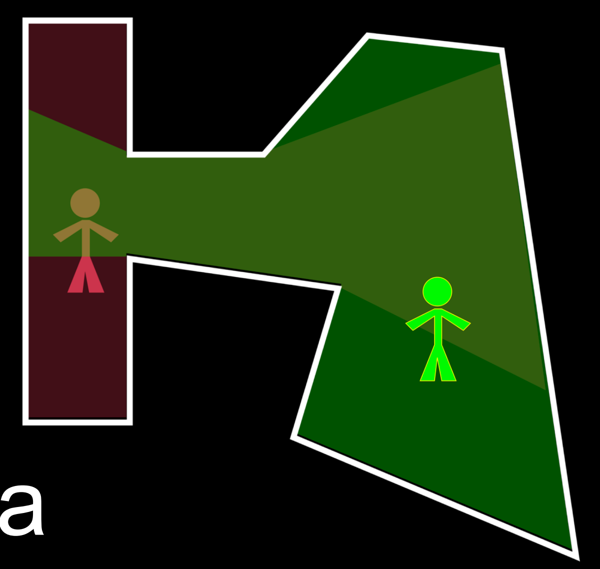

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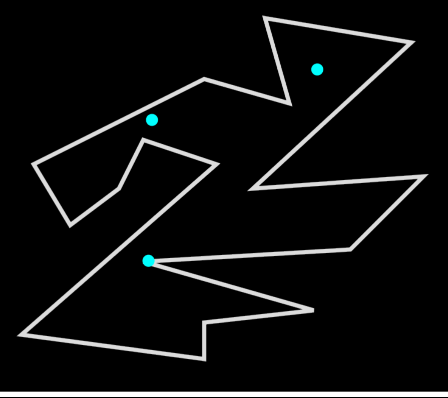
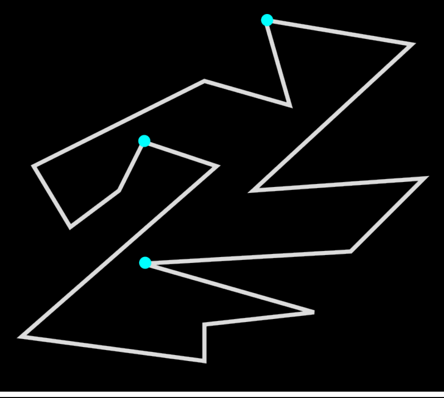
point





vertex

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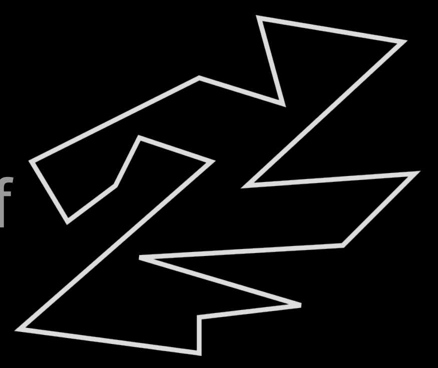
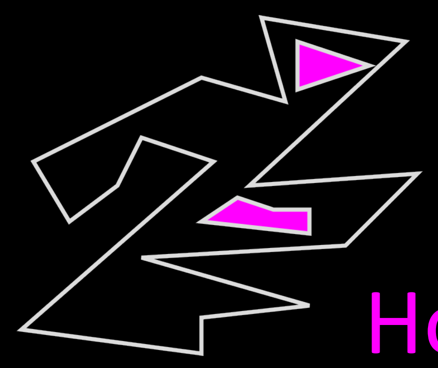


$\Longleftrightarrow$



Holes

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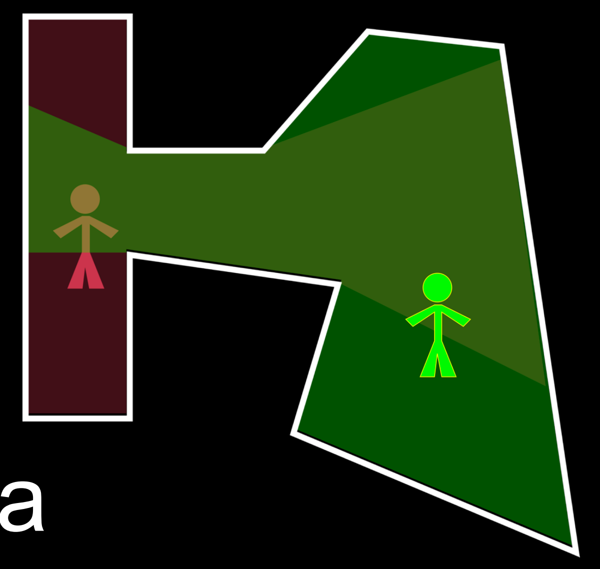
The Art Gallery Problem (AGP)



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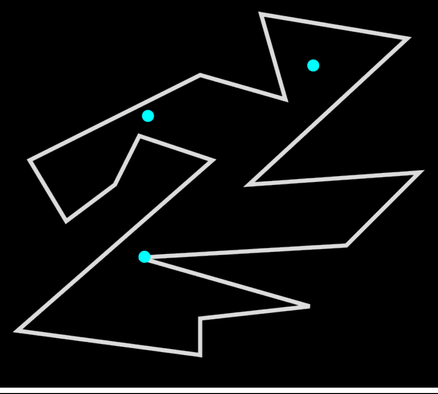
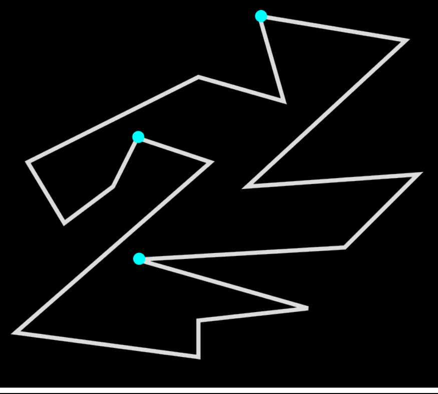
Algorithms

point





vertex

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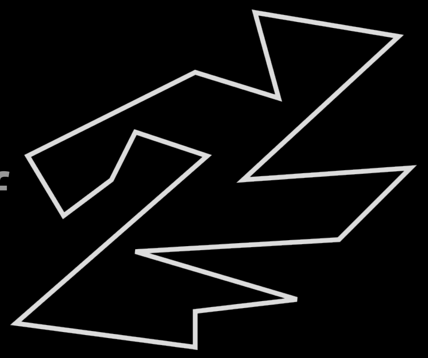

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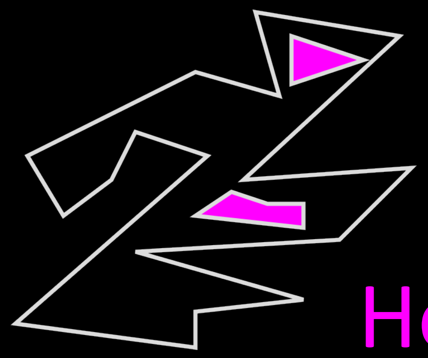

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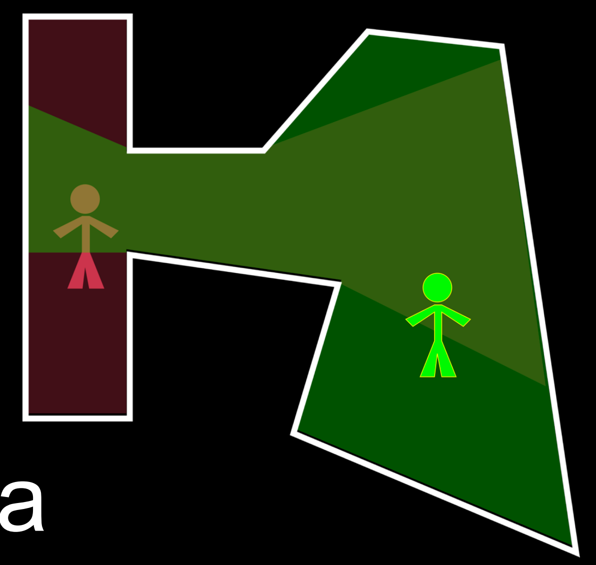

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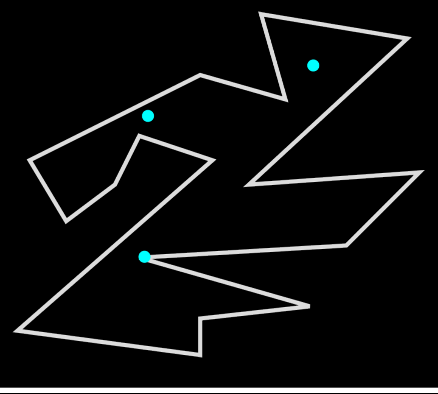
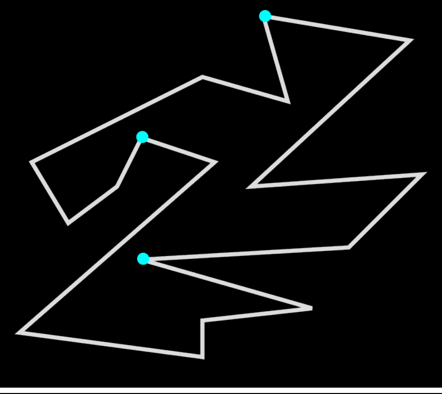
point   vertex

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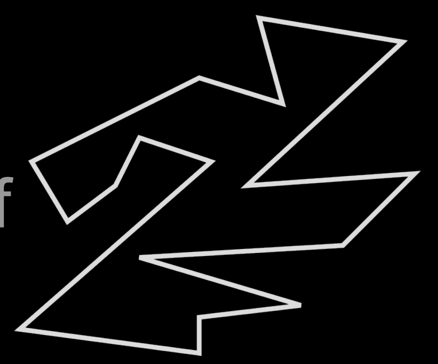
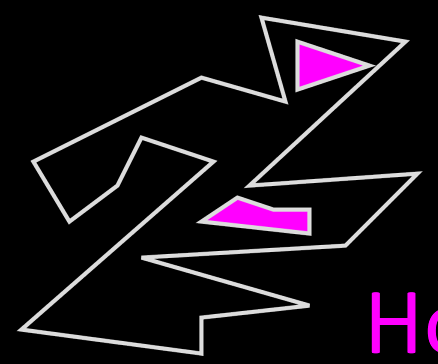
 $\Longleftrightarrow$  Holes

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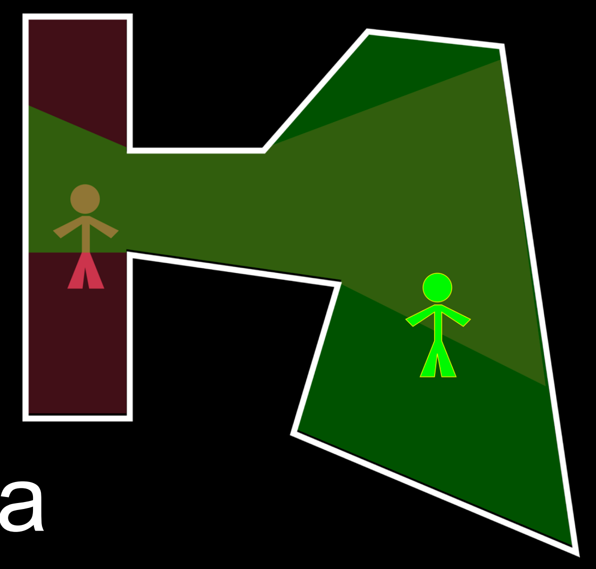

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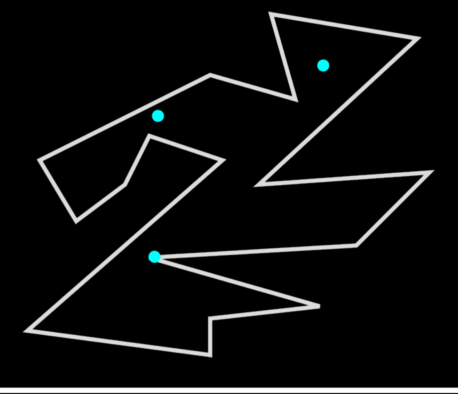
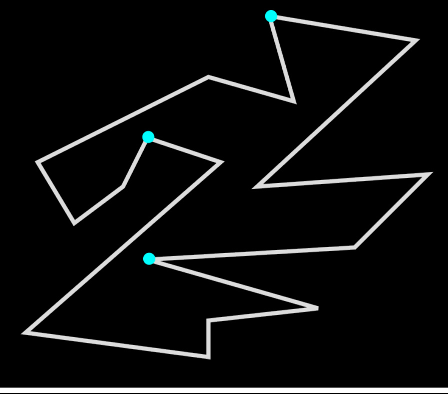
Other structural results





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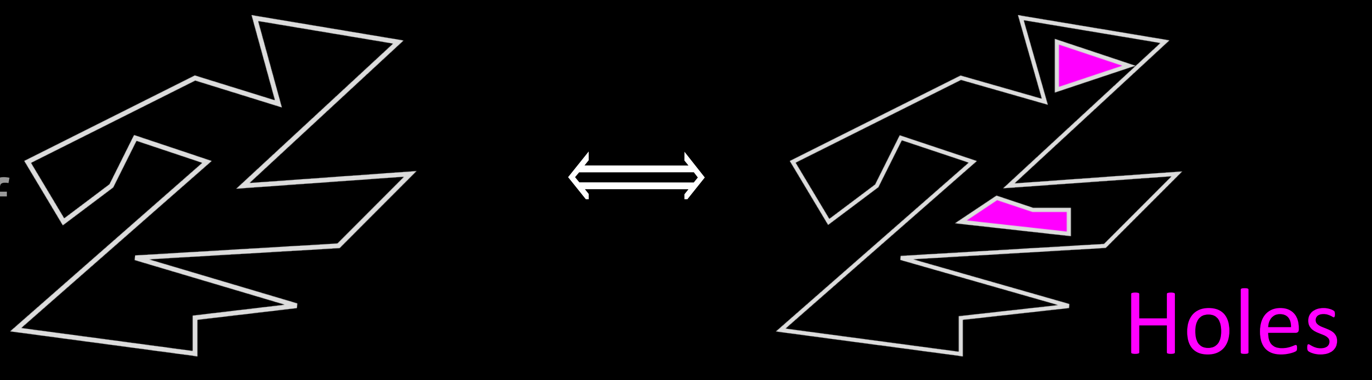
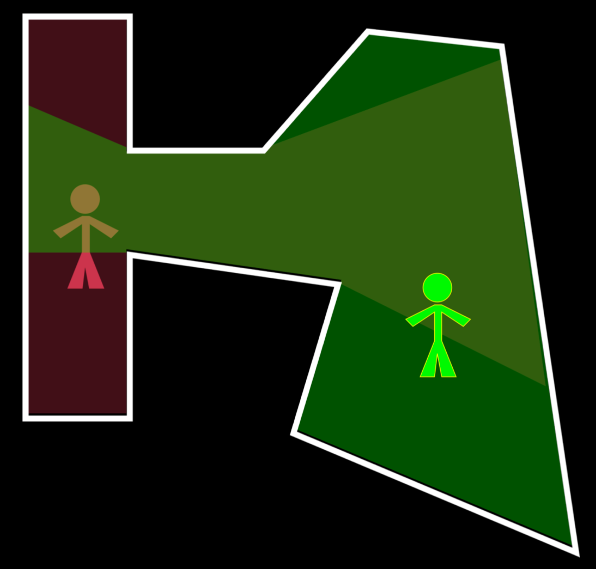


The Art Gallery Problem (AGP) and Its Variants



We can alter:

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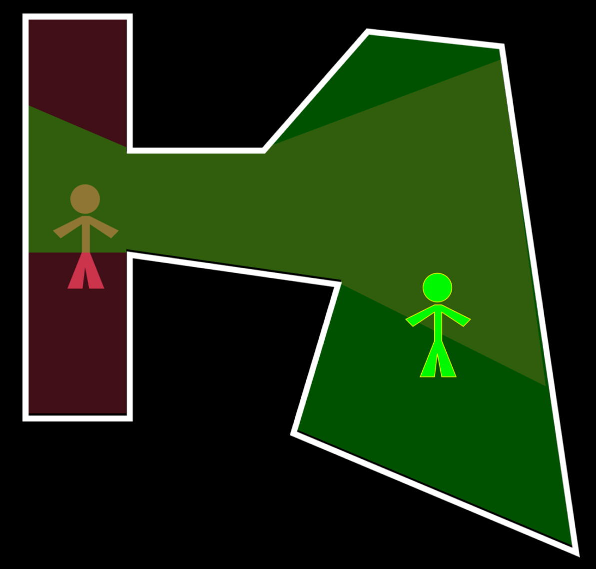

We can alter:

- Capabilities of the guards
- Environment to be guarded

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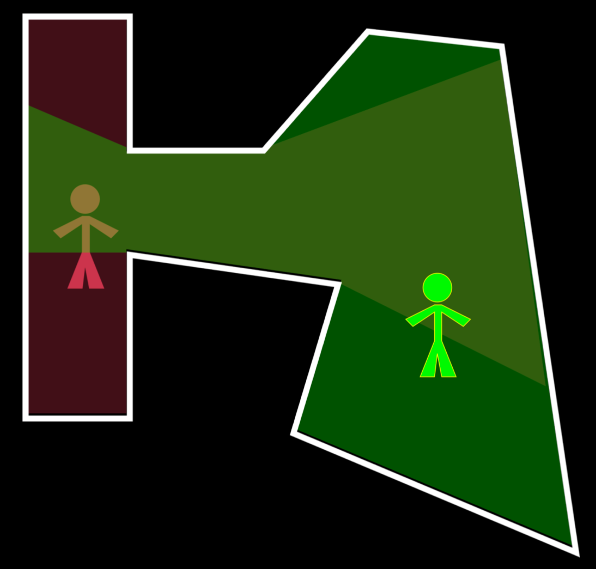

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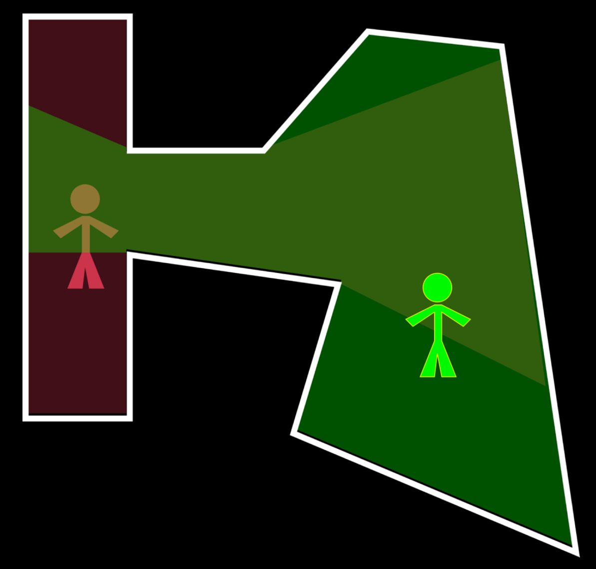

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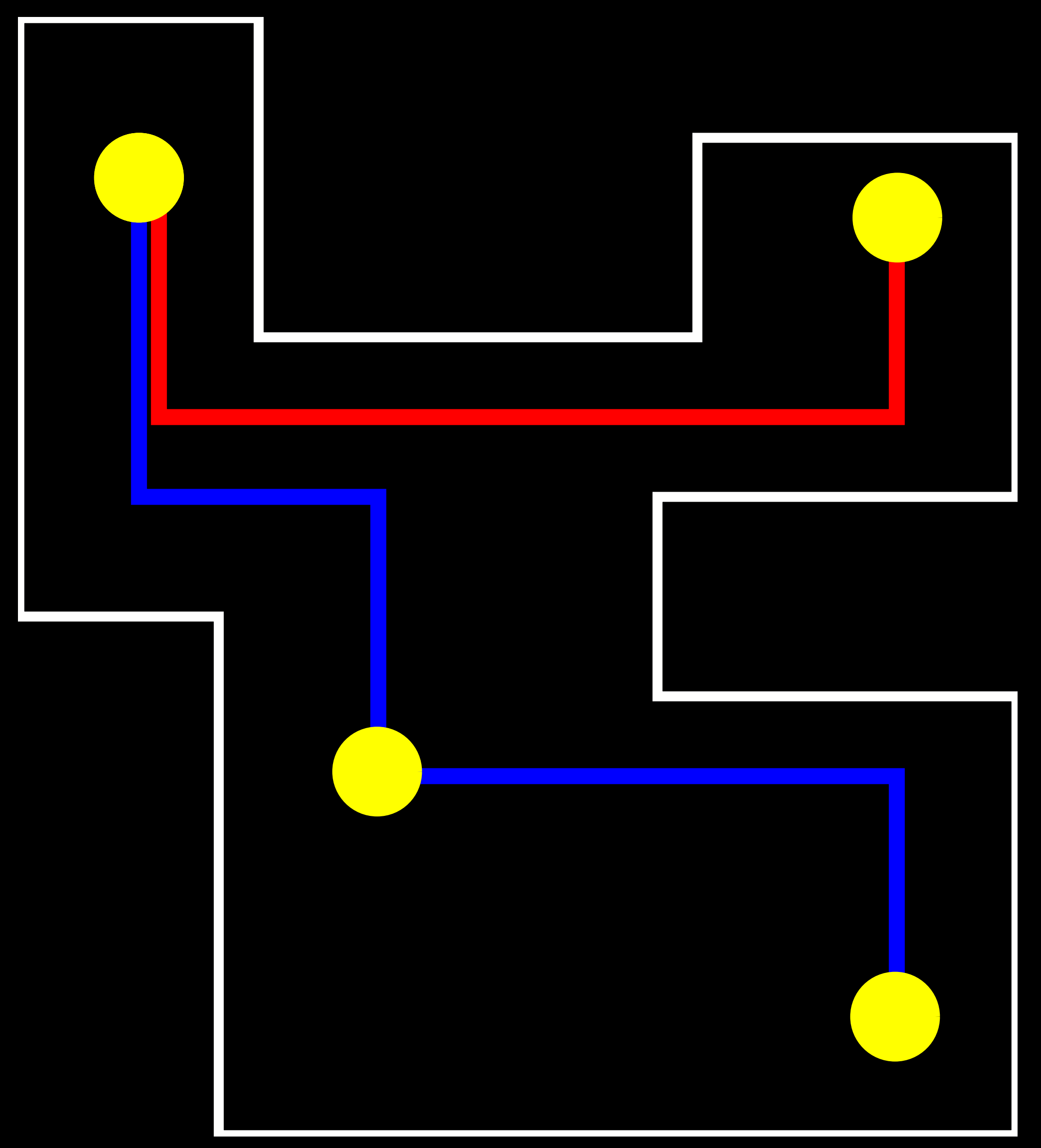
Staircase visibility/ s-visiblity:



Two points are s-visible to each other
if there exists a staircase path in P
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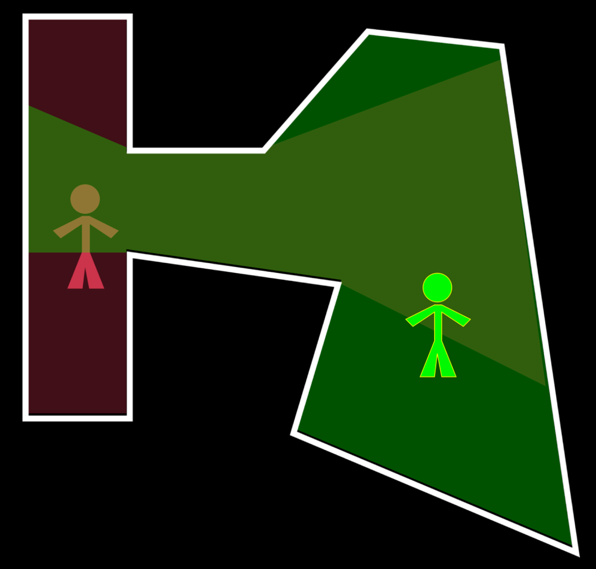
- Environment to be guarded

The Art Gallery Problem (AGP) and Its Variants



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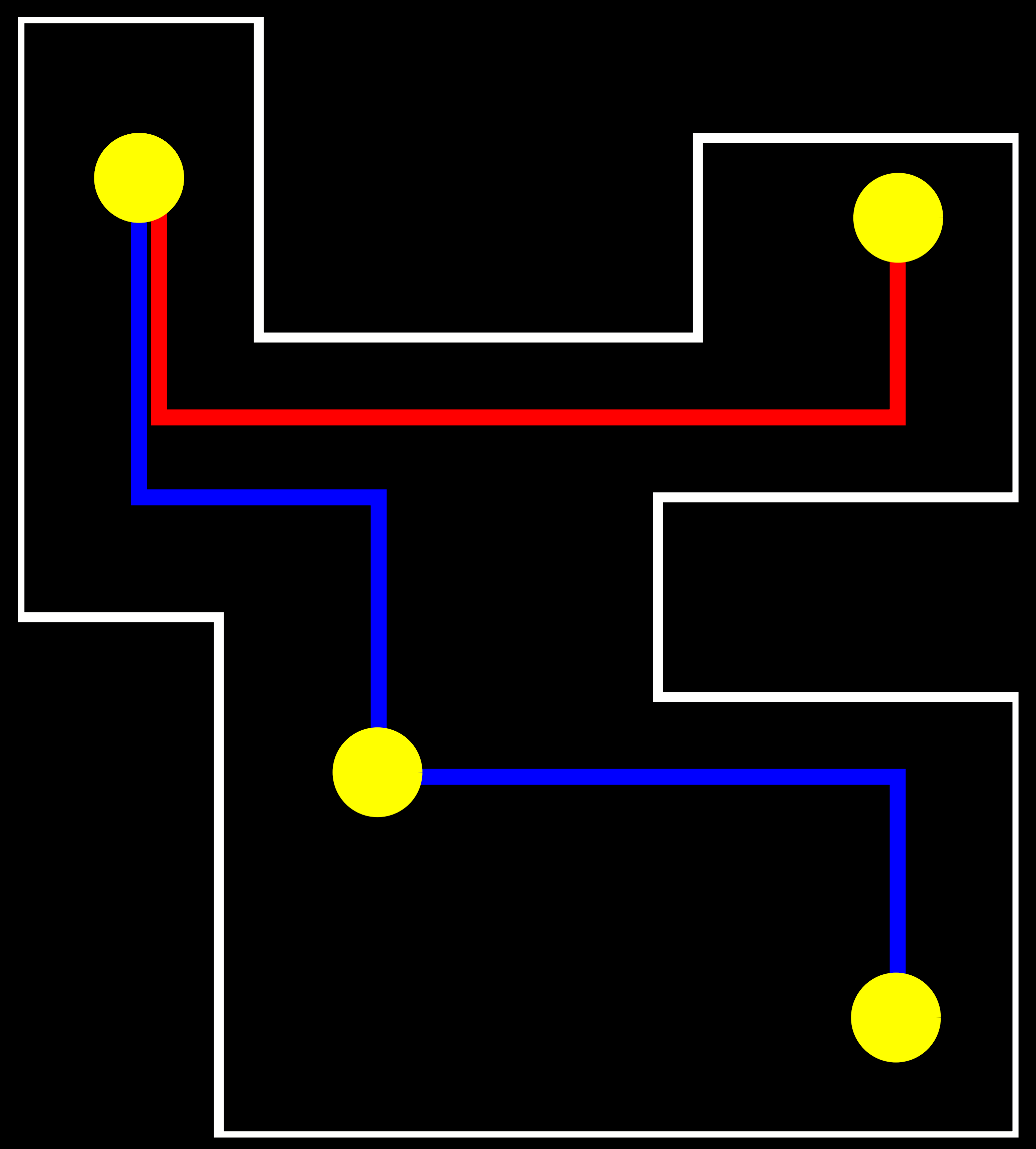

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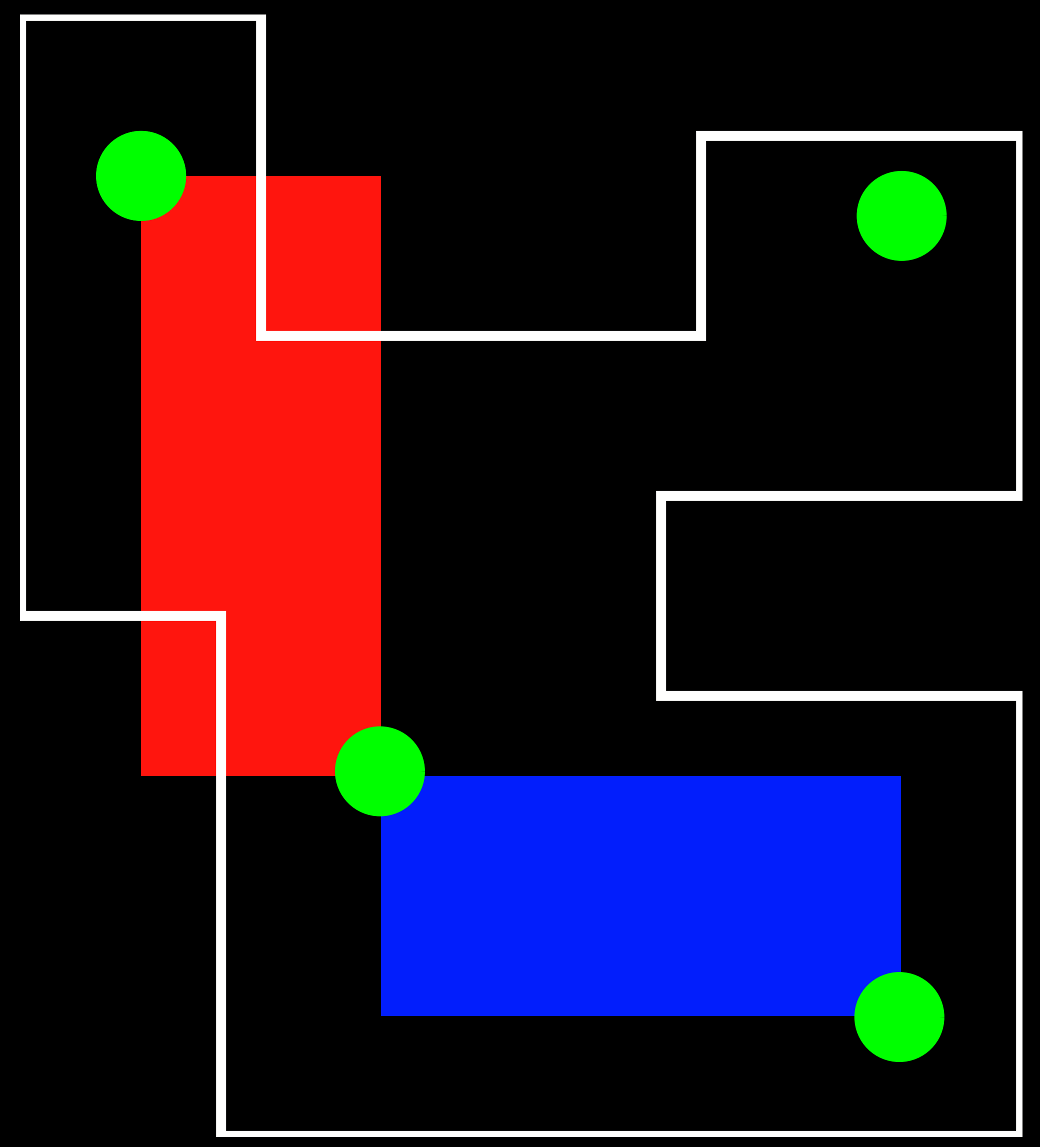

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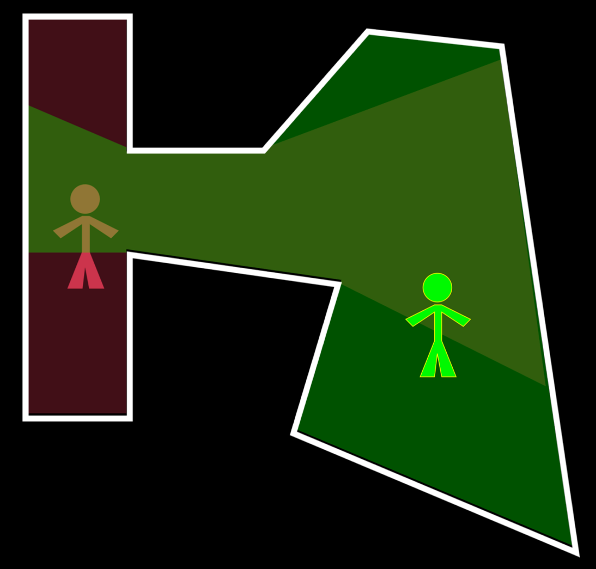
The Art Gallery Problem (AGP) and Its Variants



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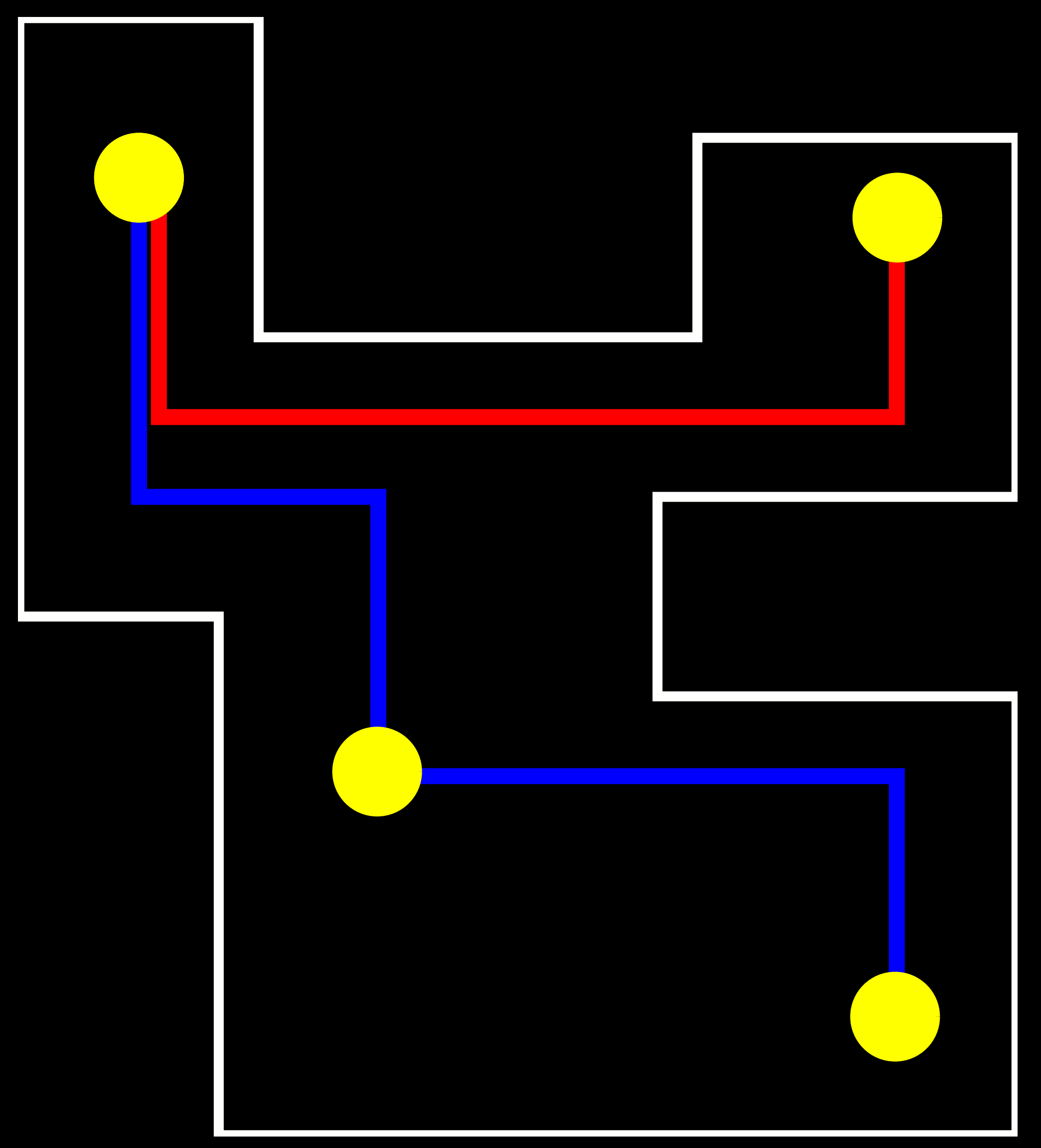

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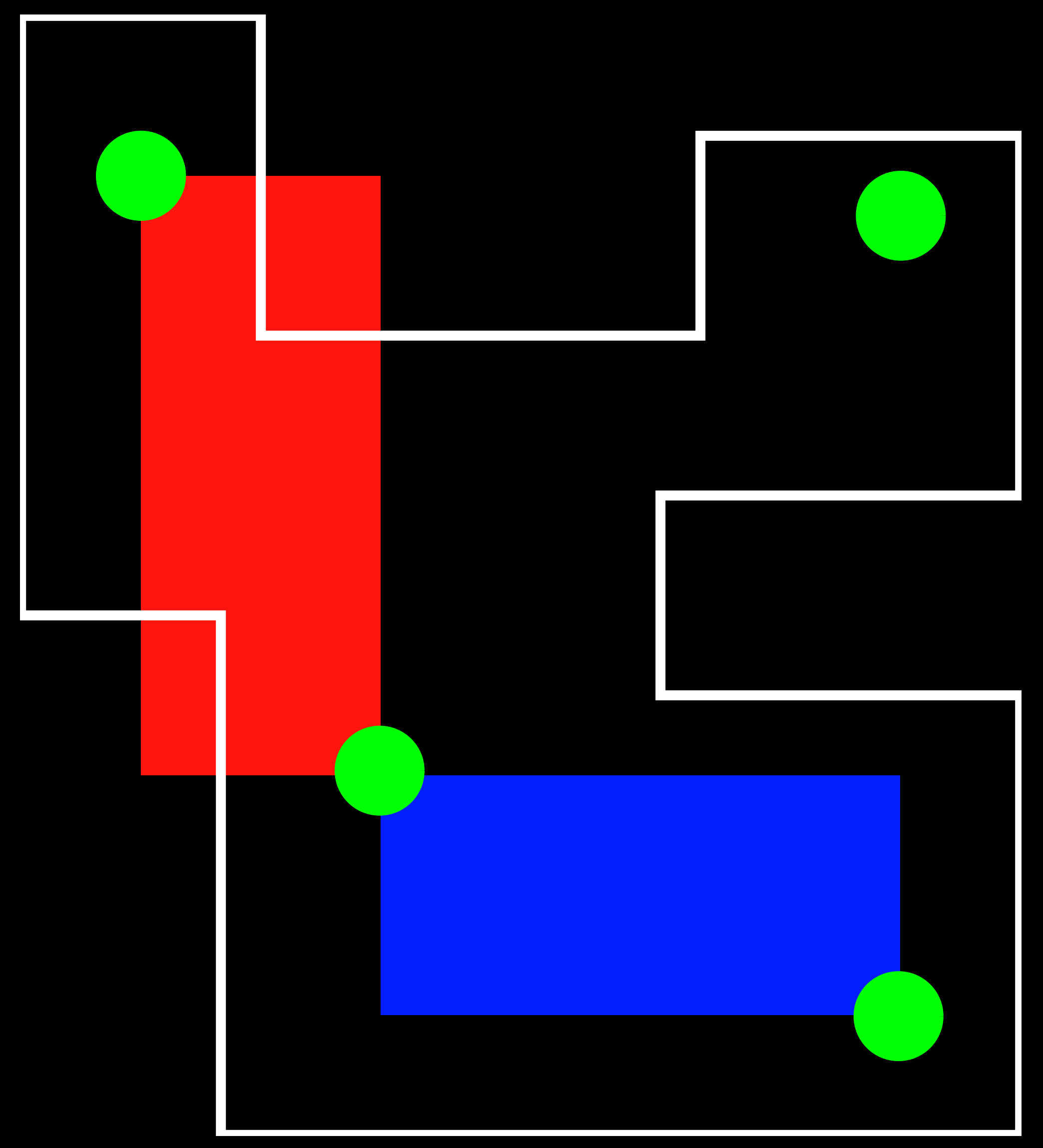

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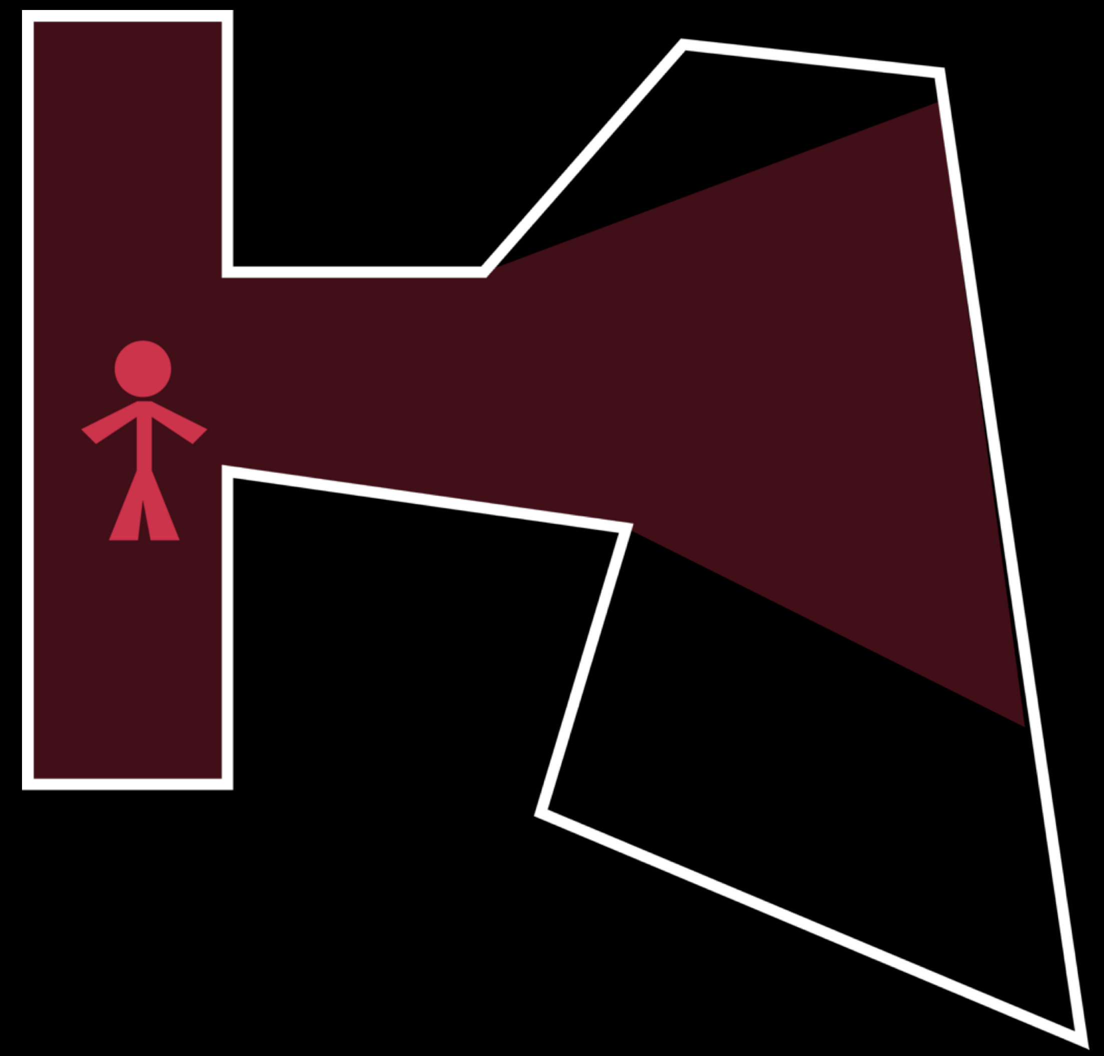
k-transmitter:



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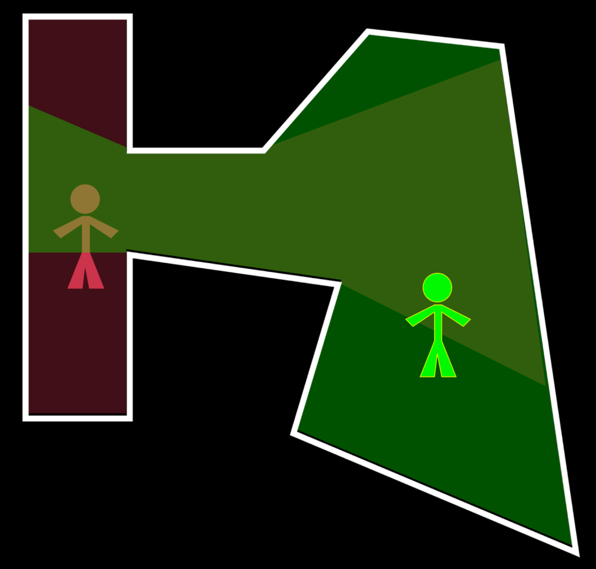

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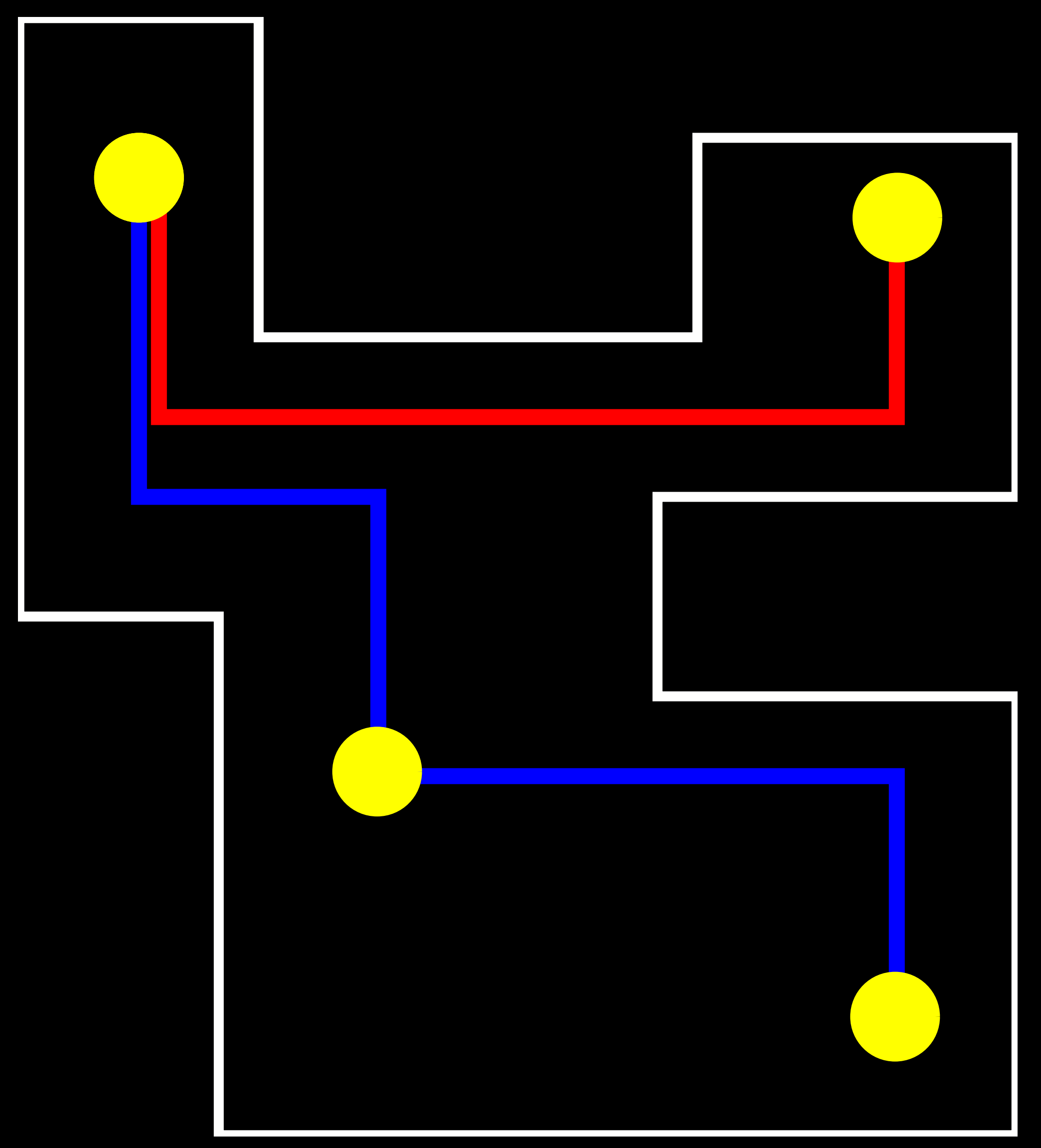

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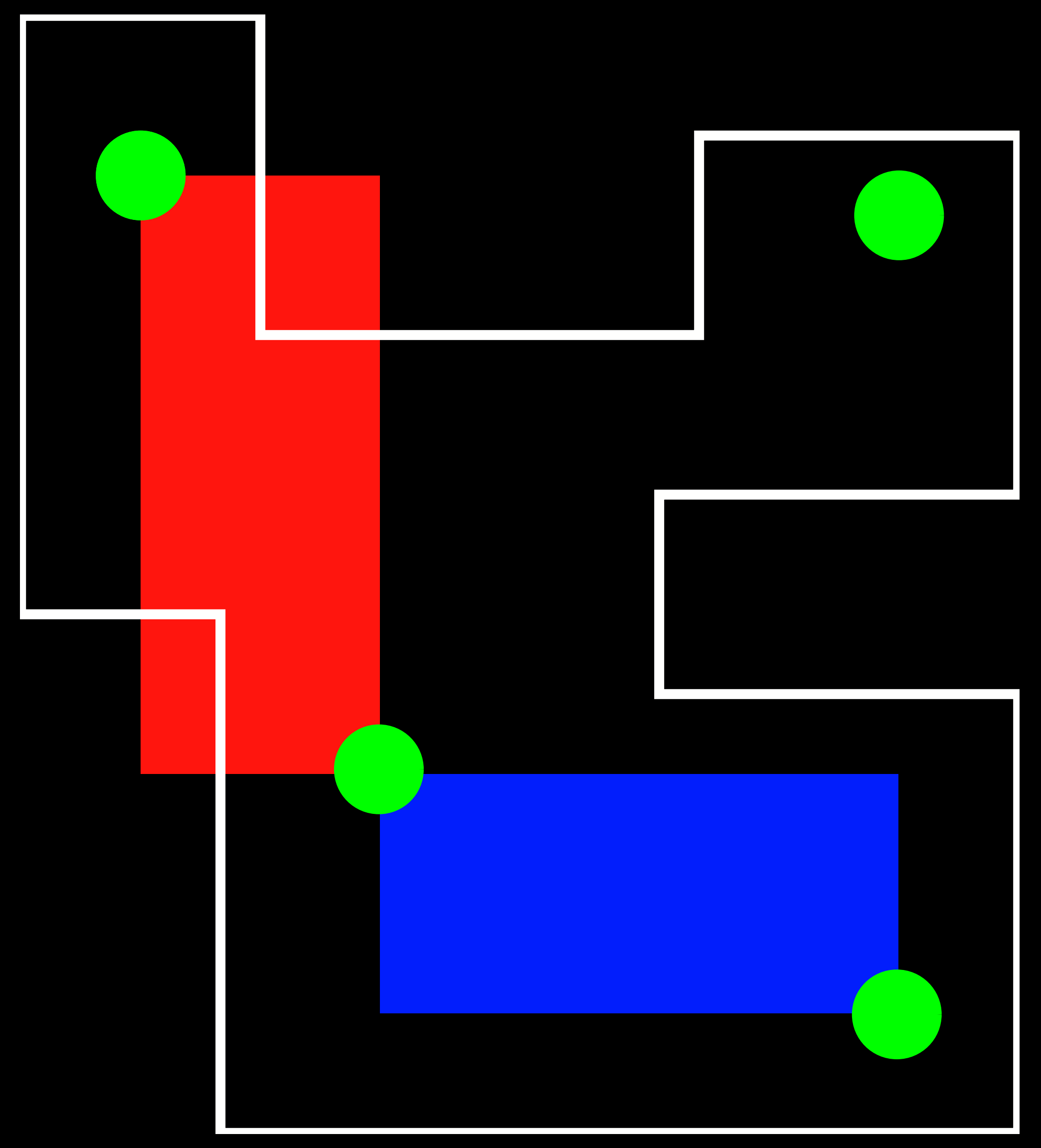

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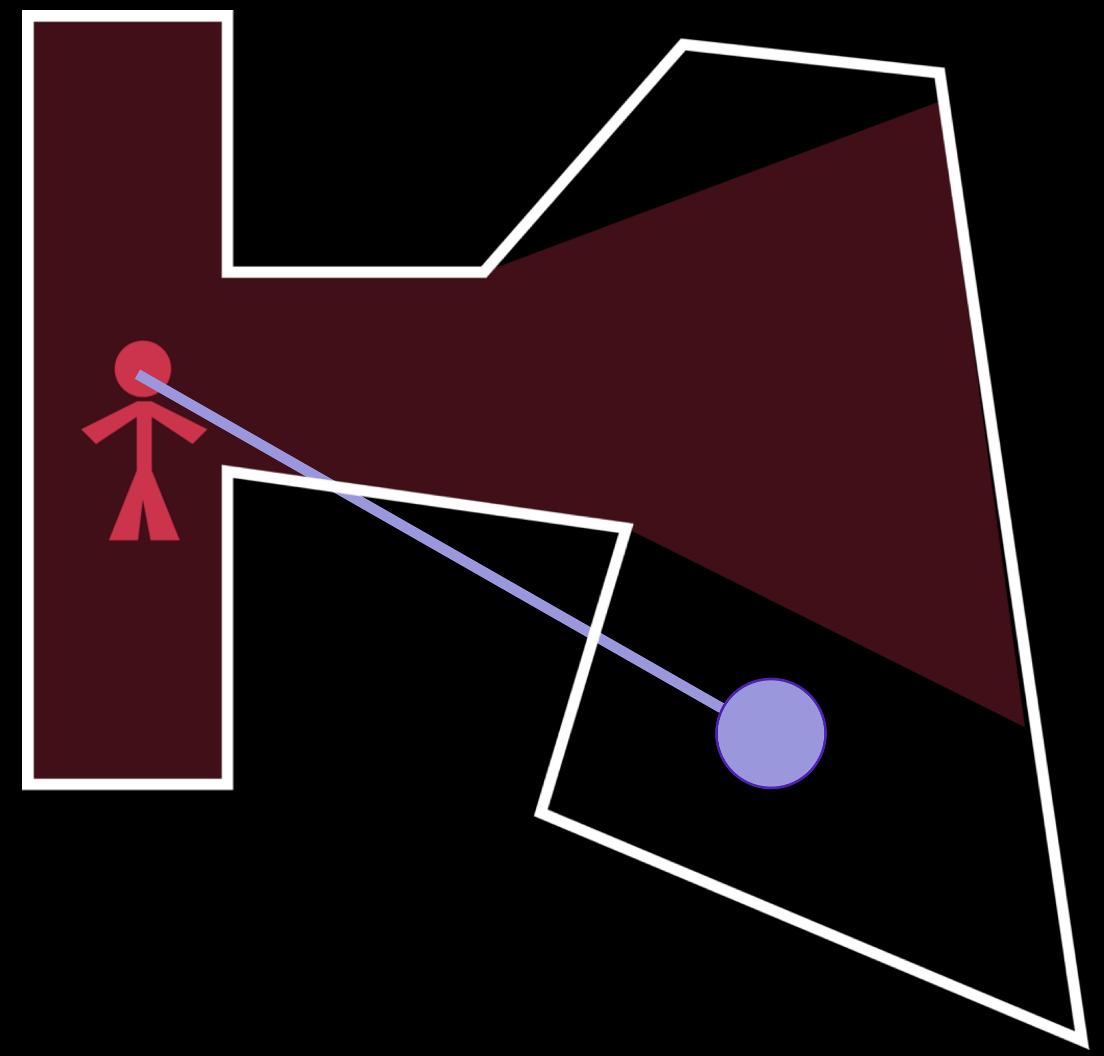

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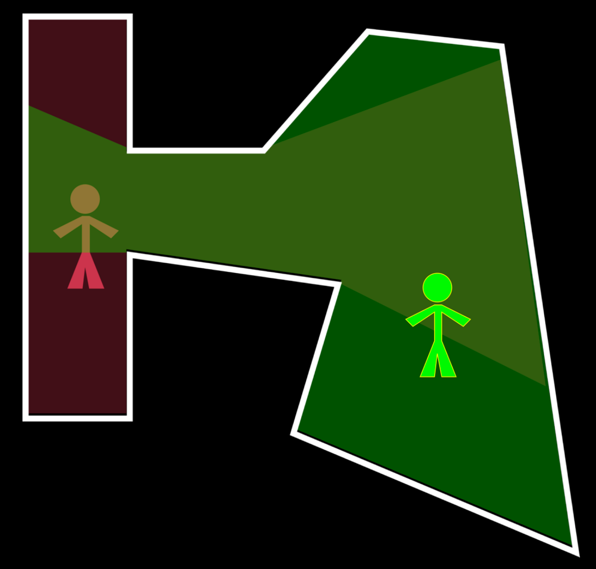

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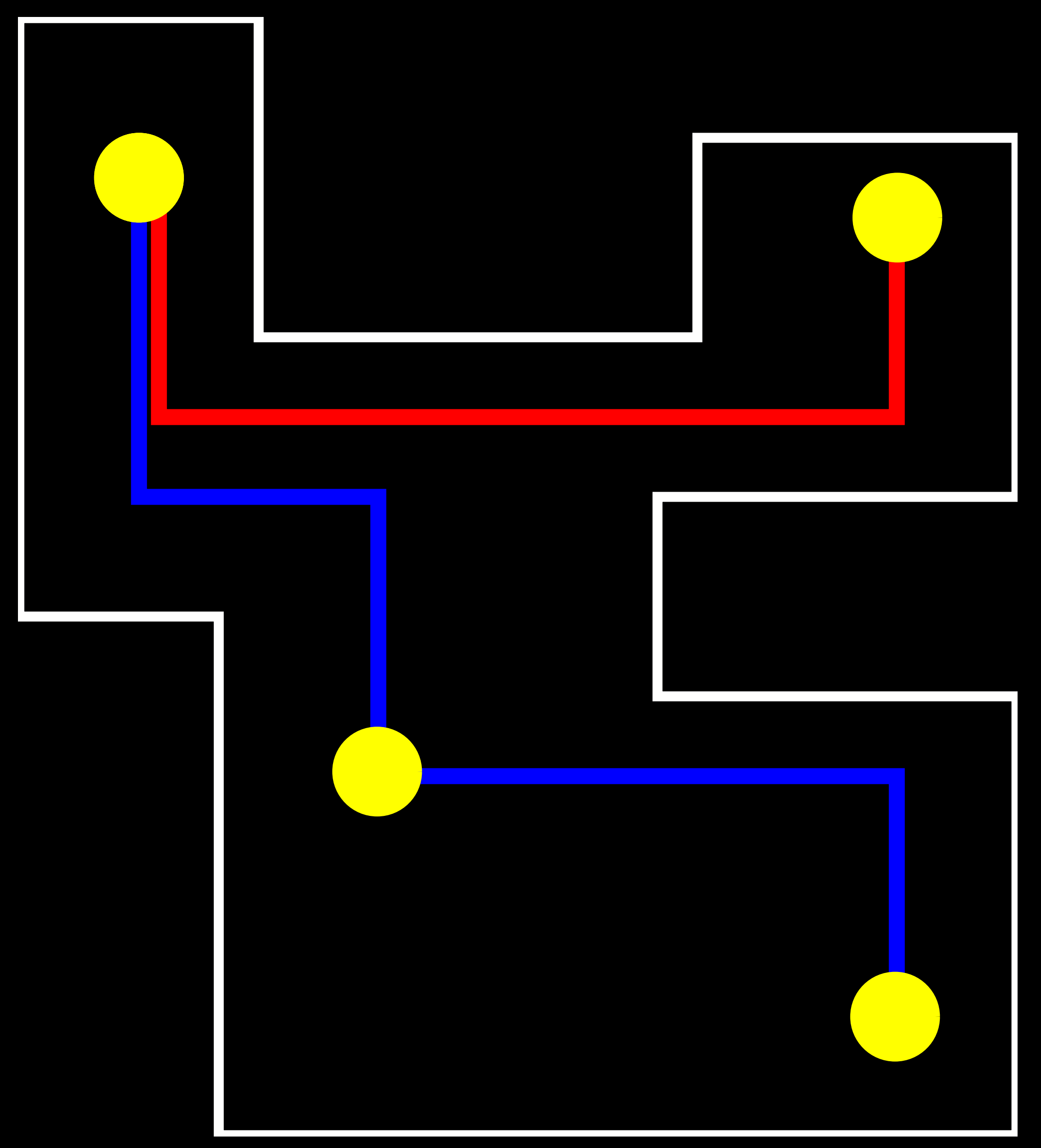

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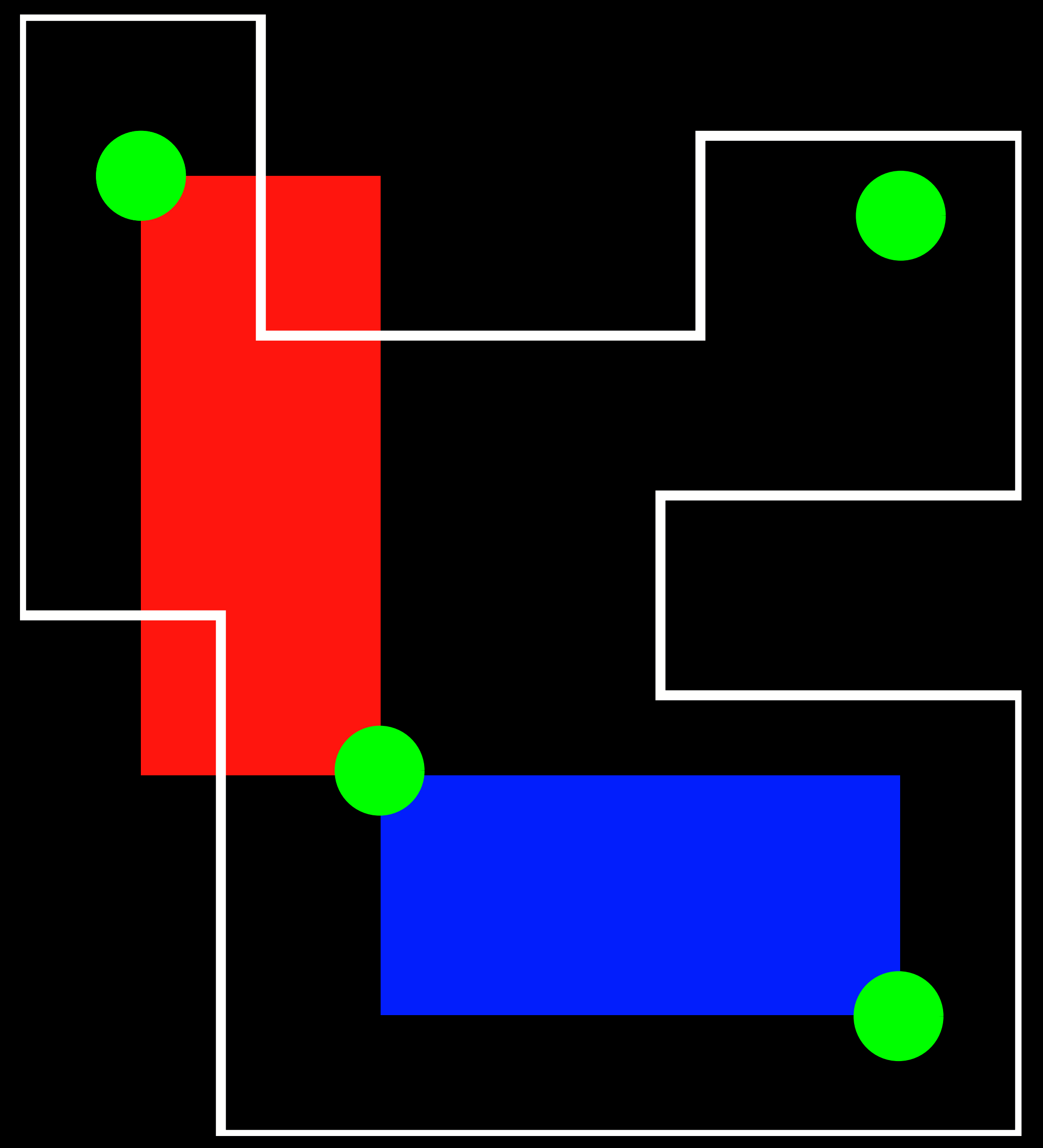

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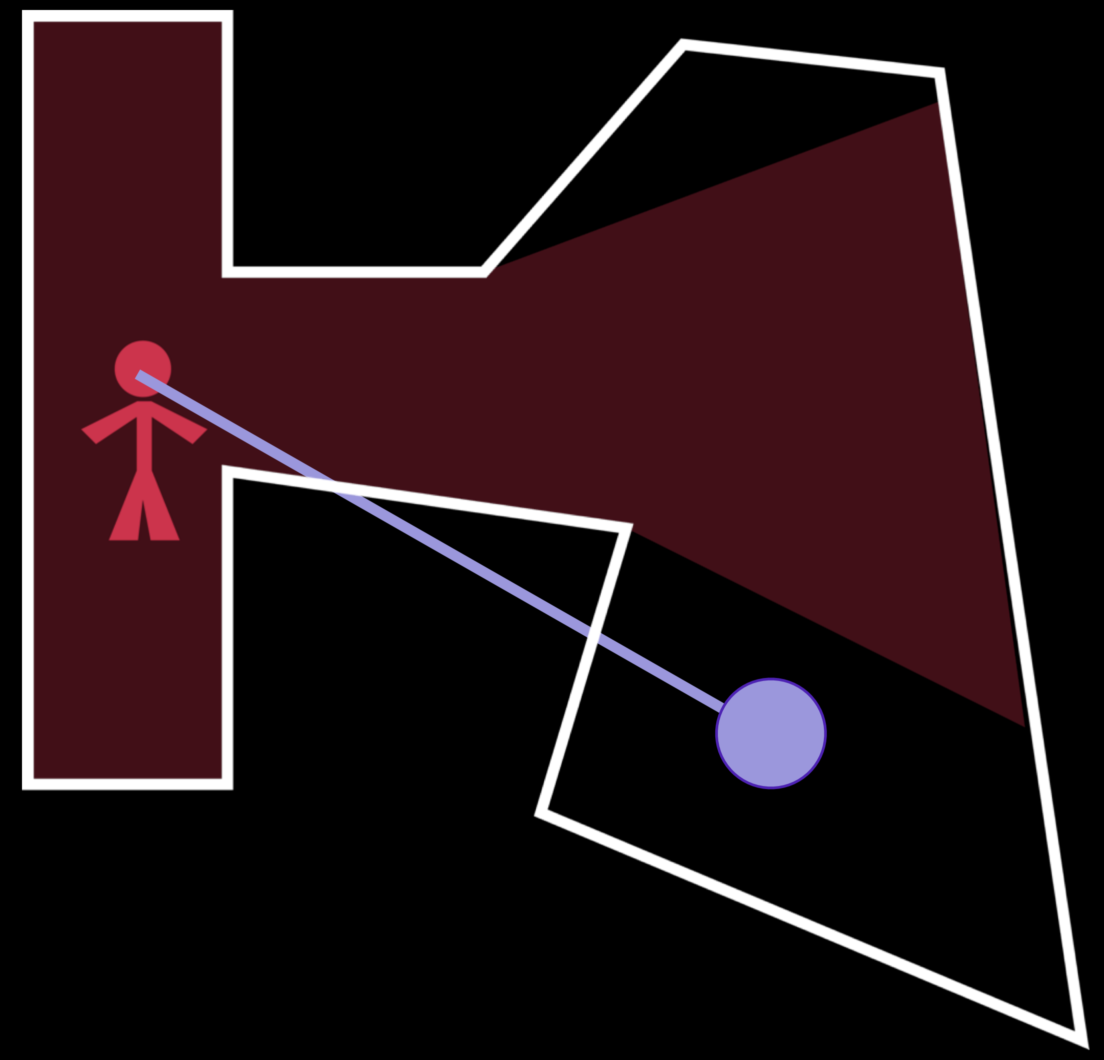
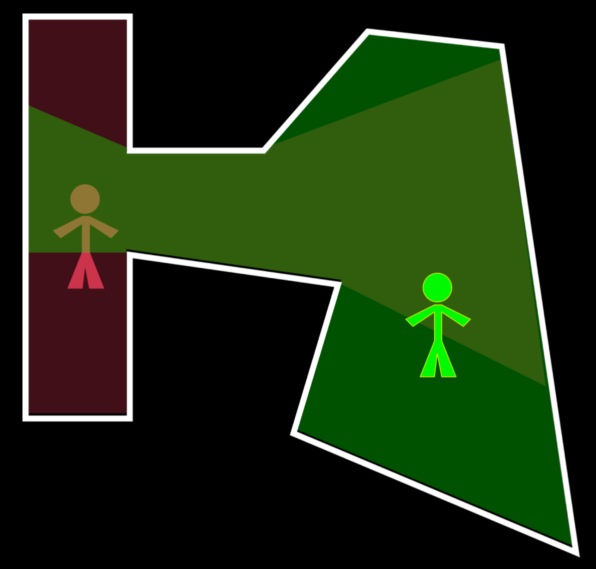


Line crosses at most 2 walls
⇒visible from the 2-transmitter

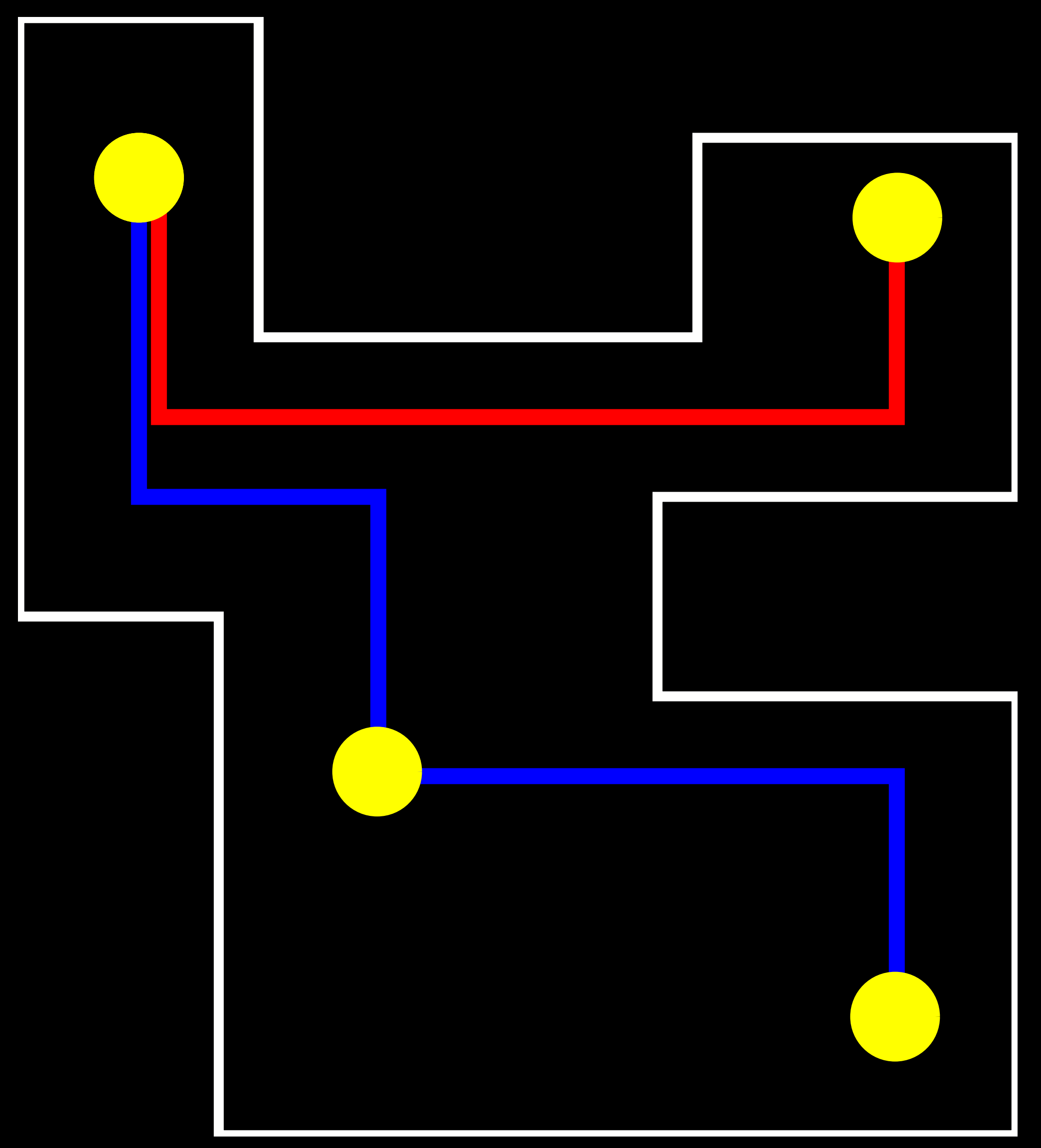
The Art Gallery Problem (AGP) and Its Variants



We can alter:

- Capabilities of the guards
- Environment to be guarded

Staircase visibility/ s-visiblity:



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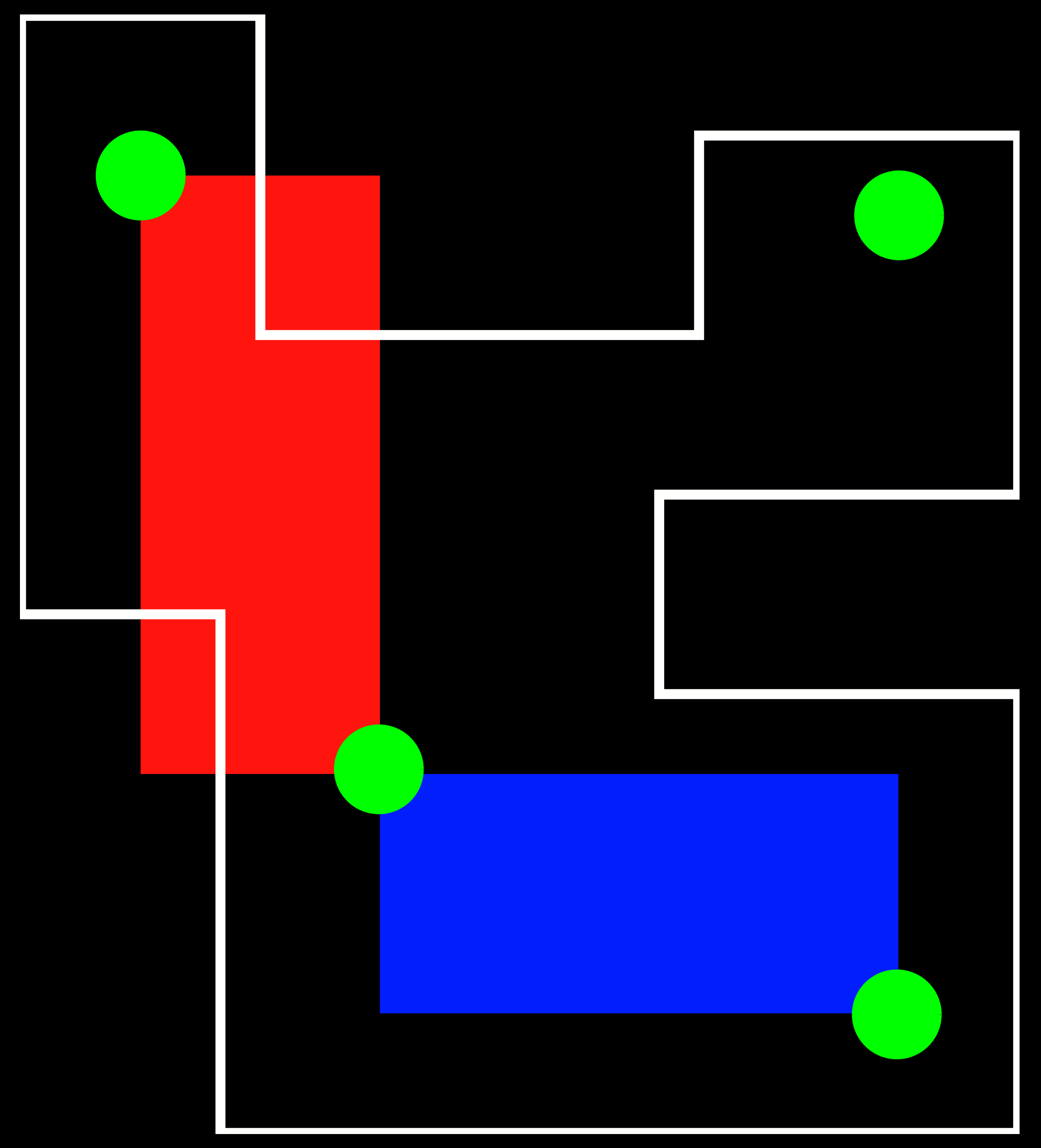

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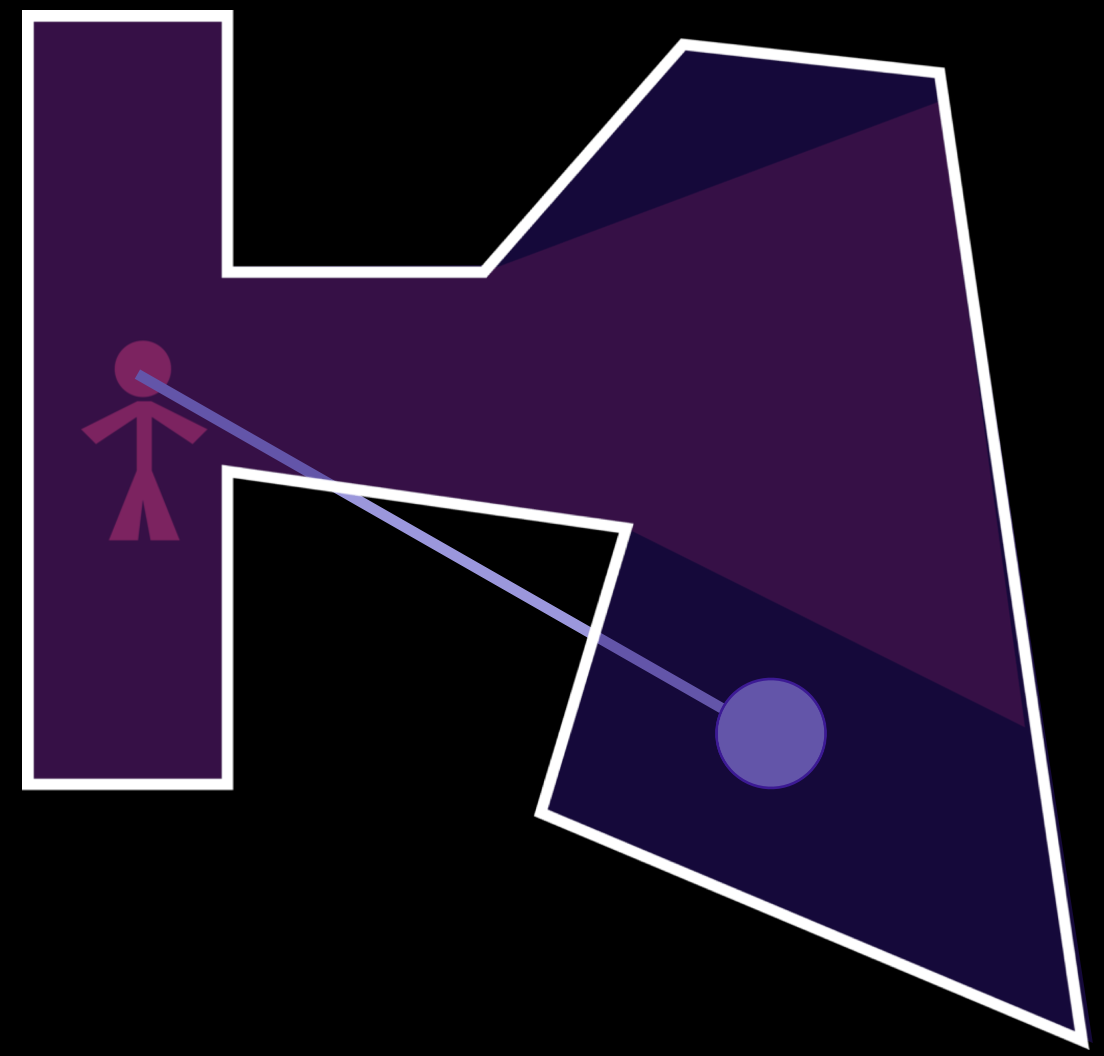

Line crosses at most 2 walls
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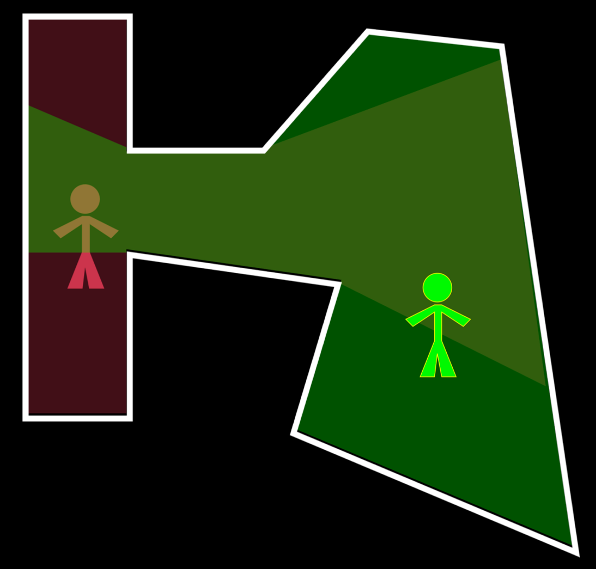

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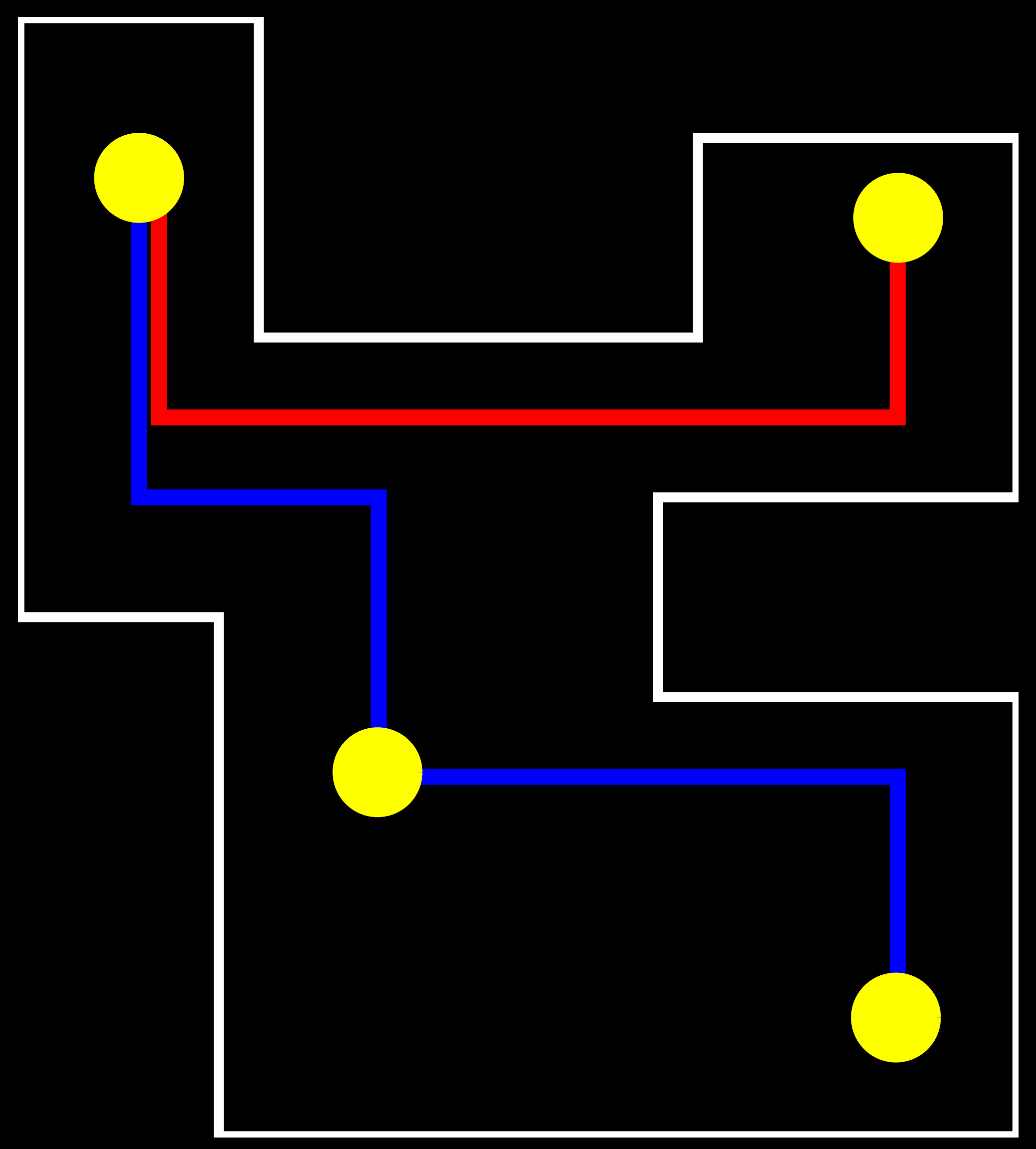

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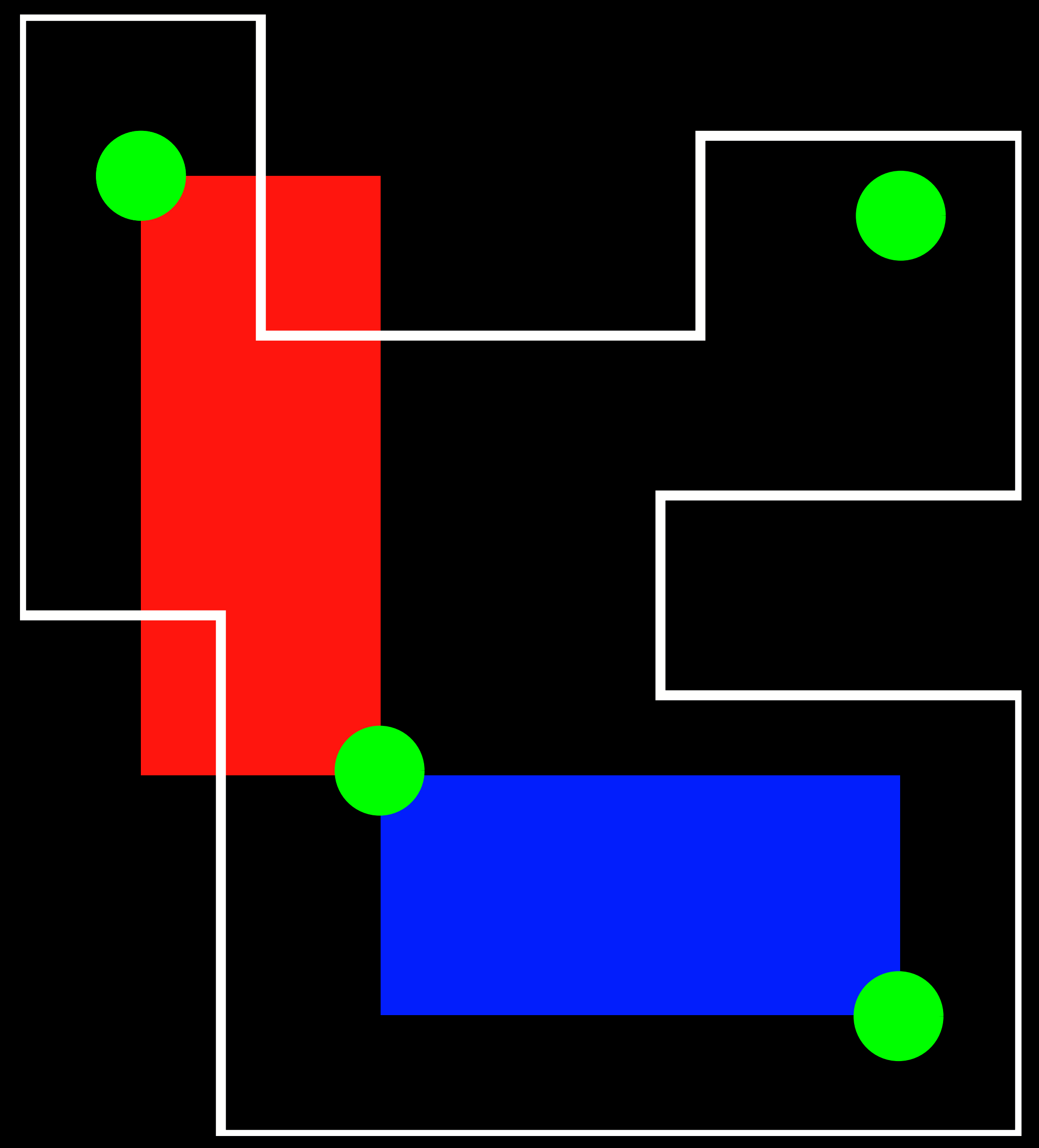

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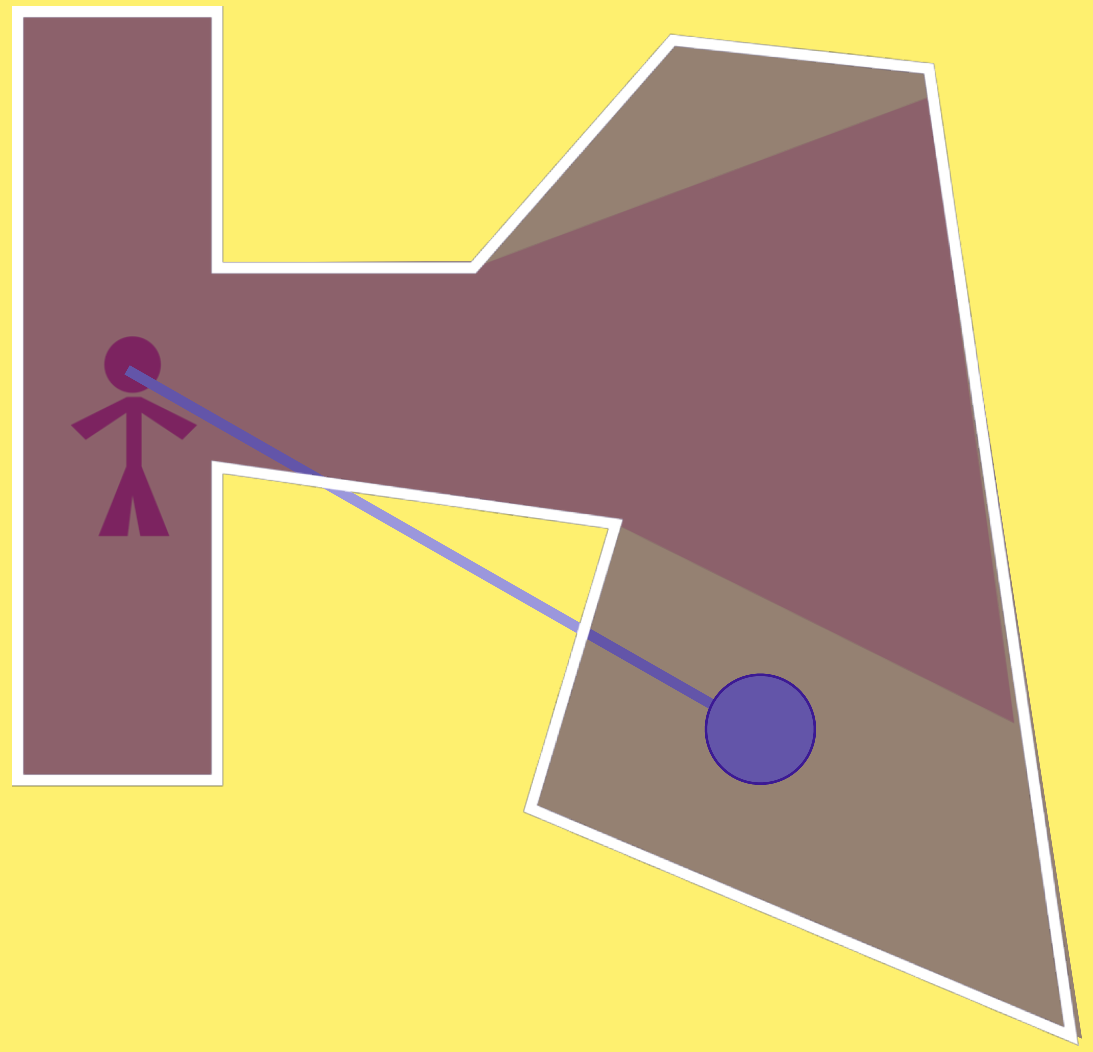

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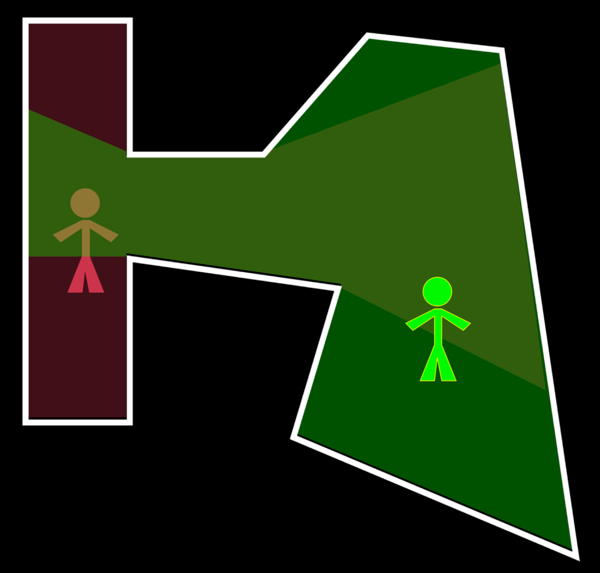

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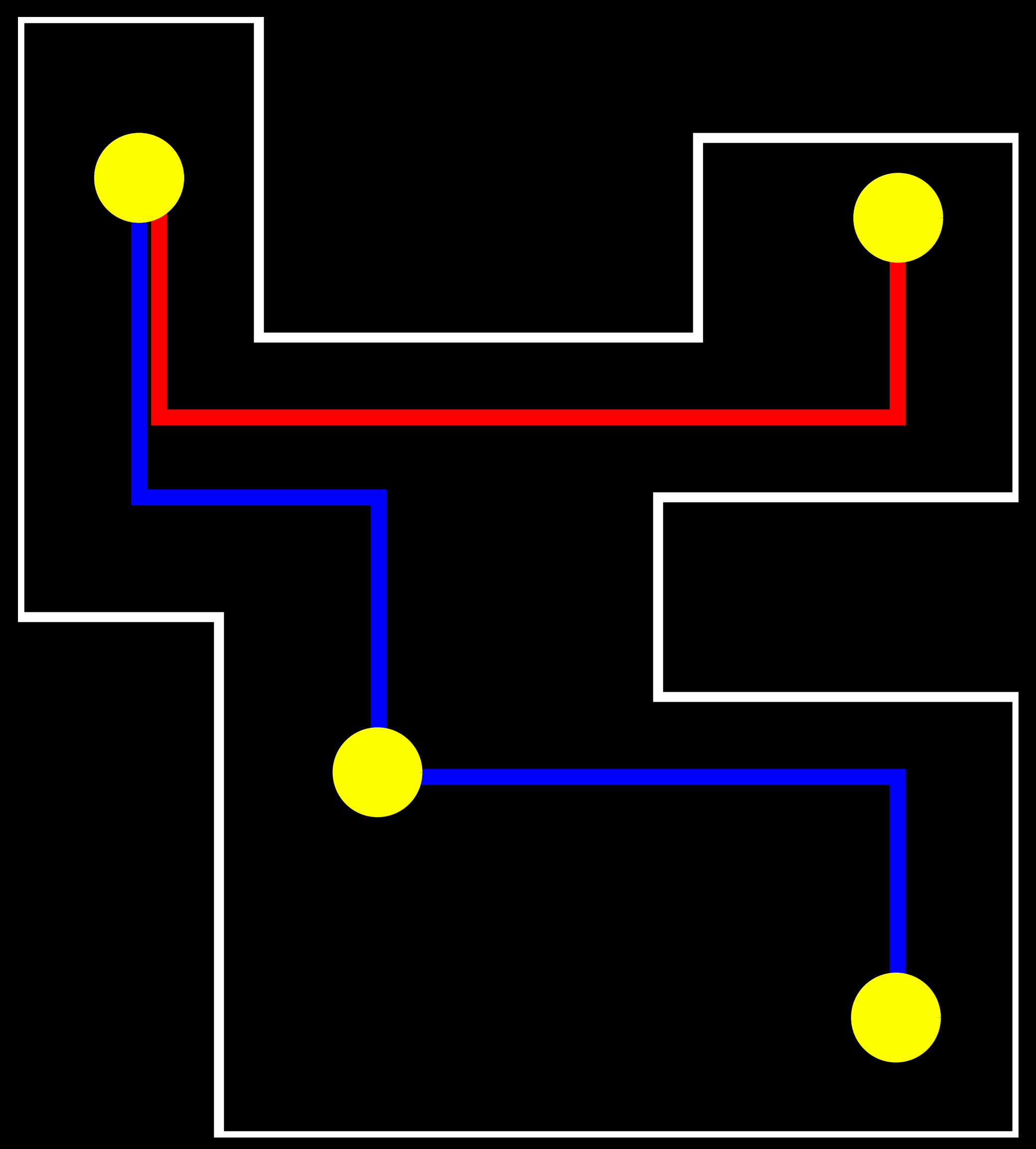

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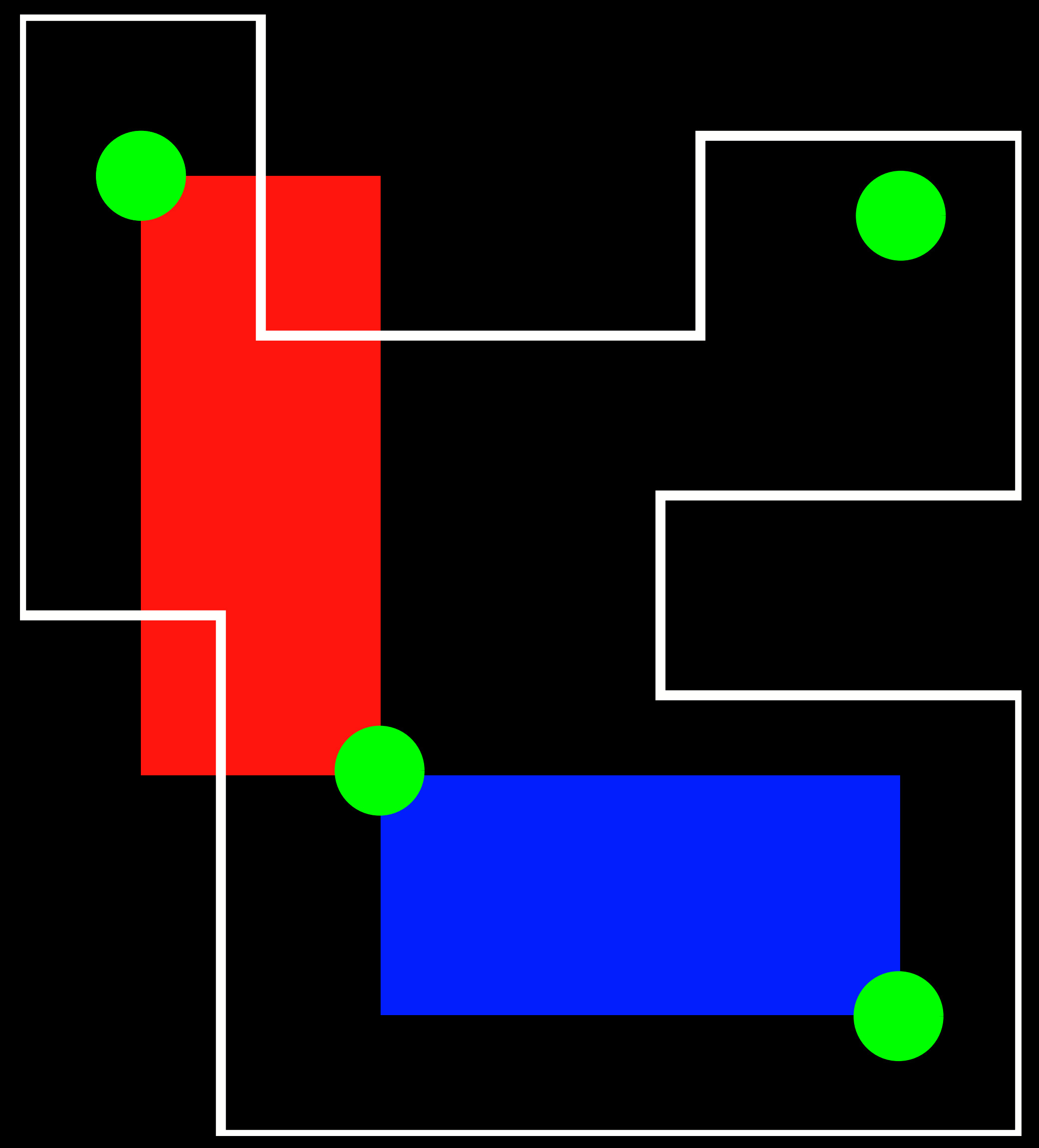

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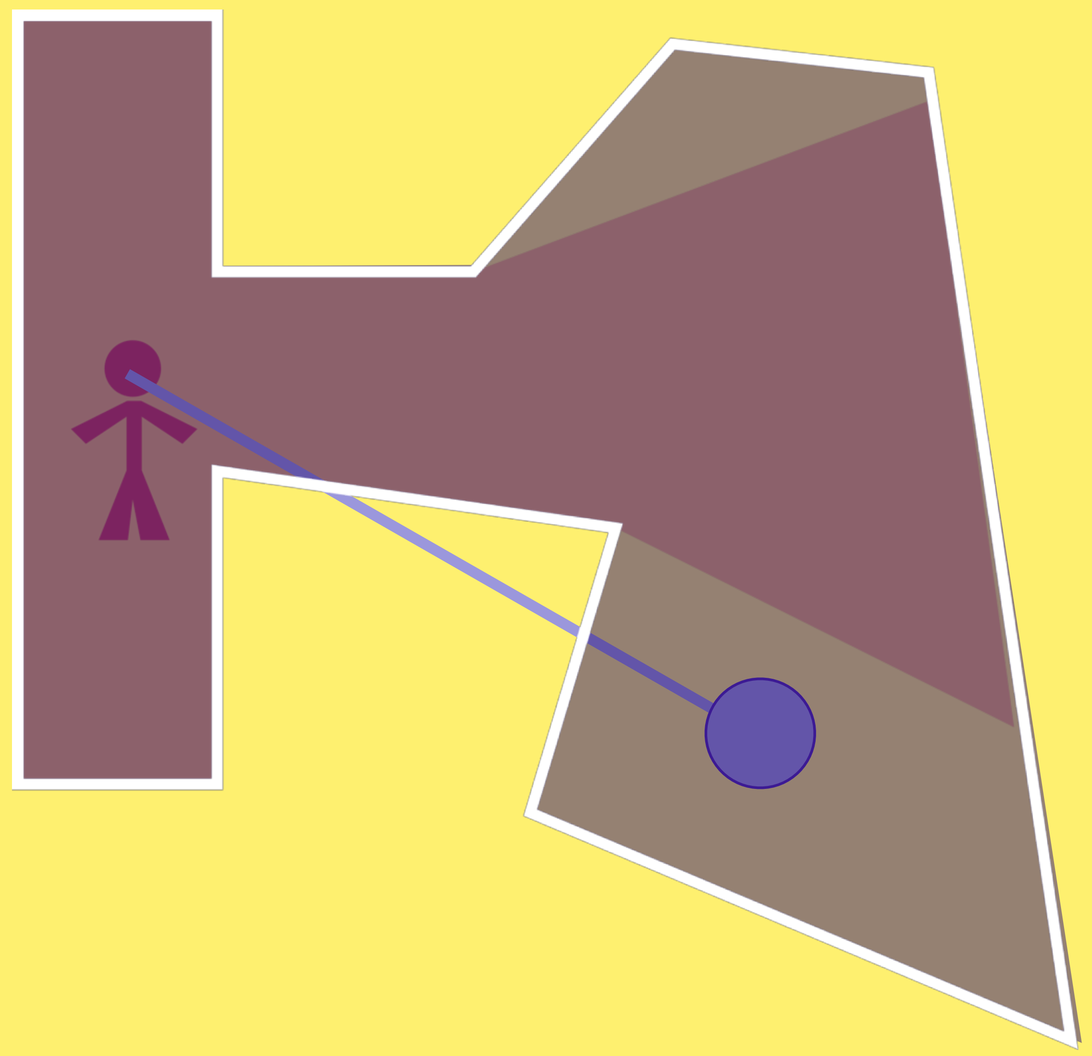

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Formally: a point *p* is **2(k)-visible** from a point *q*, if the line segment *pq* intersects *P* in at most two **(k)** connected components.

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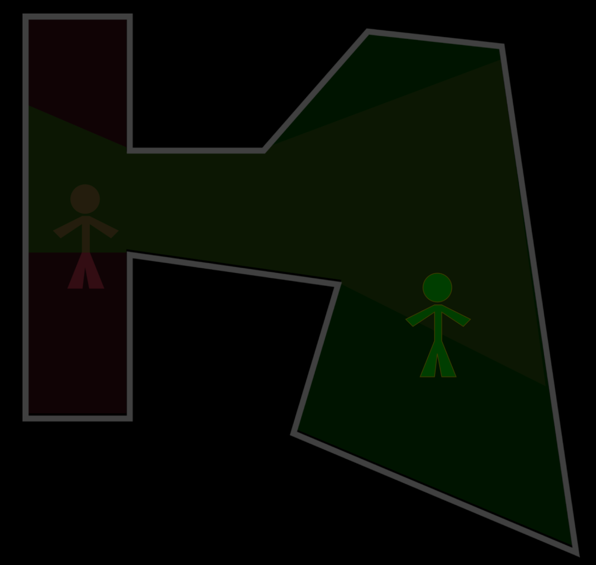

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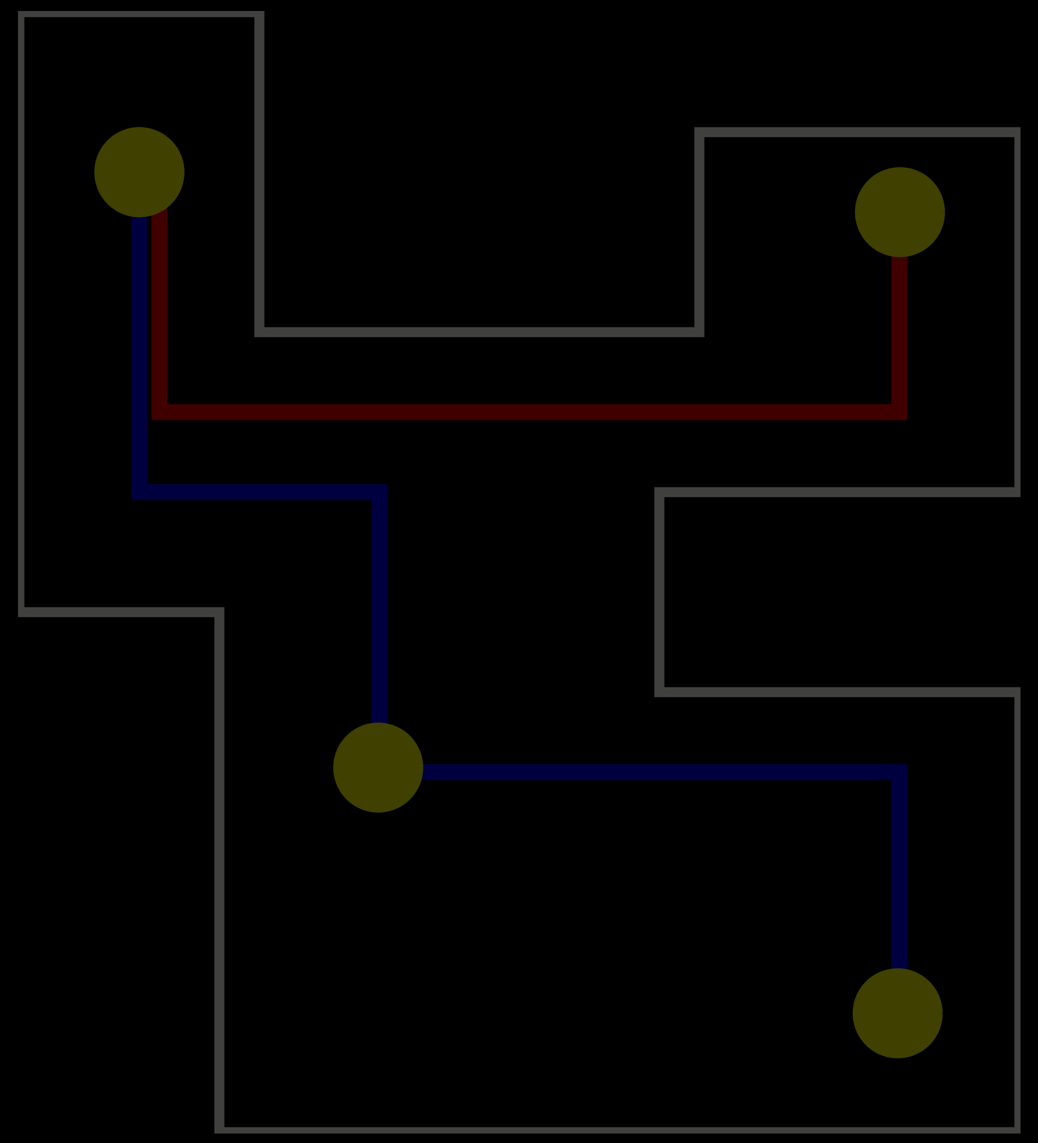

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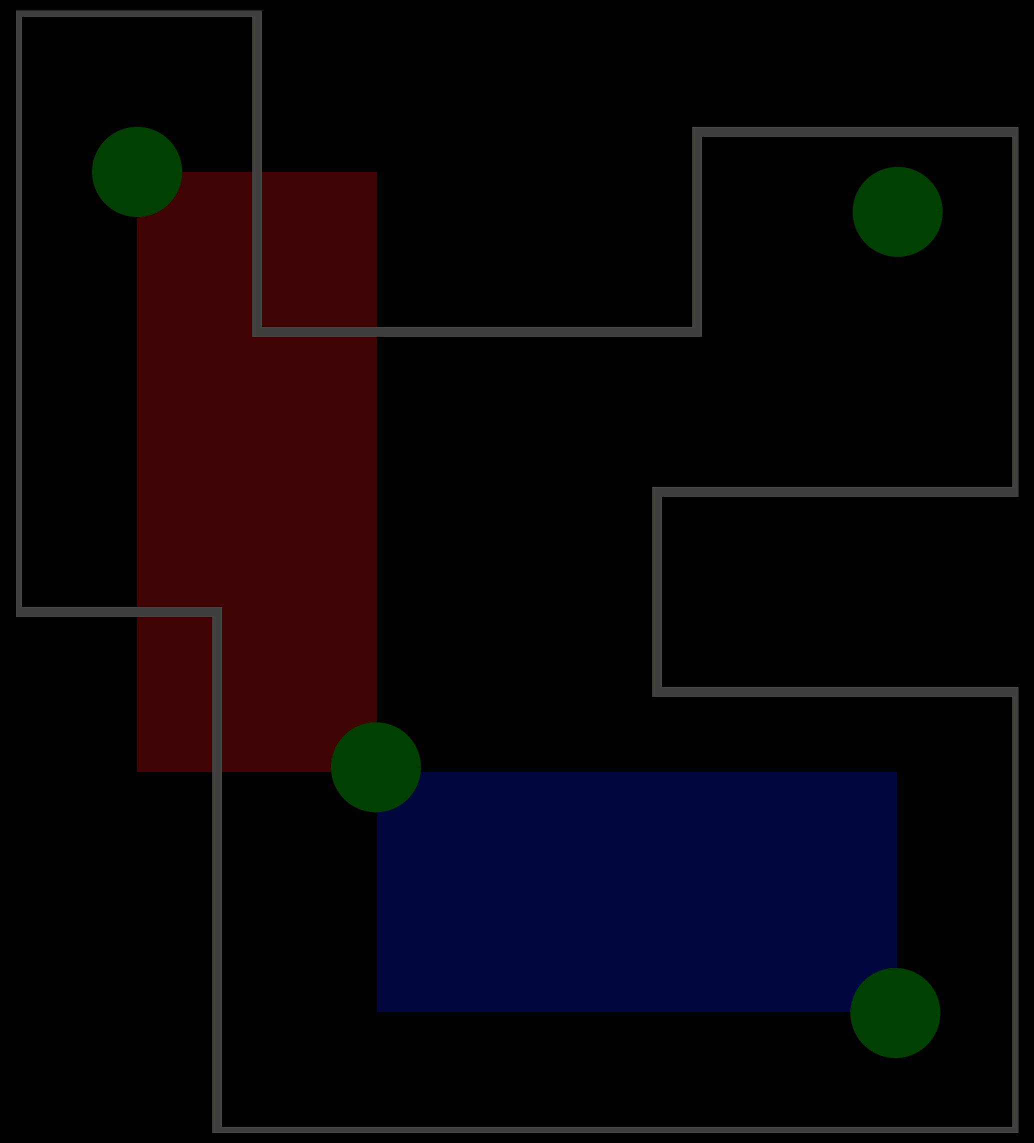

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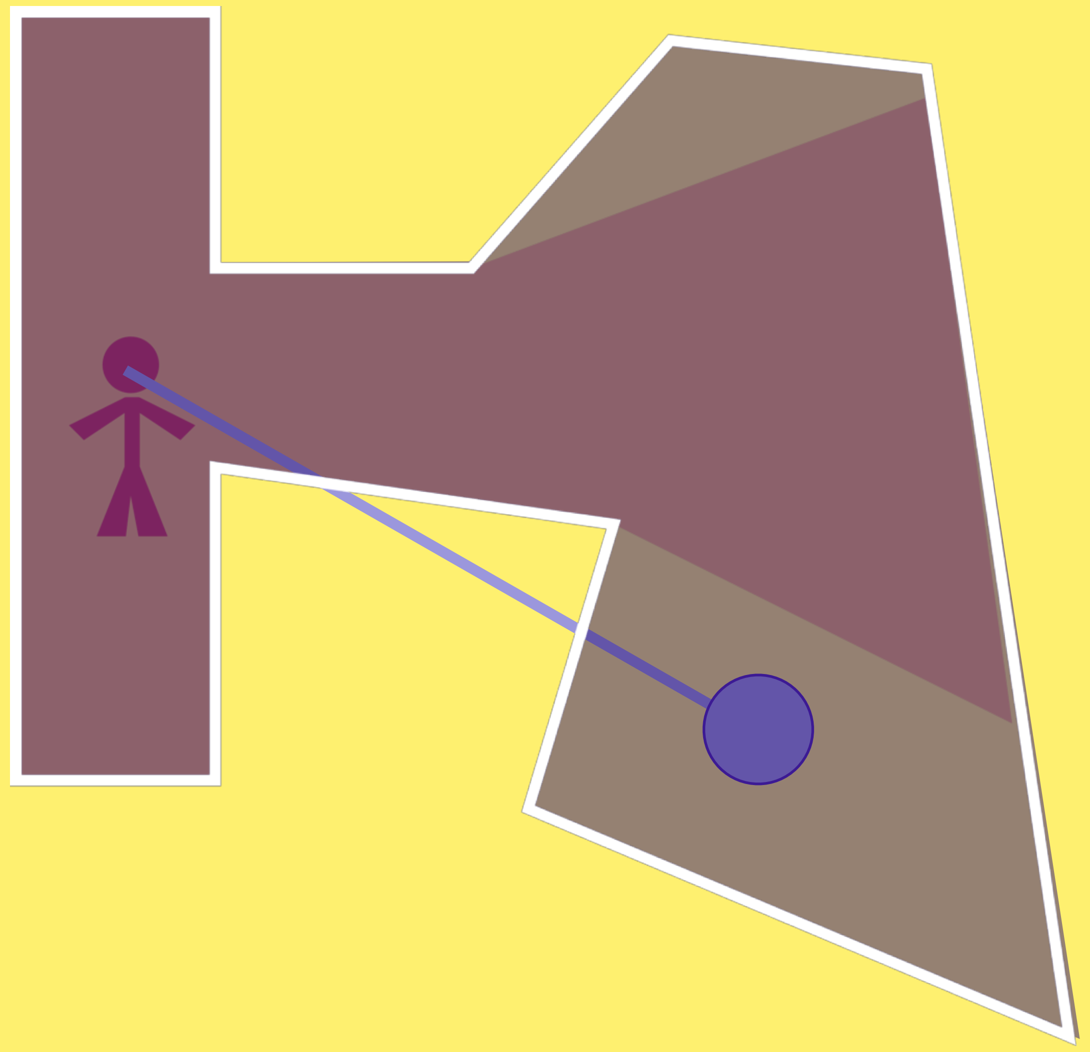

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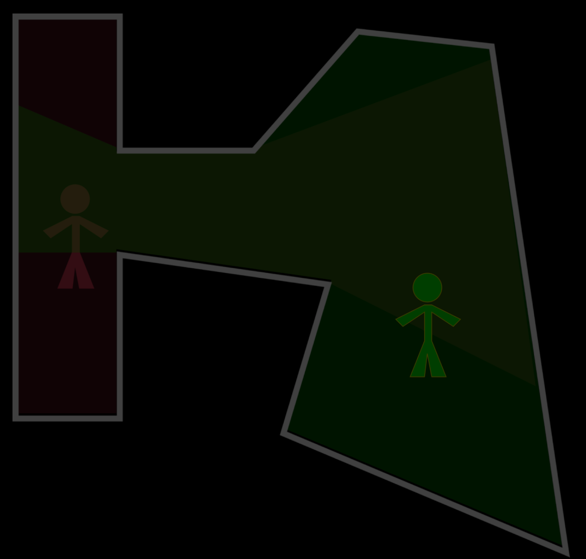

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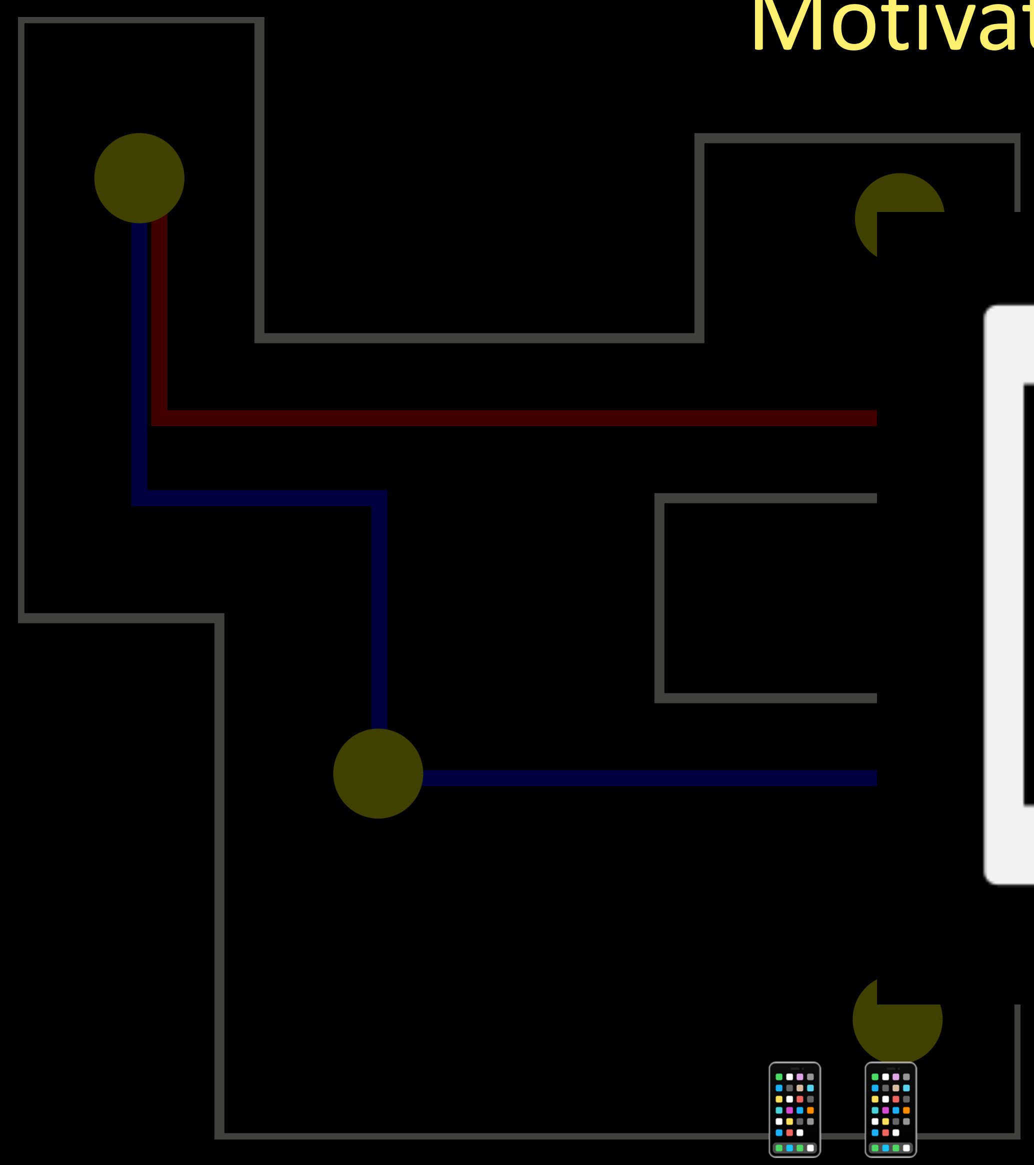
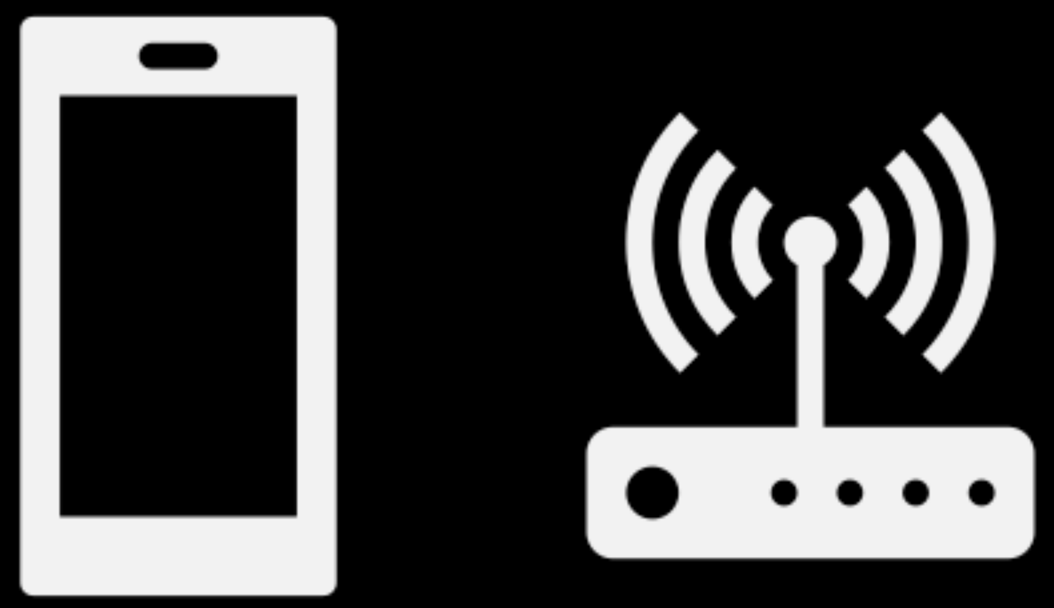
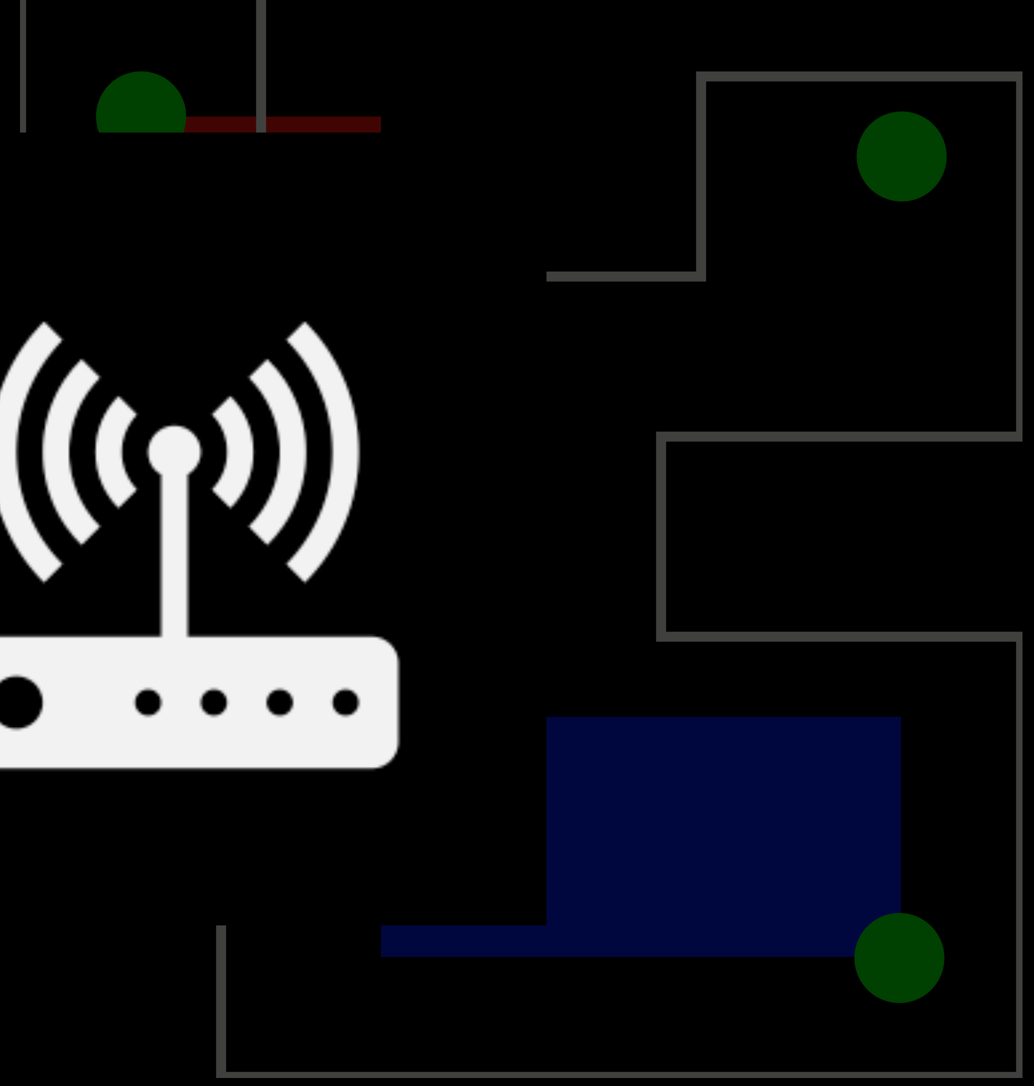
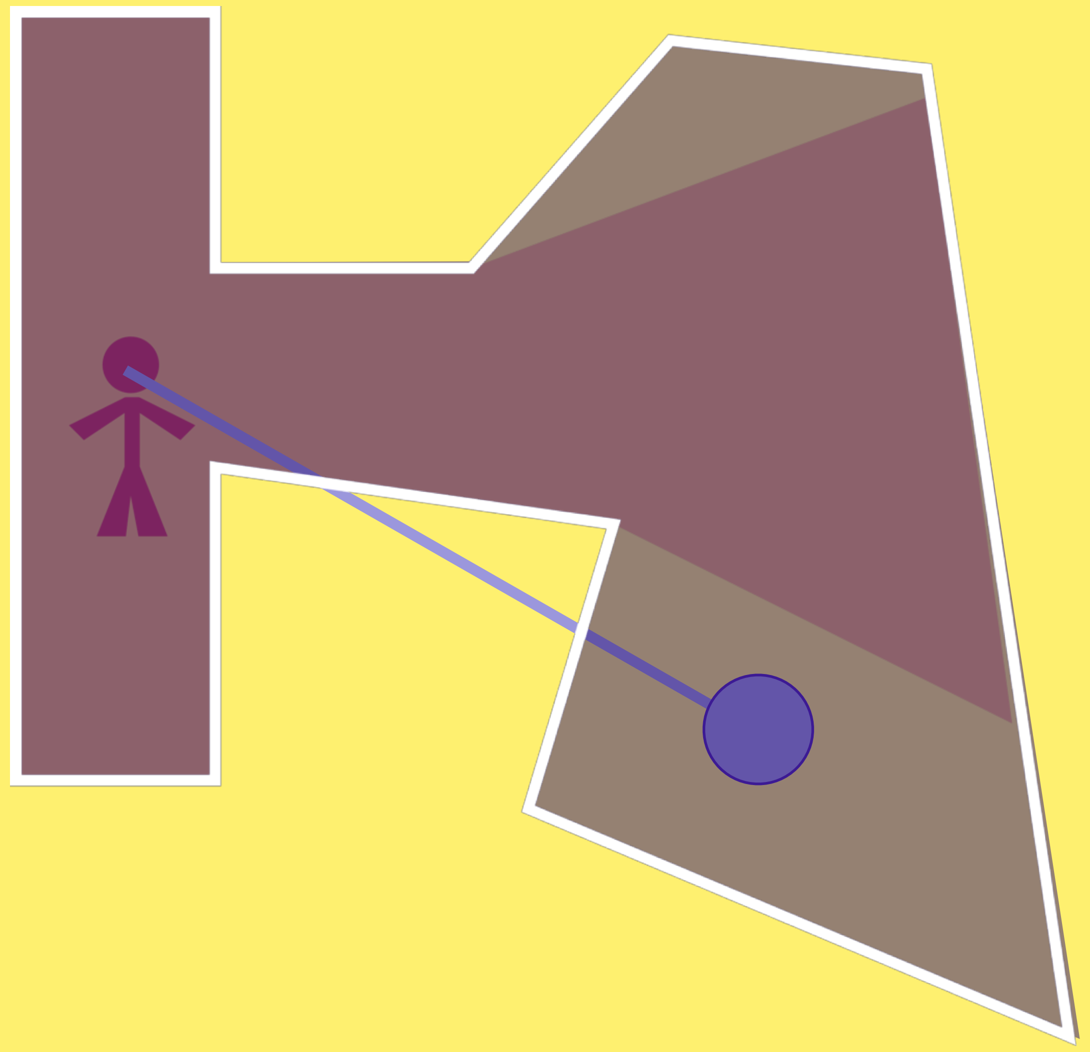
Motivated by wireless communication:









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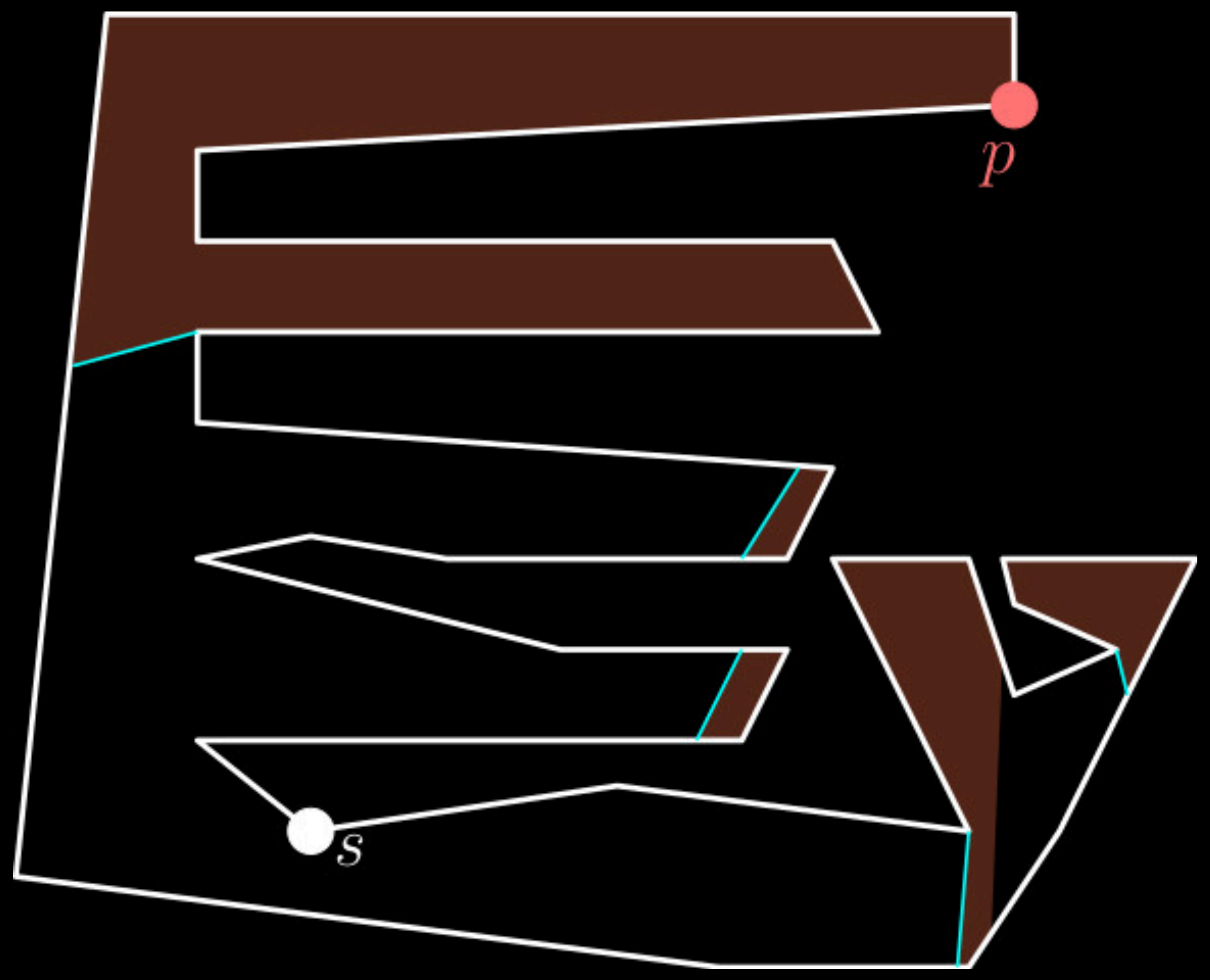
k-Transmitters

k-/2-Transmitter



2VR(p)/kVR(p) can have O(n) connected components.

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Modem illumination of monotone polygons ☆

Oswin Aichholzer [a,1], Ruy Fabila-Monroy [b,2], David Flores-Peñaloza [c,3], Thomas Hackl [a,4], Jorge Urrutia [d,5], Birgit Vogtenhuber [a,*,1]

Covering orthogonal polygons with sliding k-transmitters ☆

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Computing the k-Visibility Region of a Point in a Polygon

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Point k-Transmitter | Edge k-Transmitter | Sliding k-Transmitter

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Abstract

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Combinatorics and complexity of guarding polygons with edge and point 2-transmitters ☆

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Art Gallery Theorems for k-Transmitters

Tight bounds | Few transmitters

*from 0-transmitters | Low k

Art Gallery Theorems for k-Transmitters

Point and edge k-transmitters	Lower bound	Upper bound
Simple n-gons	$\lfloor n/5 \rfloor$ for k=2 [4]	$\lfloor n/3 \rfloor$ for k=2 * O(n/k) k-transmitters [5]
Monotone n-gons	$\lceil (n-2)/(2k+3) \rceil$ [1]	$\lceil (n-2)/(2k+3) \rceil$ [1]
Monotone orthogonal n-gons	$\lceil (n-2)/(2k+4) \rceil$ for k=1, k even [1] $\lceil (n-2)/(2k+6) \rceil$ k≥3 odd [1]	$\lceil (n-2)/(2k+4) \rceil$ for k=1, k even [1] $\lceil (n-2)/(2k+6) \rceil$ k≥3 odd [1]
Ortogonal (2m)-gon		m even: Single (m-1)-transmitter; m odd: Single m-transmitter [2]
Spiral n-gons		$\lfloor n/4 \rfloor$ for k=2 [3]
Arrangement of lines in the plane	Single $\lceil 2n/3 \rceil$-transmitter [2] Two $\lceil n/2 \rceil$-transmitters [2]	Single $\lceil 2n/3 \rceil$-transmitter [2] Two $\lceil n/2 \rceil$-transmitters [2]
d-dim Euclidean space \w n convex obstacles		Single (dn+1)/(d+1)-transmitter [6]
Plane with obstacles		$\lceil (5n+6)/12 \rceil$ 1-tr for n disjoint line segments [3]
Simple n-gons	$\lfloor n/6 \rfloor$ for k=2 [4]	$\lfloor 3n/10 \rfloor$+1 for k=2 *
Monotone n-gons	$\lceil (n-2)/9 \rceil$ for k=2 [4]	$\lceil (n-2)/8 \rceil$ for k=2 [4]
Monotone orthogonal n-gons	$\lceil (n-2)/10 \rceil$ for k=2 [4]	$\lceil (n-2)/10 \rceil$ for k=2 [4]
Orthogonal n-gons	$\lfloor (3n+4)/16 \rfloor$ for k=2 [4]	$\lceil (n-2)/10 \rceil$ for k=2 *

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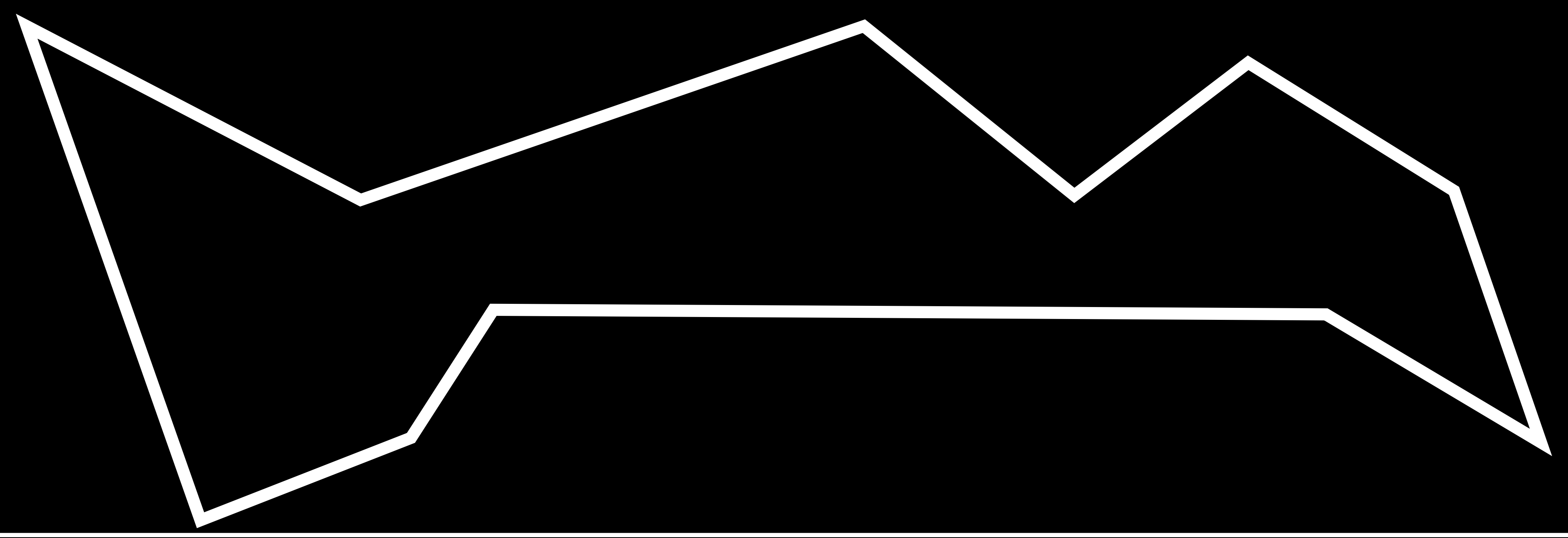
Central concept used for monotone polygons: Splitting Lemma from [Aichholzer et al., 2009]



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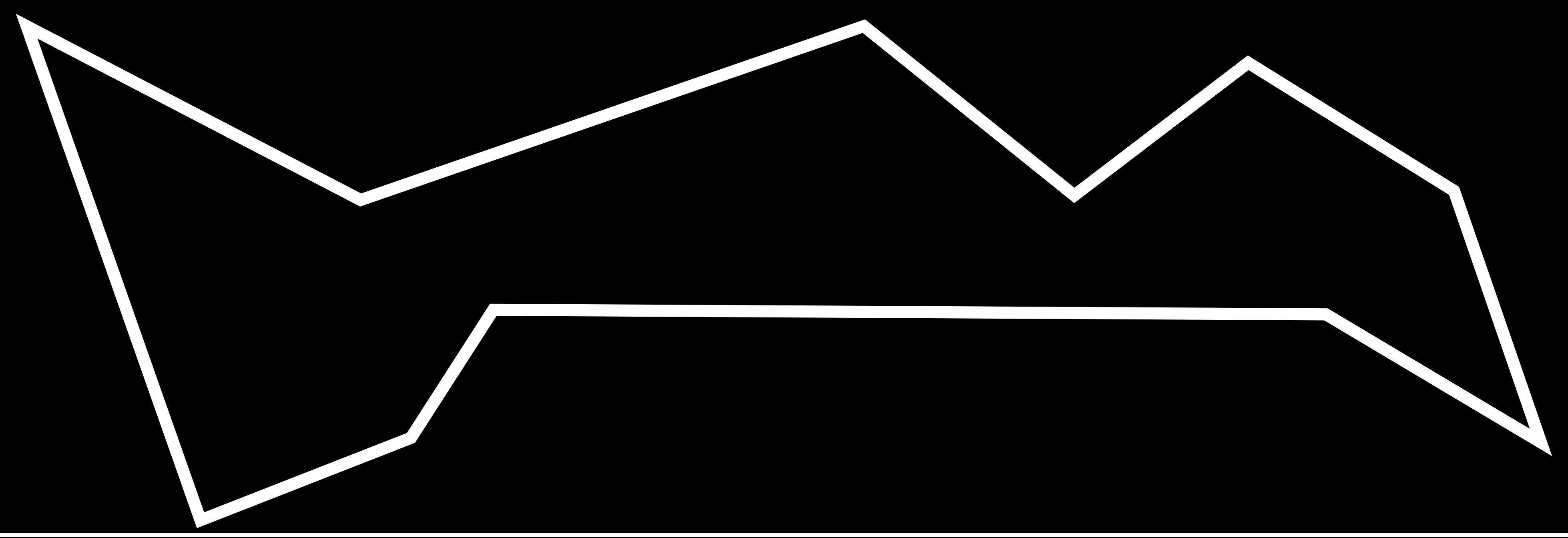

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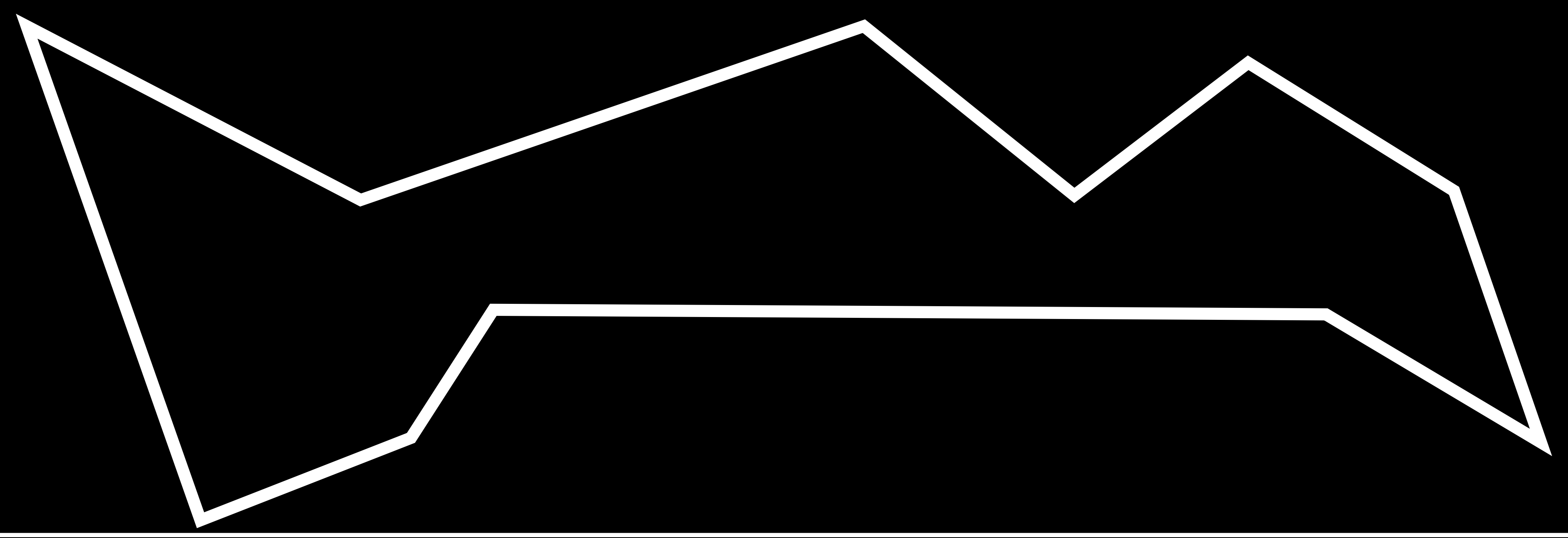

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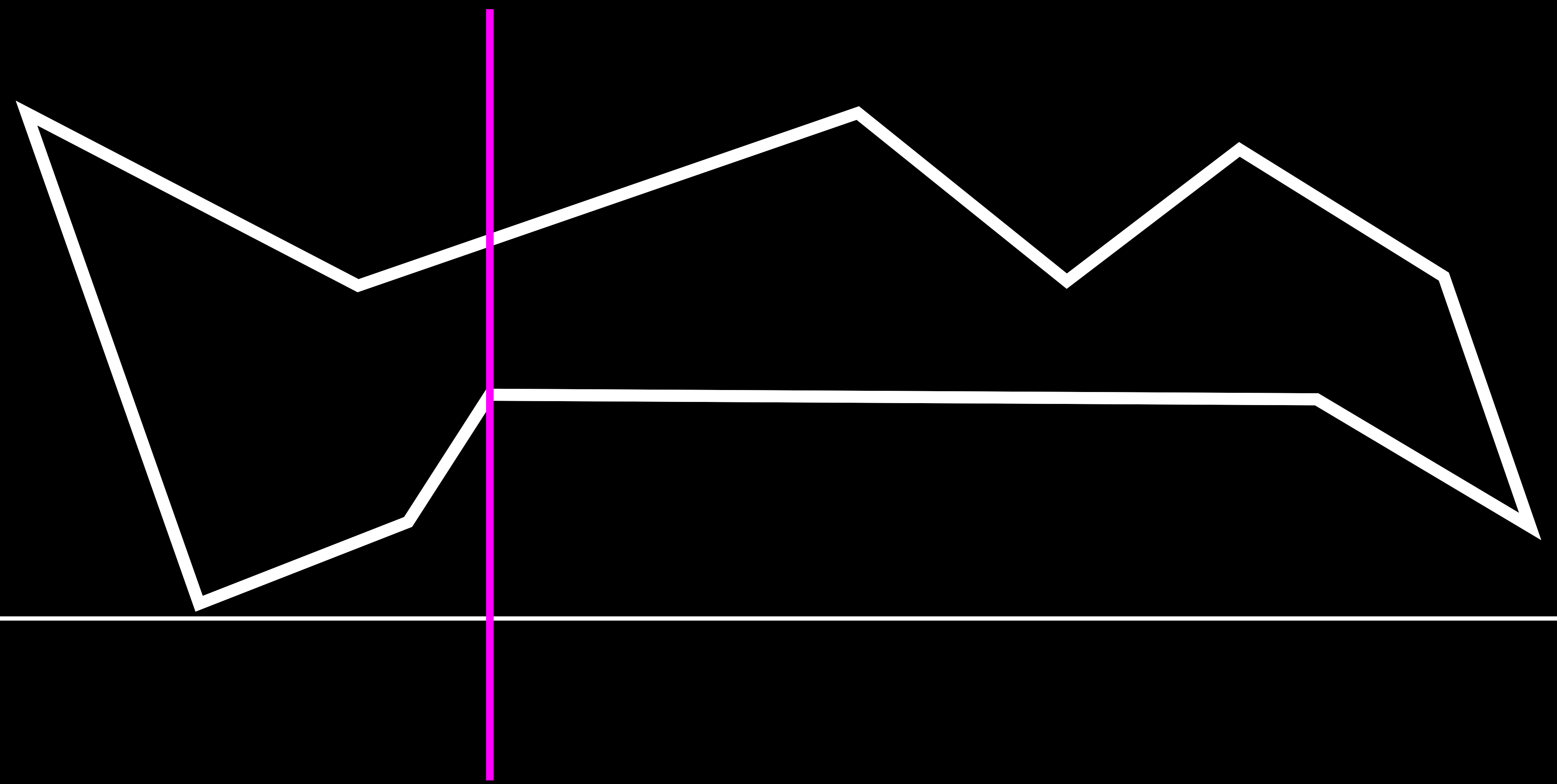

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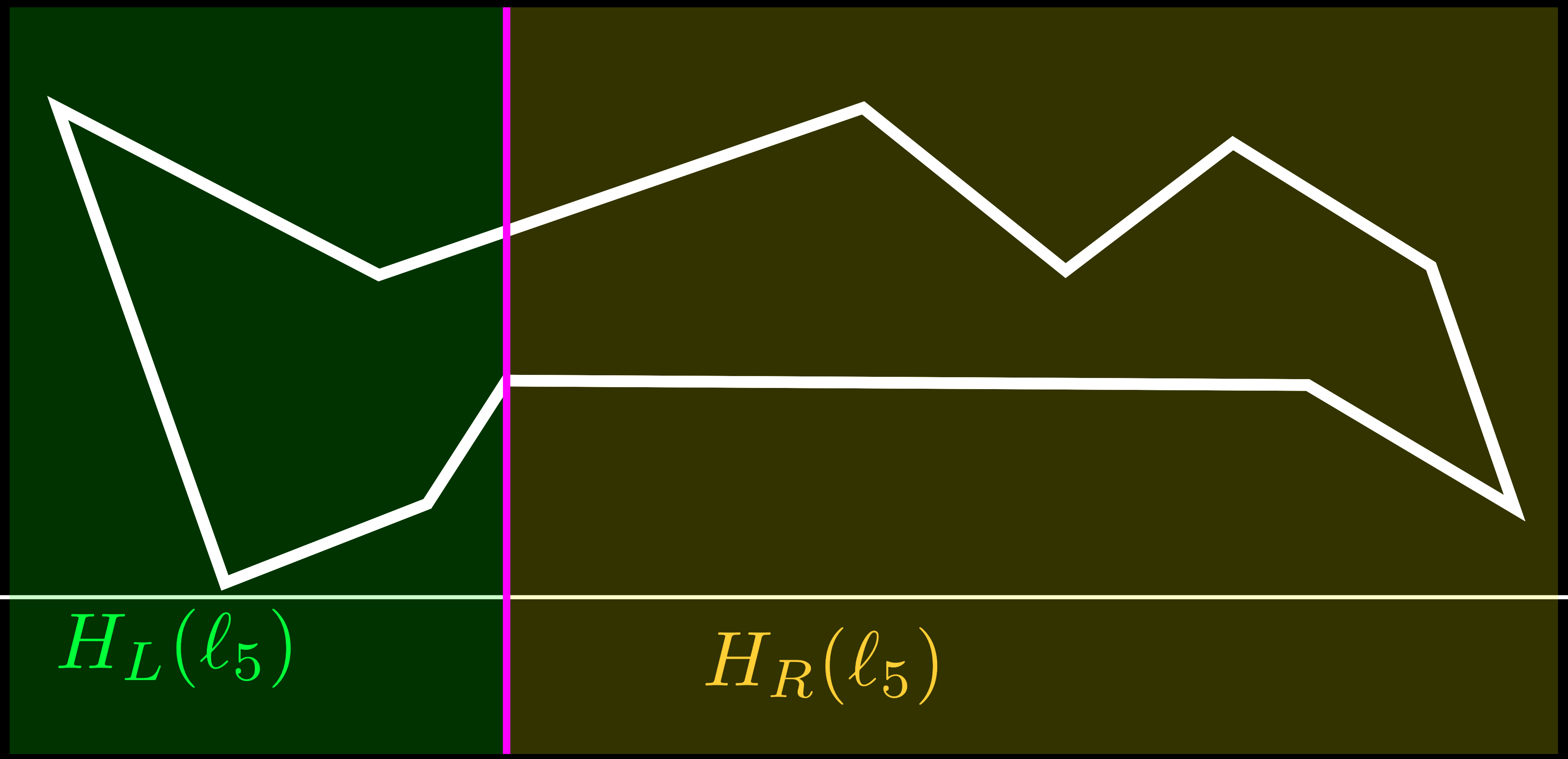

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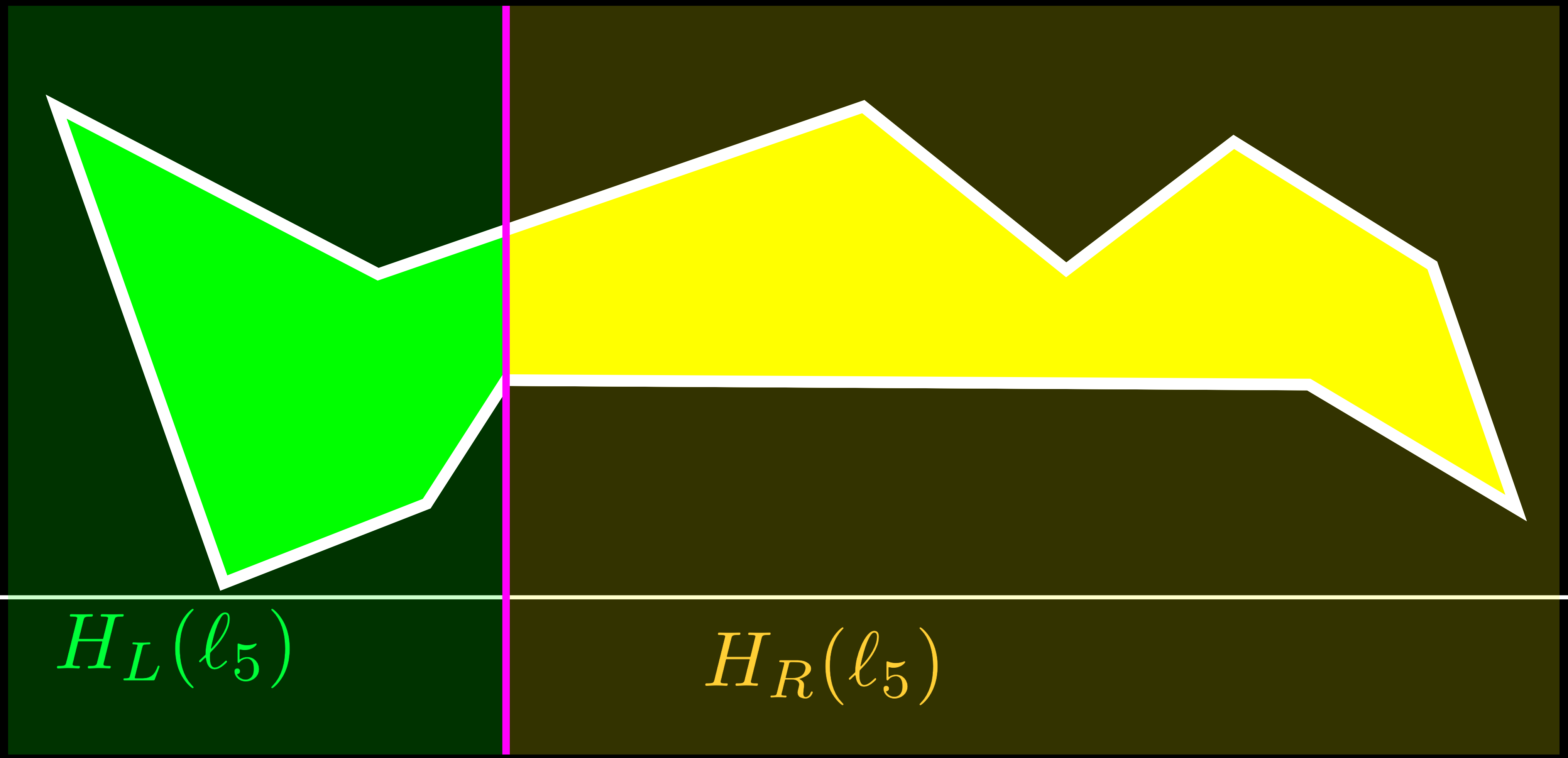

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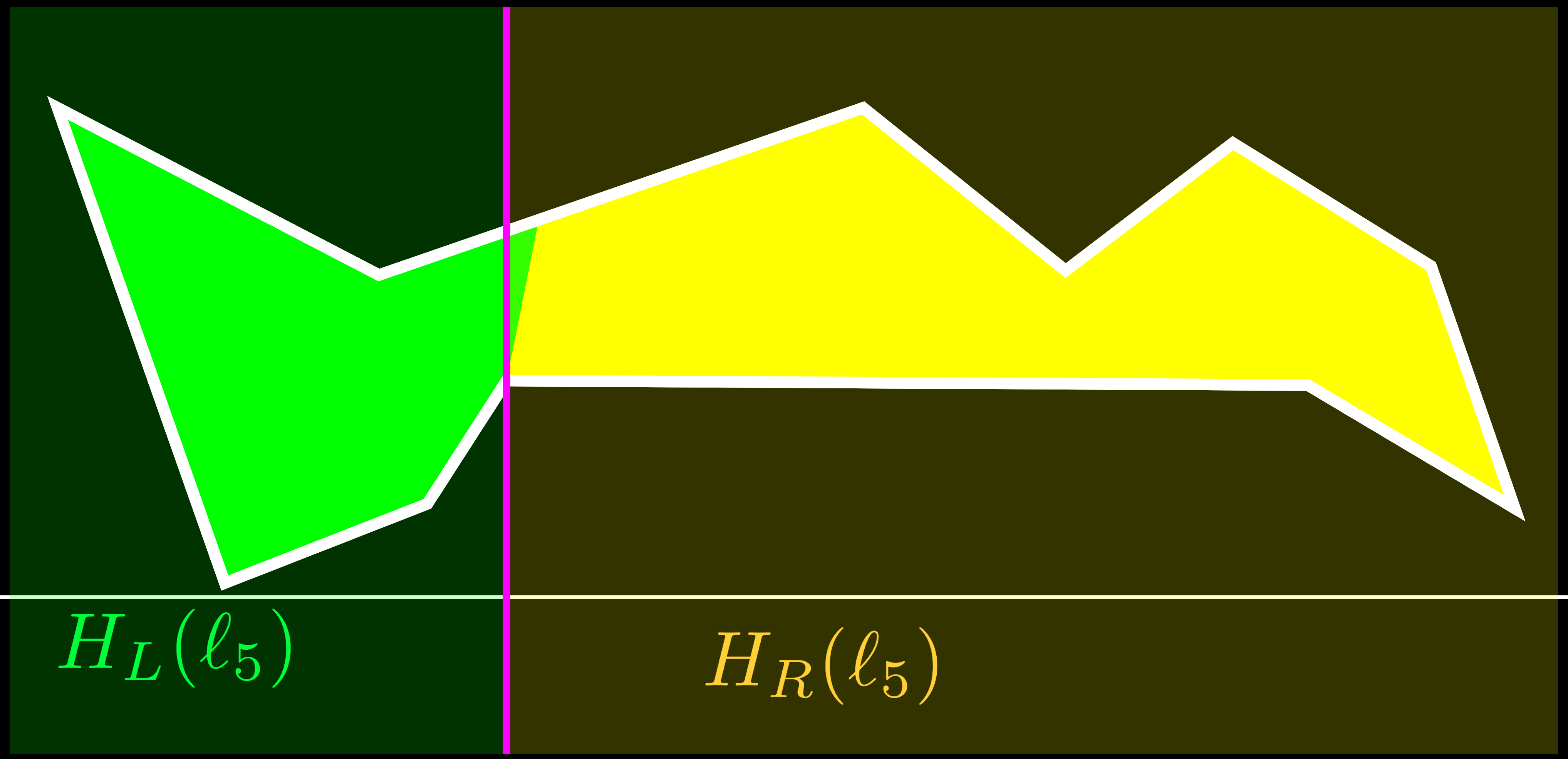

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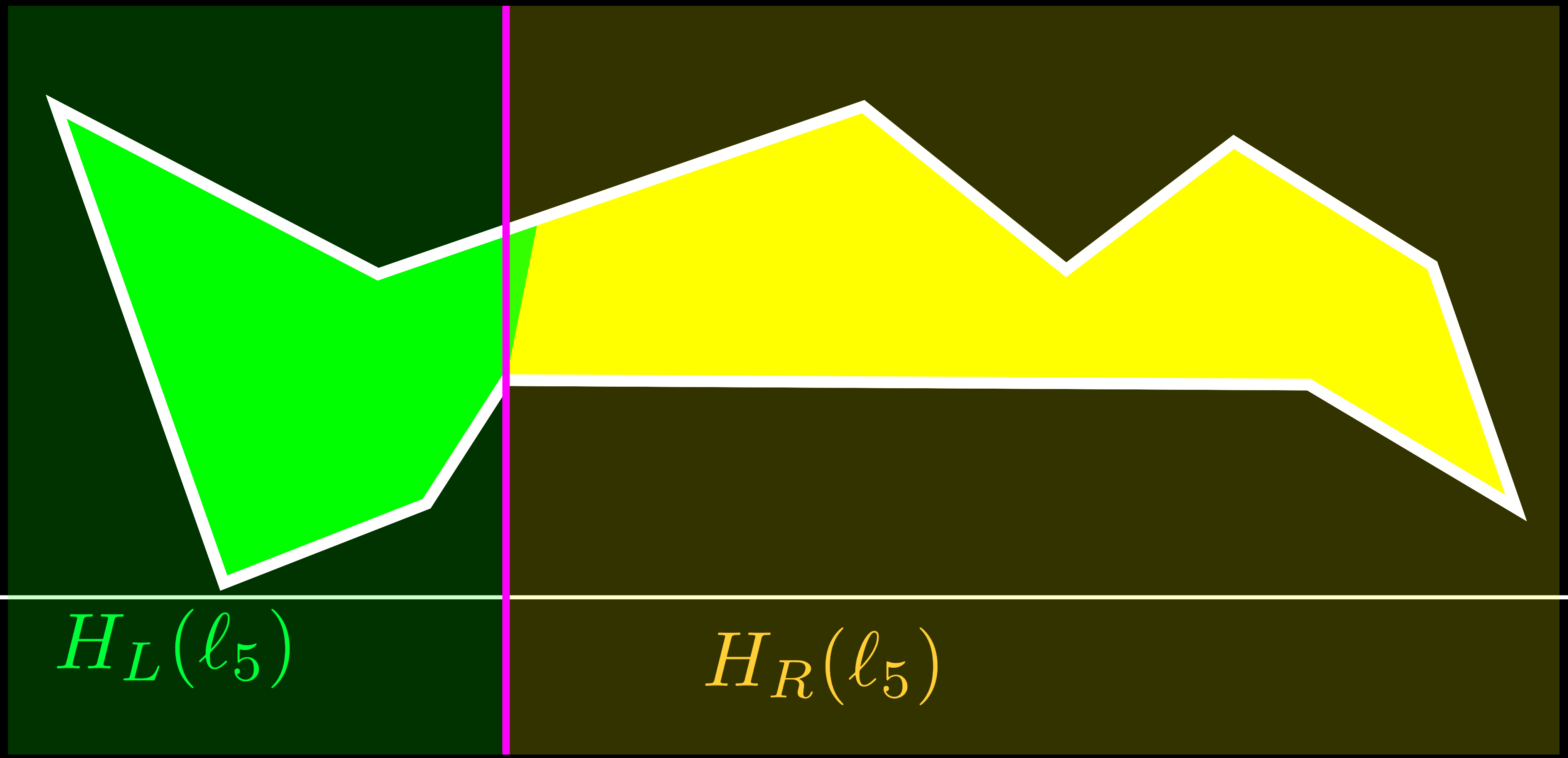
✤ $P \cap H_L(\ell_i)$ contains i edges of P.



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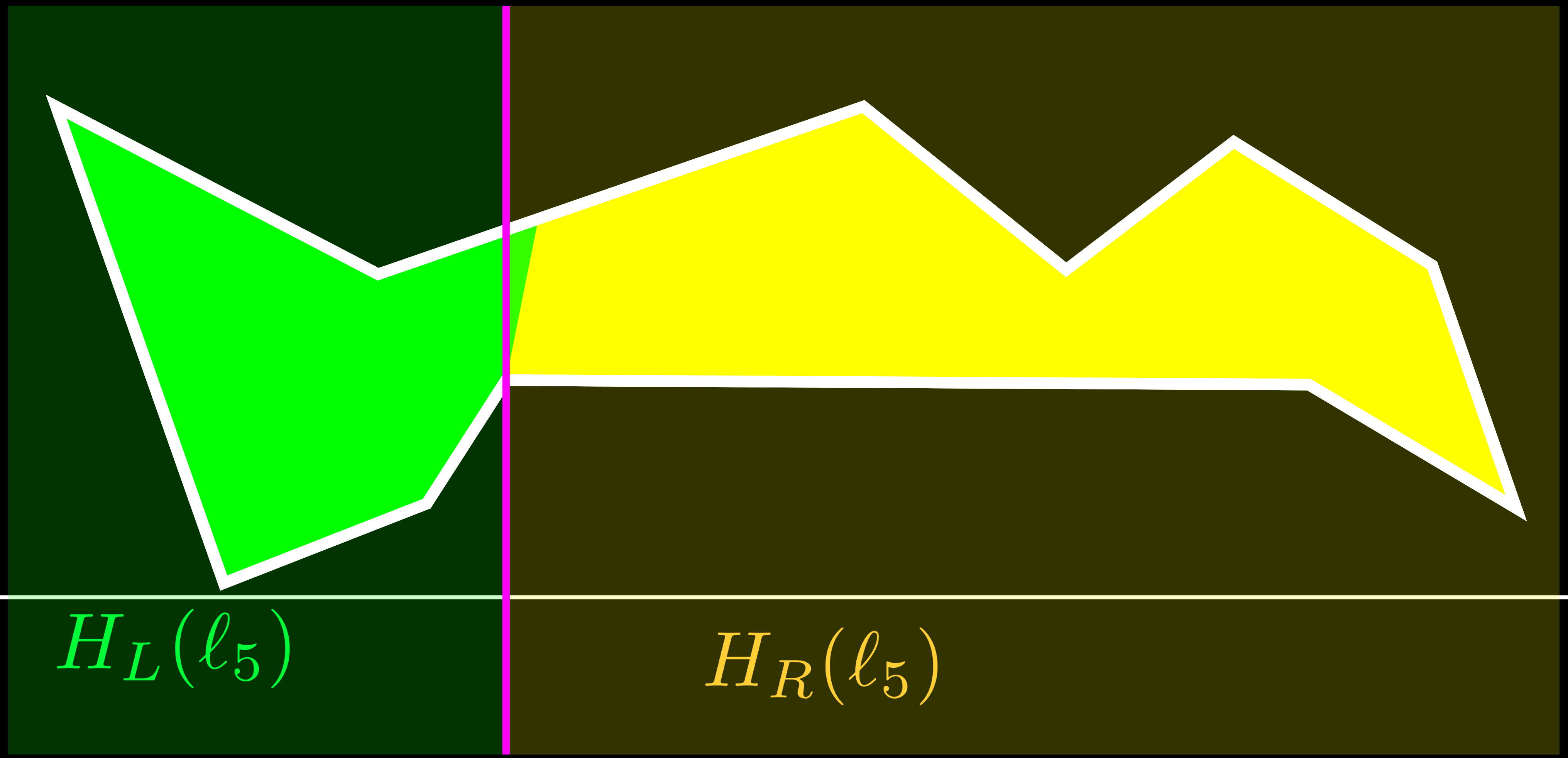

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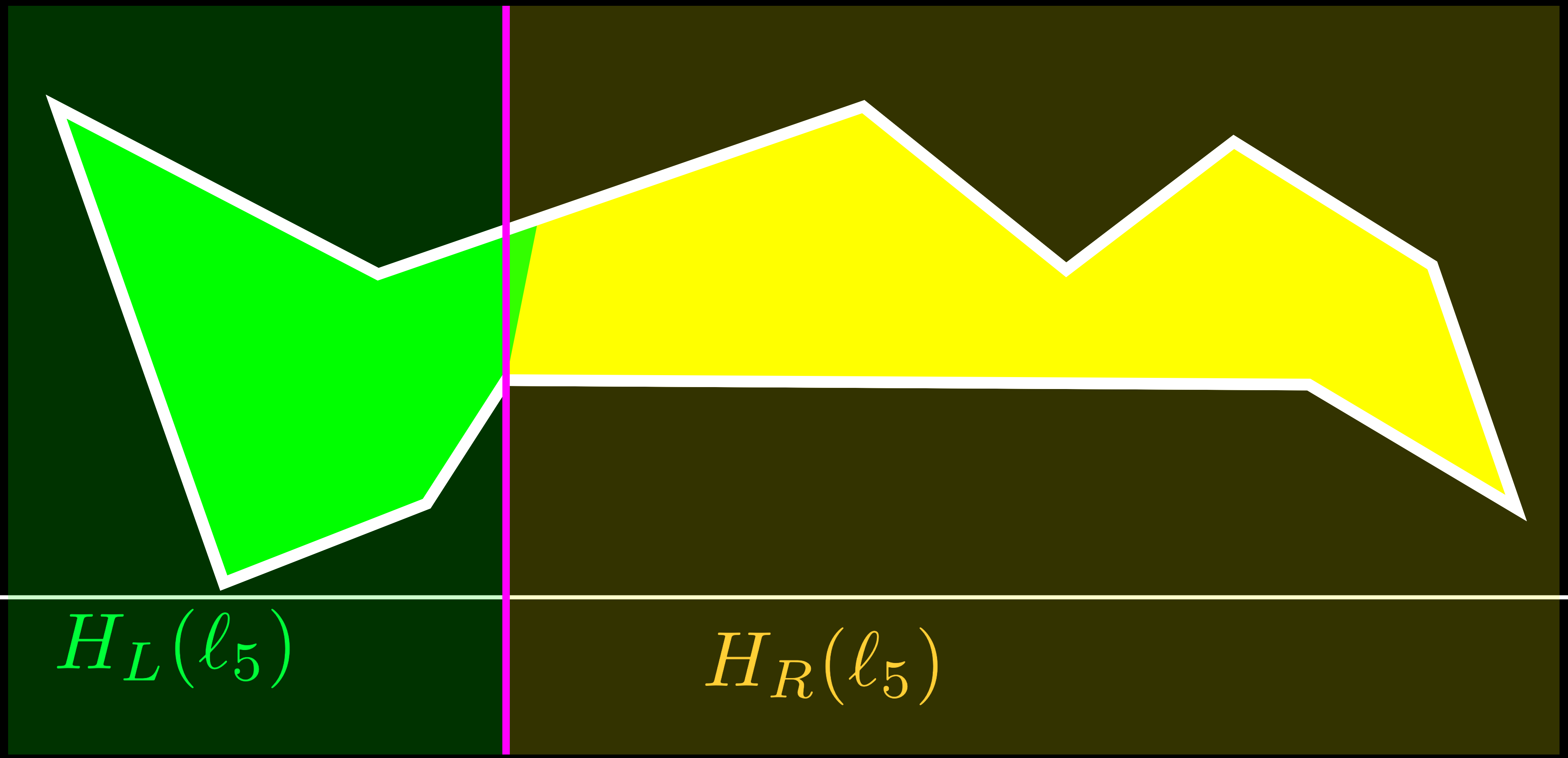
- If both $P \cap H_L(\ell_i)$ and $P \cap H_R(\ell_i)$ are illuminated, then P is illuminated



Open Problem: k-Transmitter Combinatorics

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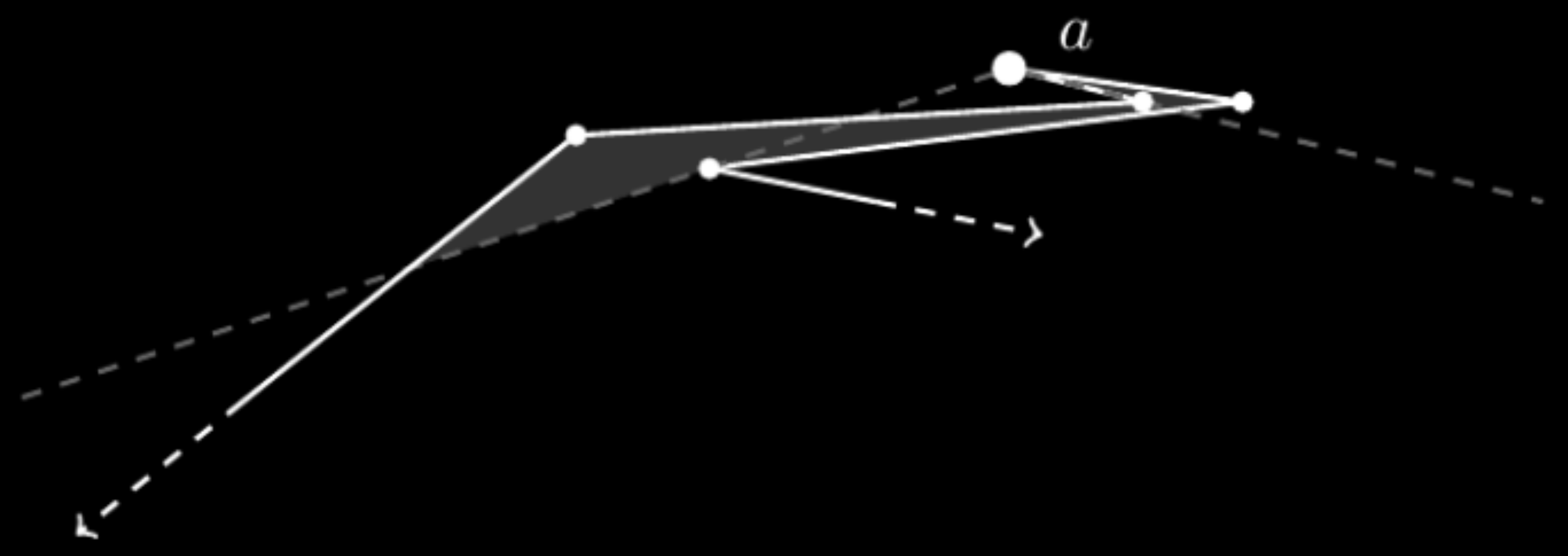

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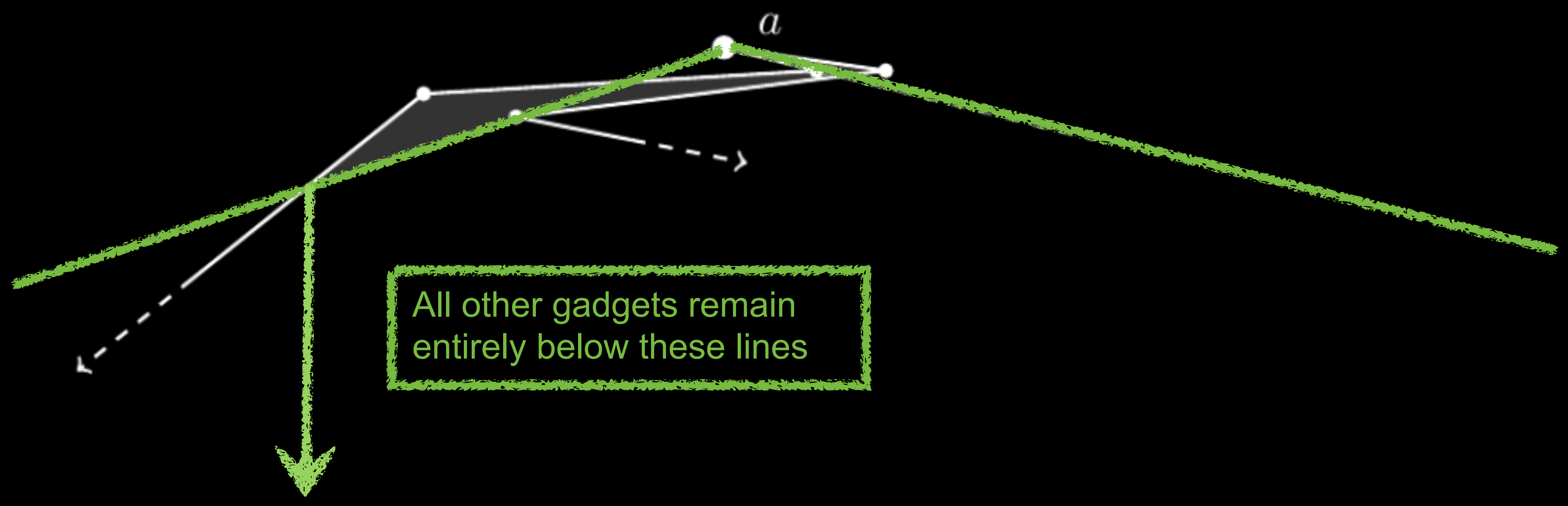

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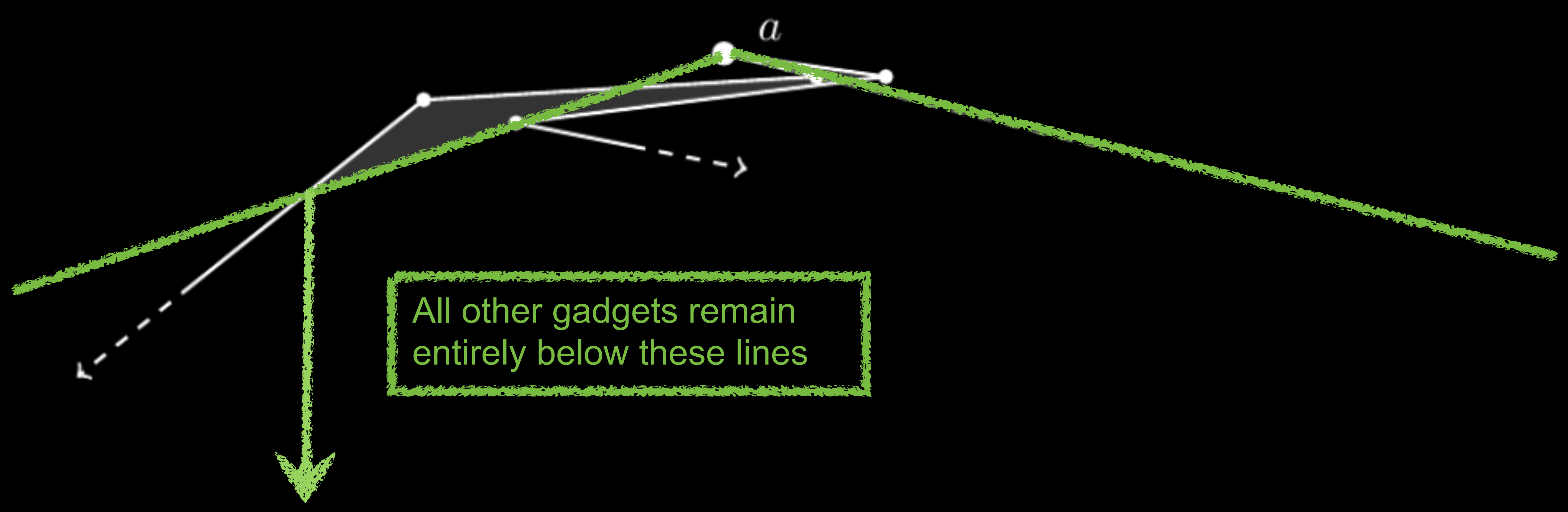

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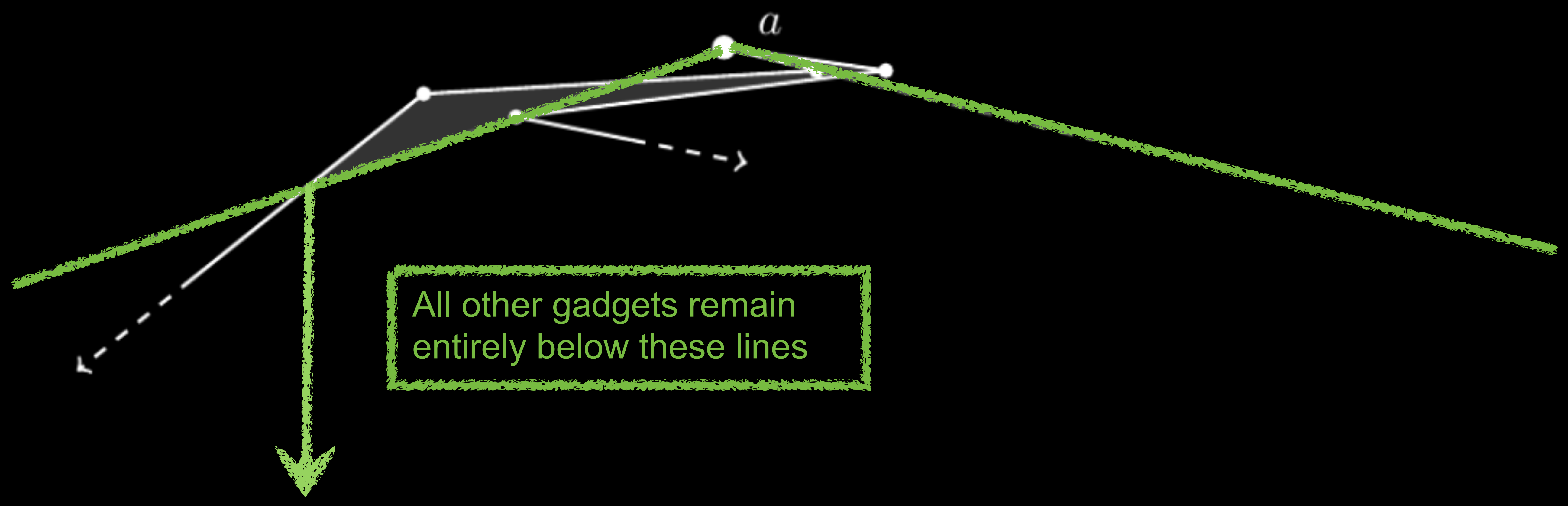

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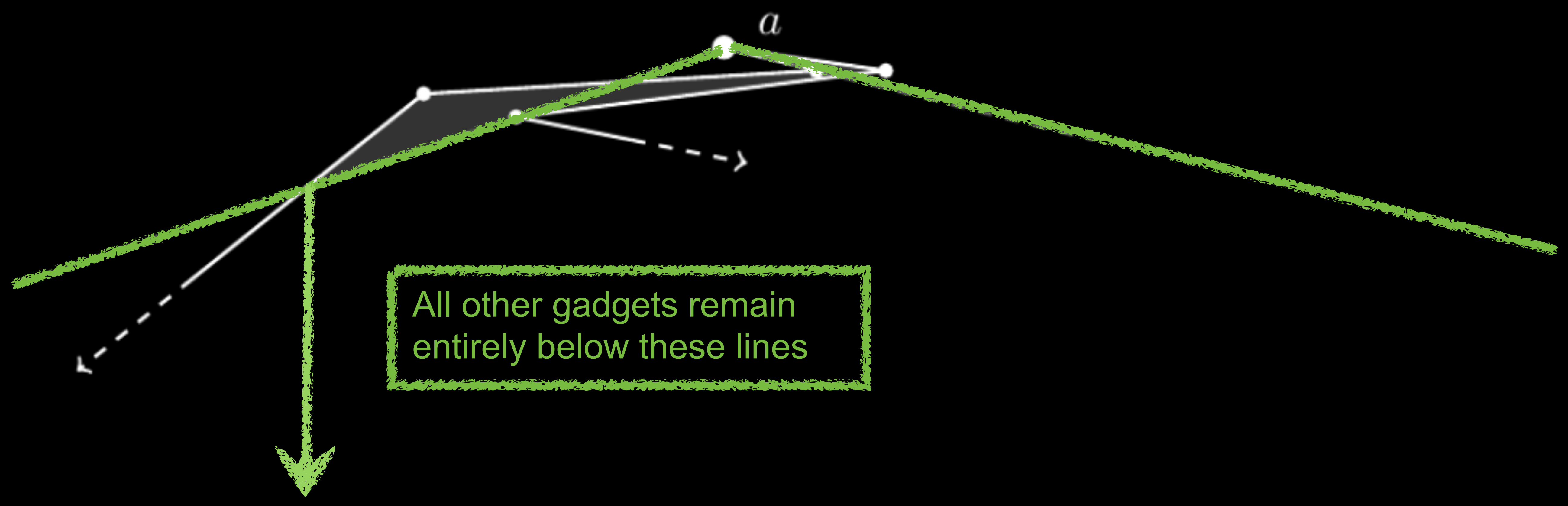

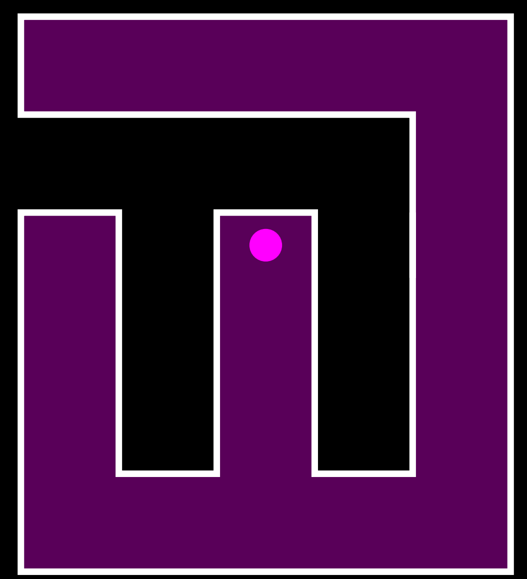

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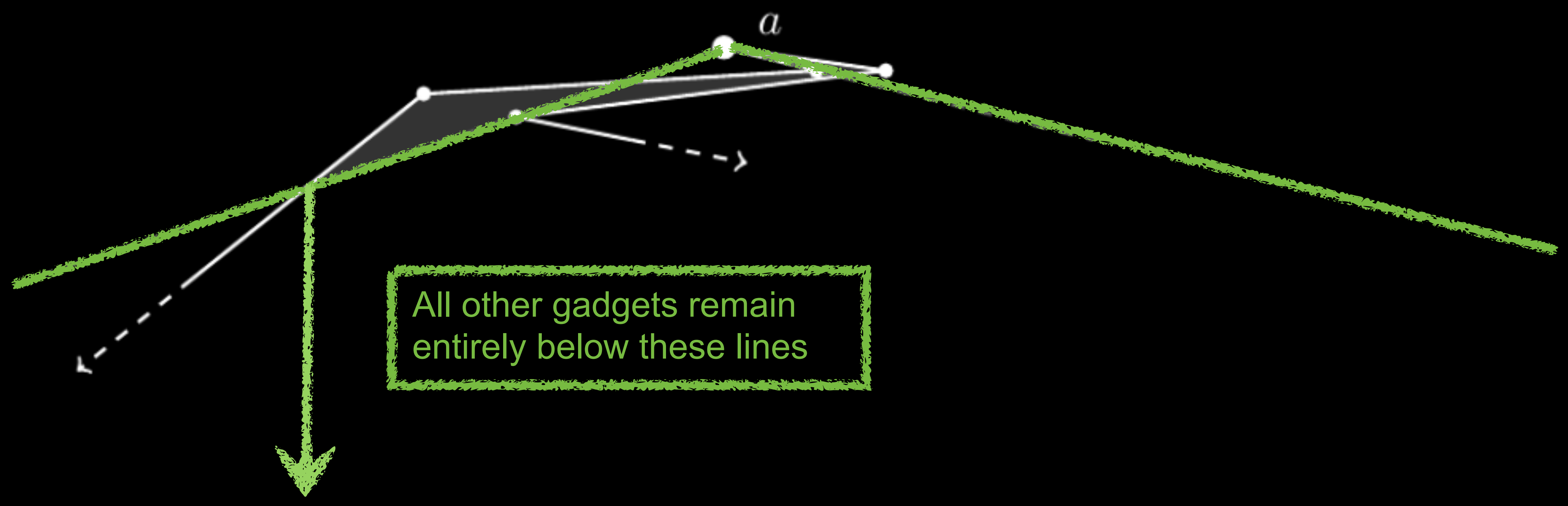

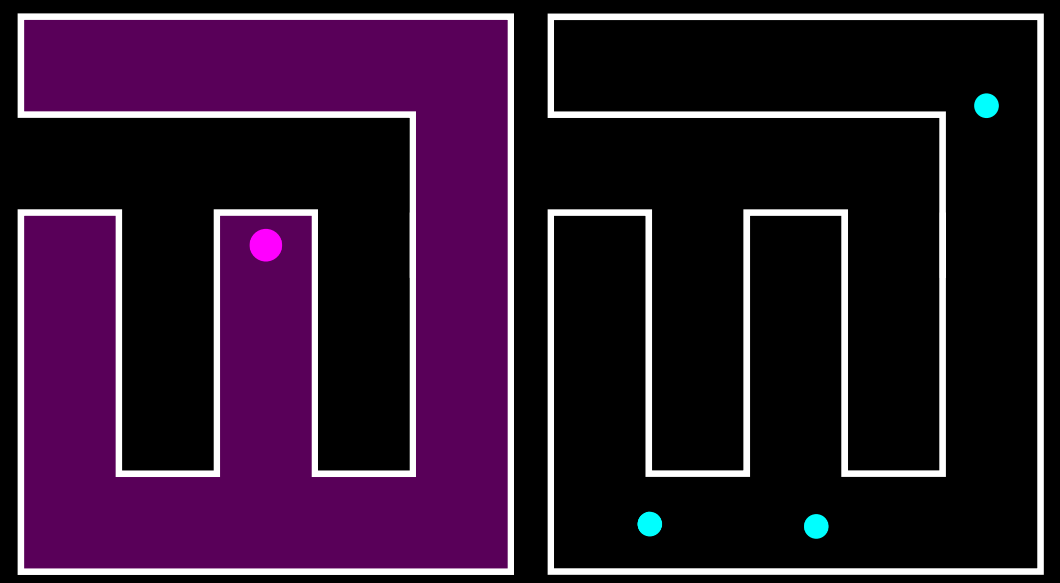

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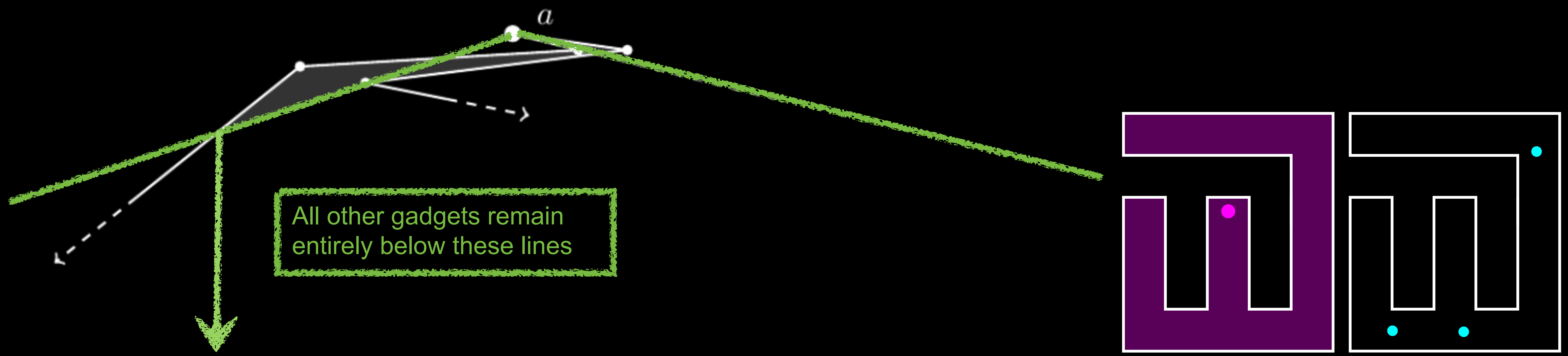

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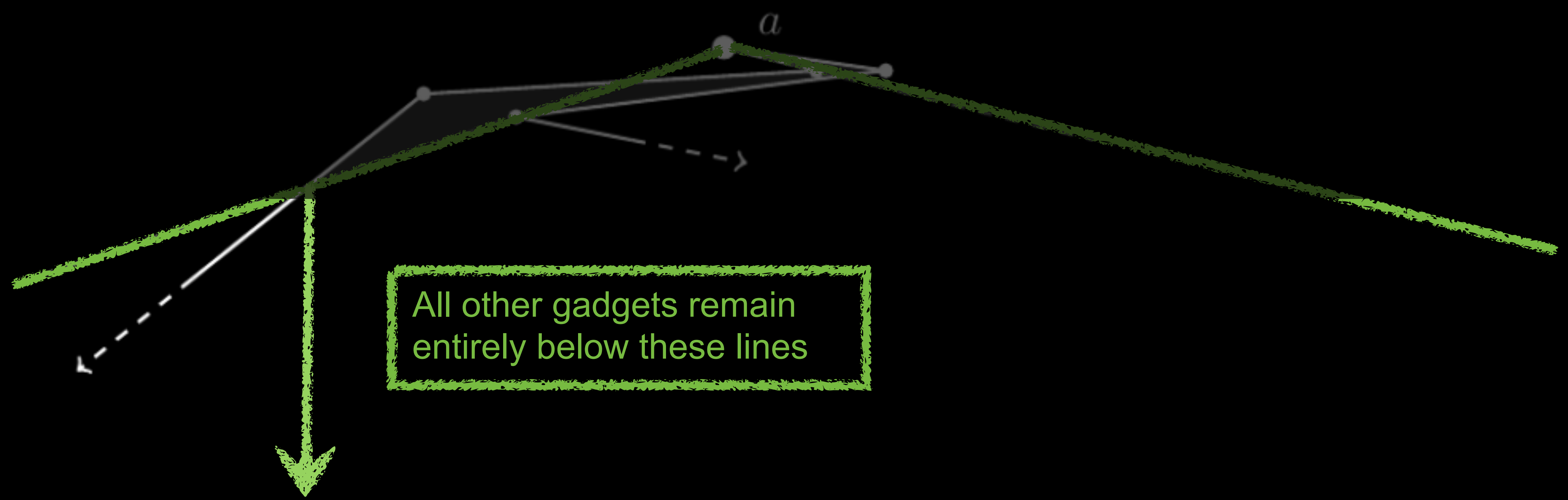

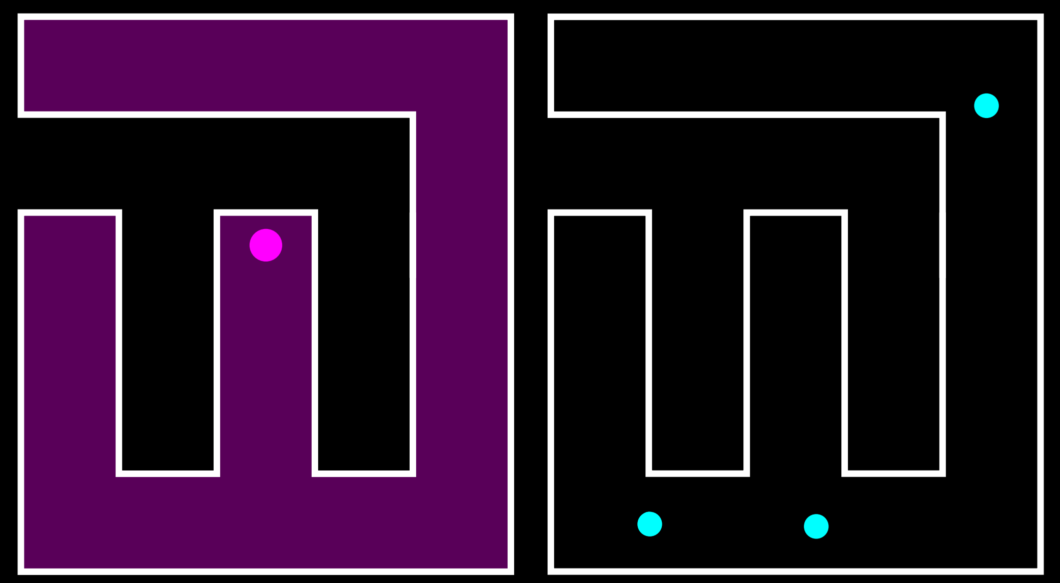

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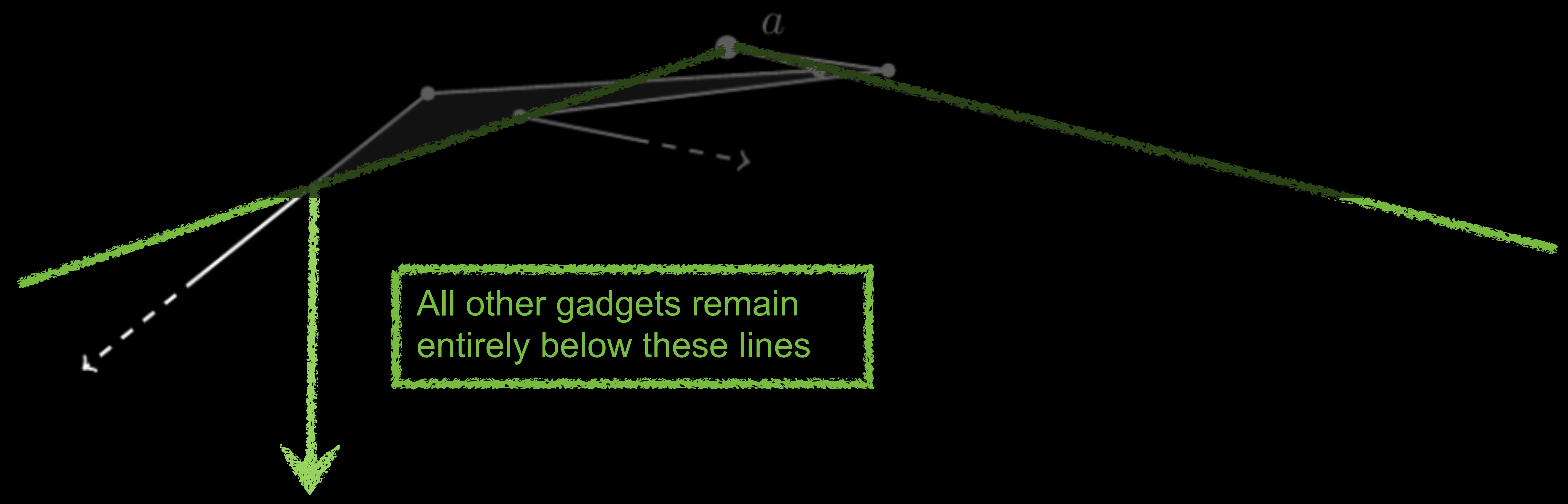

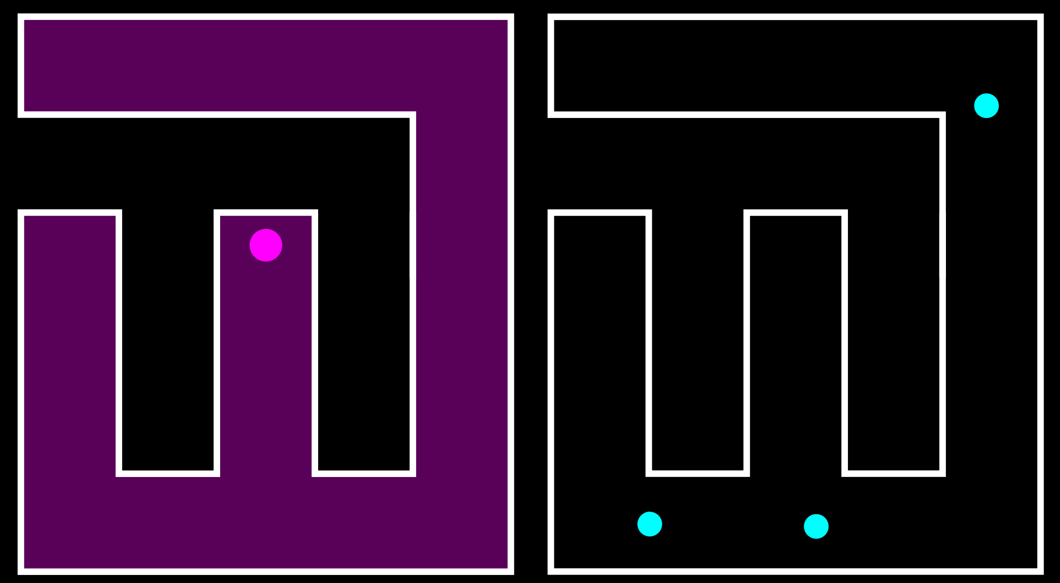

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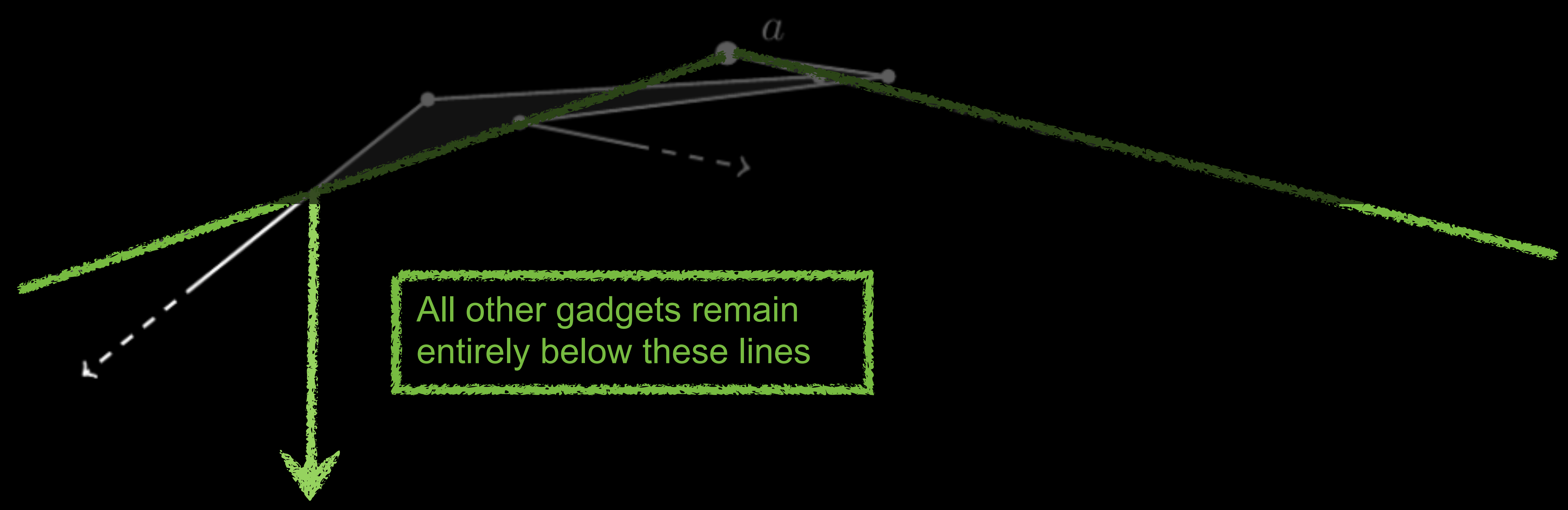

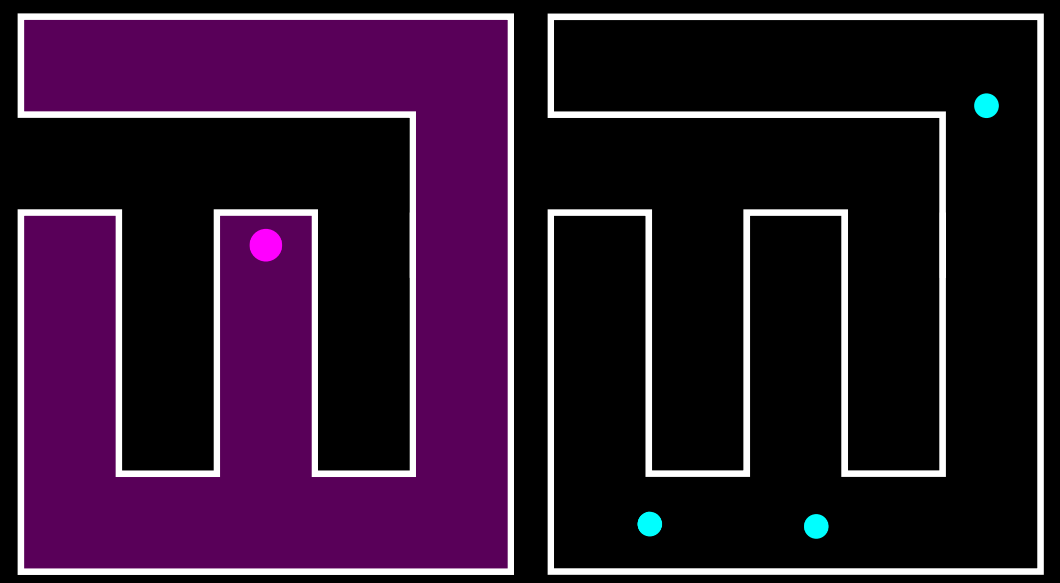

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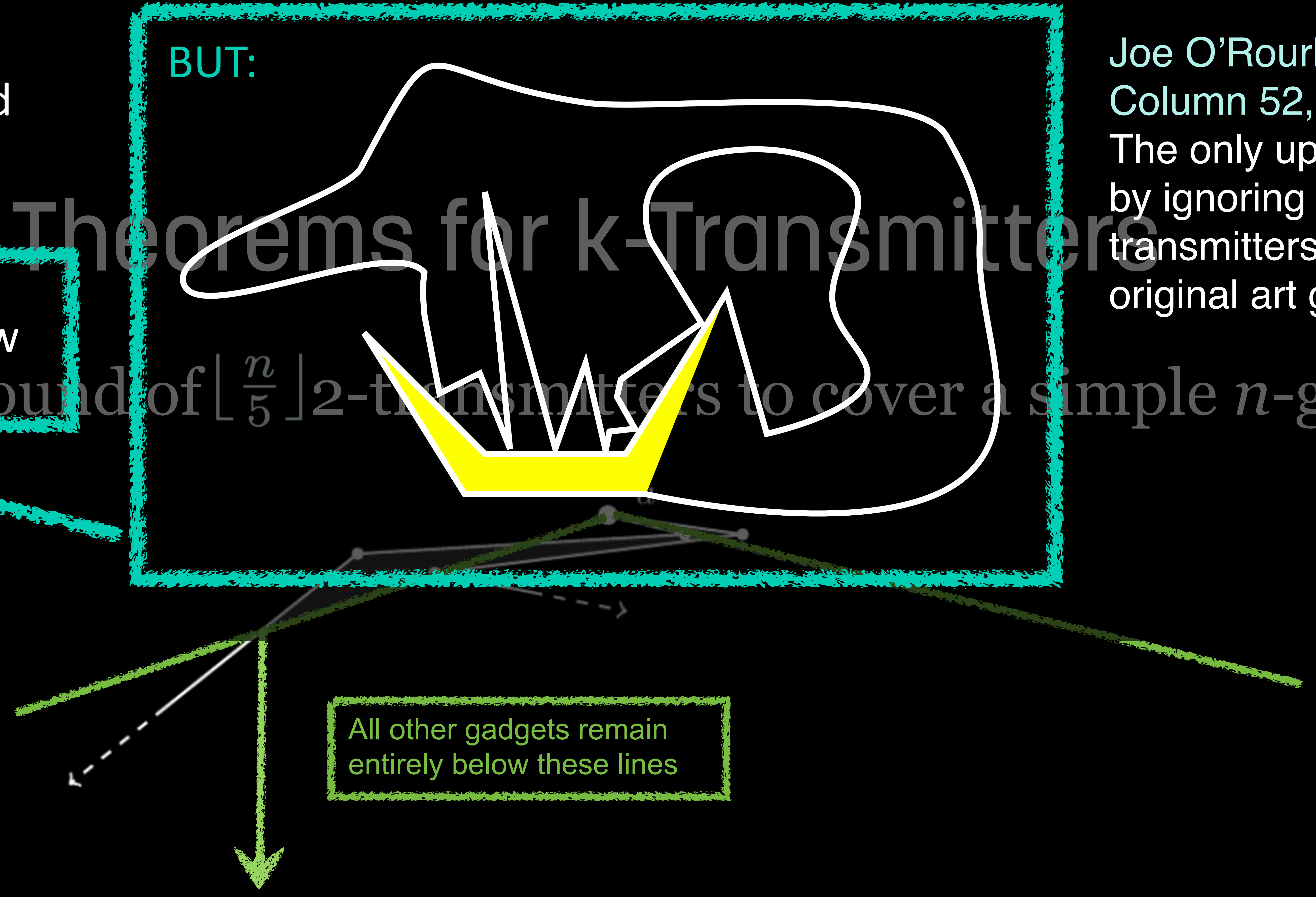

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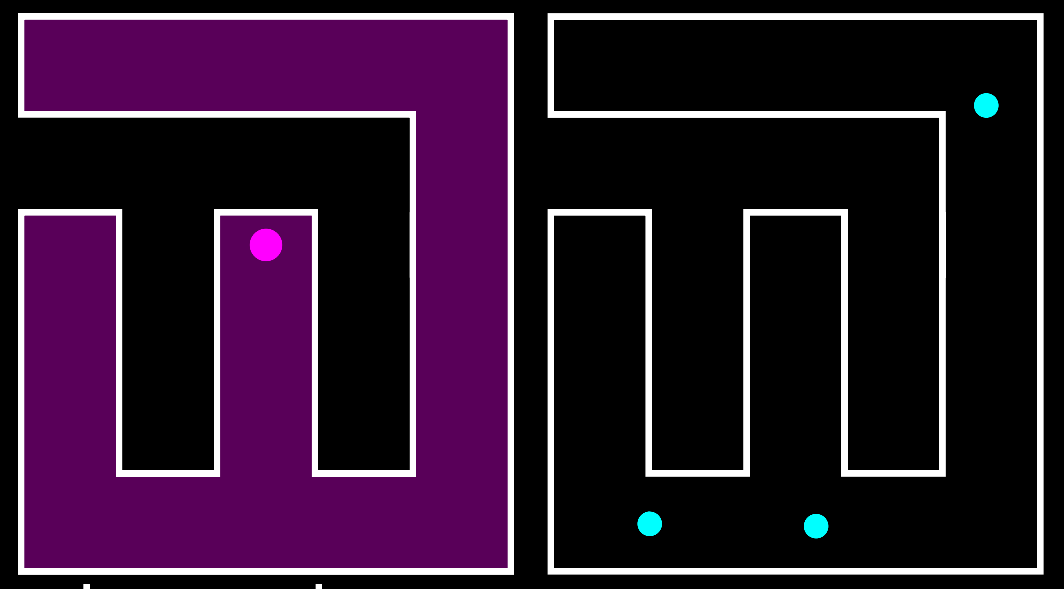

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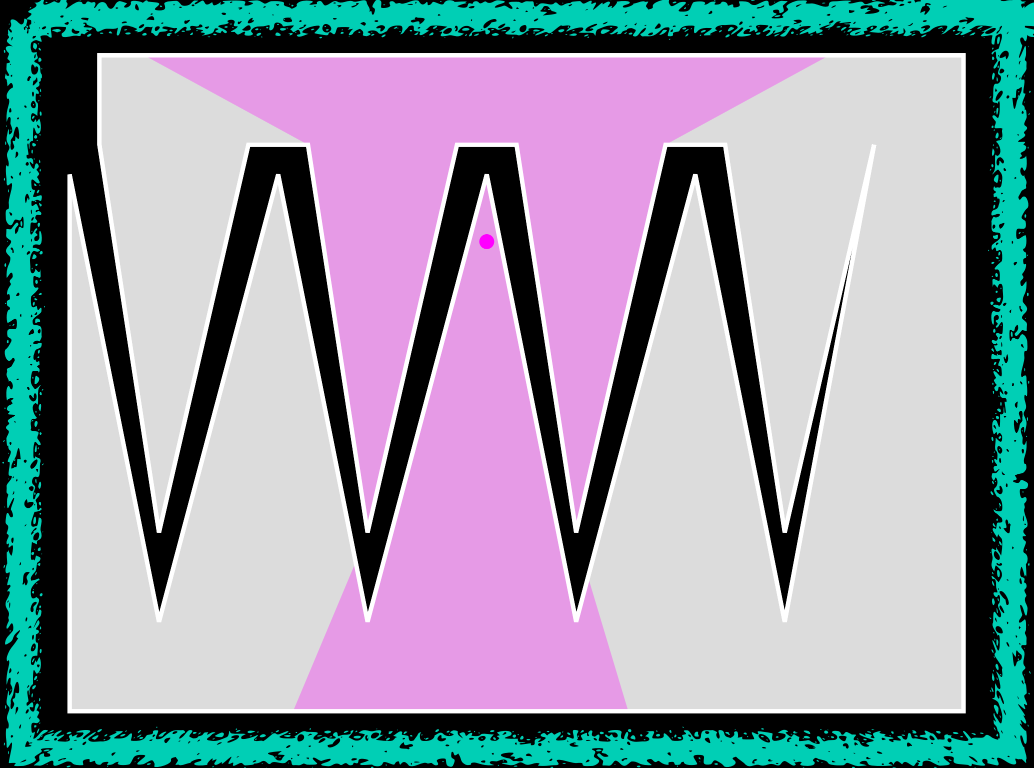
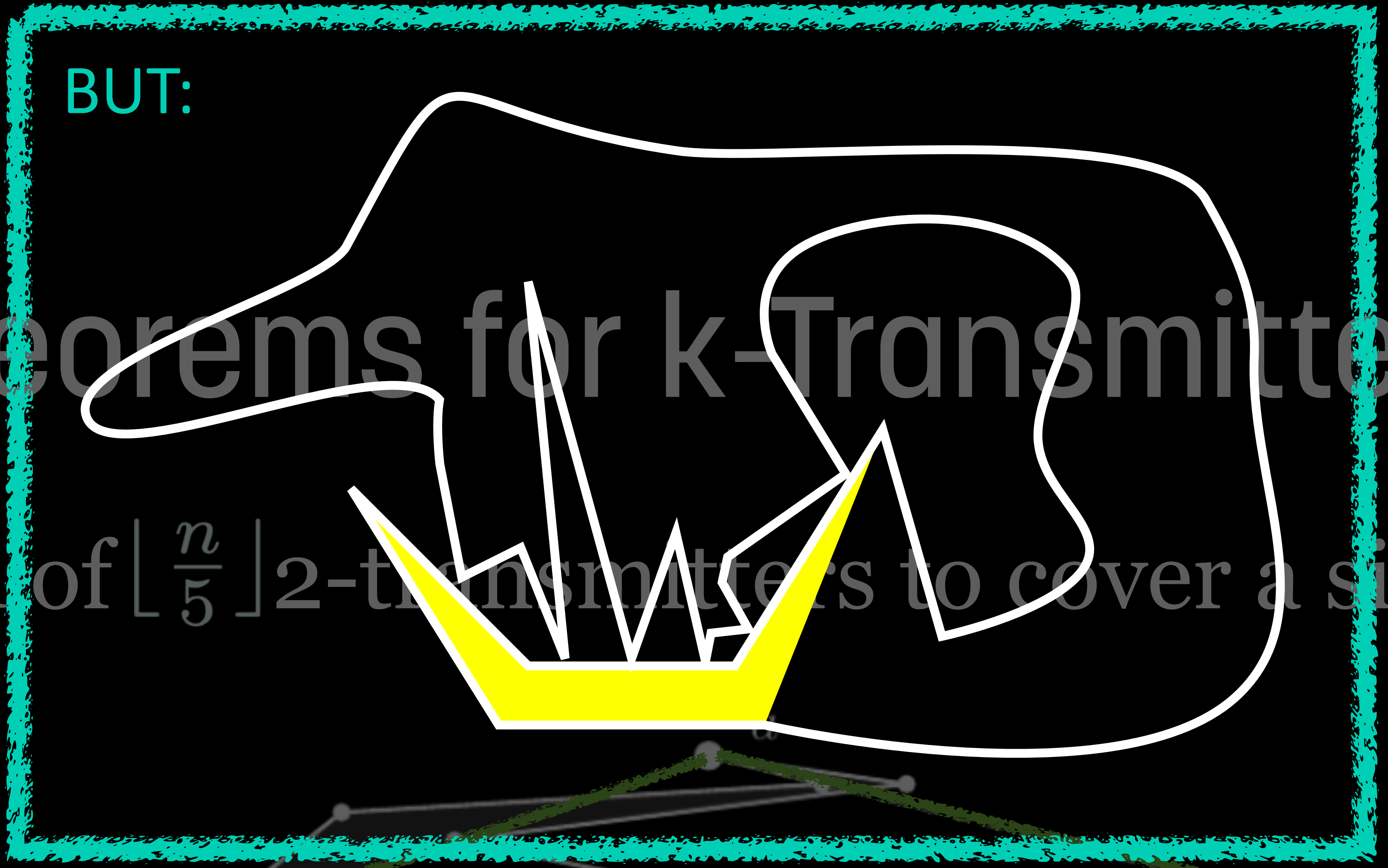
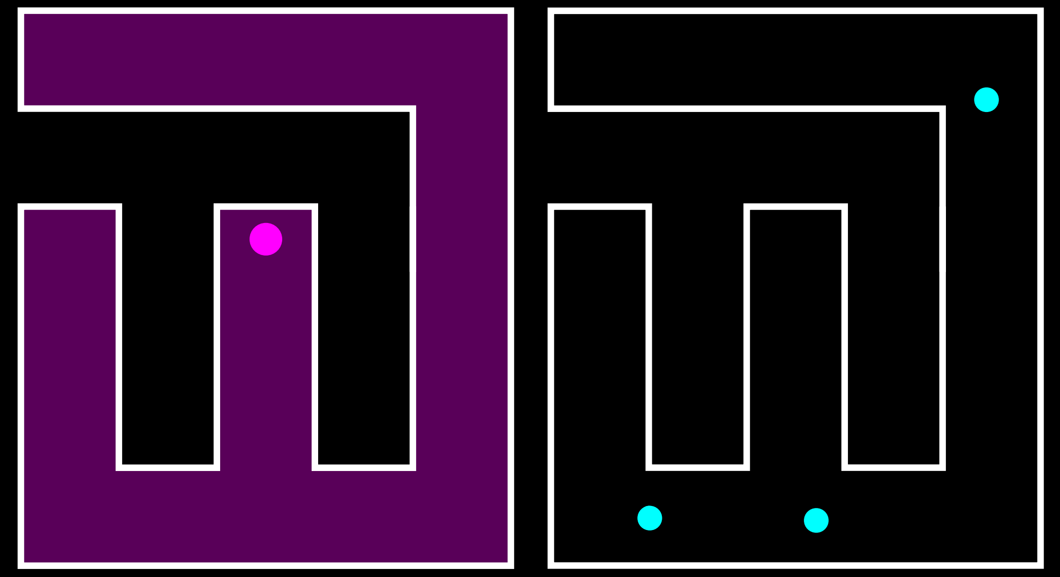




All other gadgets remain entirely below these lines



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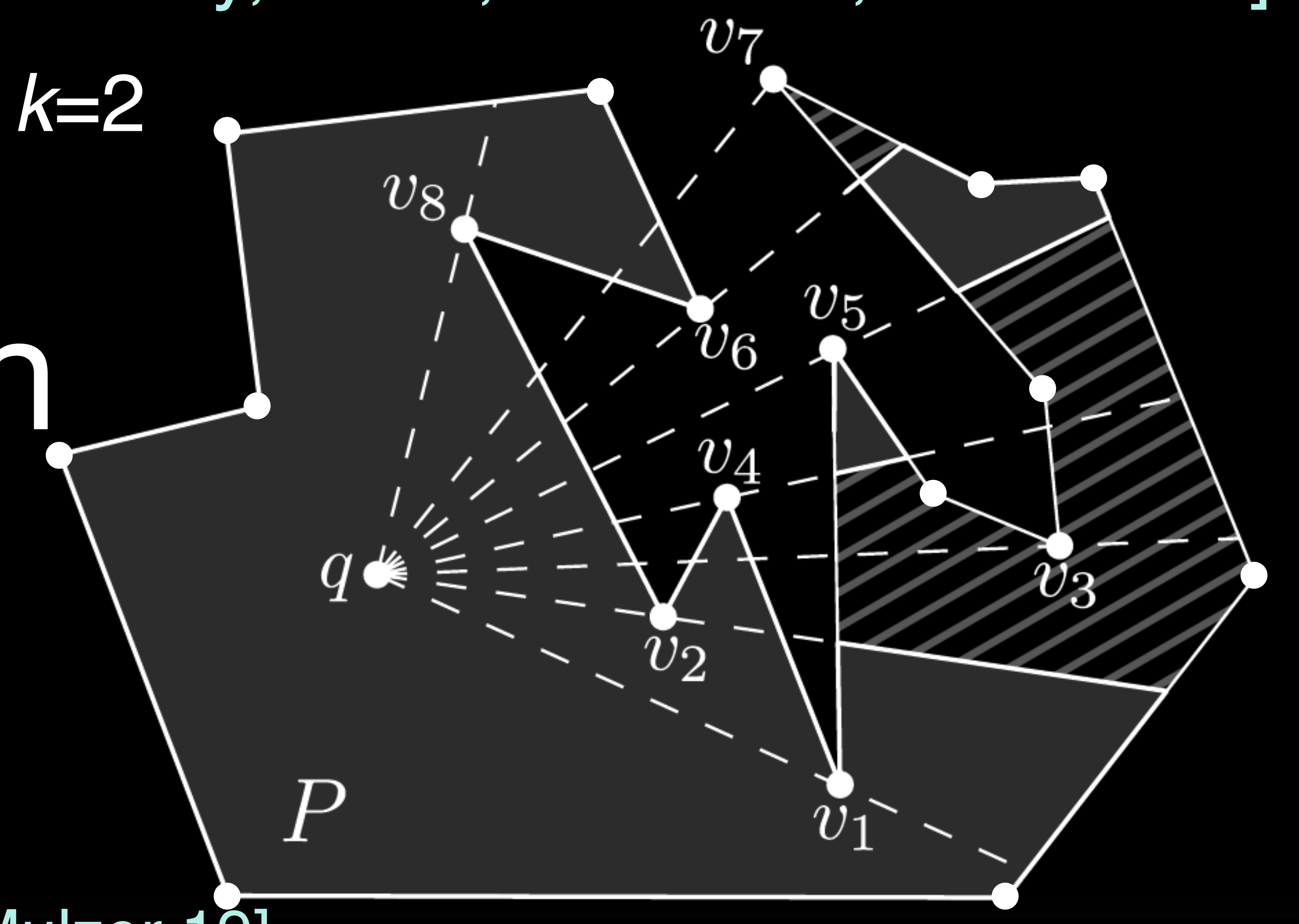

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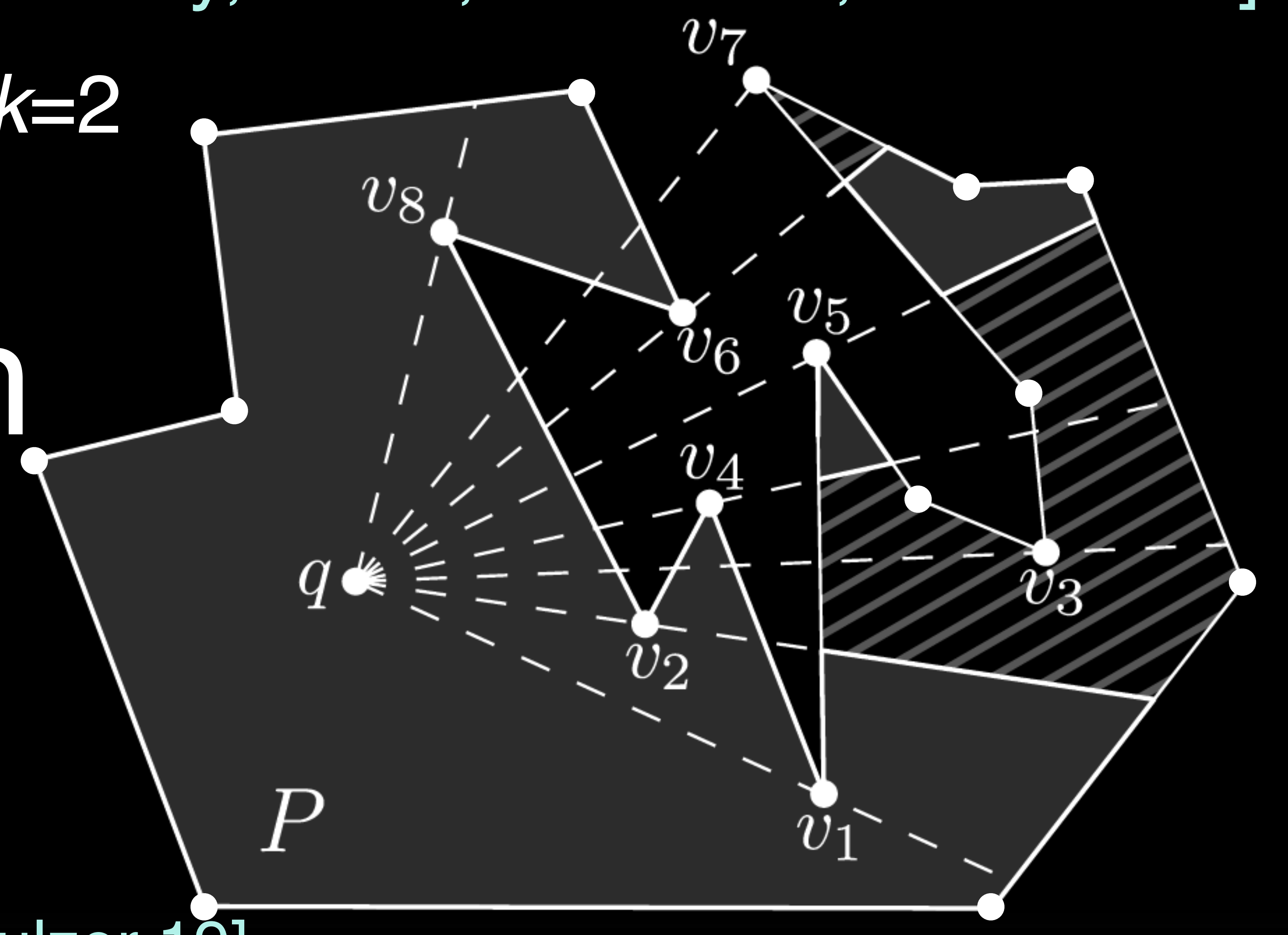

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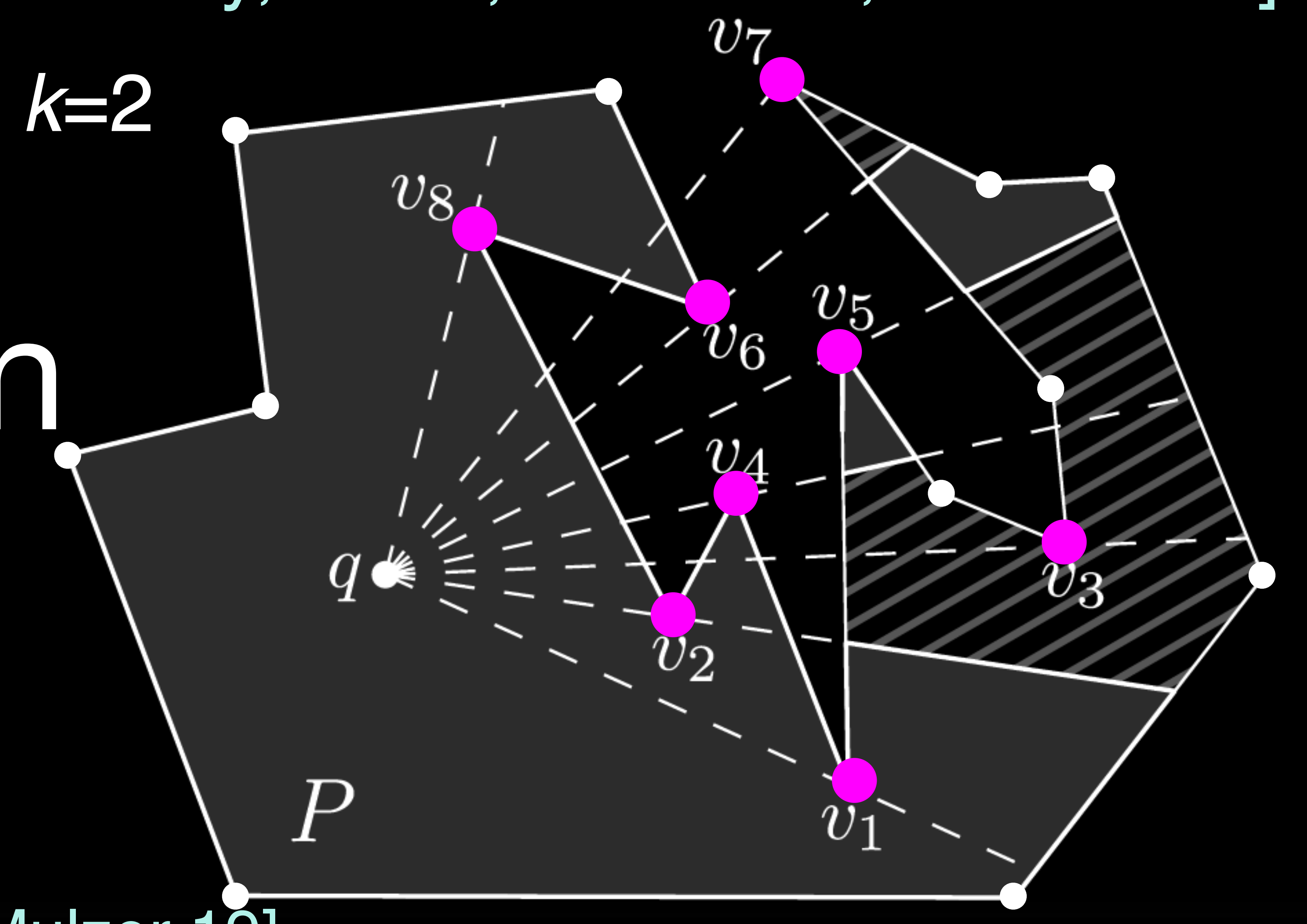

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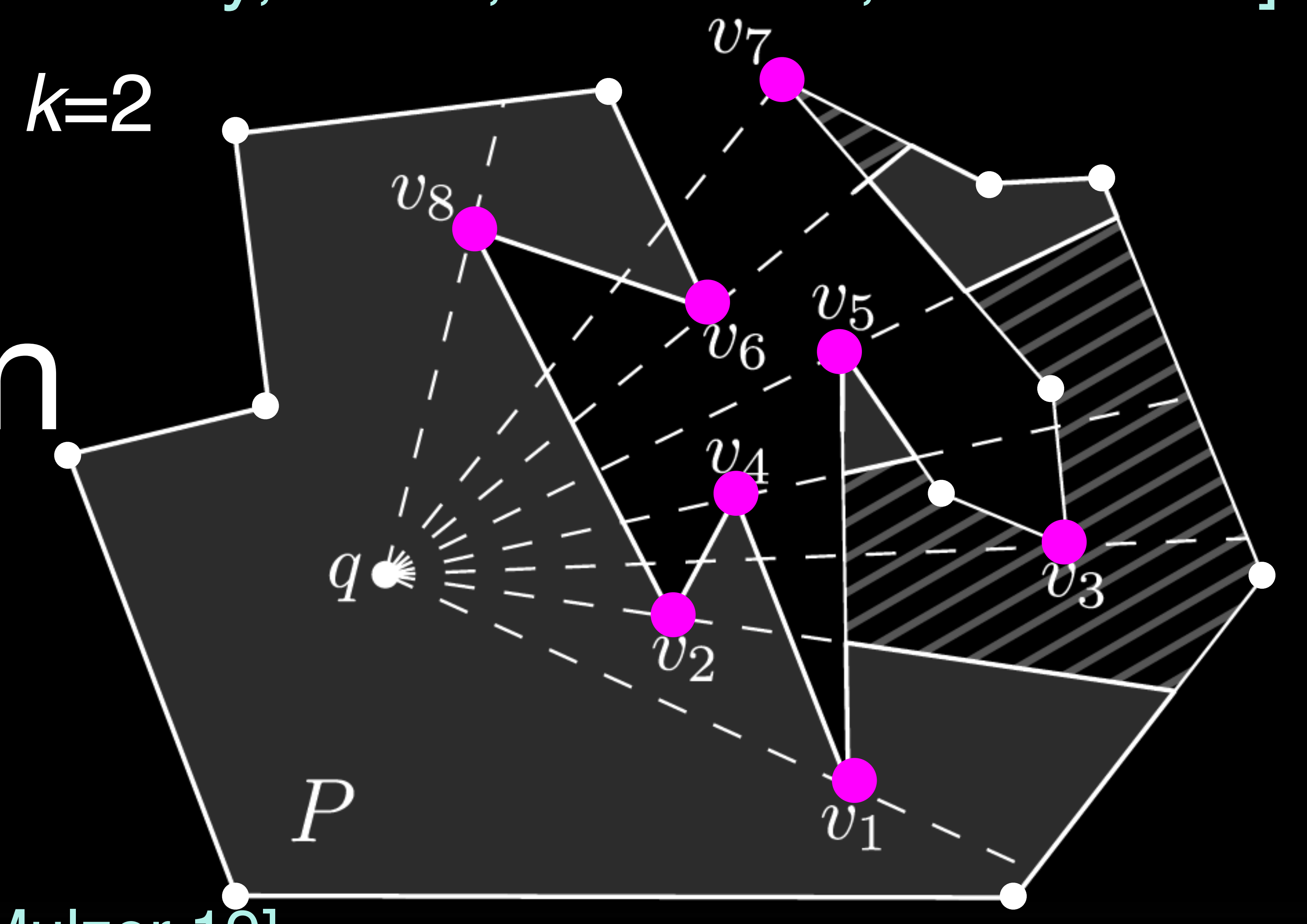

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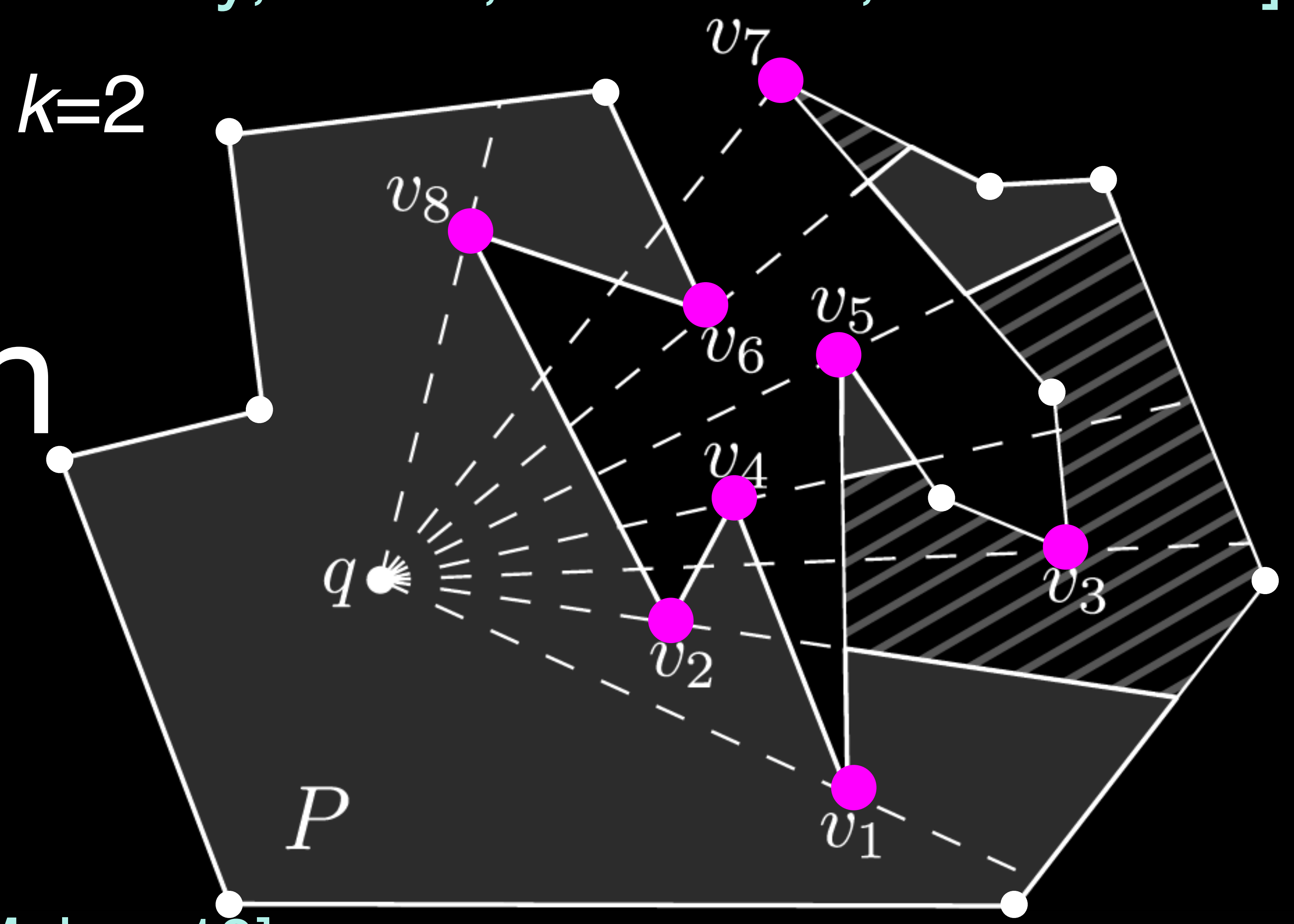

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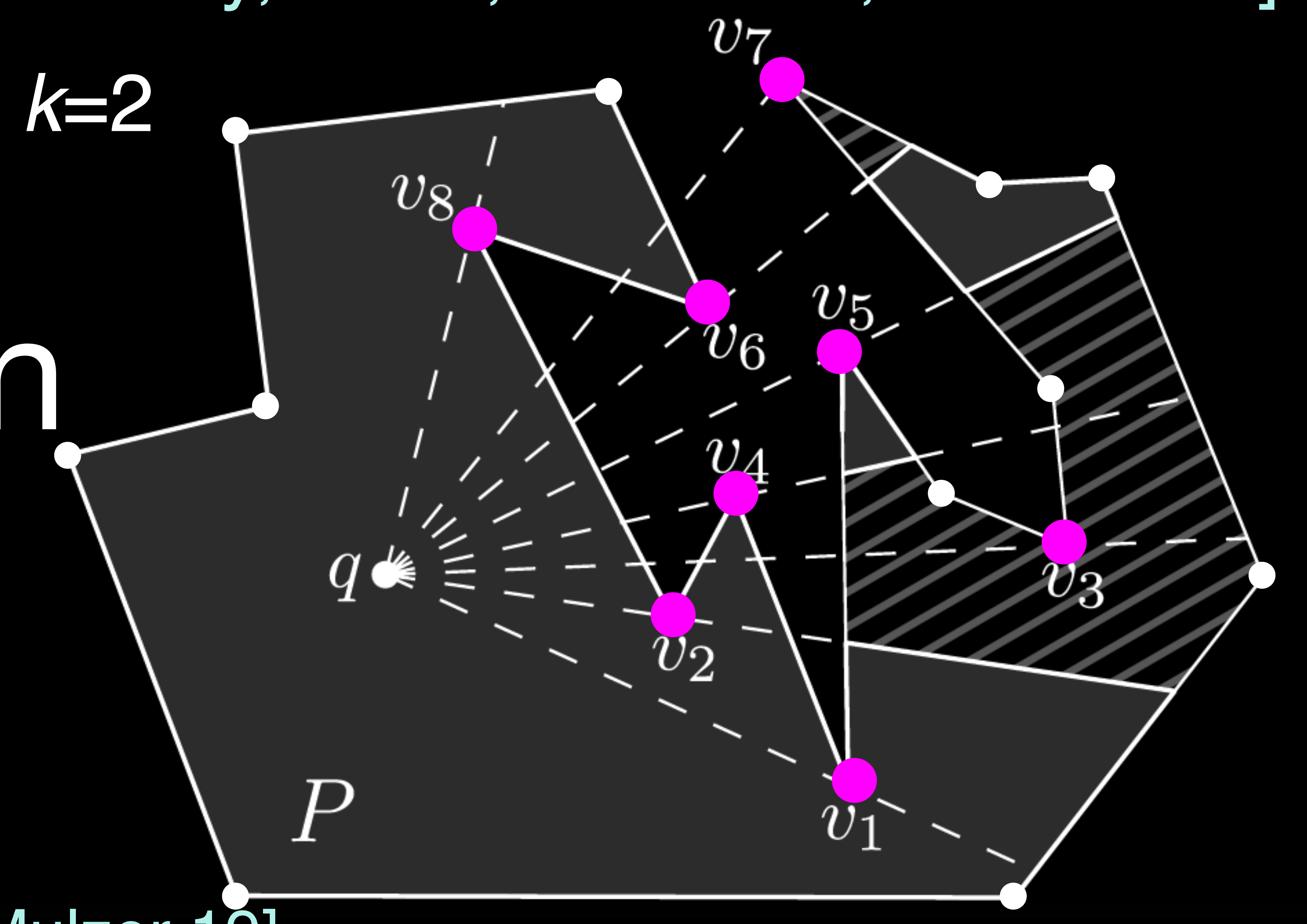

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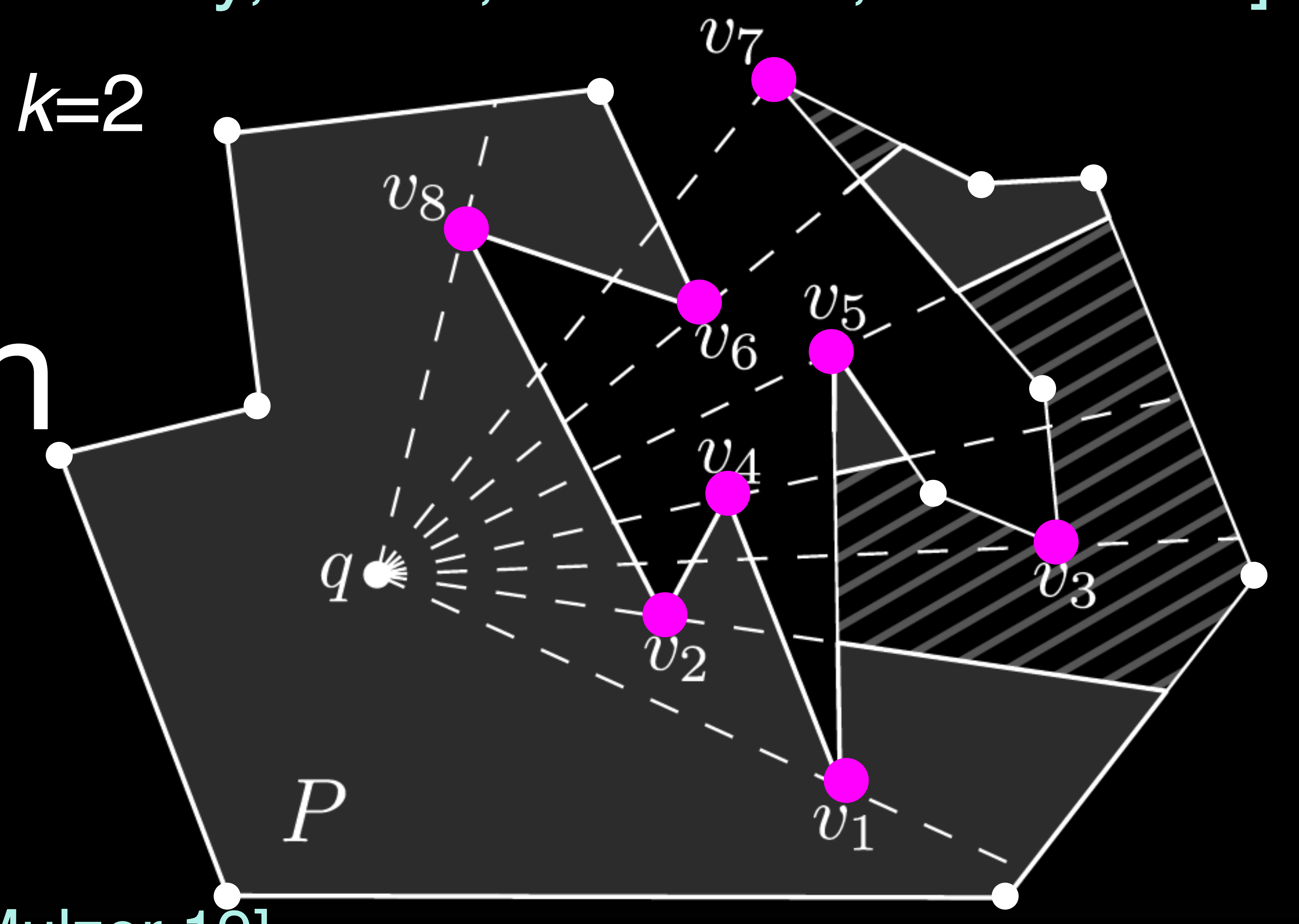

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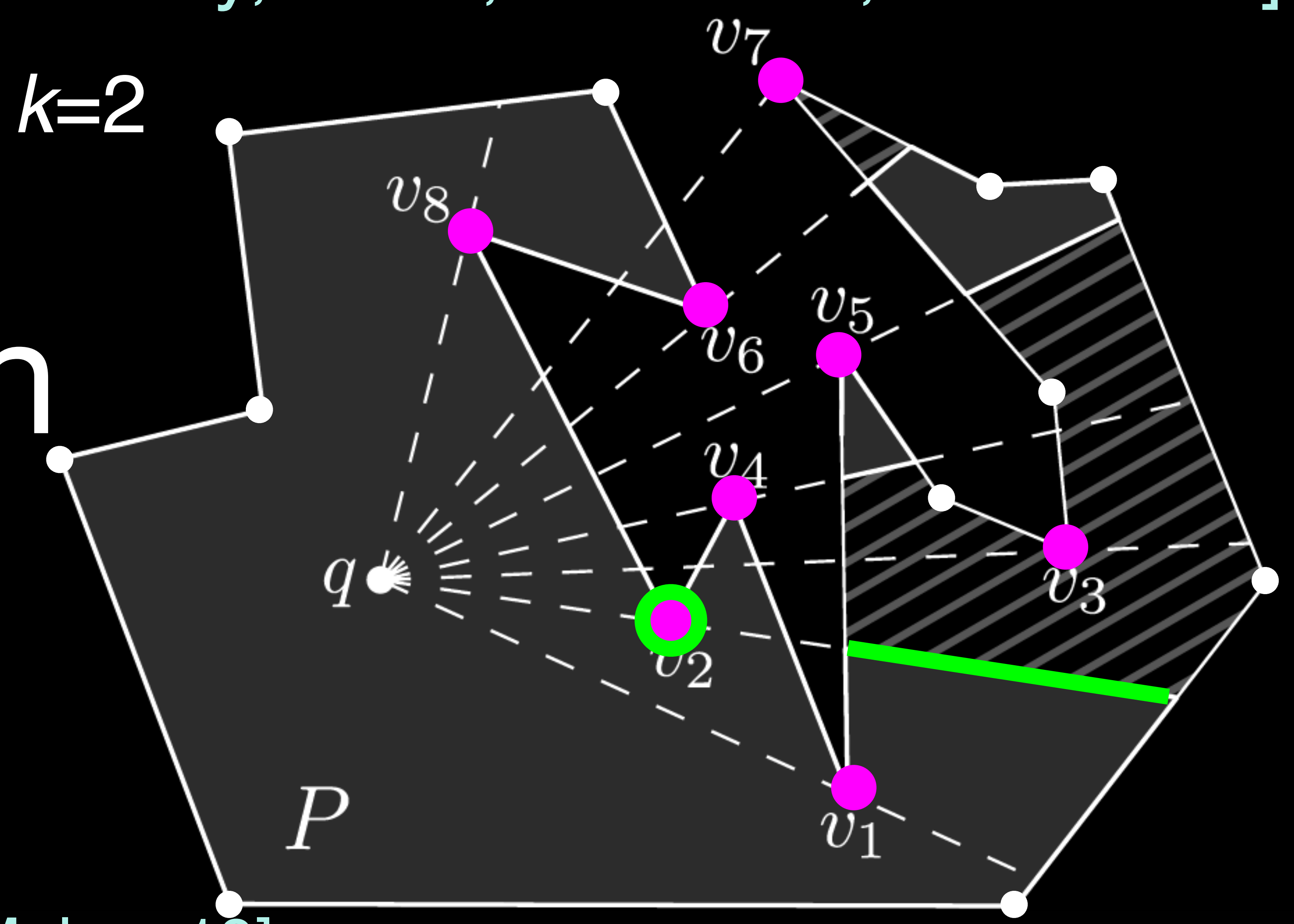

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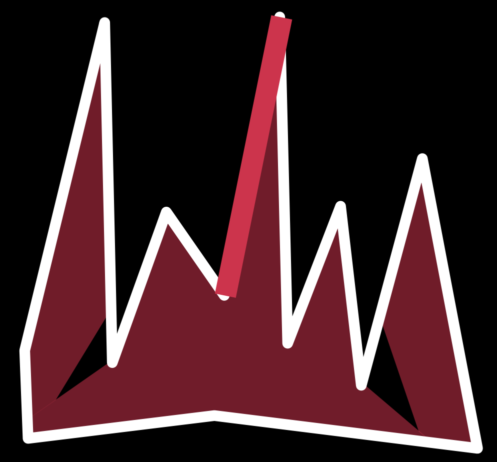

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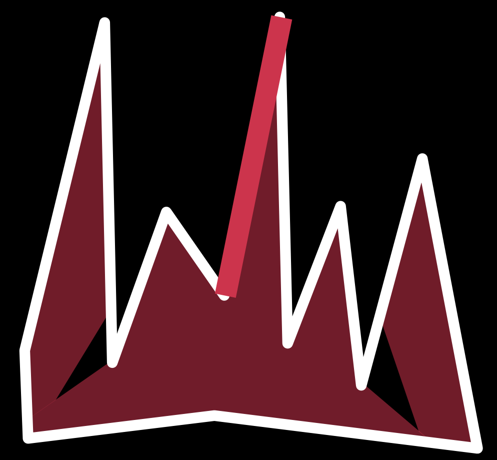

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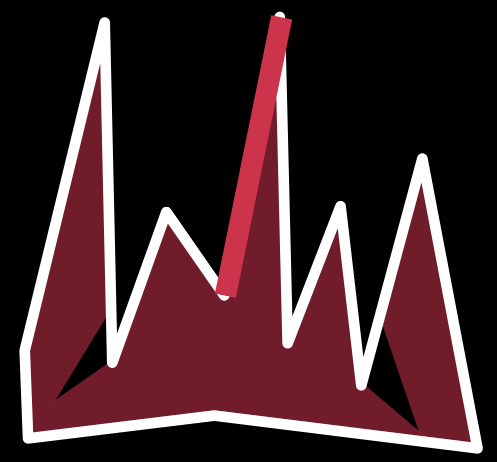

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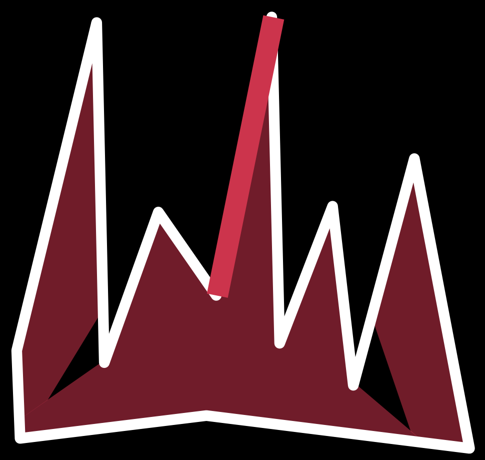

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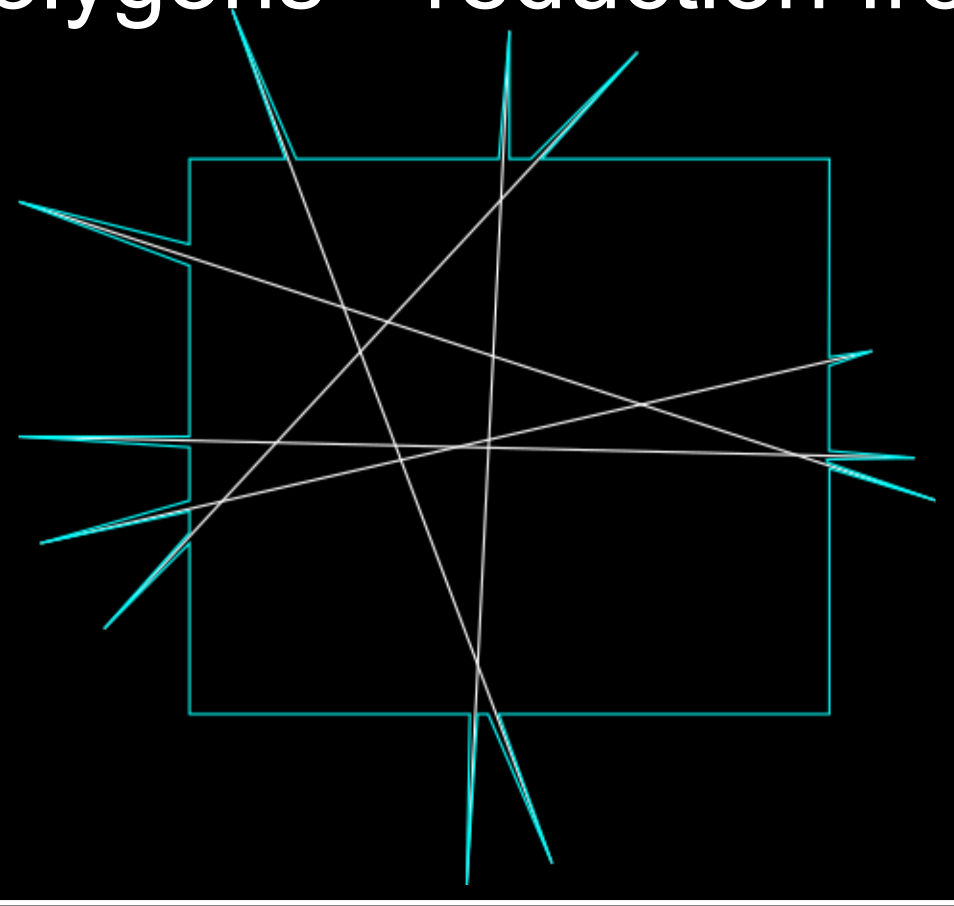

Usual spike box

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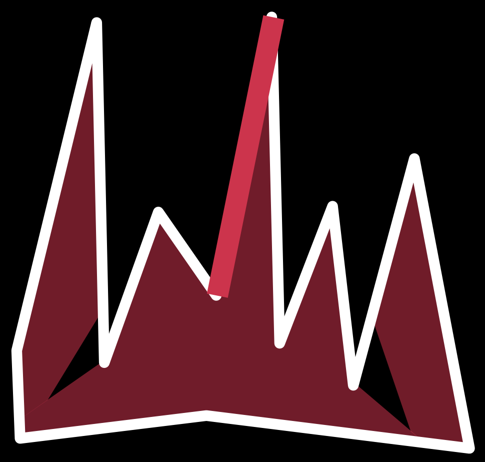

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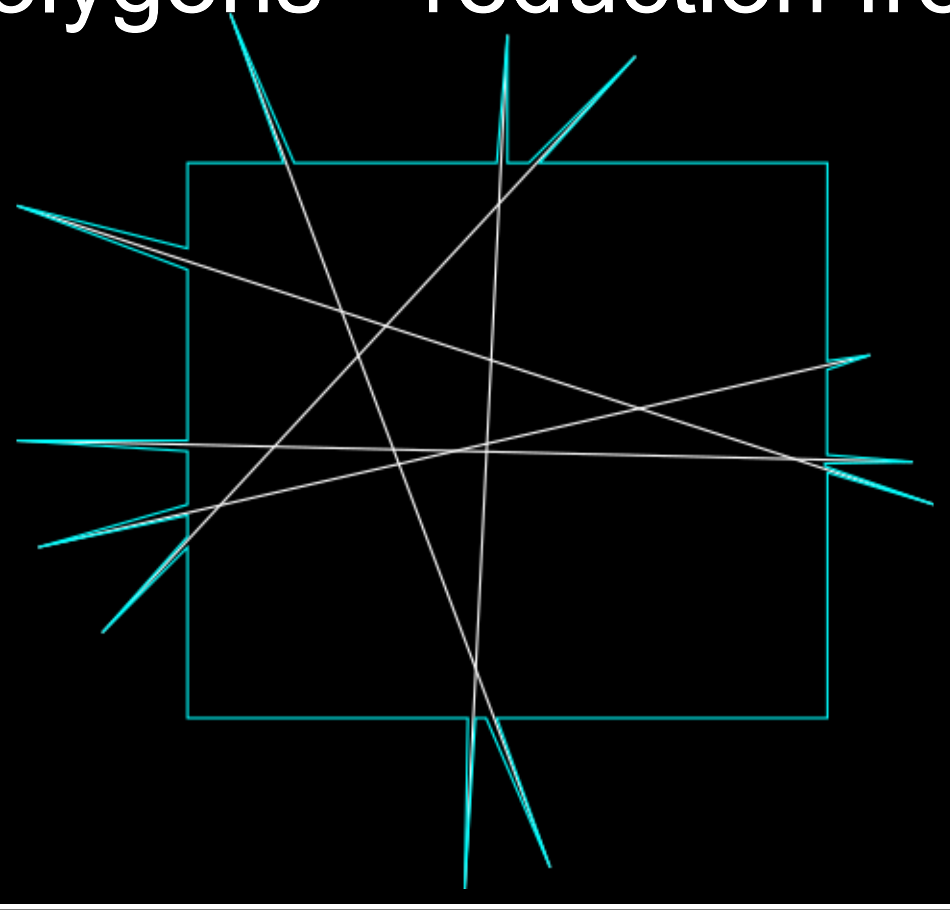
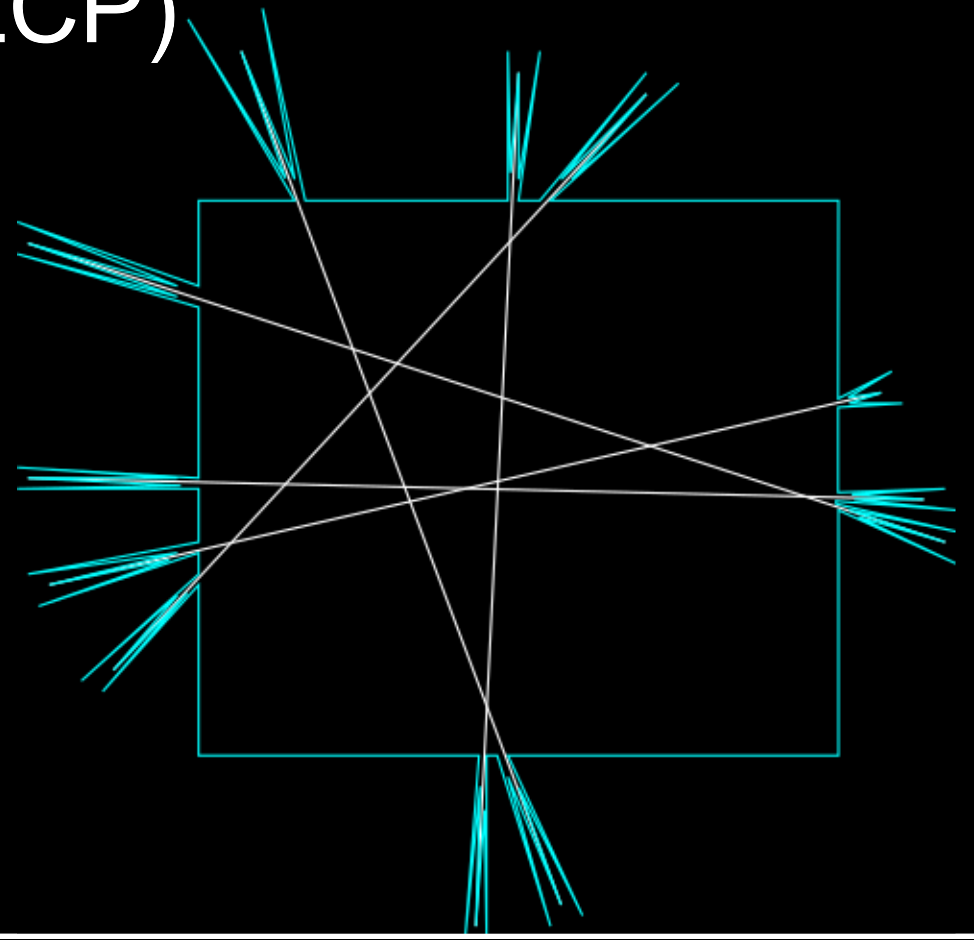


Usual spike box
➜ Modify slightly by adding “crowns”



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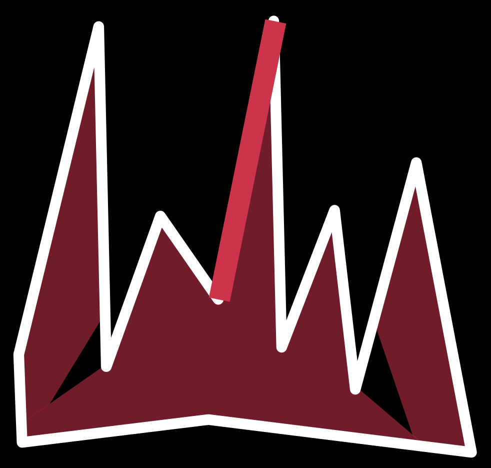

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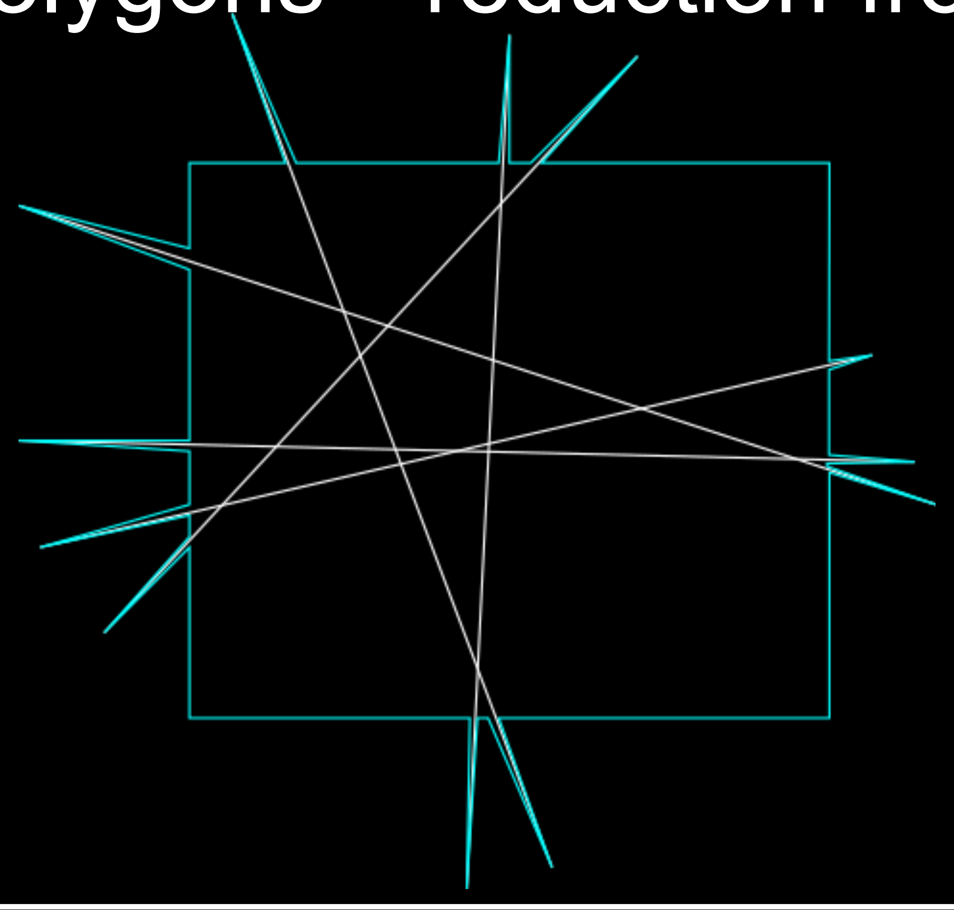

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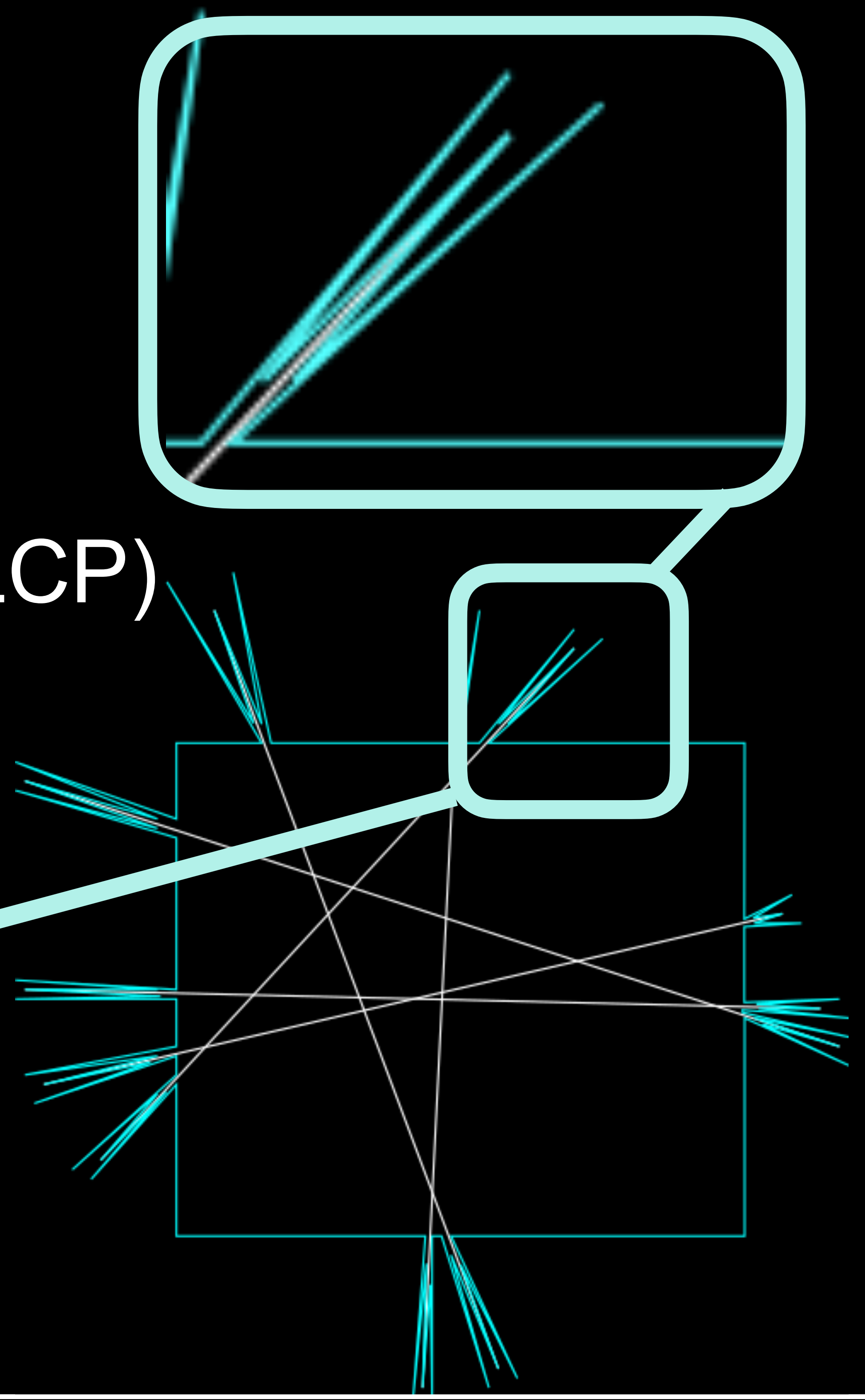

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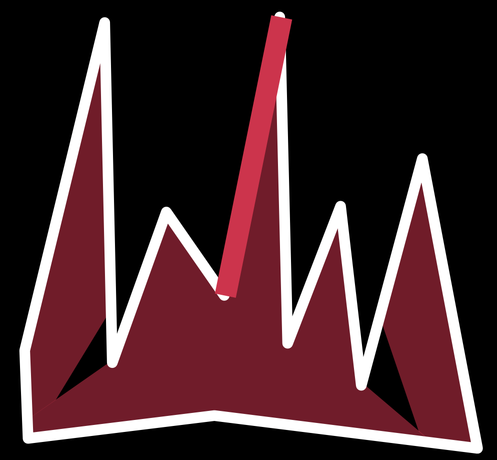

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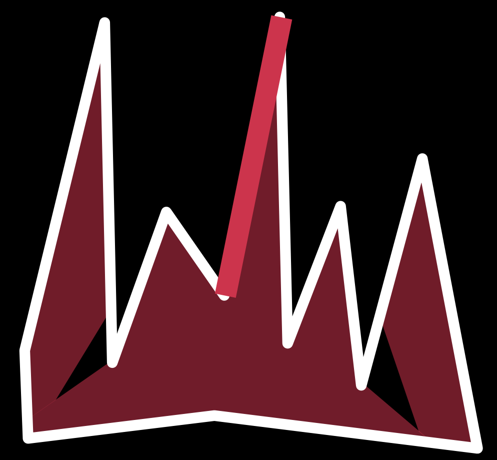

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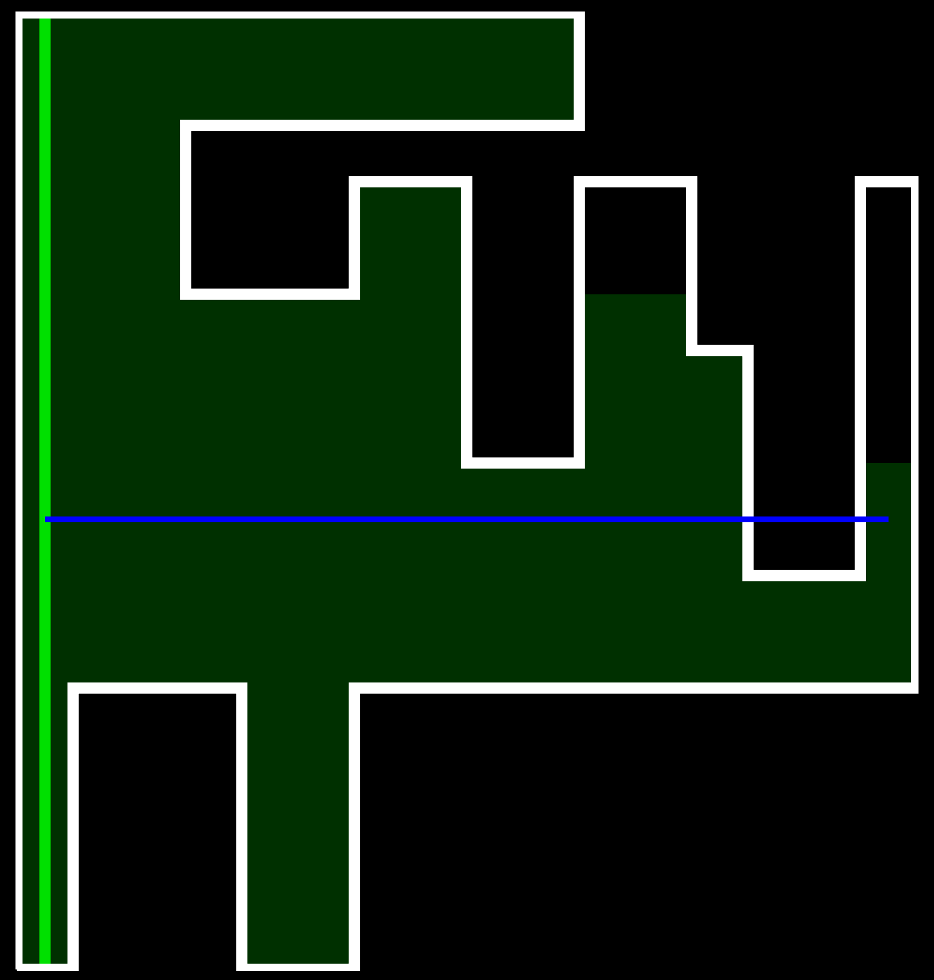

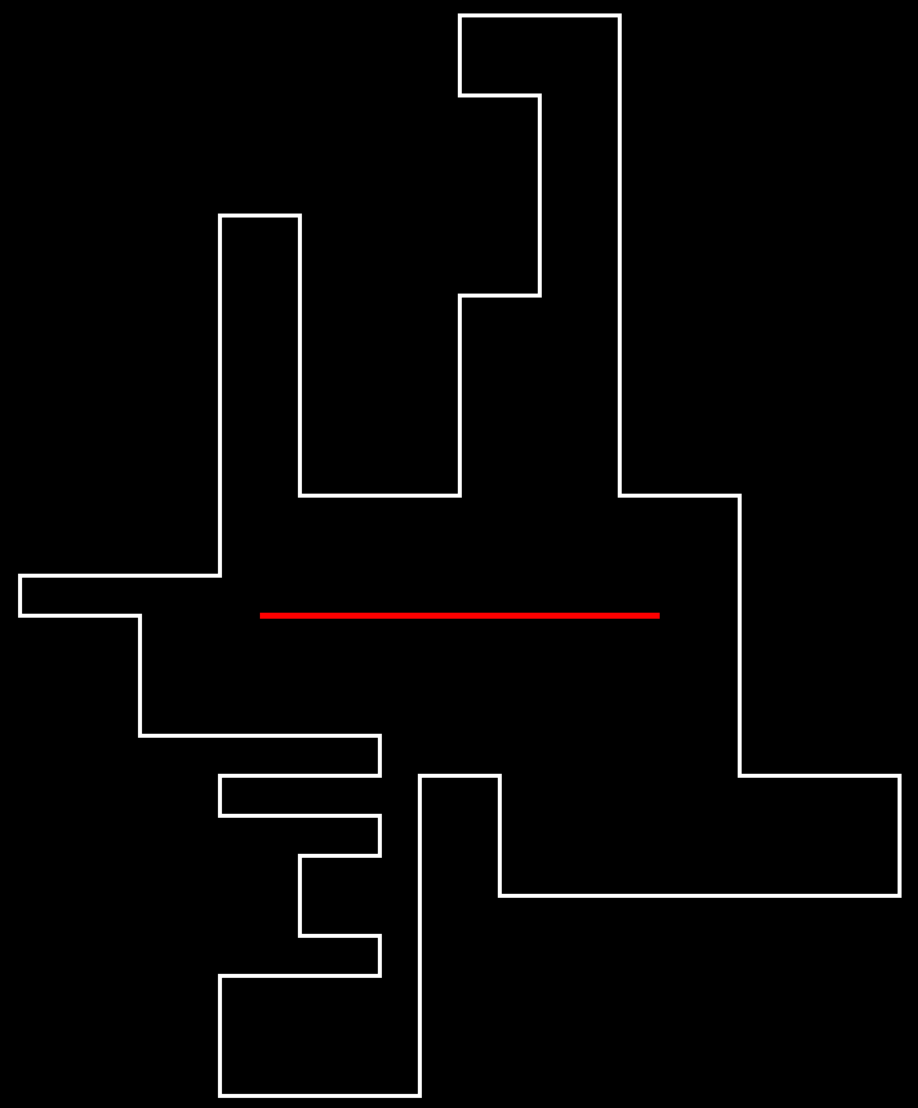

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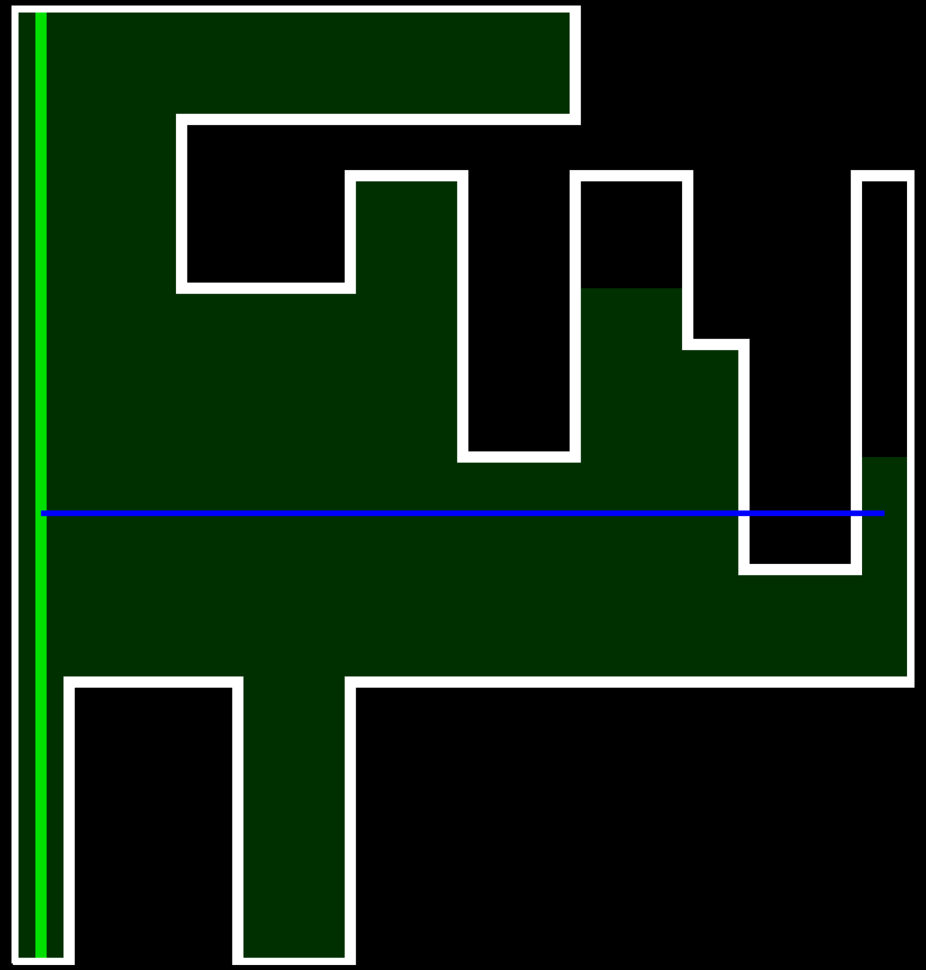

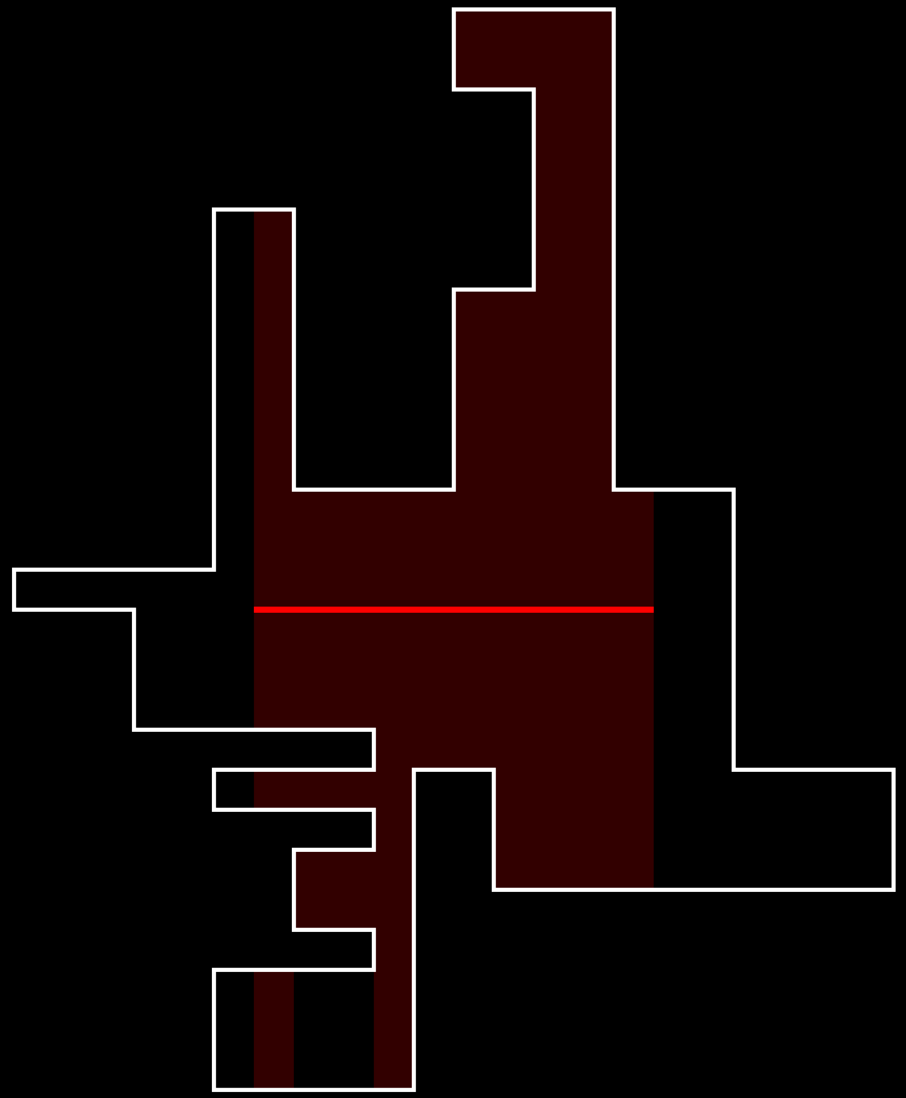

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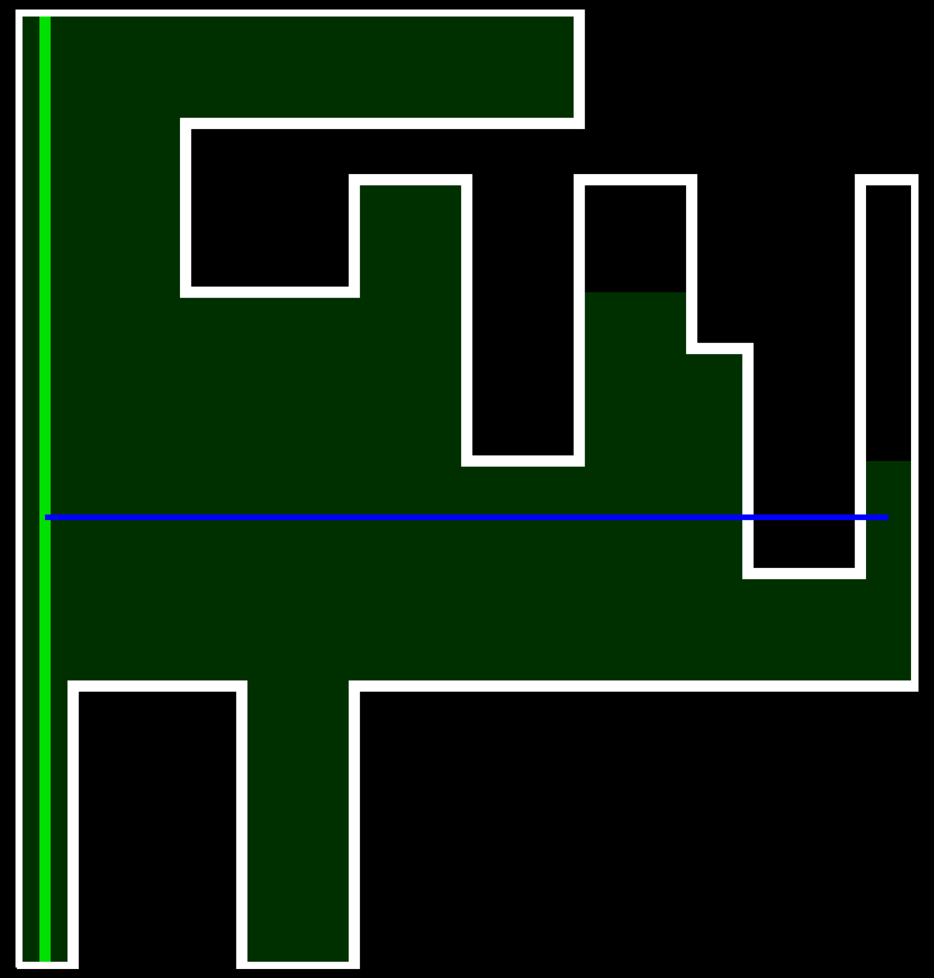

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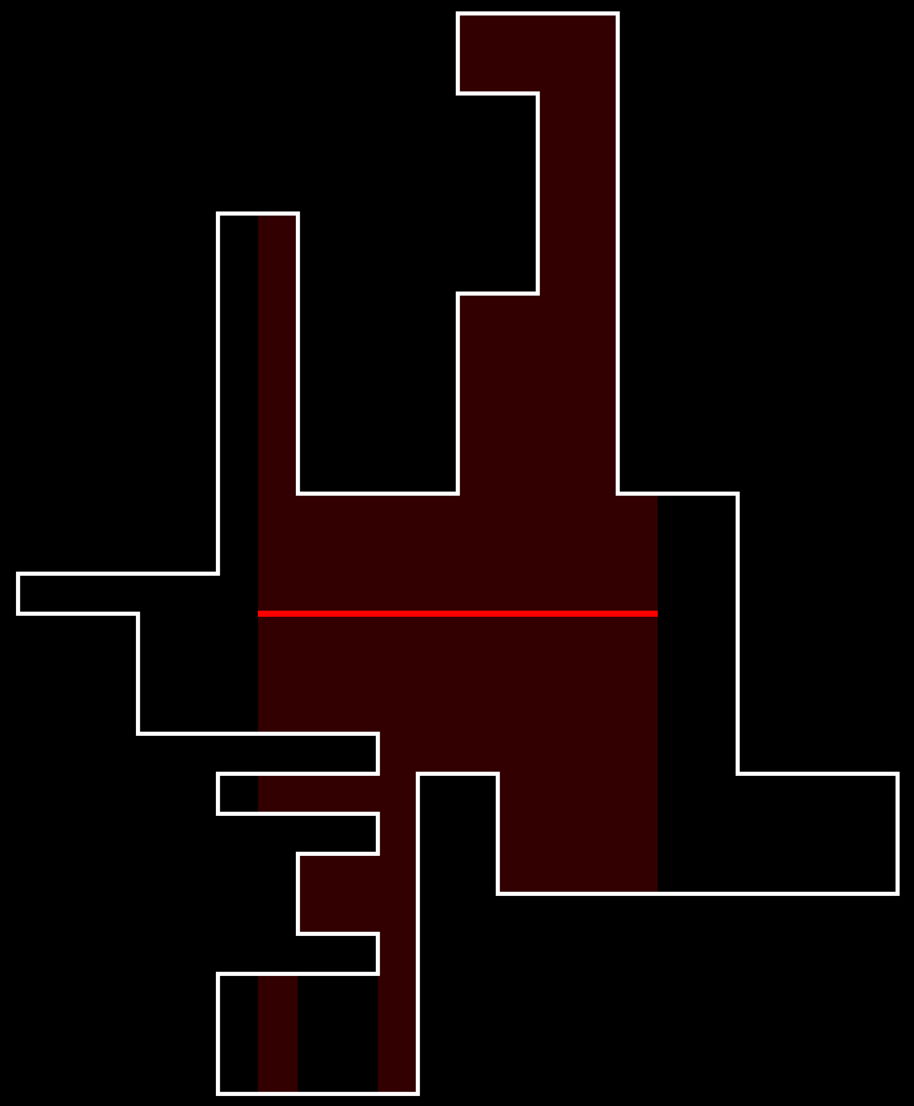

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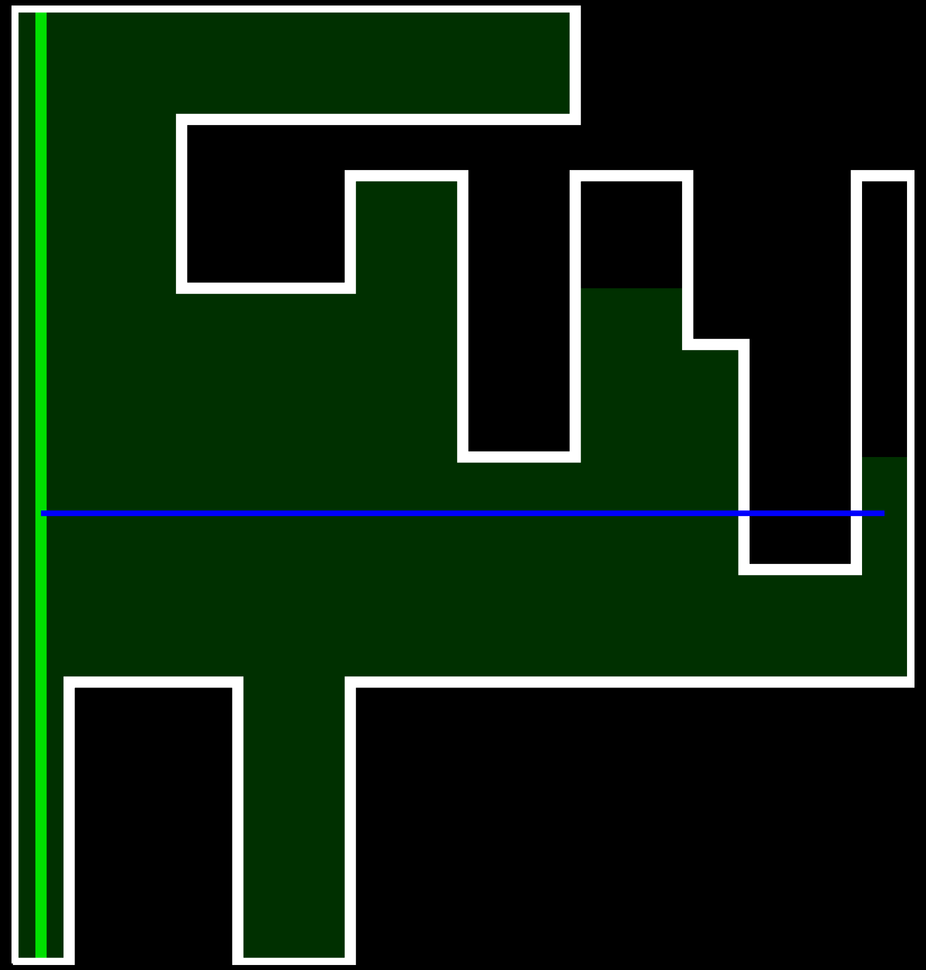

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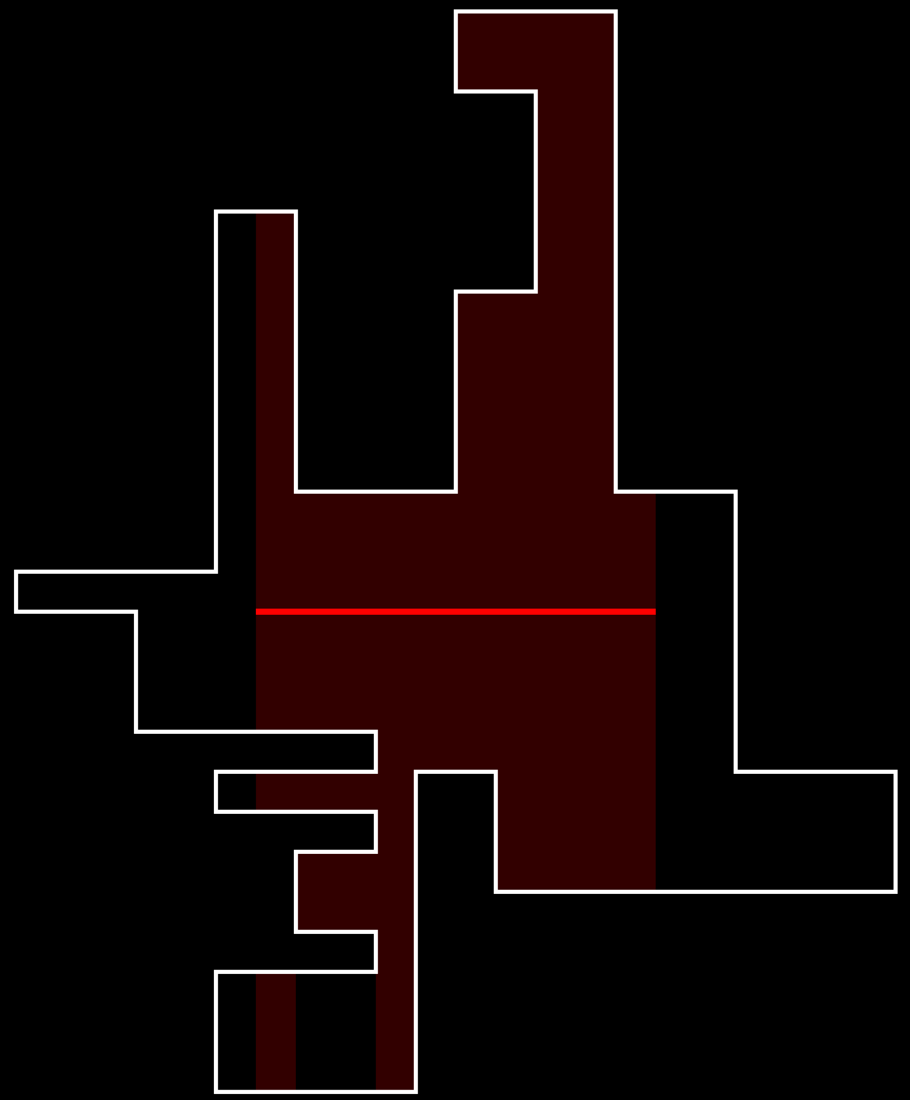

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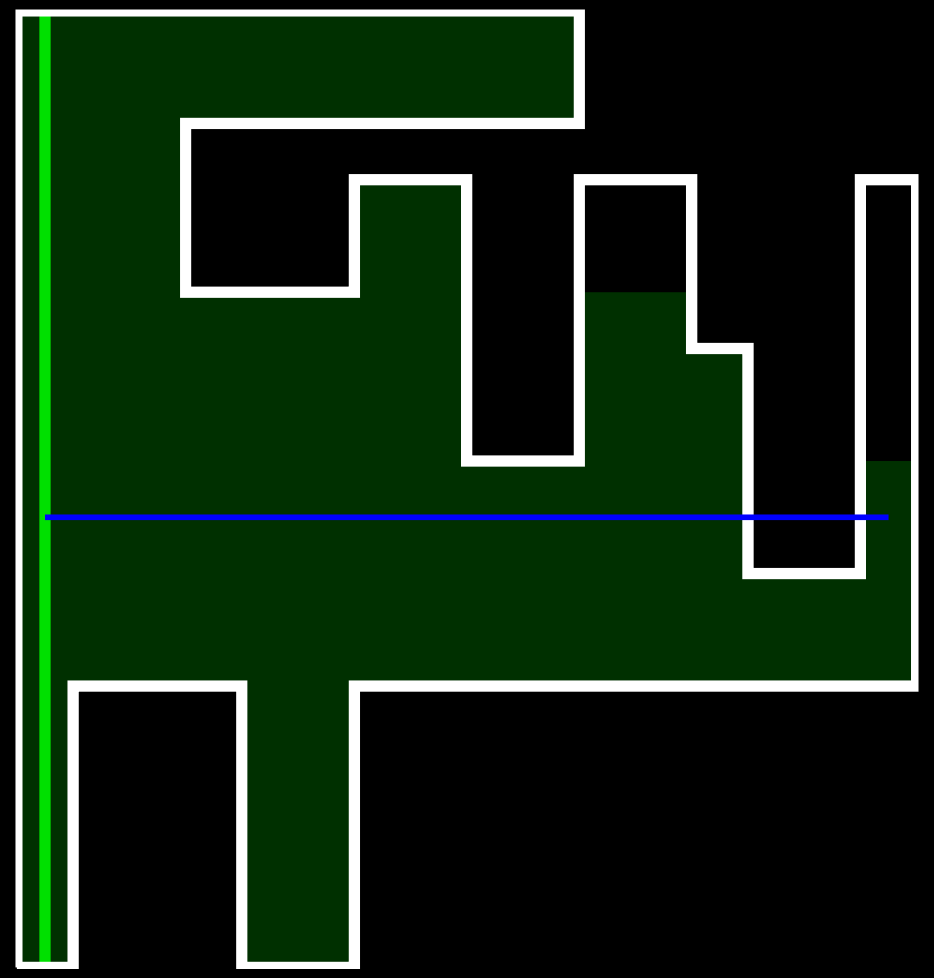

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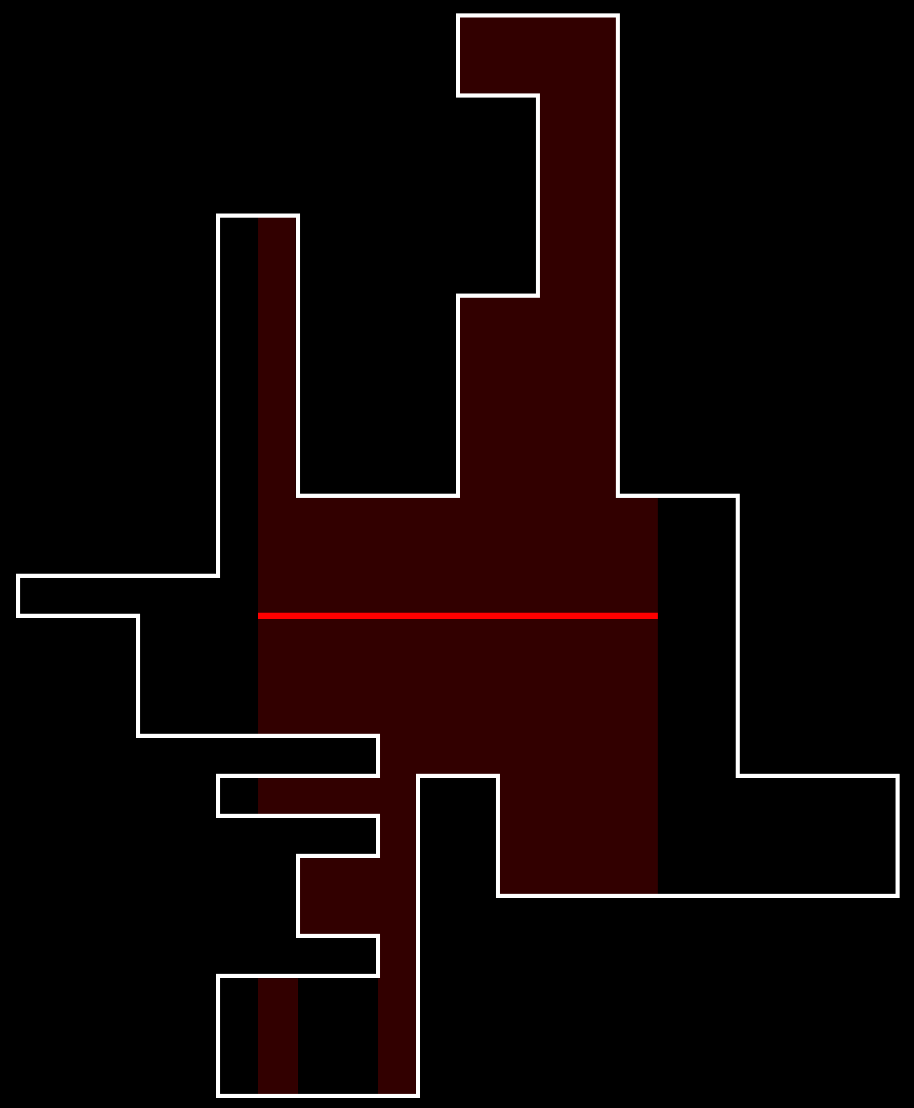

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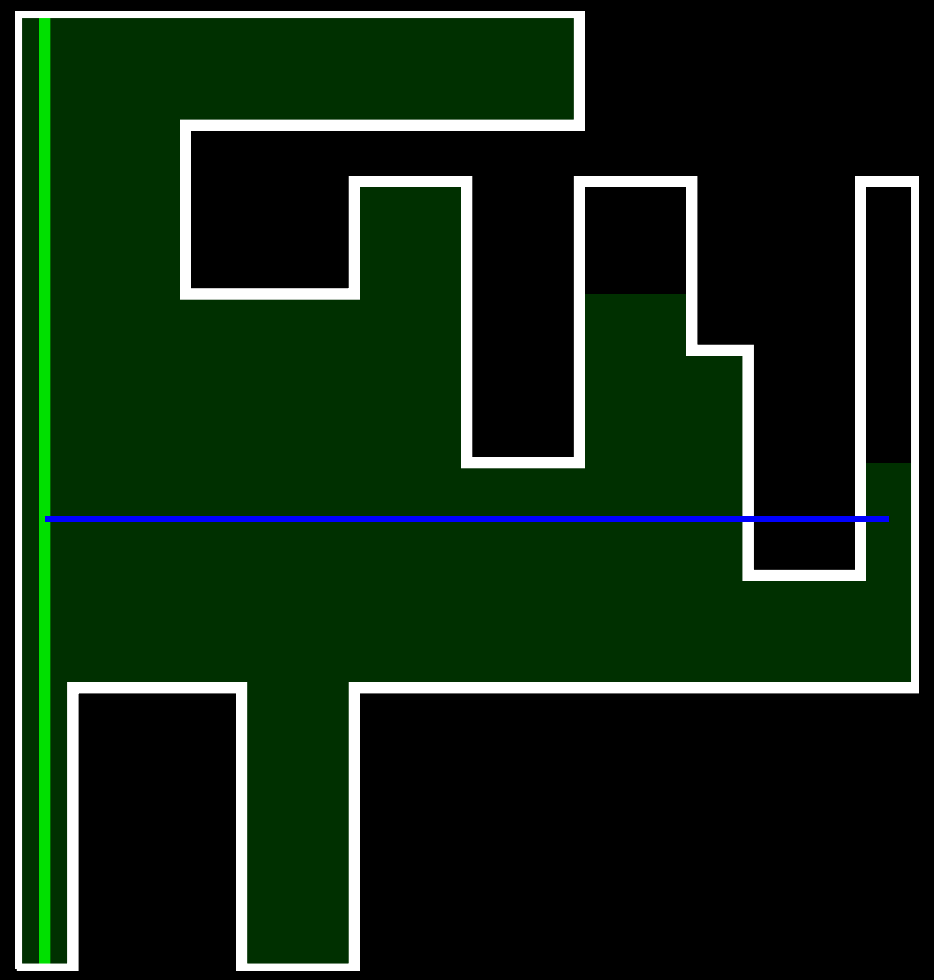

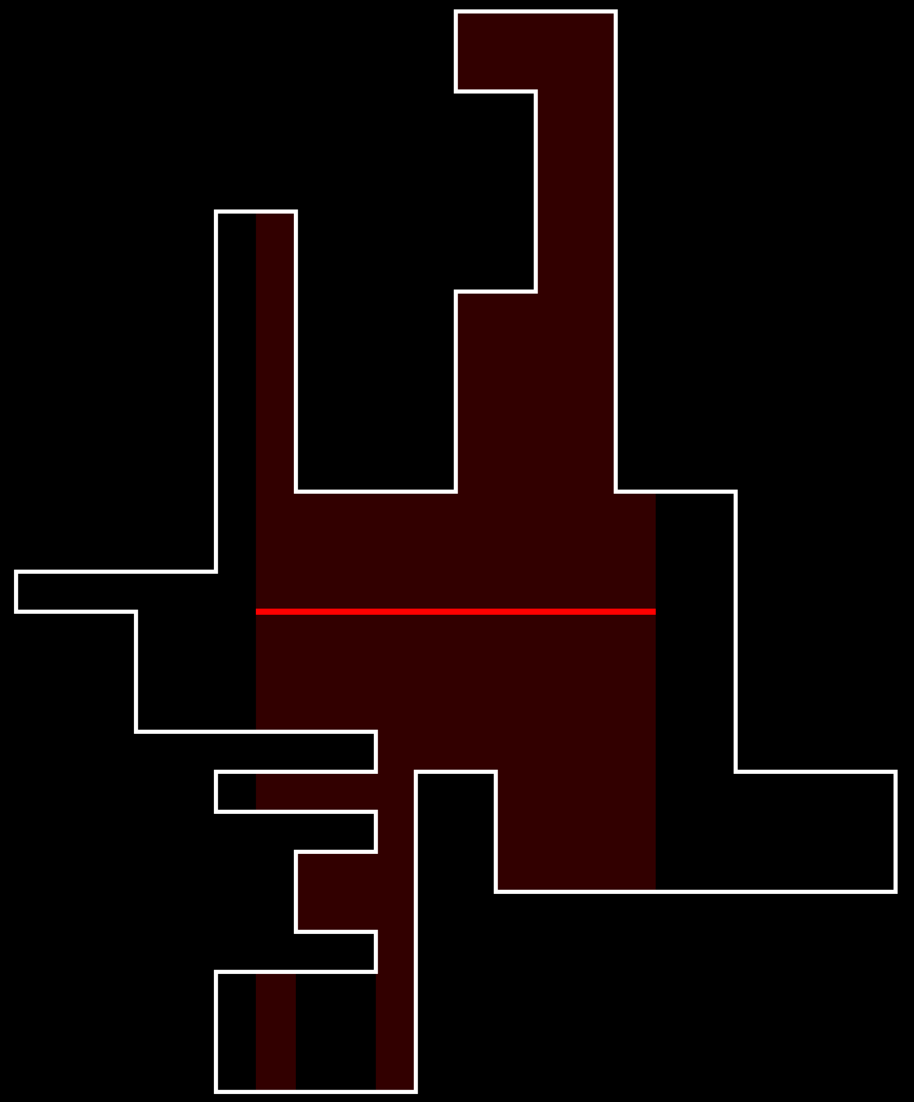

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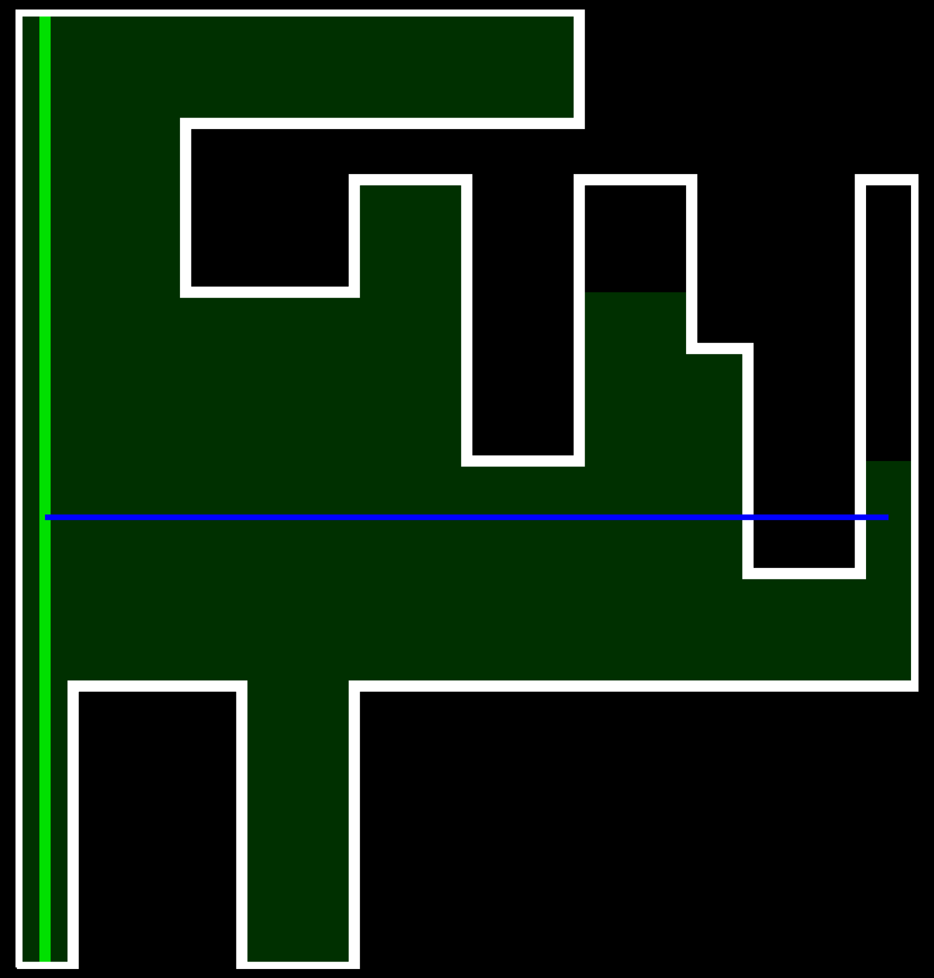

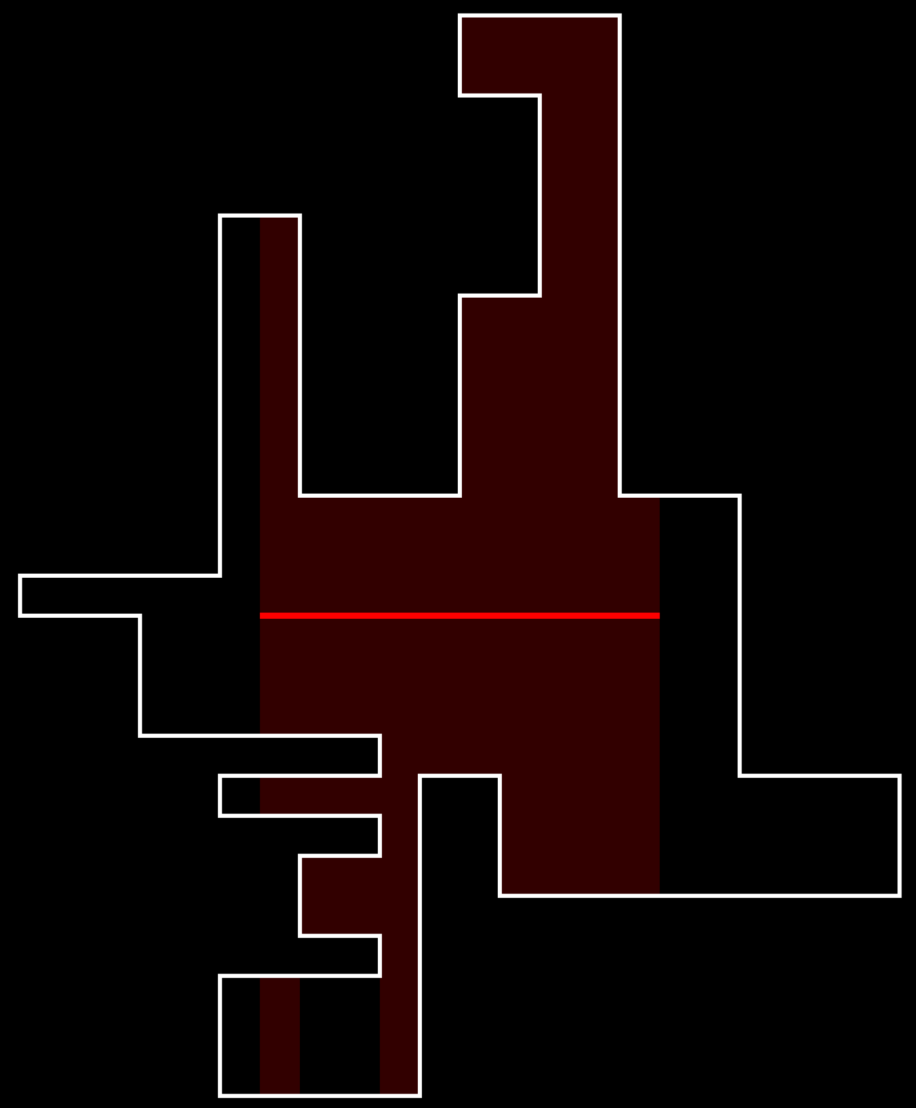

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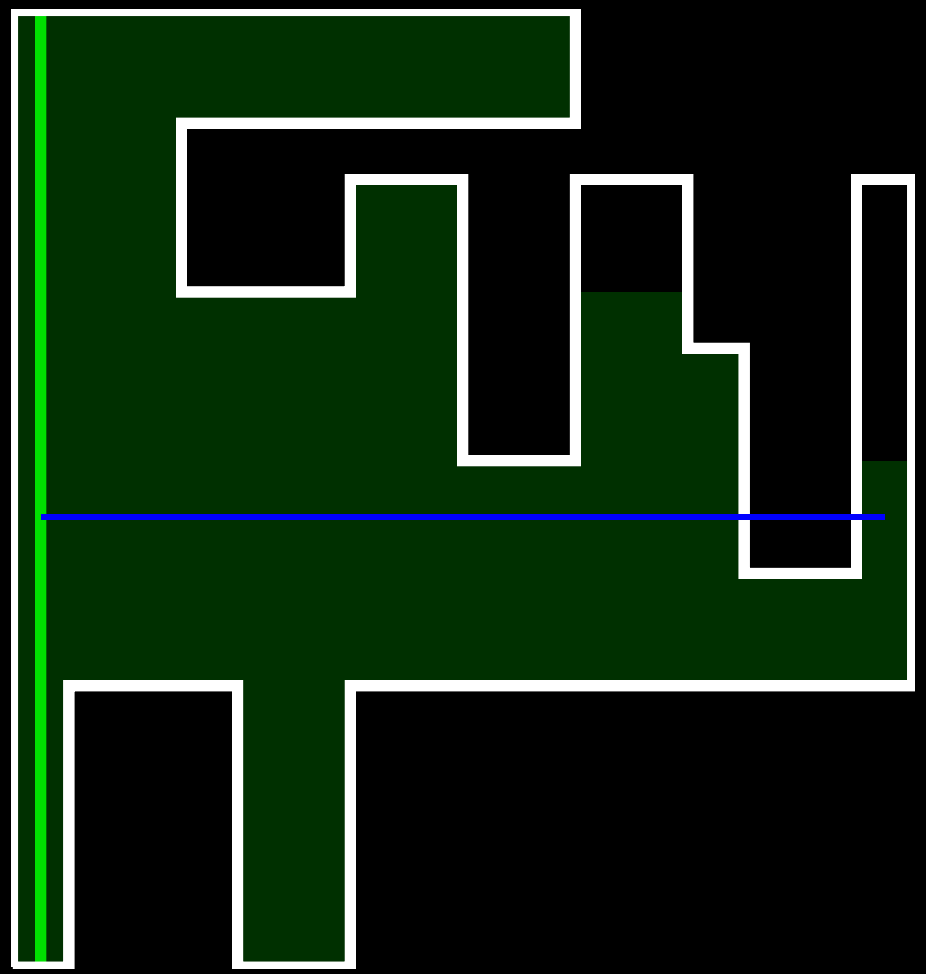

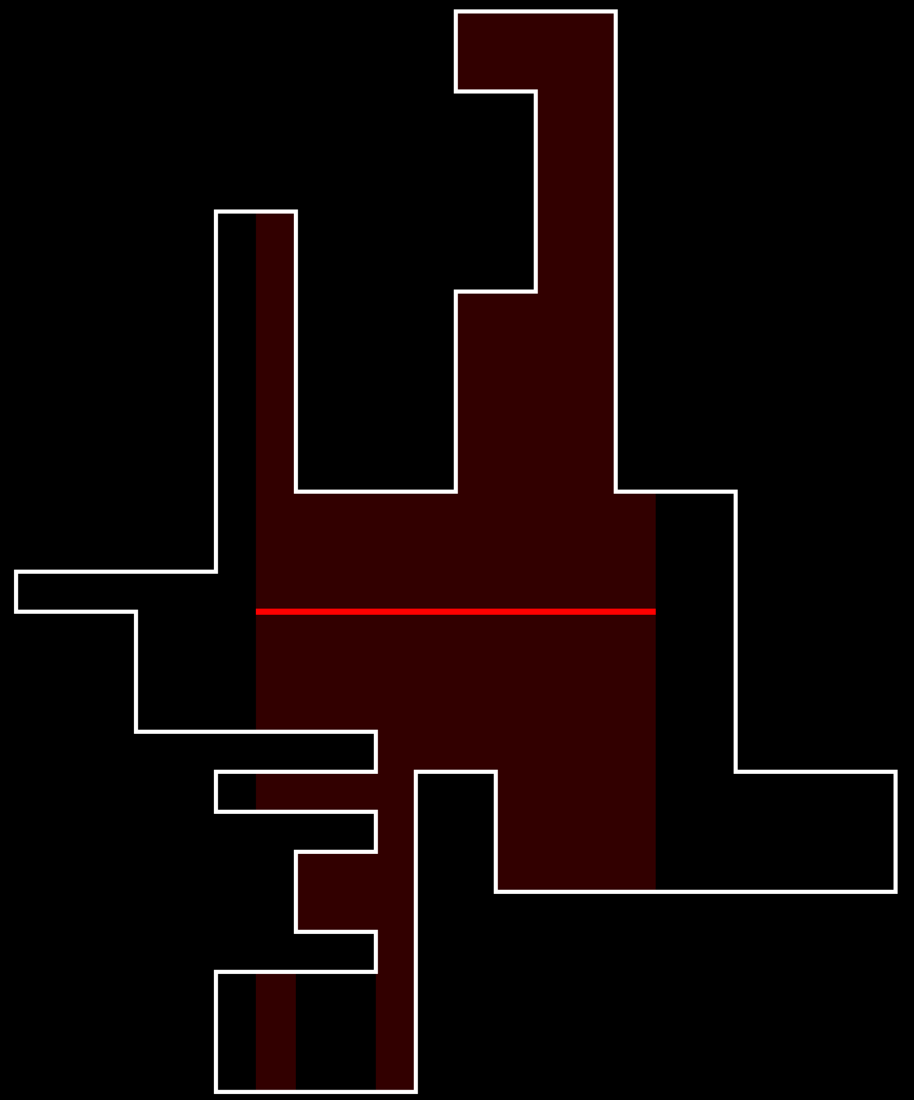

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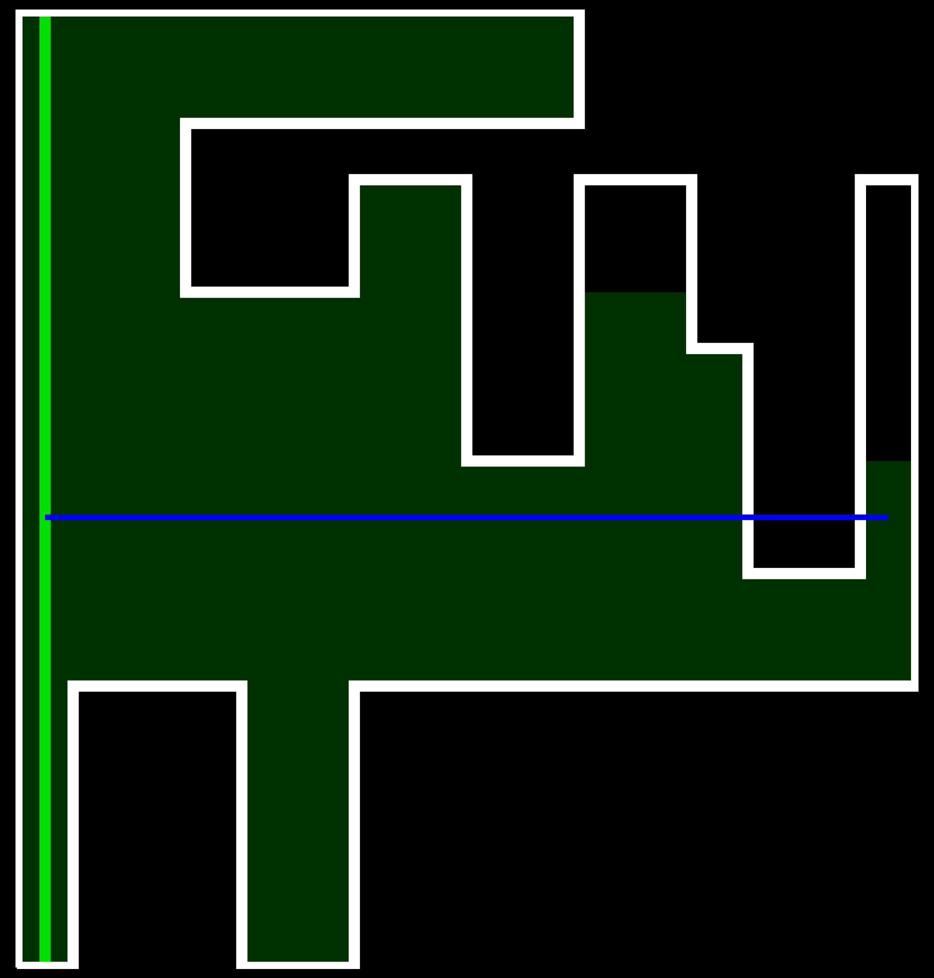

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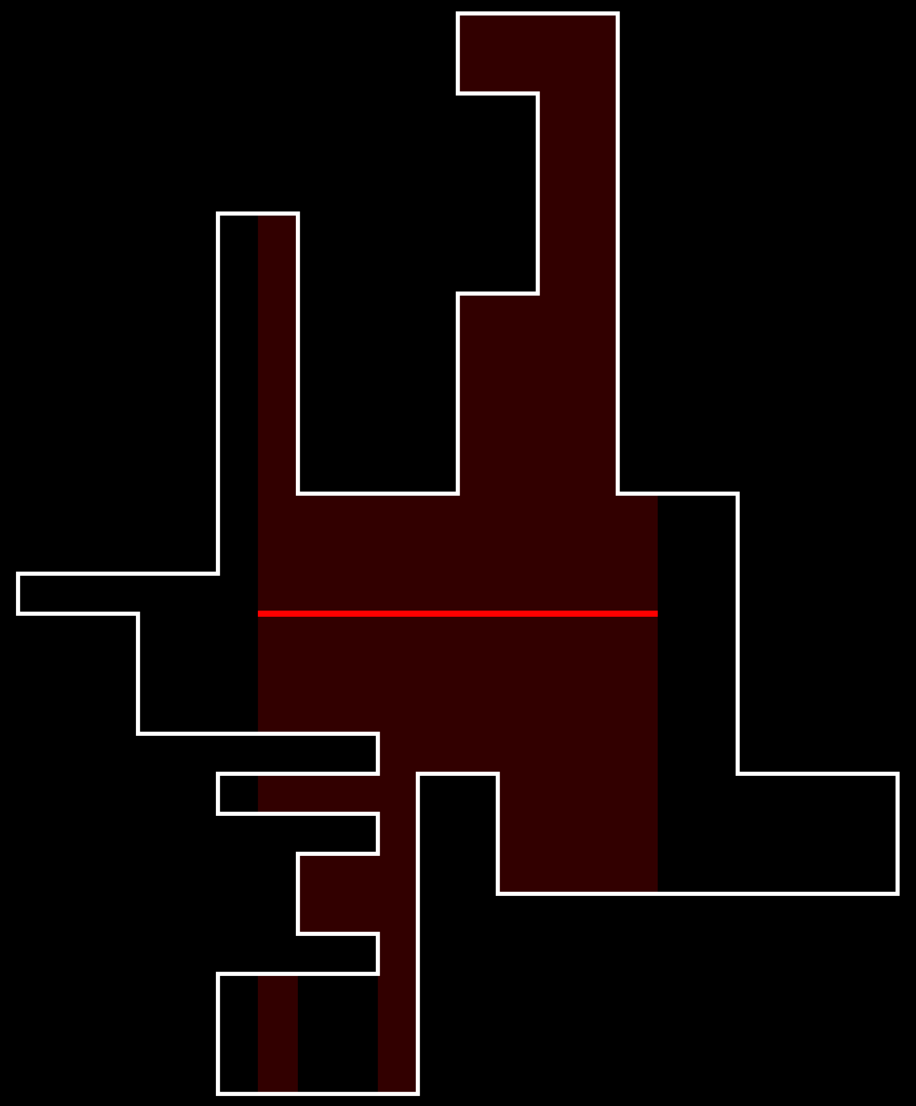

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The Watchman Route Problem (WRP)

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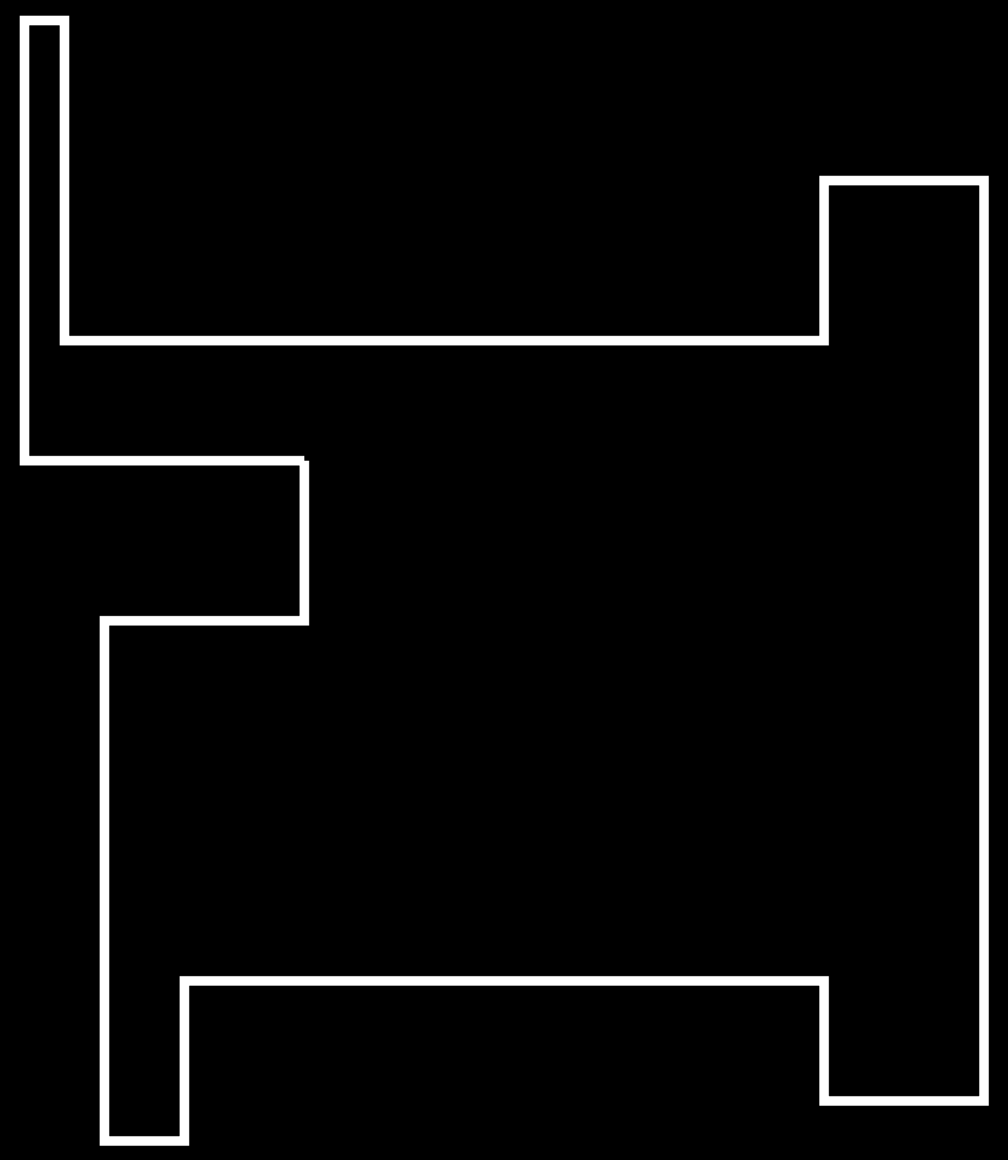

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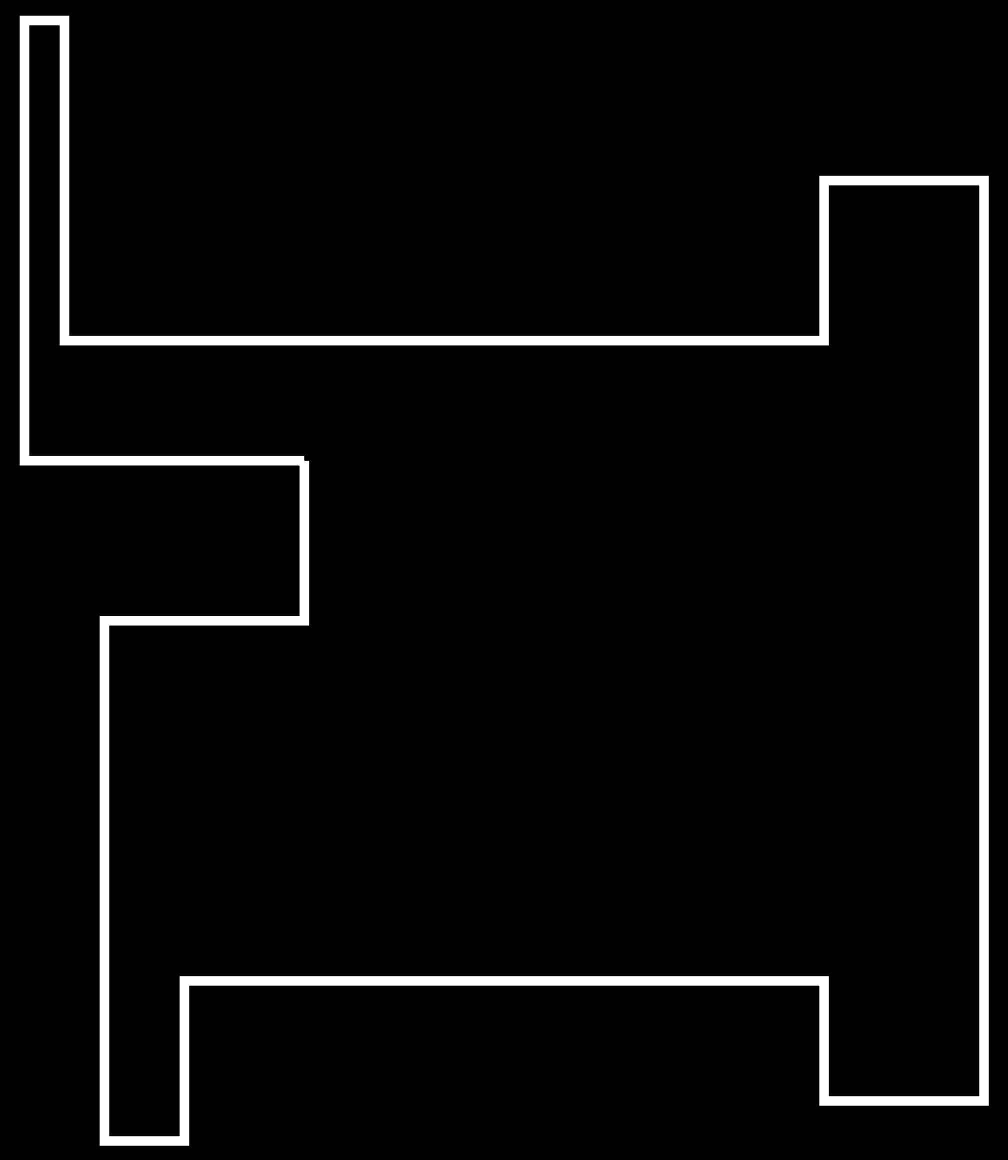

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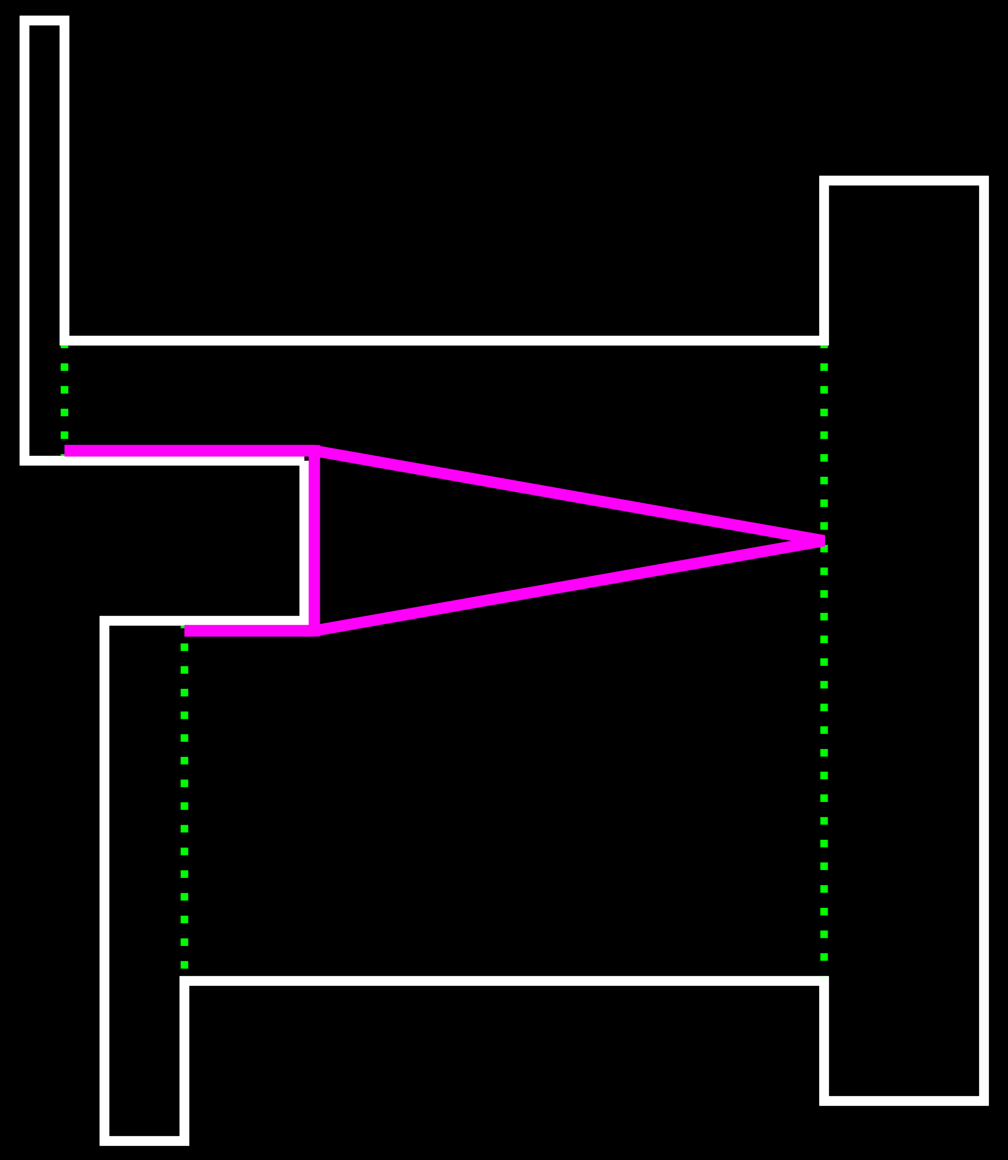

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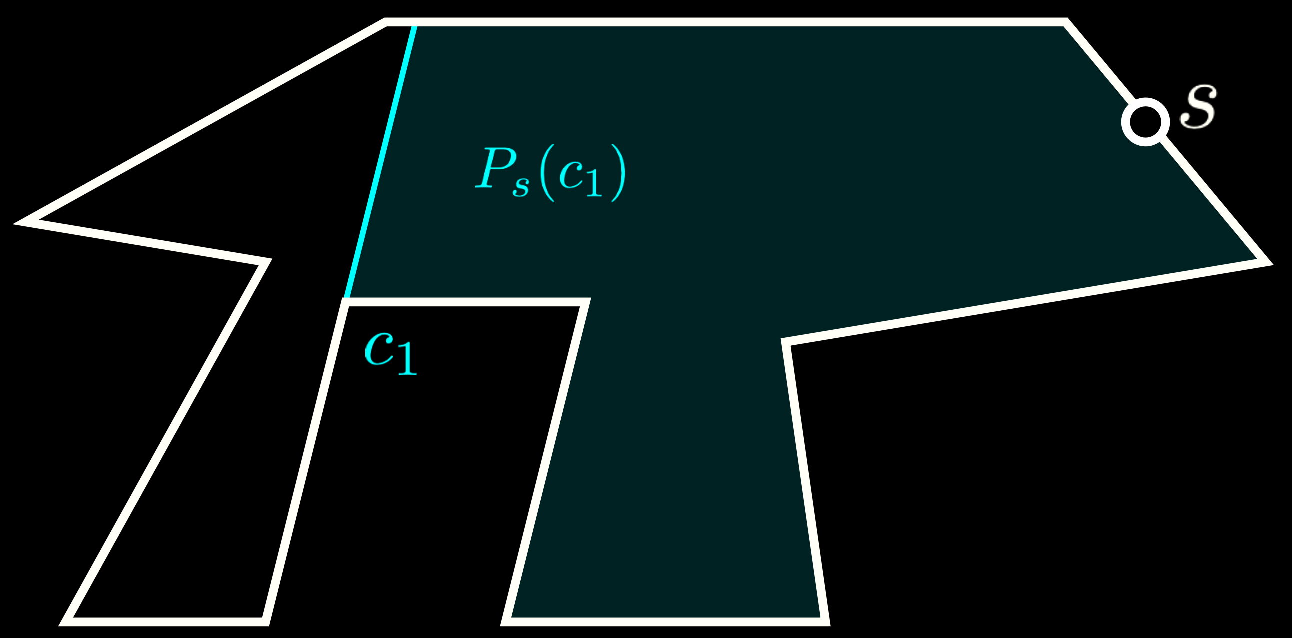

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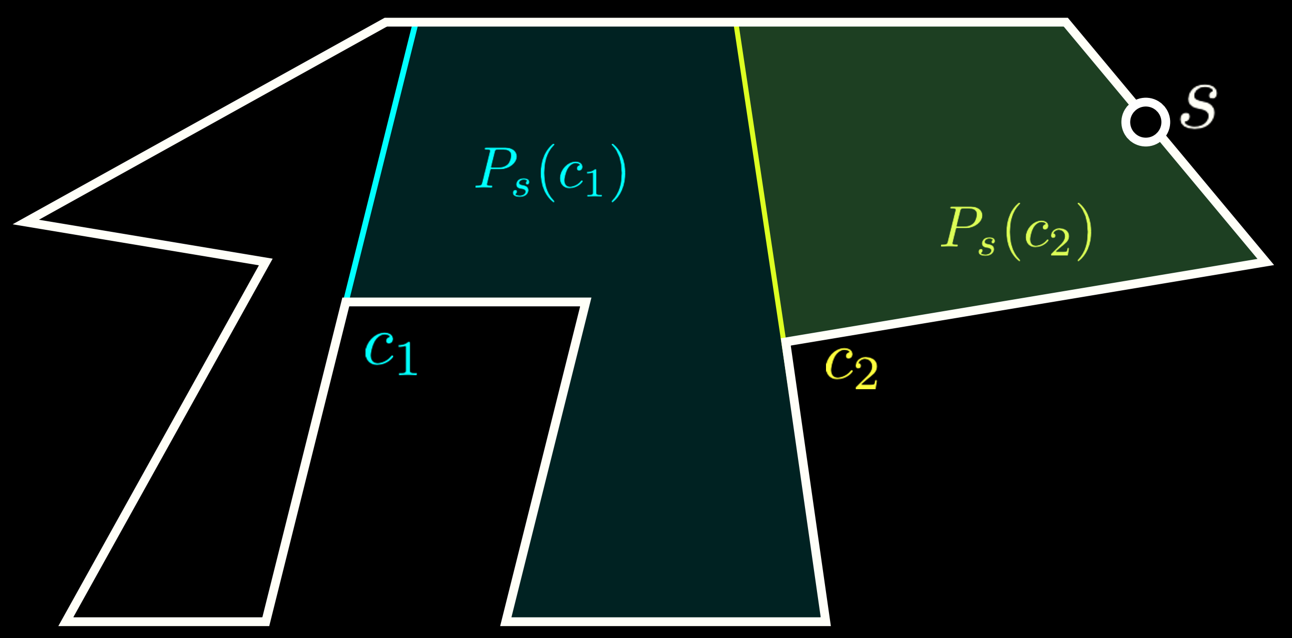

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A cut c_1 dominates c_2 if $P_s(c_2) \subseteq P_s(c_1)$

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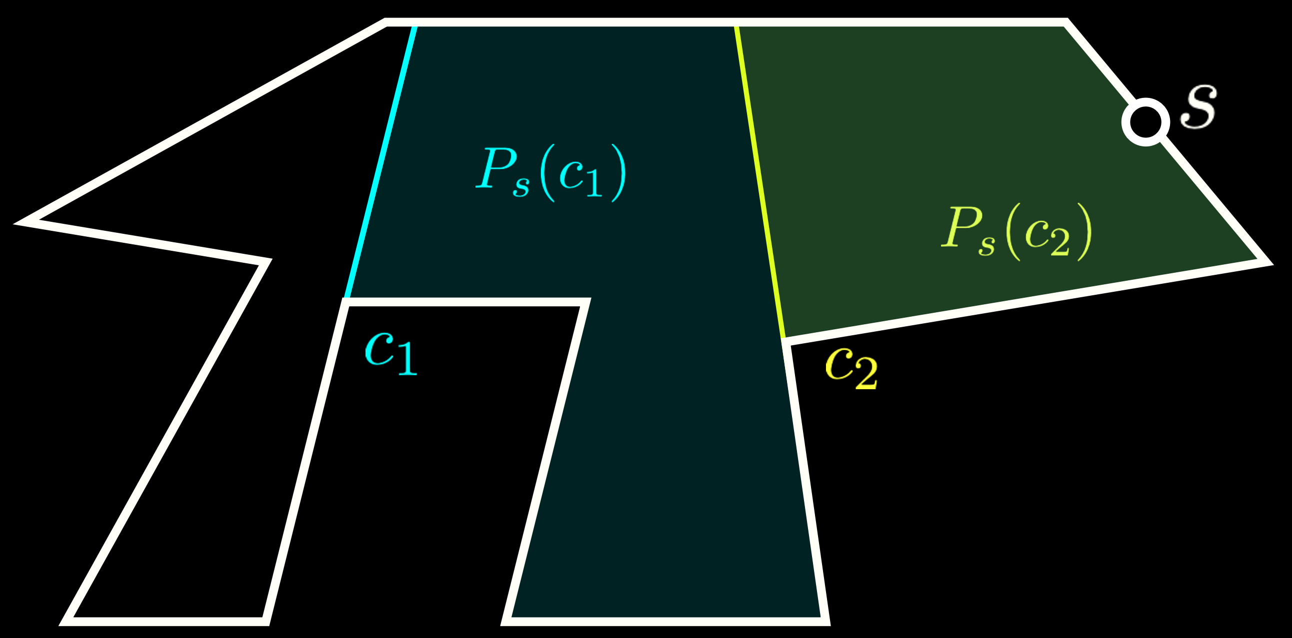

A cut c partitions polygon into two subpolygons:
$P_s(c)$—subpolygon that contains starting point s
A cut c_1 dominates c_2 if $P_s(c_2) \subseteq P_s(c_1)$
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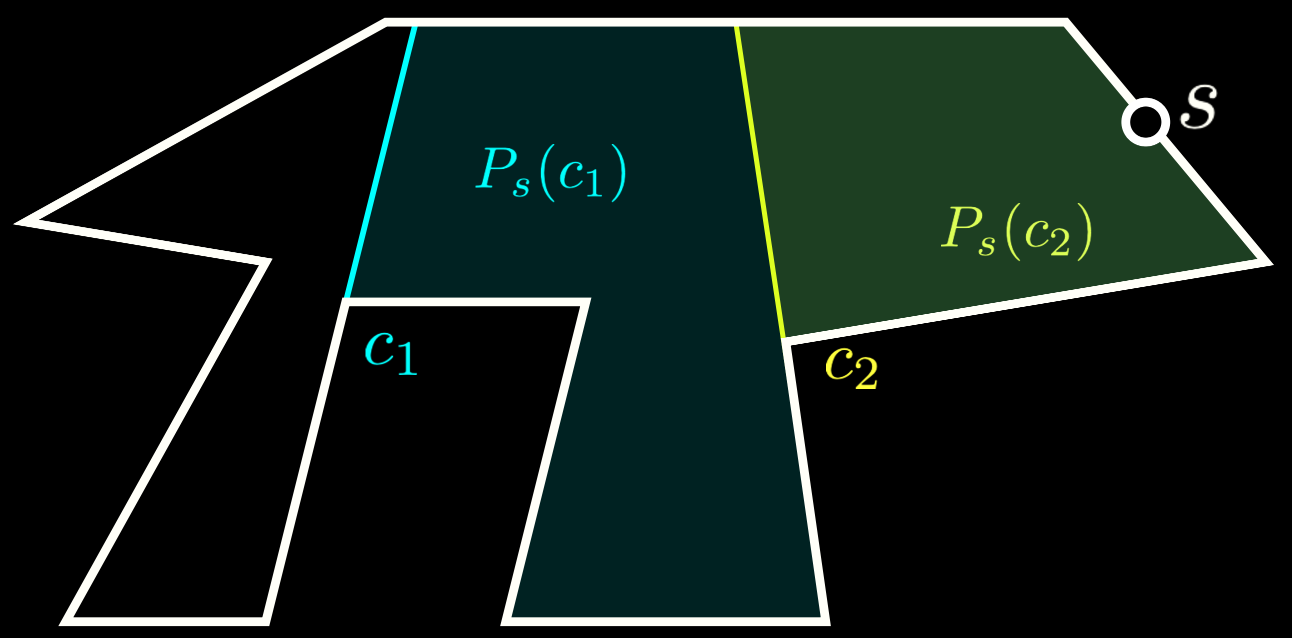

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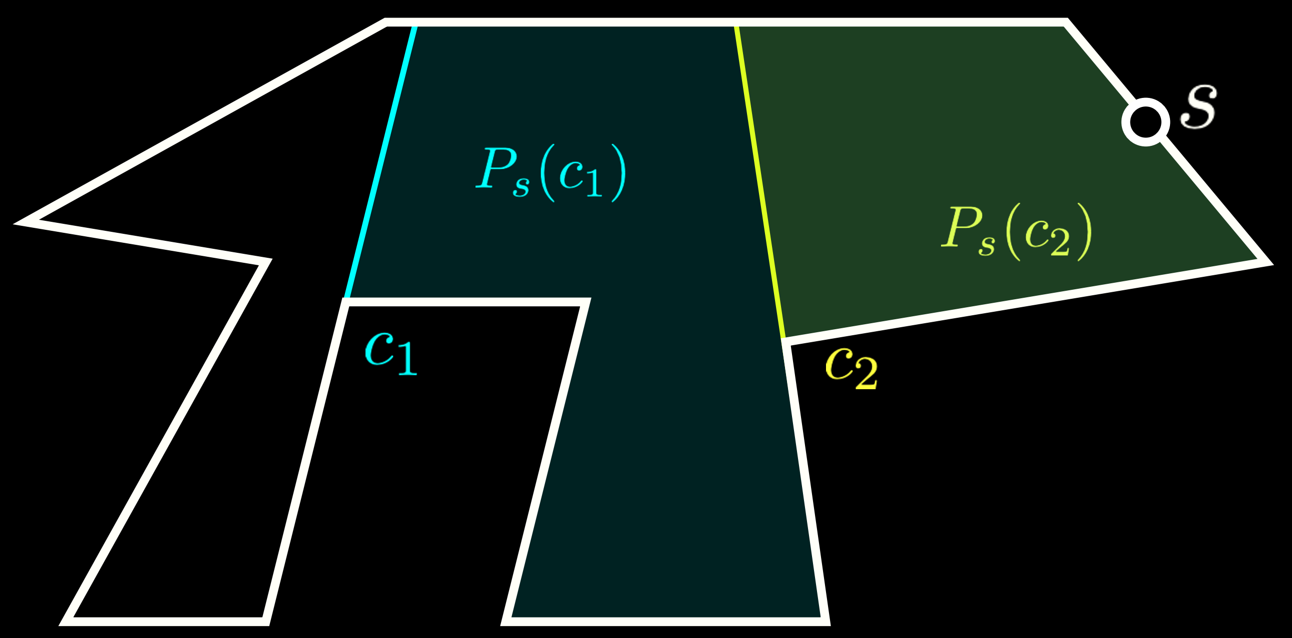

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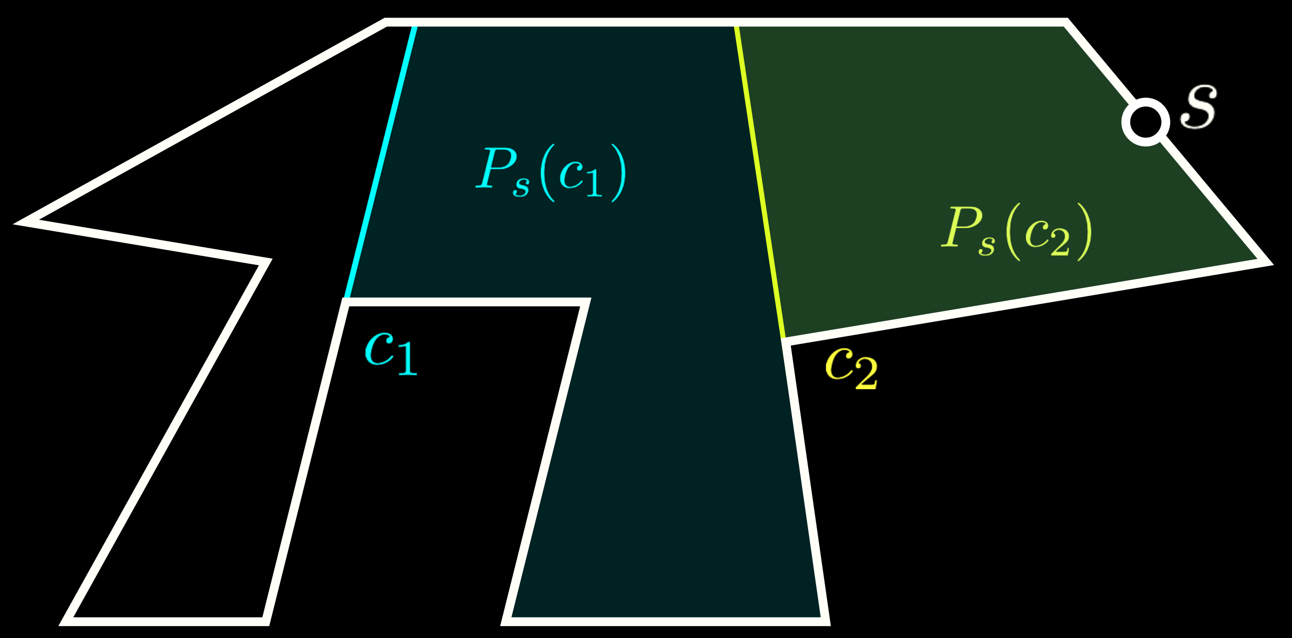
- As for the AGP, we can alter the capabilities of the watchman or the area to be guarded



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k-Transmitter Watchman Routes

[Bengt J. Nilsson, S., 2023]

k-Transmitter Watchman Routes

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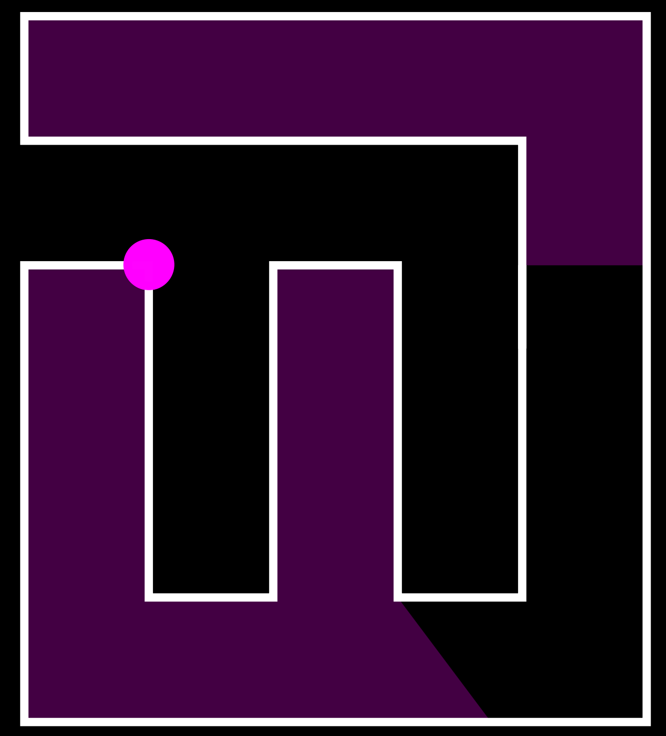
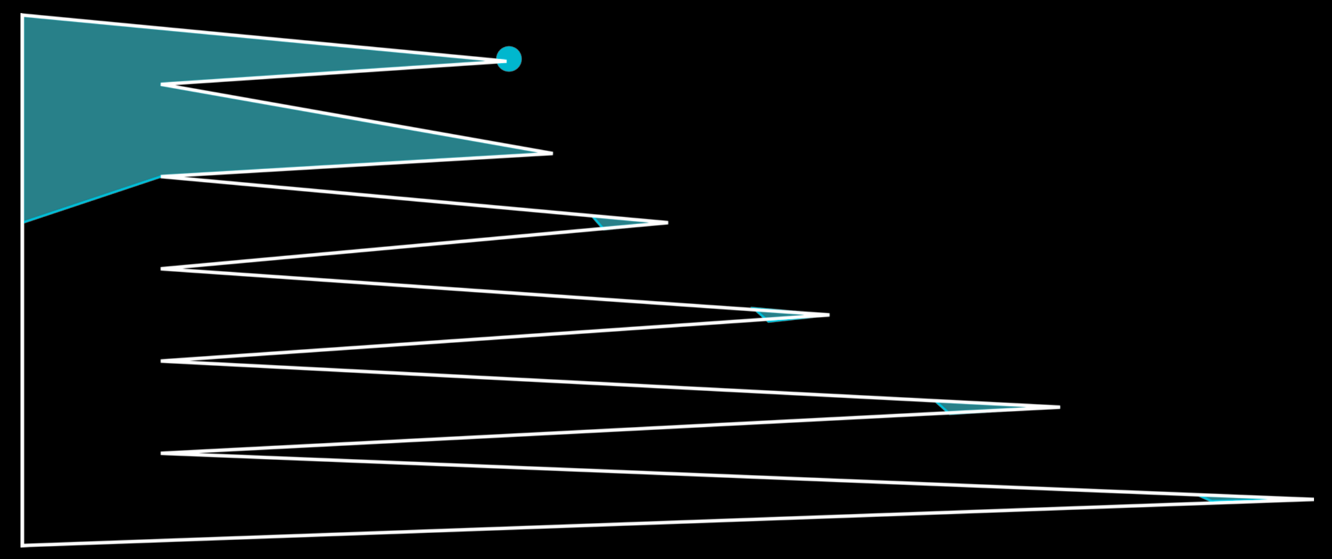
- Extensions do not translate to *k*-transmitters for $k \geq 2$ (no longer local!)





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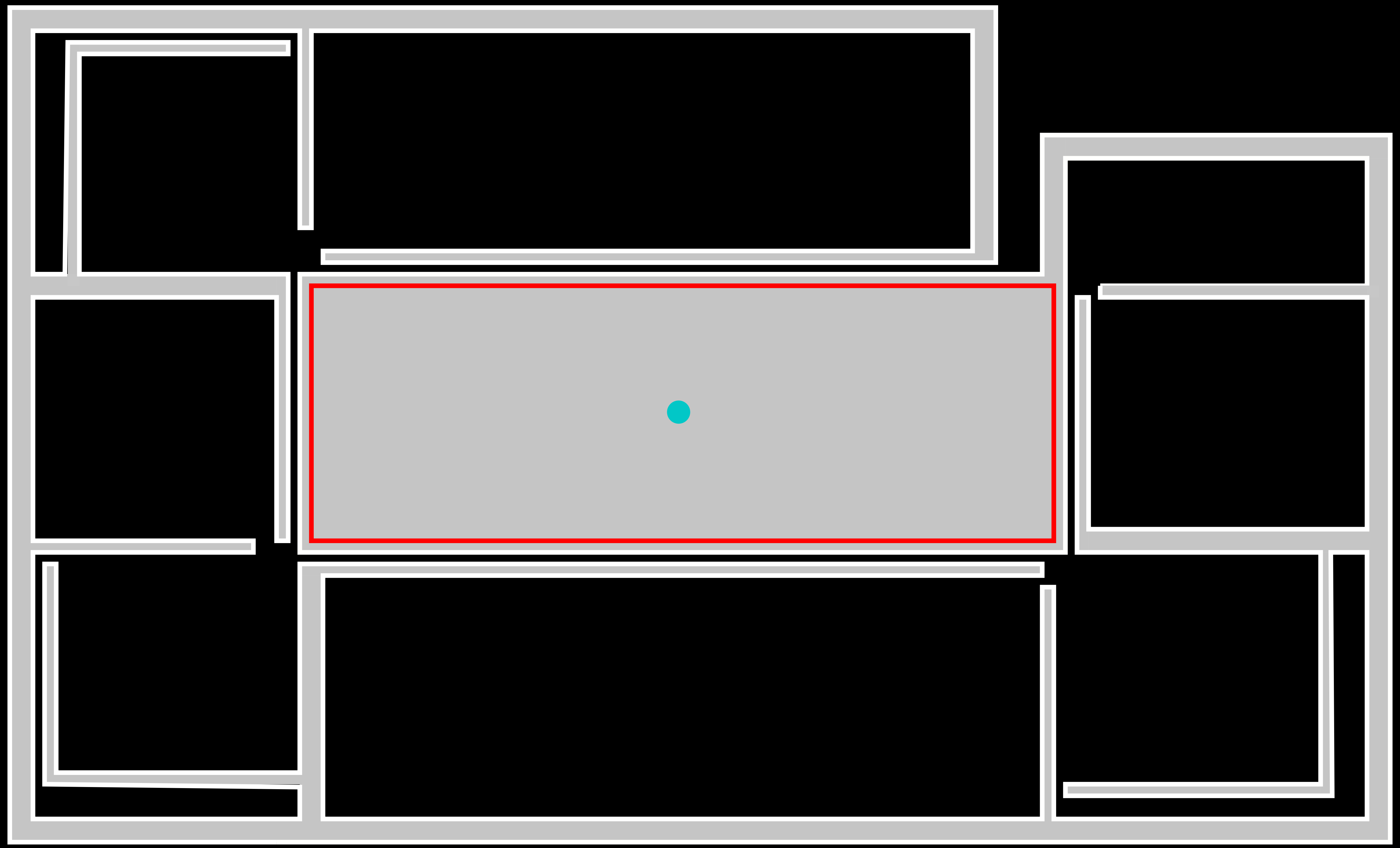
Even for a tour in a simple polygon seeing the boundary is not enough:



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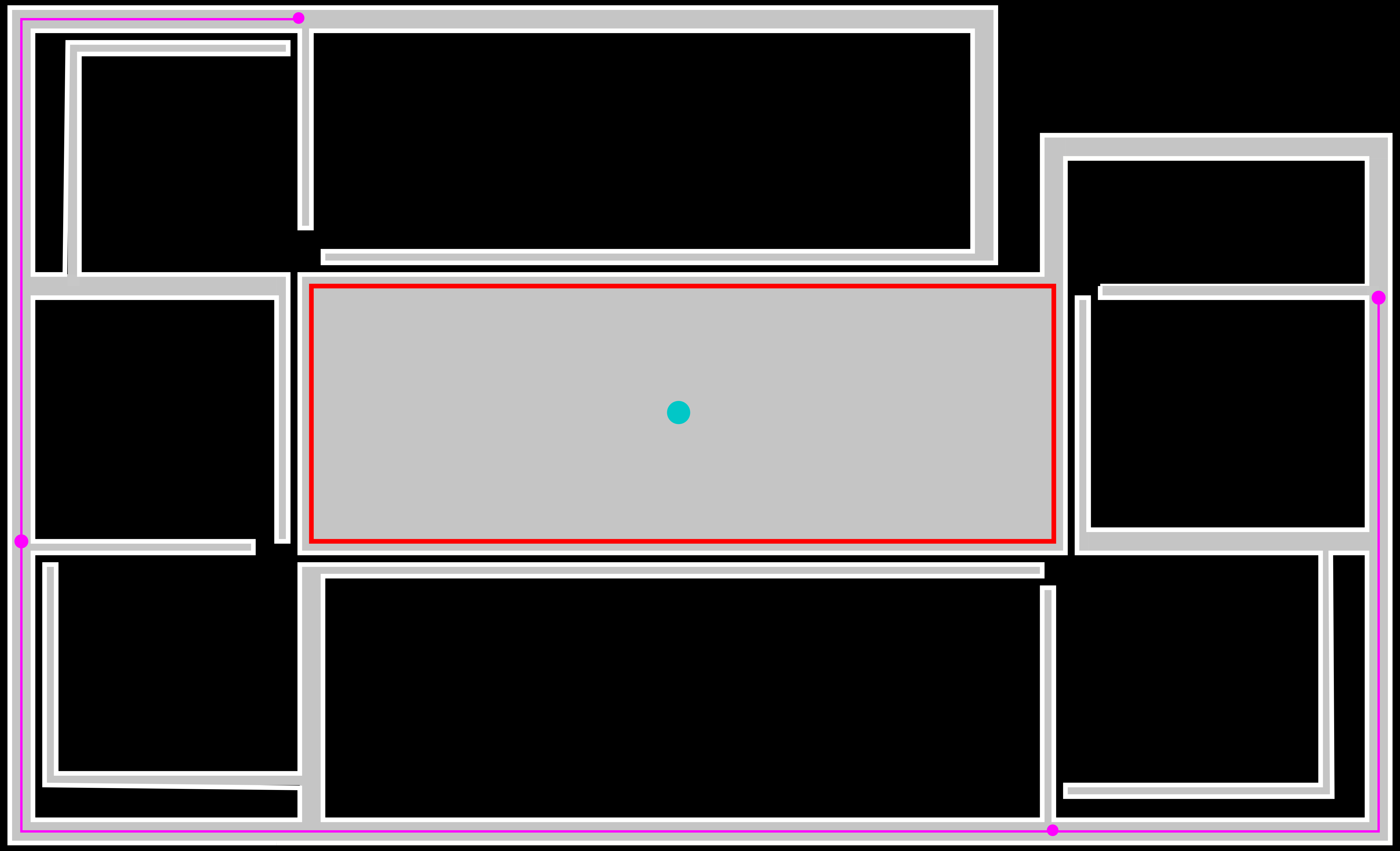

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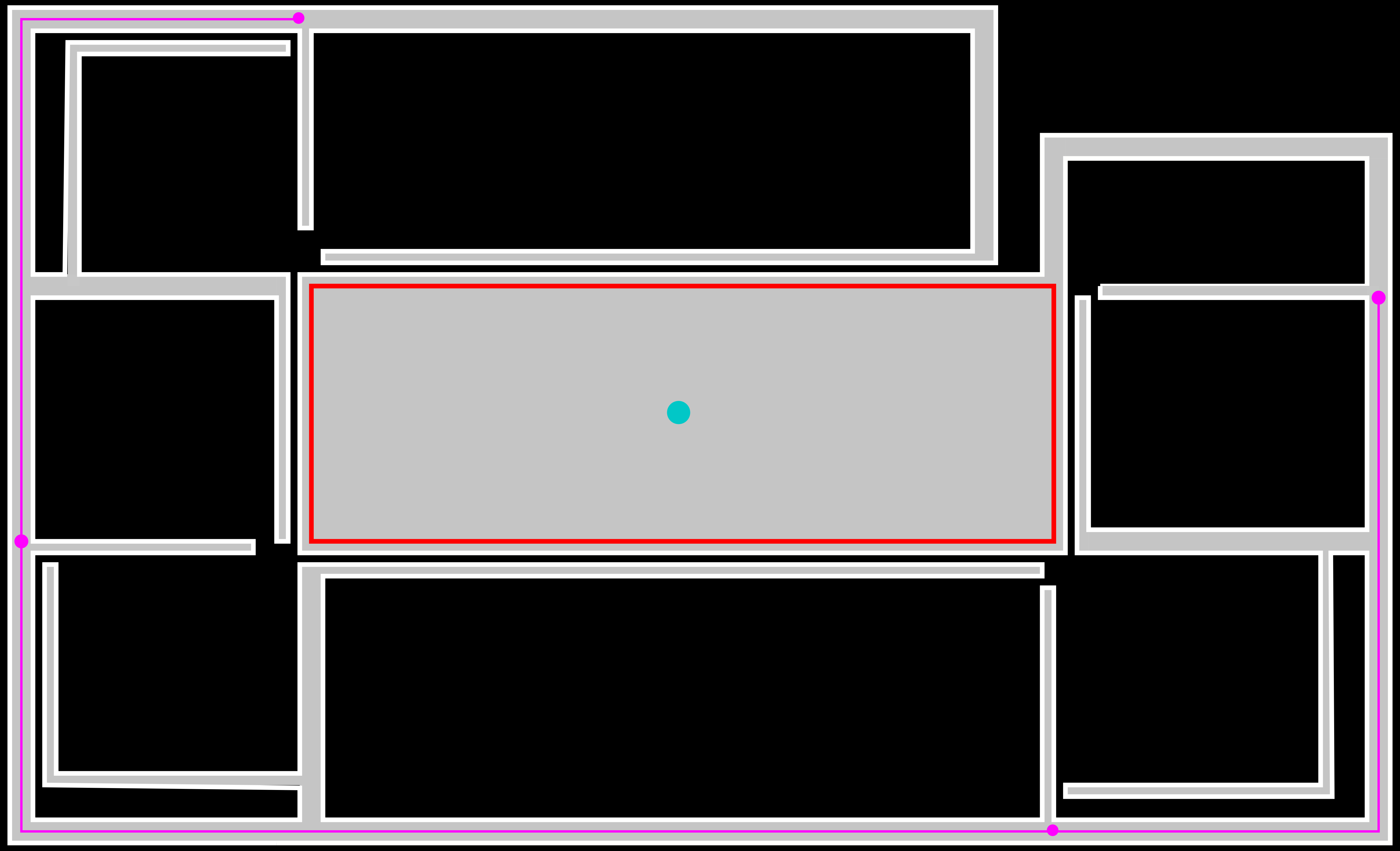

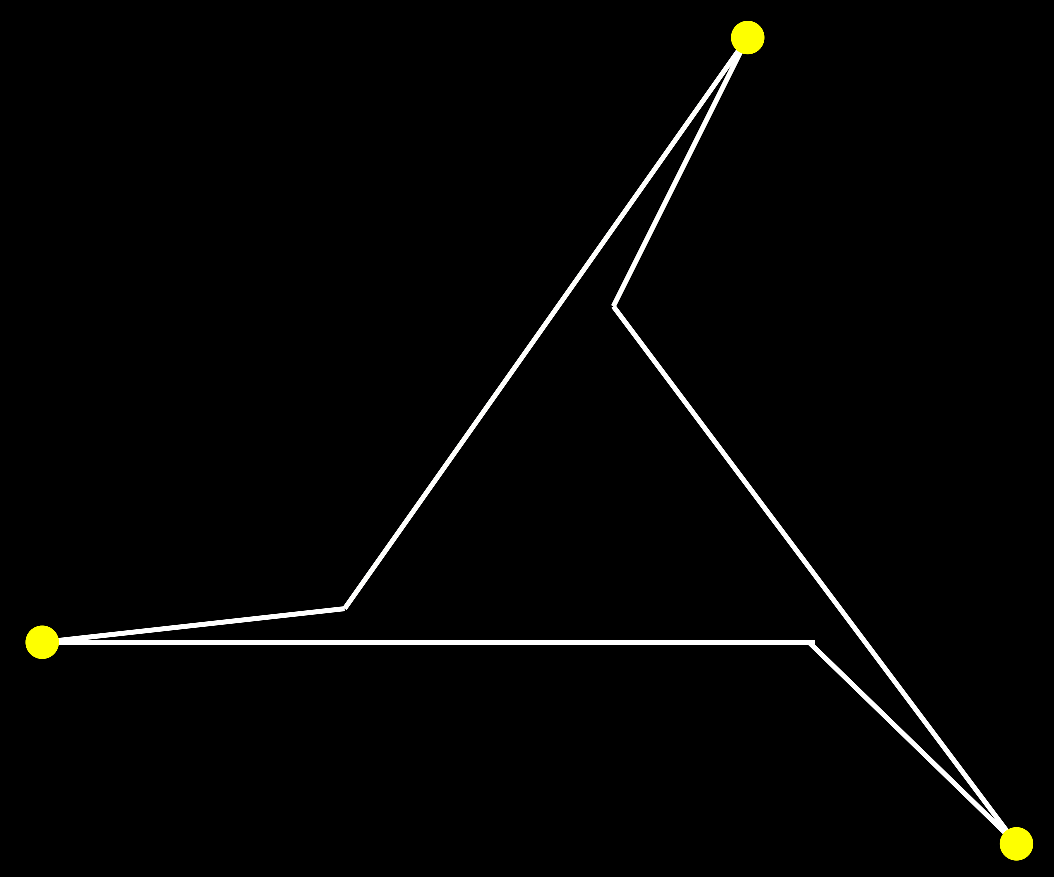

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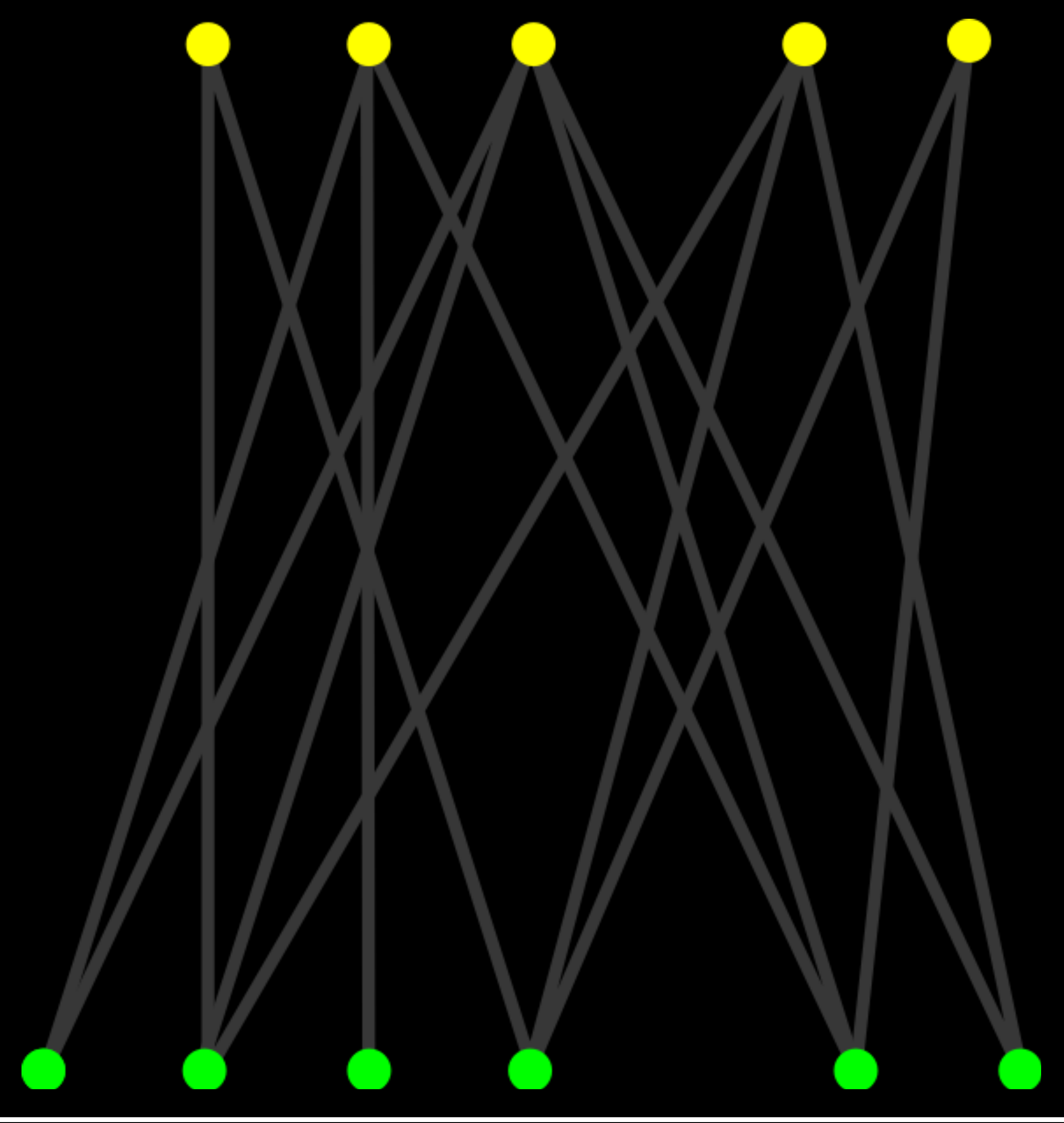

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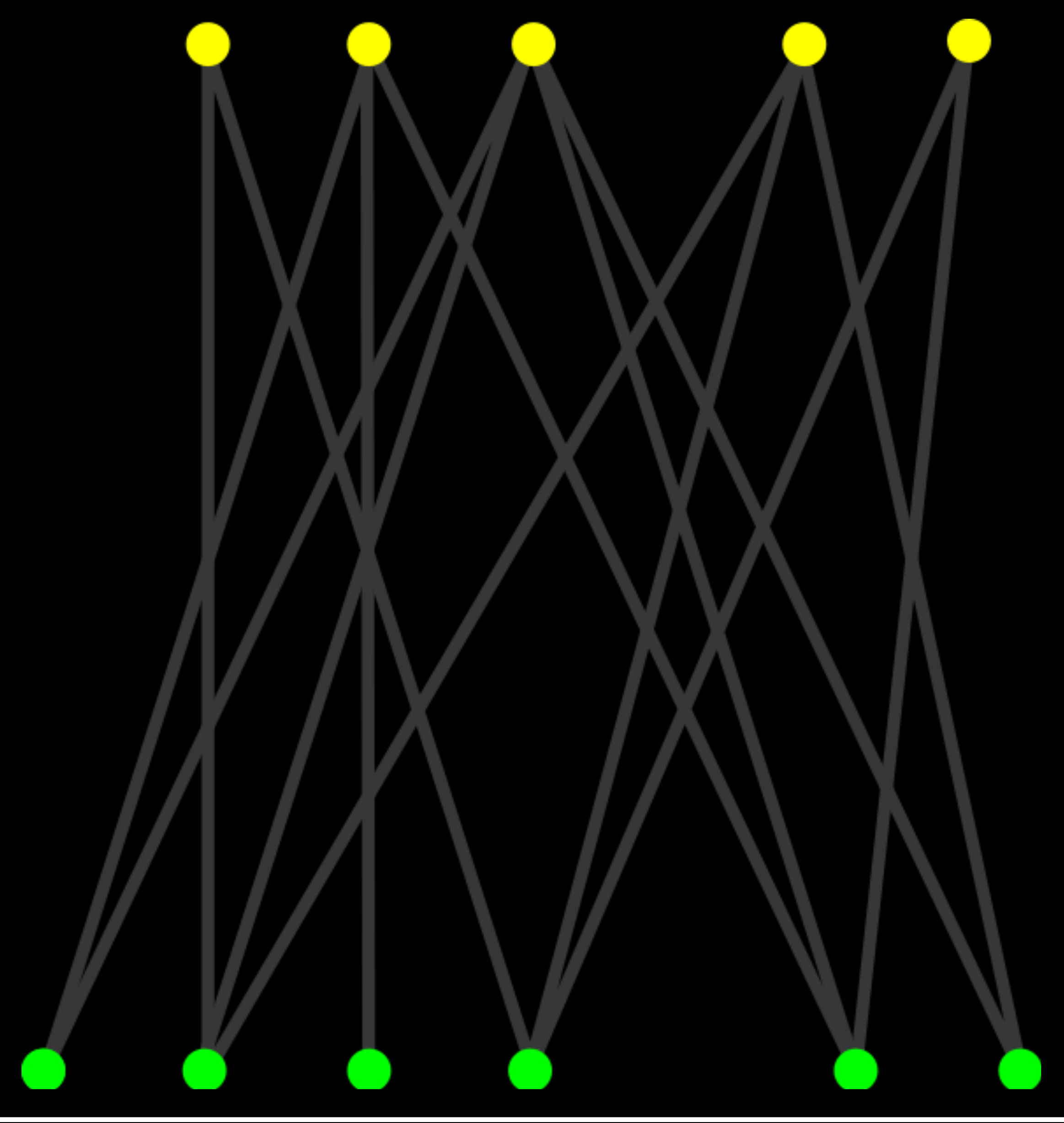

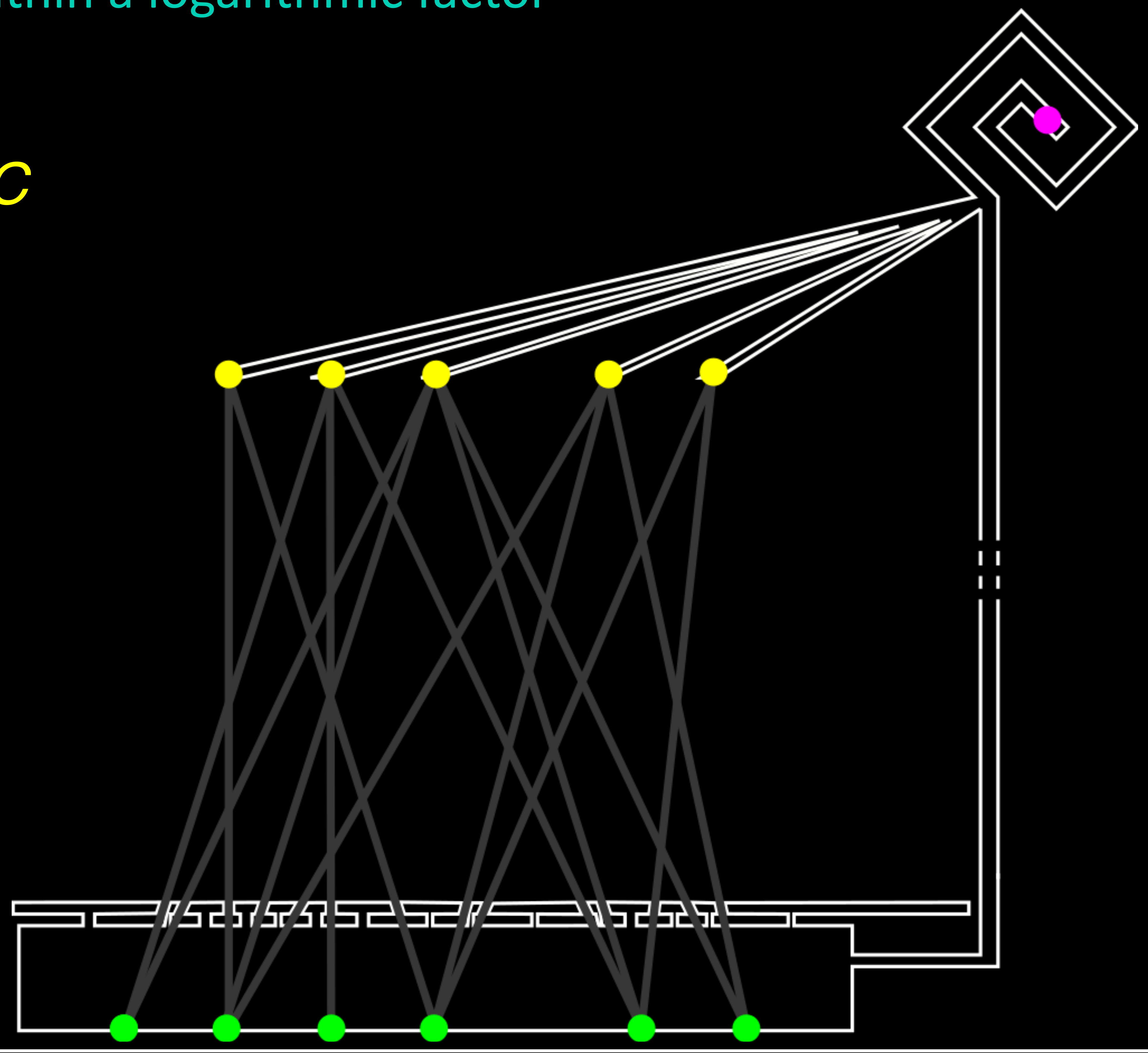

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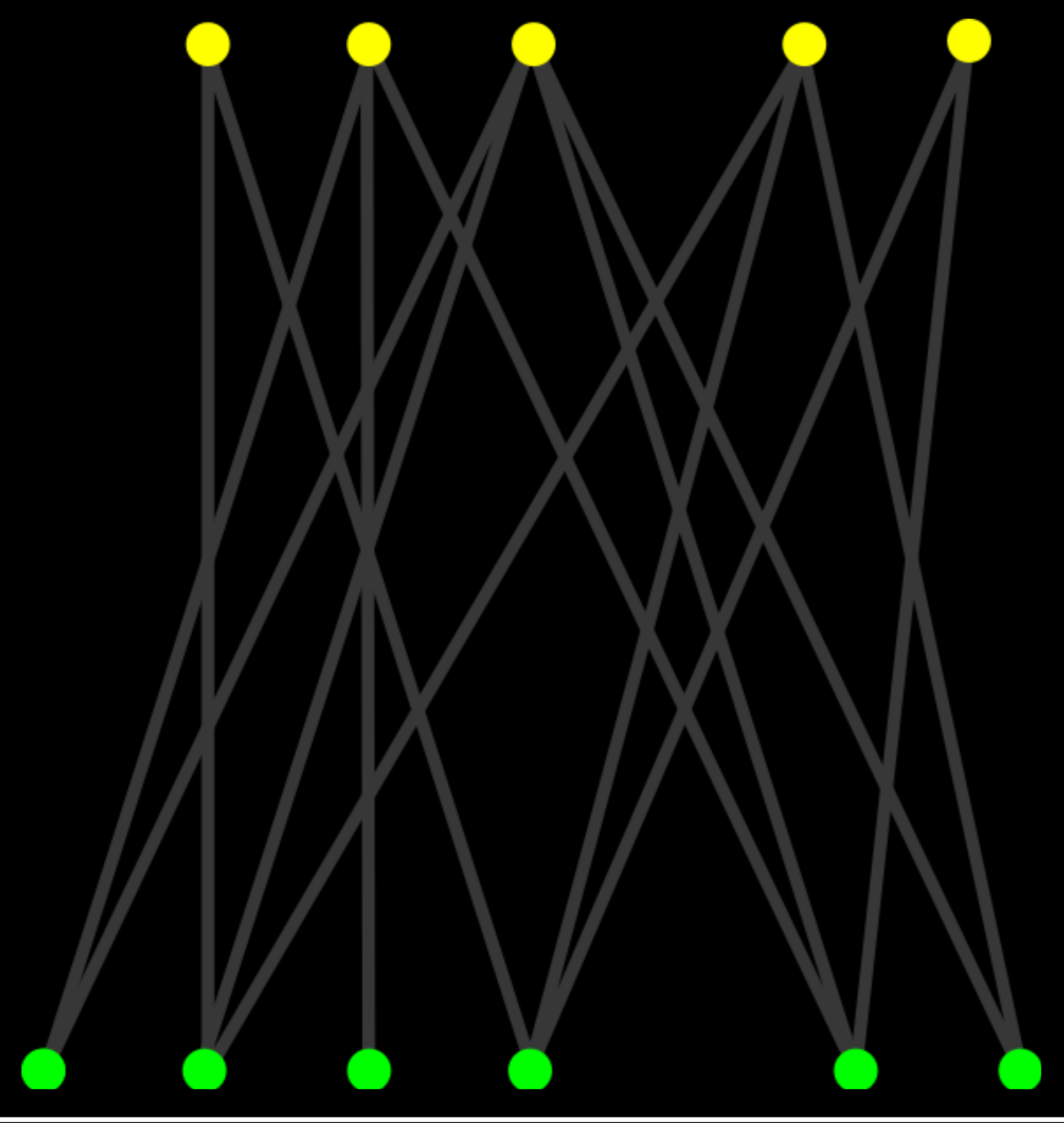

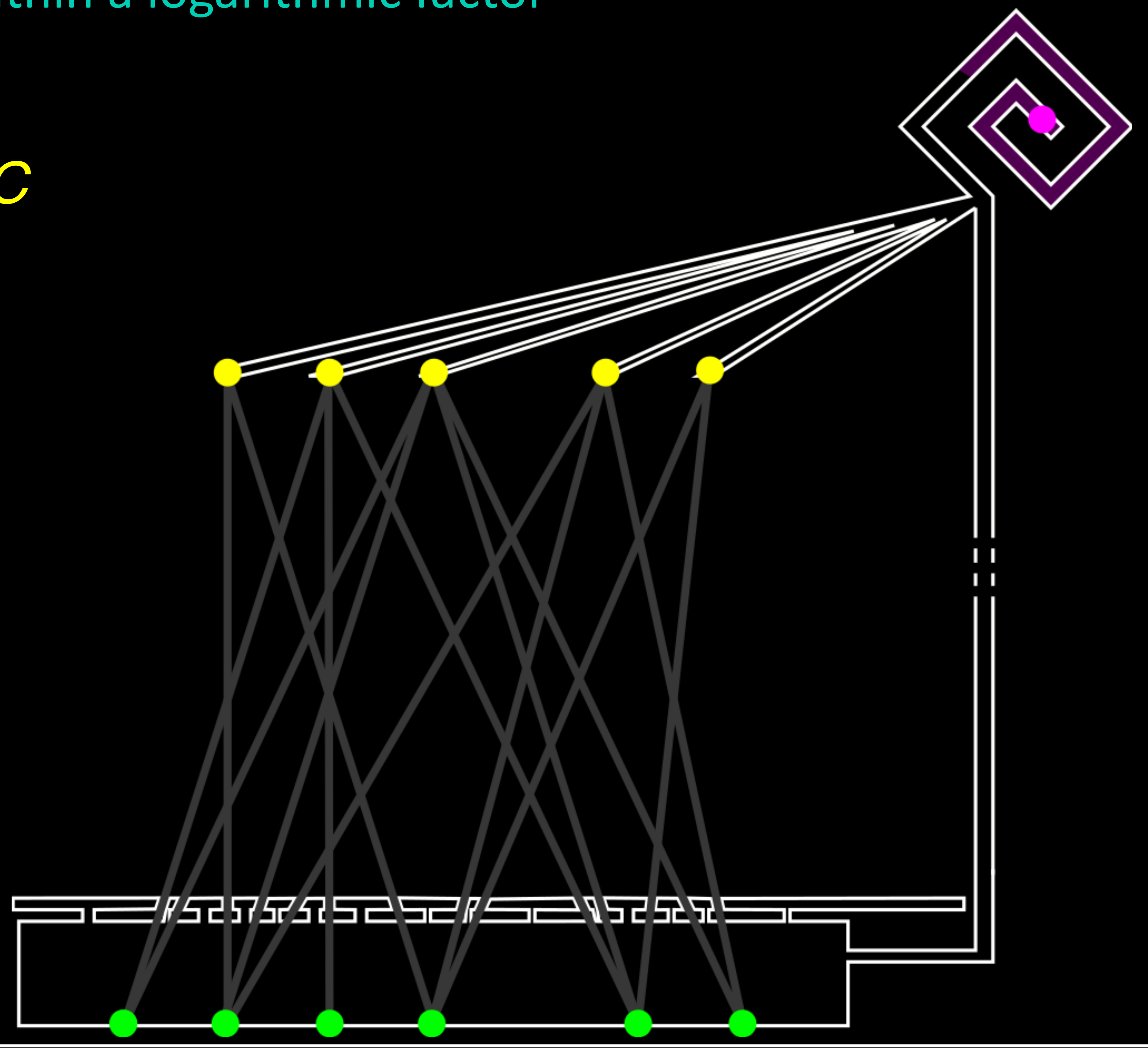

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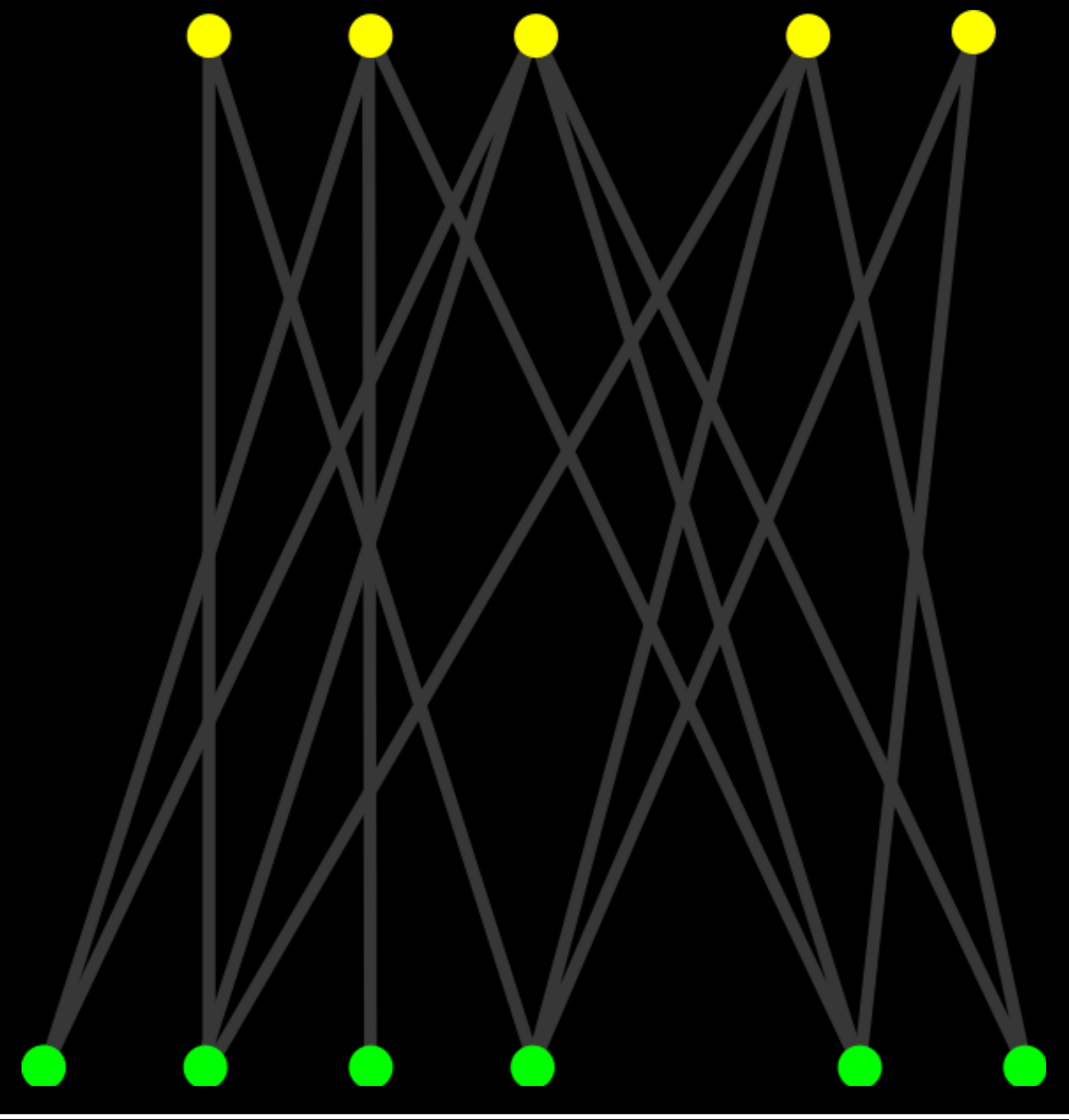

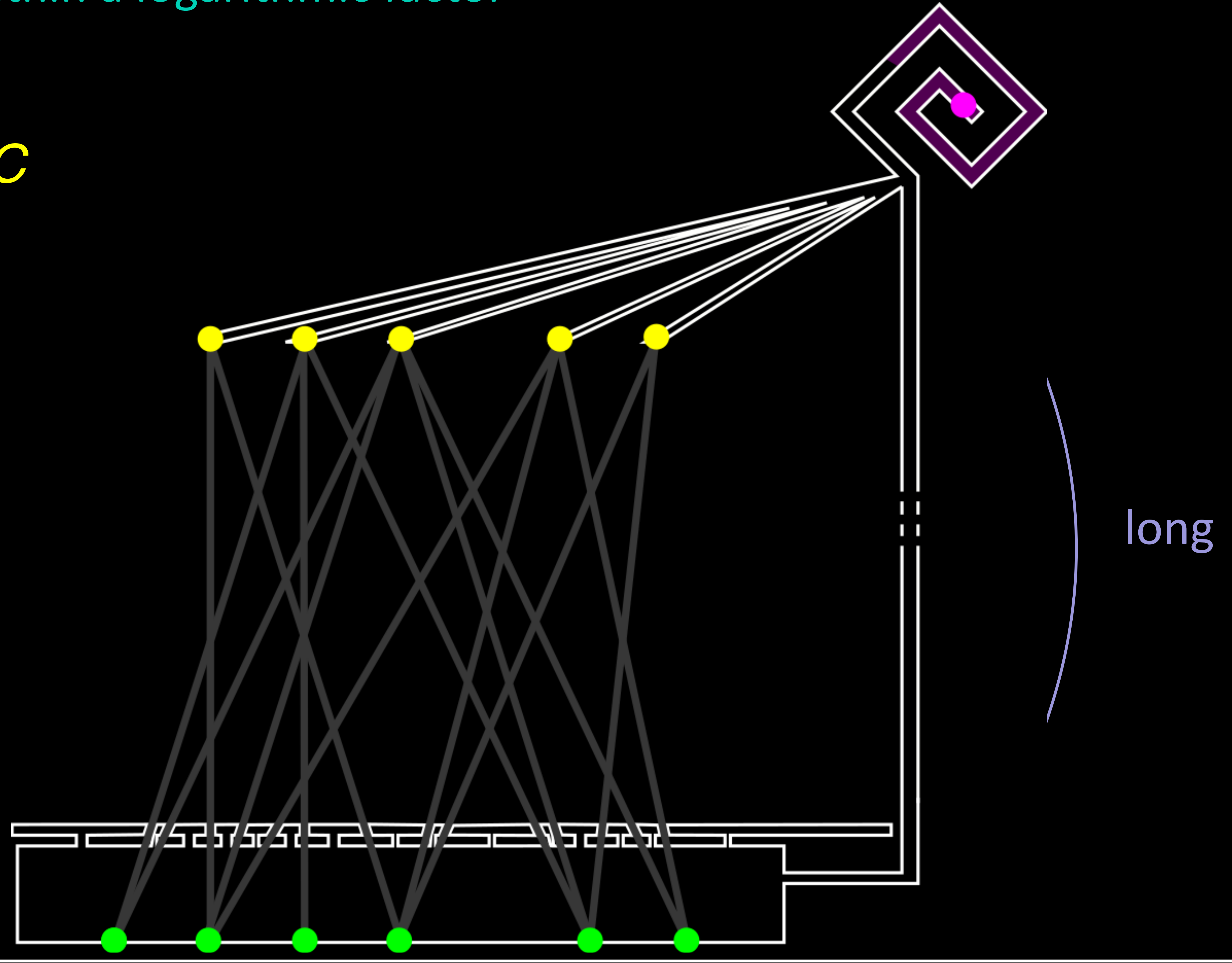

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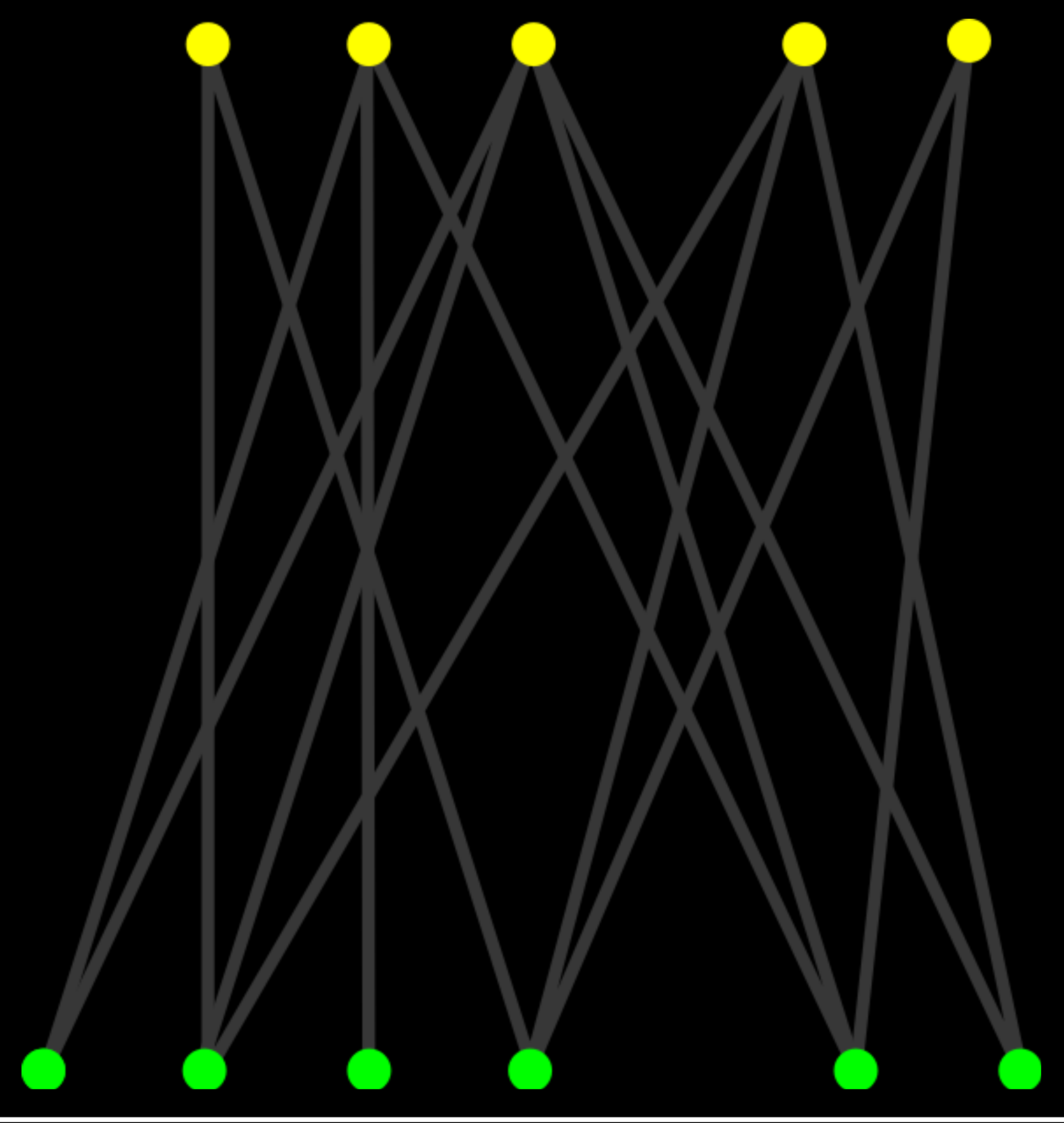

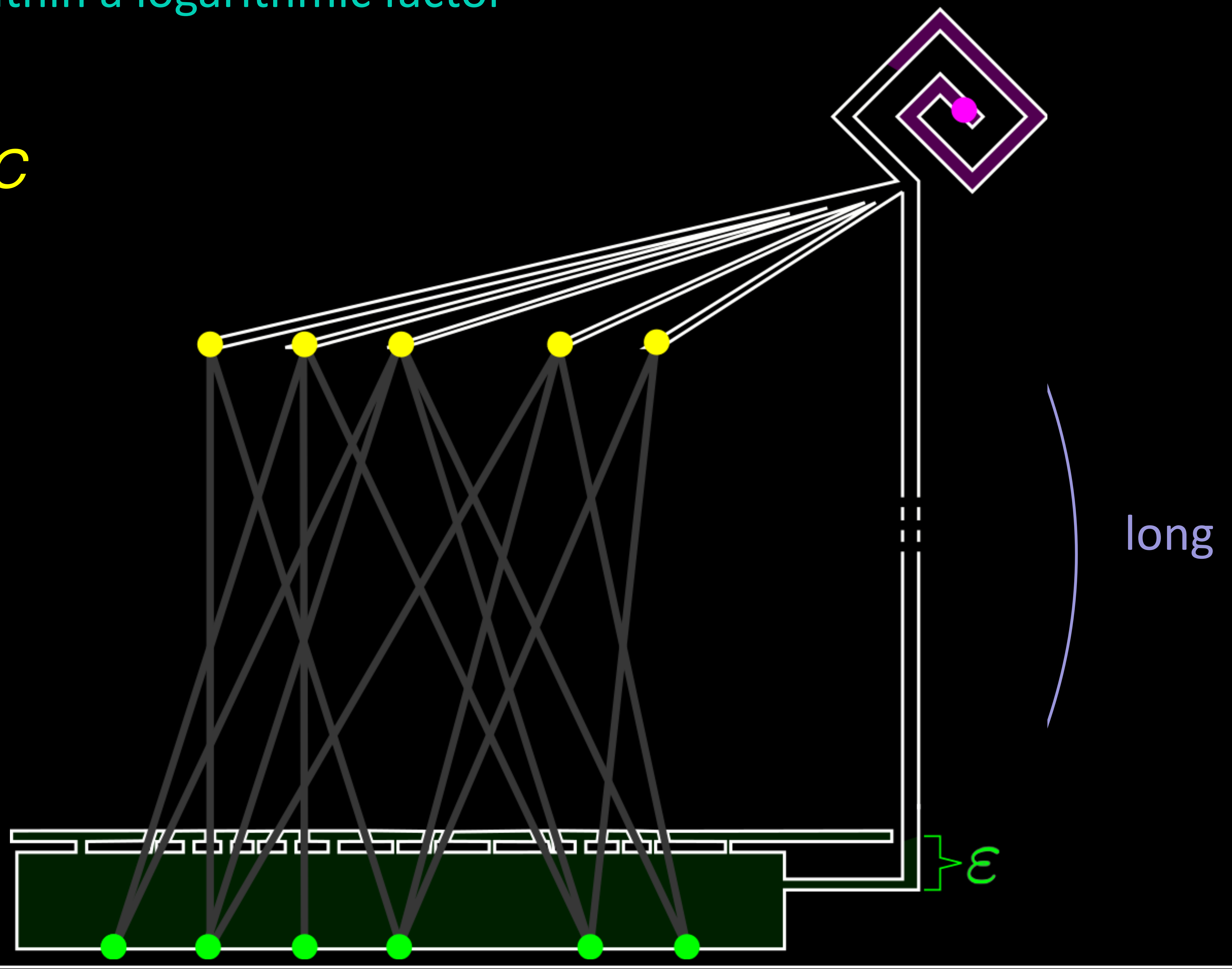

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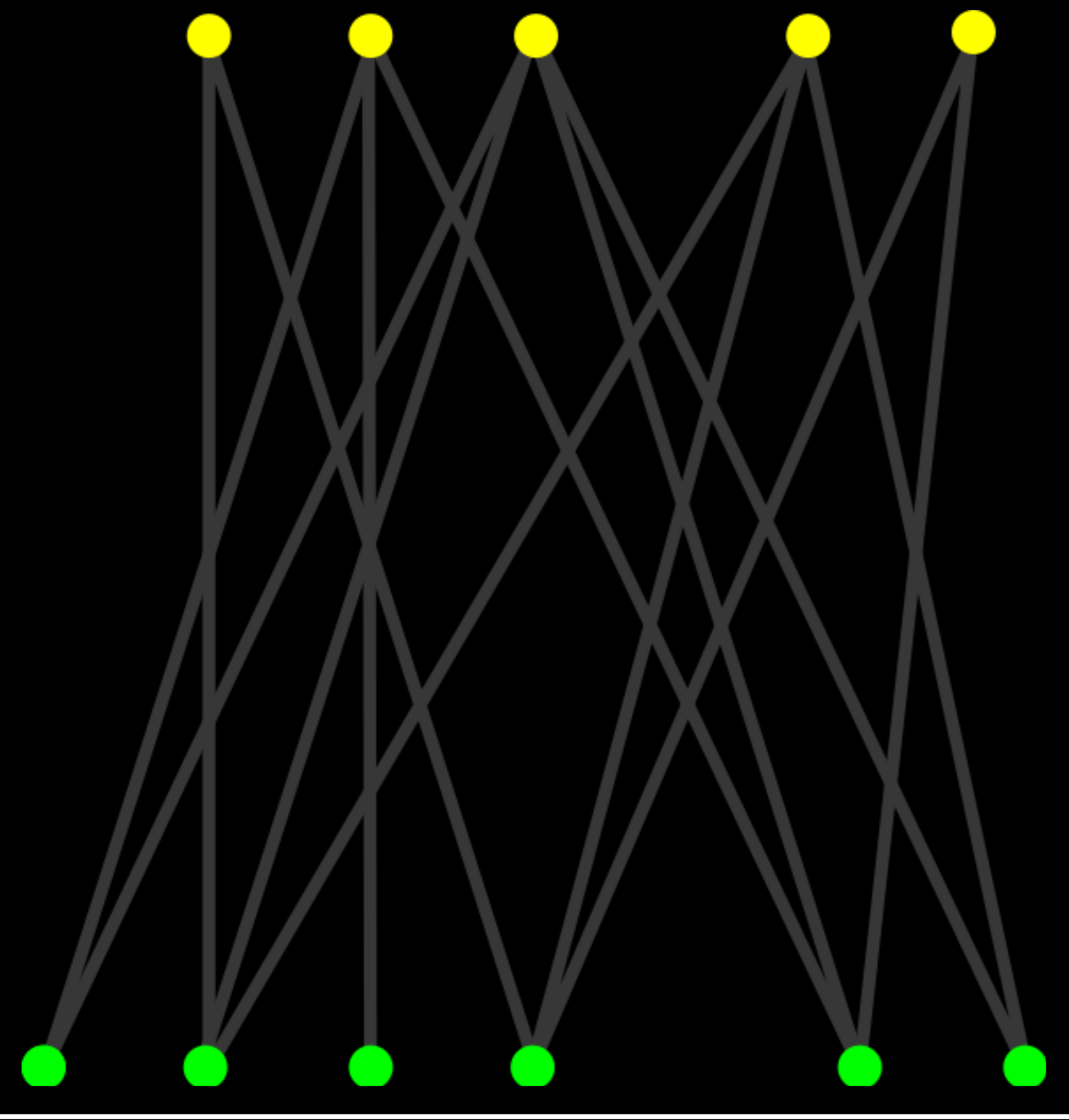

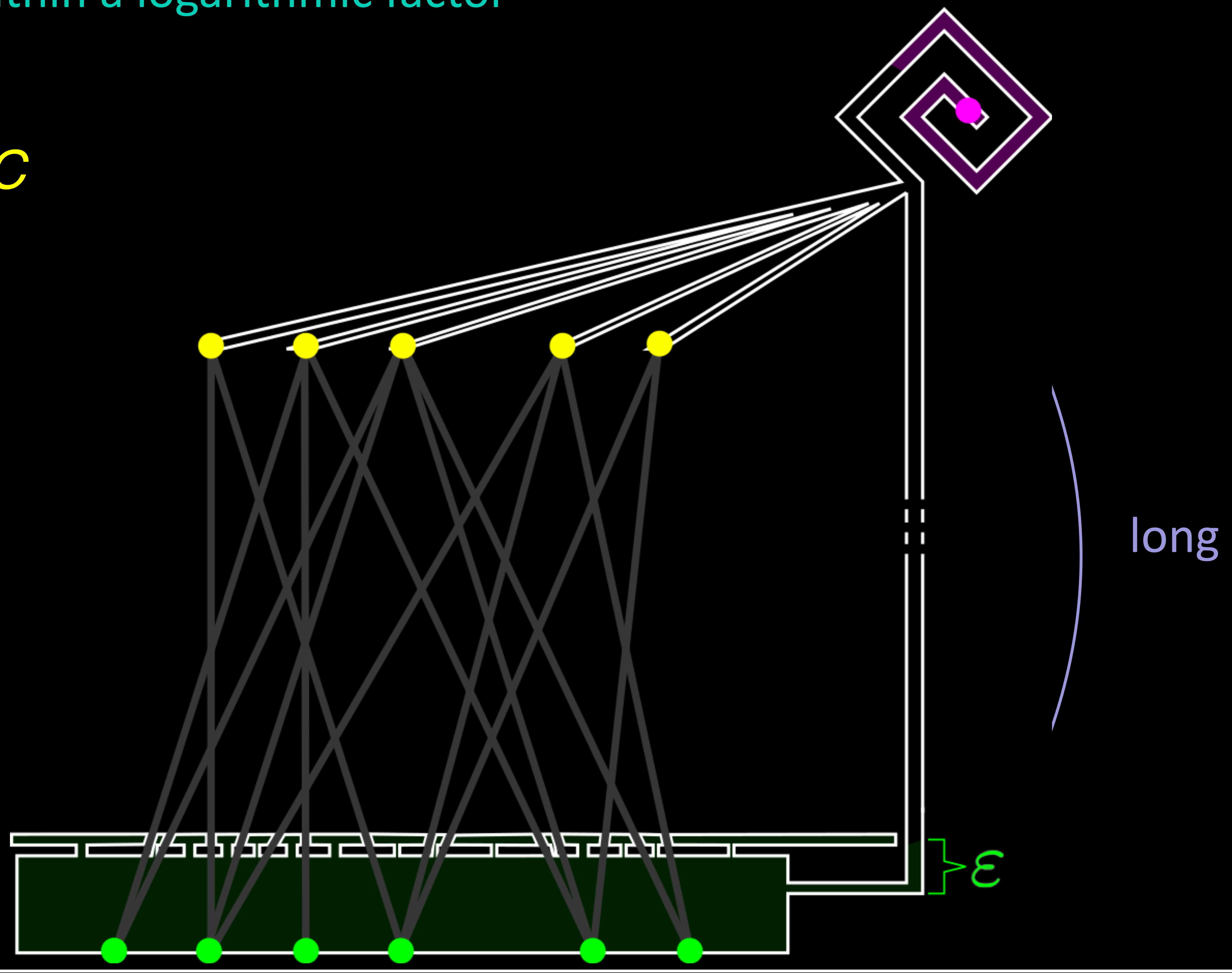

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Corollary: The same holds for *k*-TrWRP(*S,P,s*).

k-Transmitter Watchman Routes

Well, actually, for $k \geq 4$ hard to approximate even for “simpler” polygon classes (than simple polygons).

[Recent joint work with Anna Brötzner, Bengt J. Nilsson, Valentin Polishchuk]

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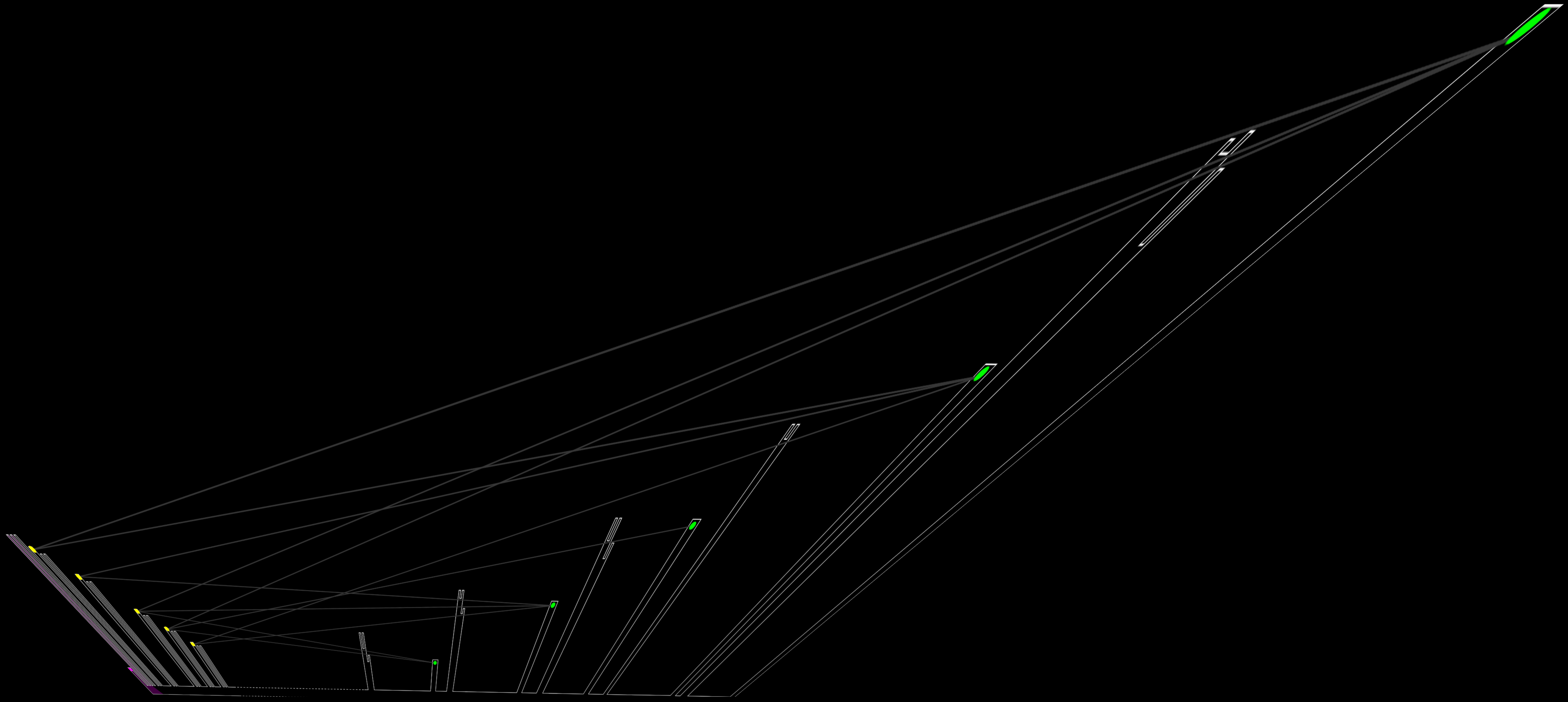
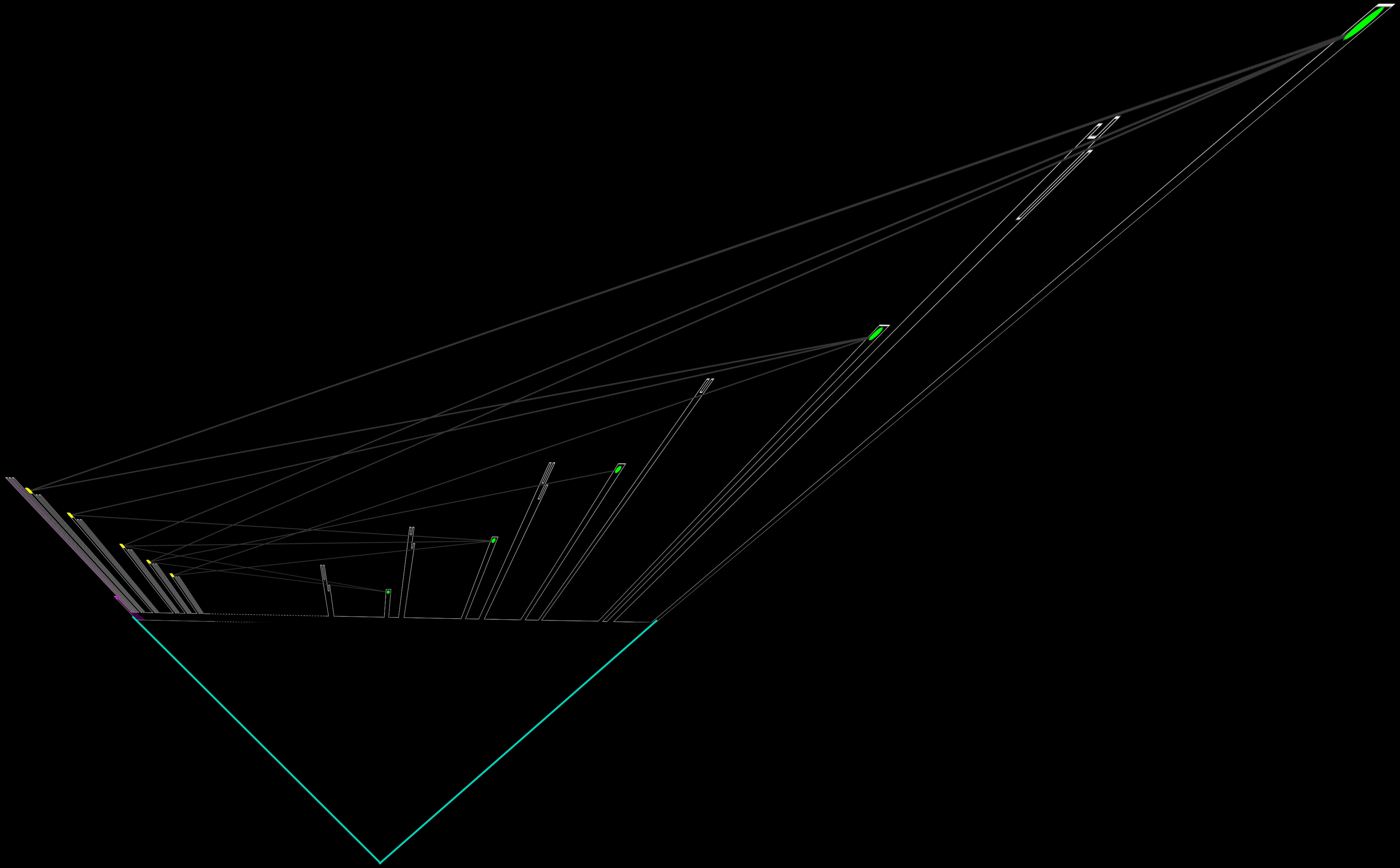
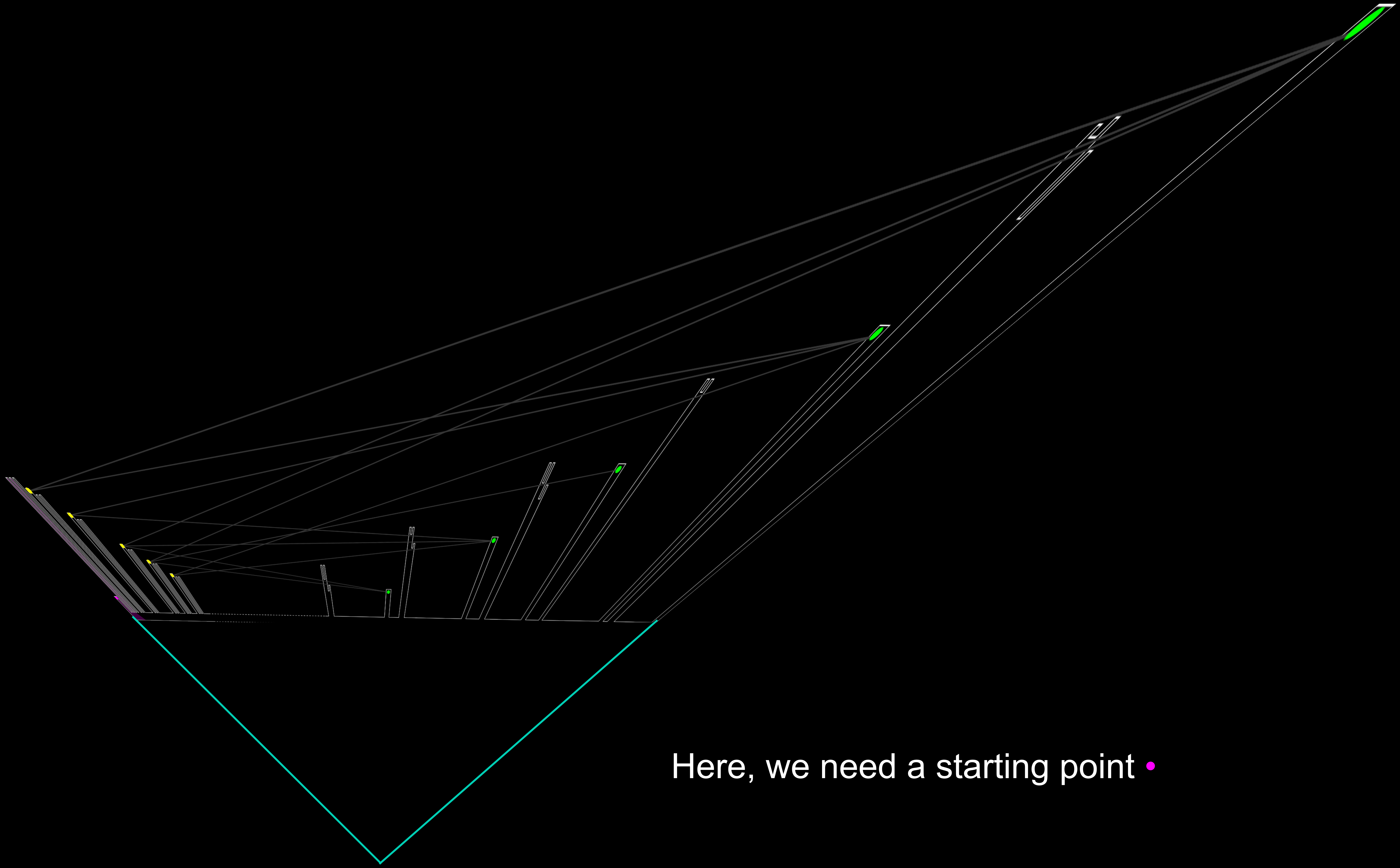
When we map a point (x, y) to $(x, y+cx)$ for a large enough constant c, we obtain a ***x-y*-monotone polygon** for which the **visibility properties are maintained**

We can even transform our histogram into a star-shaped polygon:







Here, we need a starting point •

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Approximation Algorithm for *k*-TrWRP(*S*,*P*,*s*)

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Theorem 3: Let *P* be a simple polygon with $n=|P|$. Let OPT(S,P,s) be the optimal solution for the *k*-TrWRP(S,P,s) and let R be the solution by our algorithm ALG(S,P,s). Then R yields an approximation ratio of $O(\log^2(|S|\,n)\log\log(|S|\,n)\log|S|)$.

Approximation Algorithm for *k*-TrWRP(*S*,*P*,*s*)

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- Create a candidate point for each connected component of the *k*-visibility region of each point in *S*.

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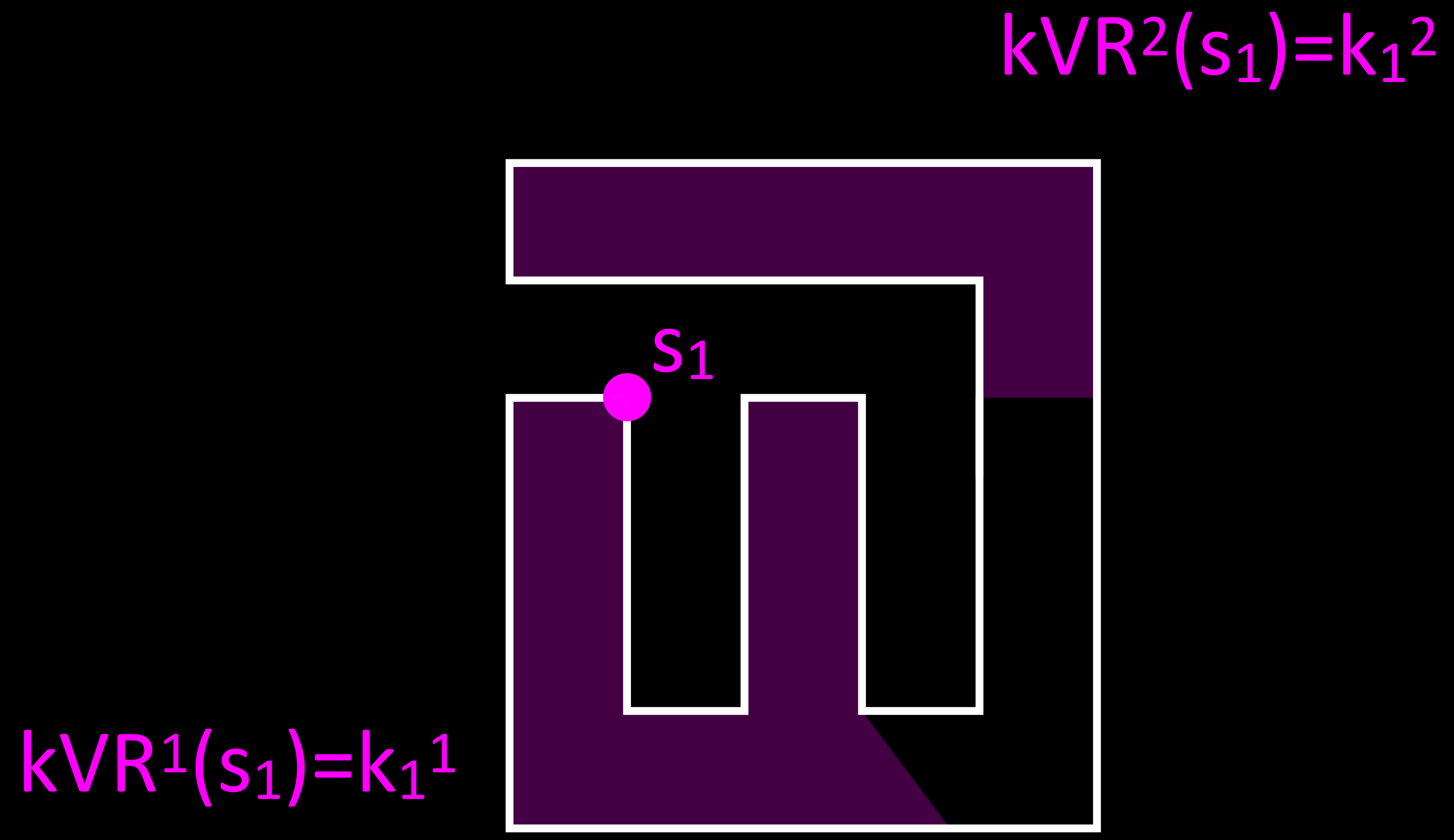

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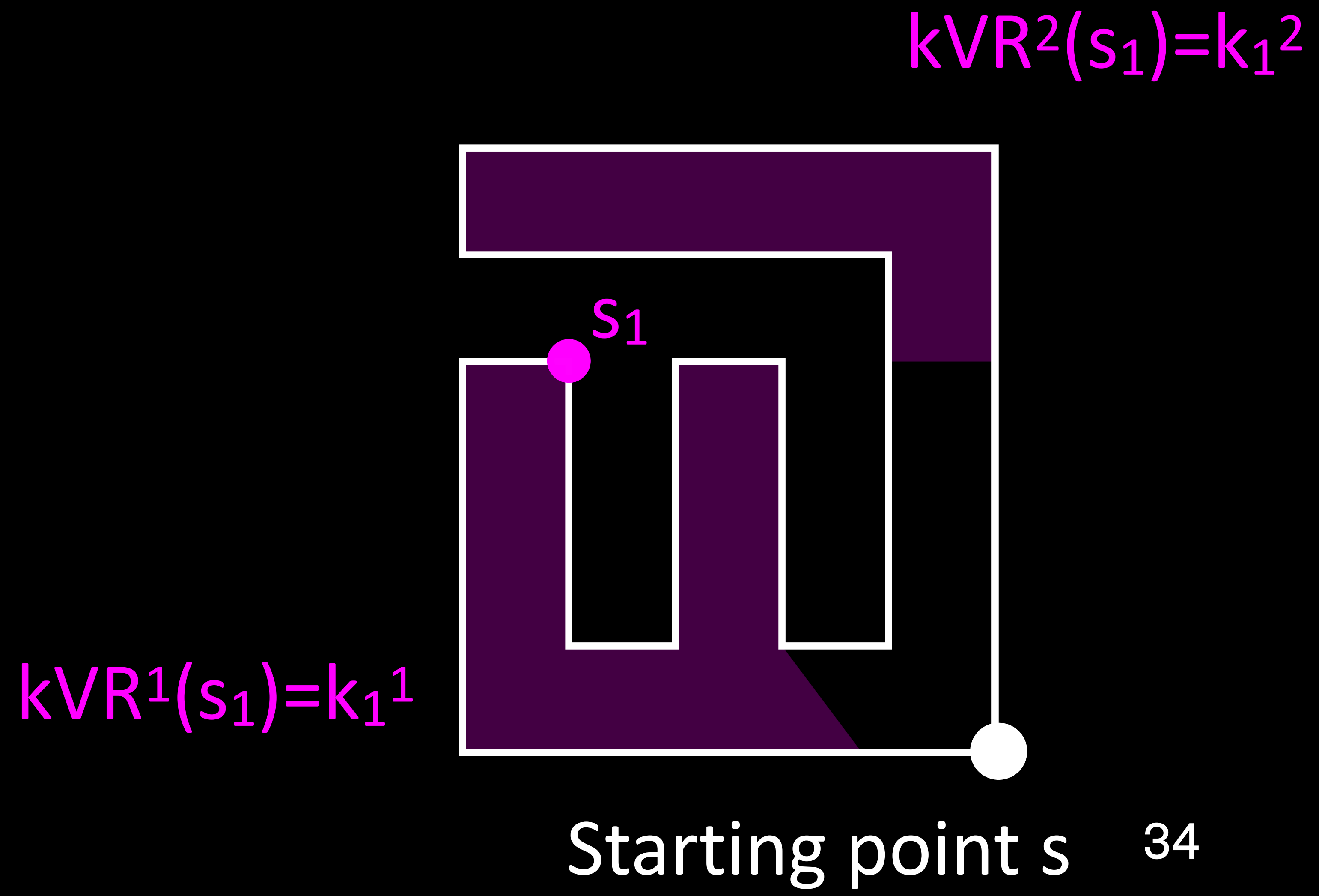

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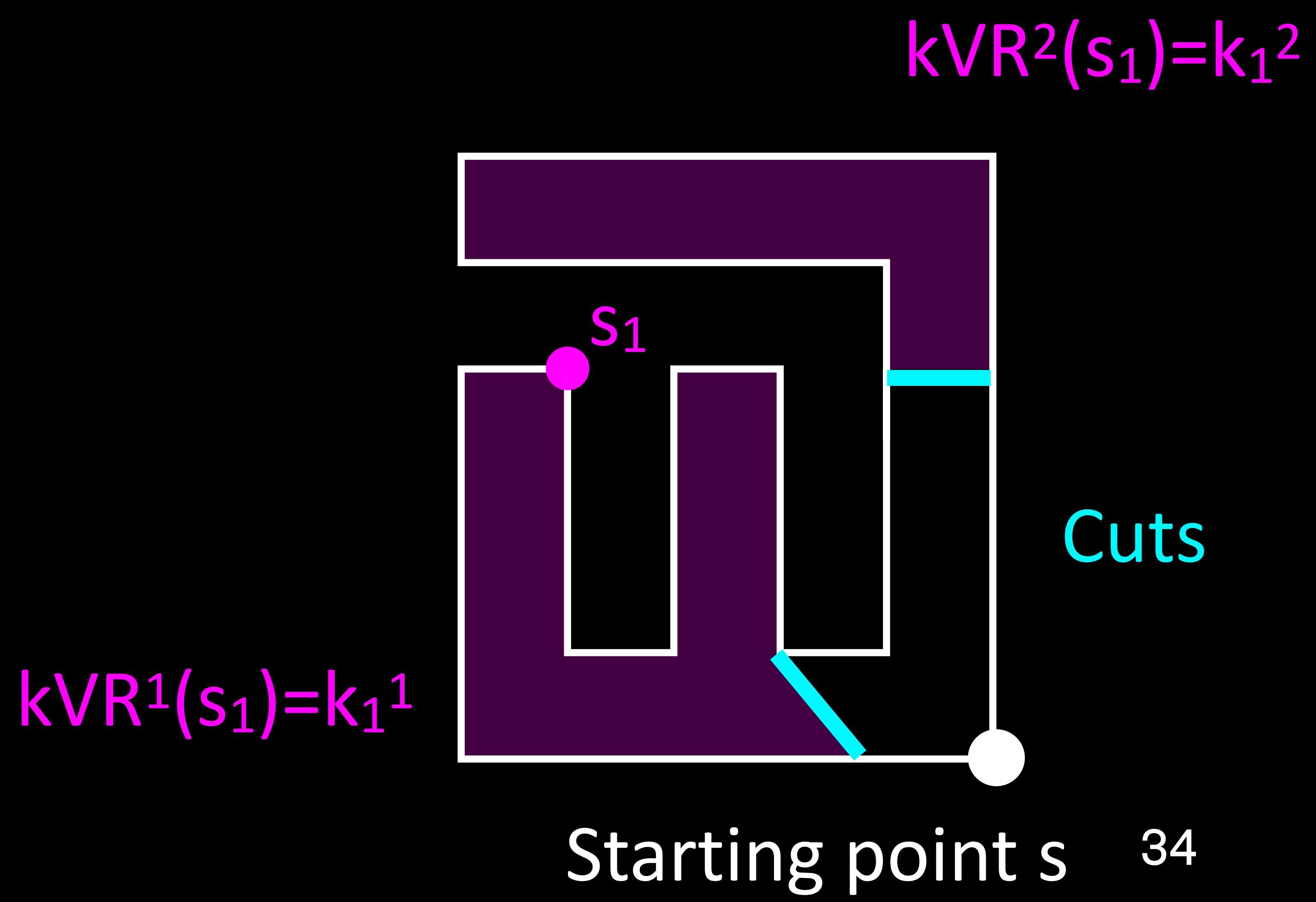

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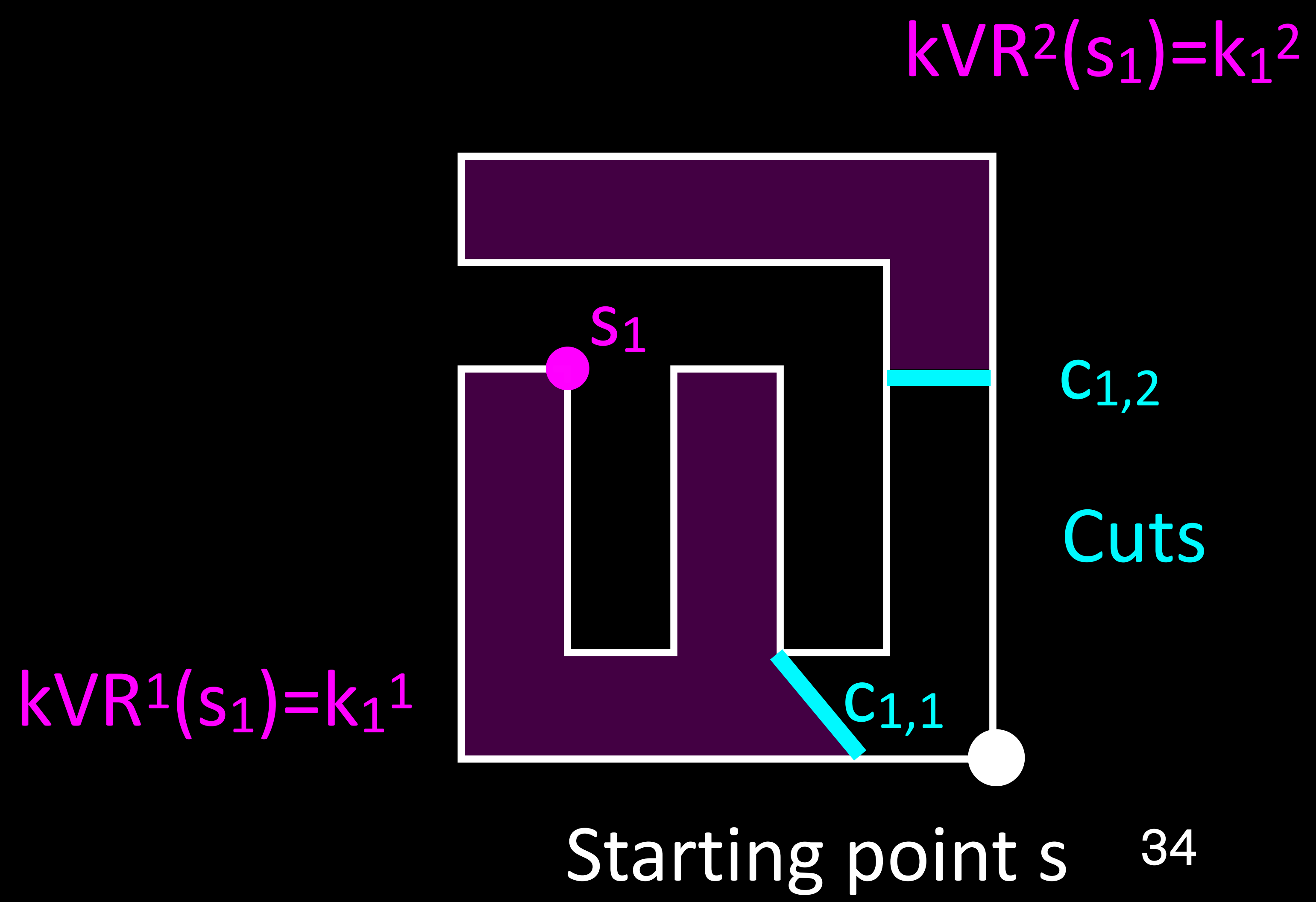

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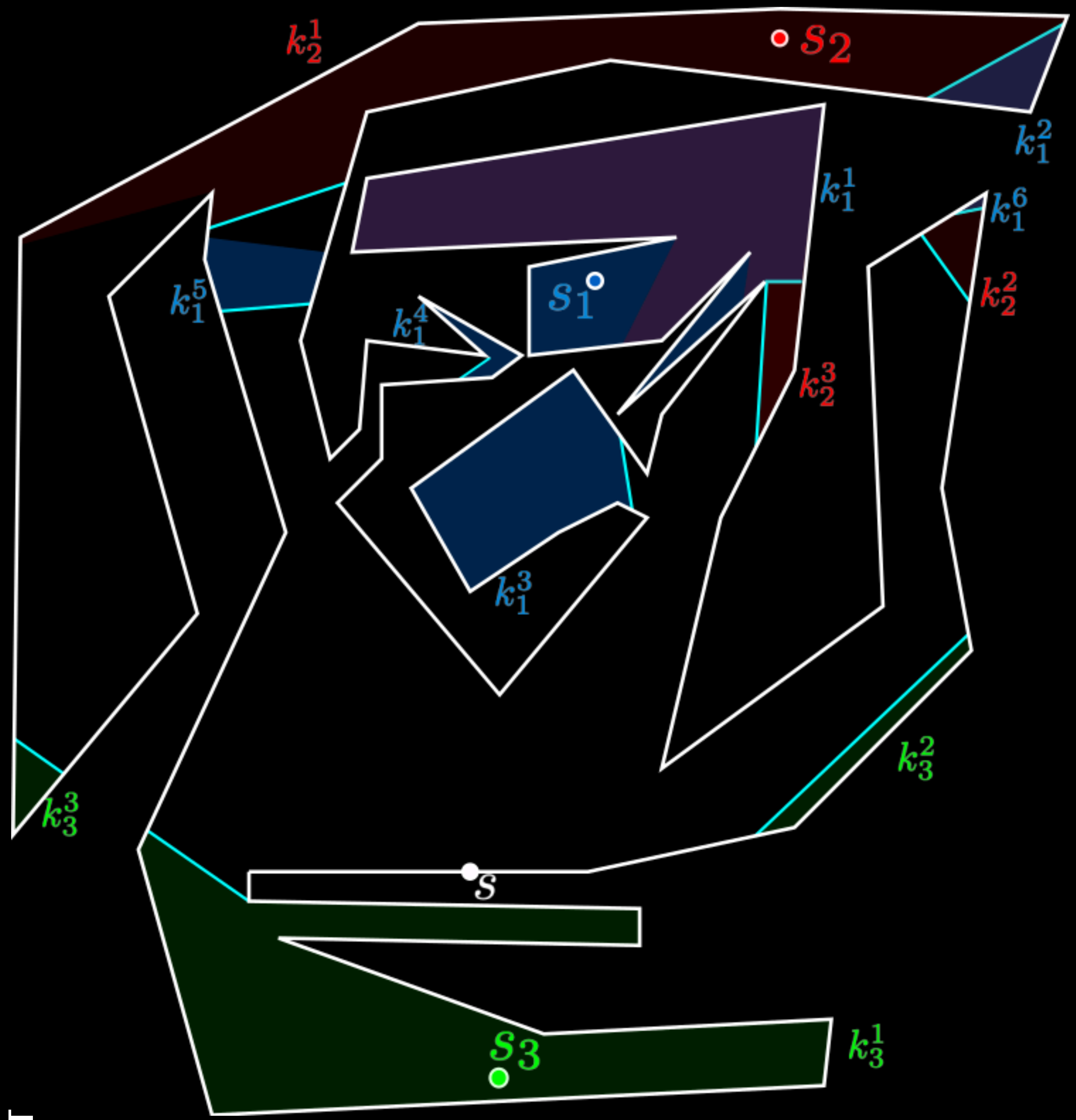

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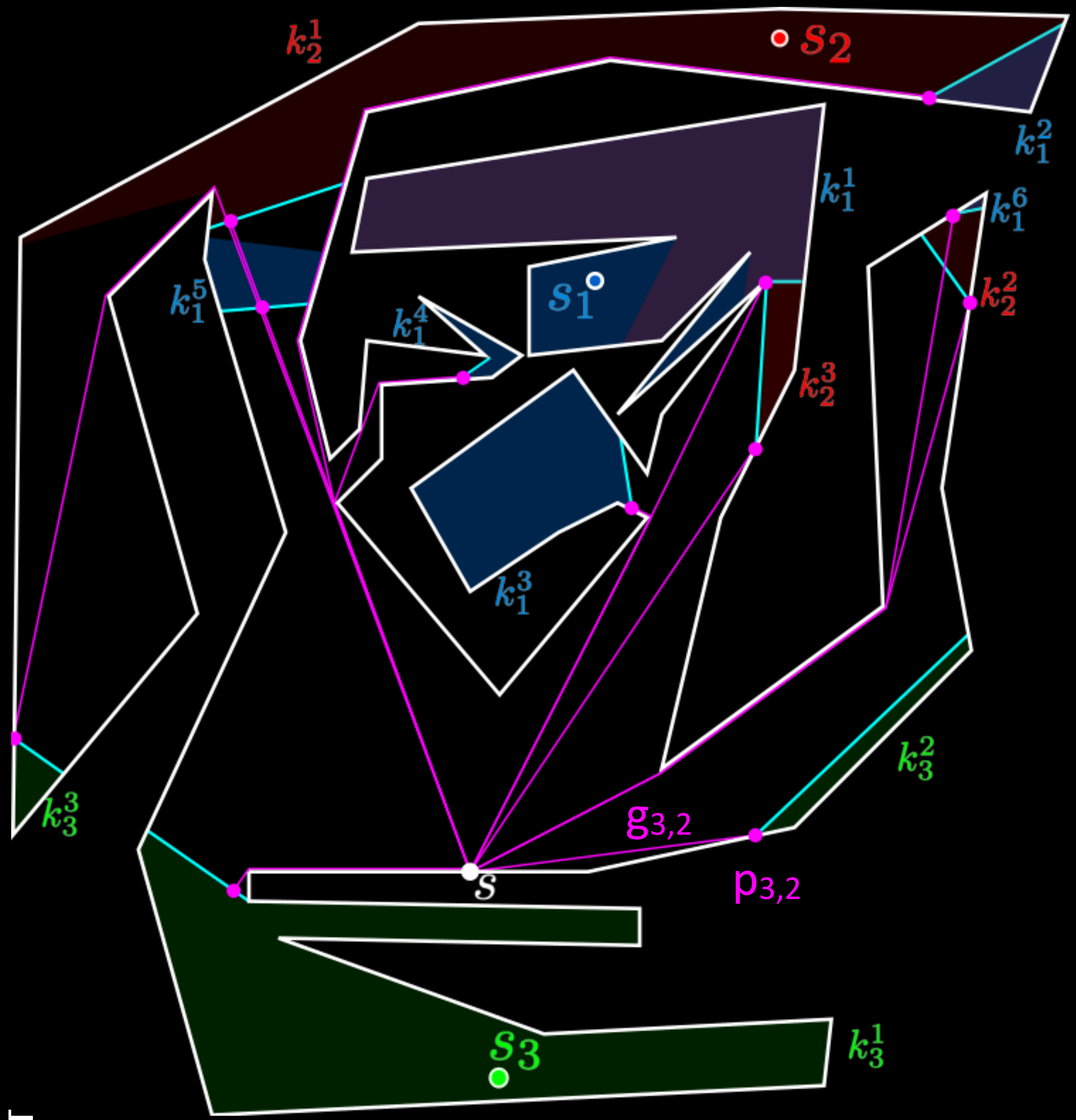

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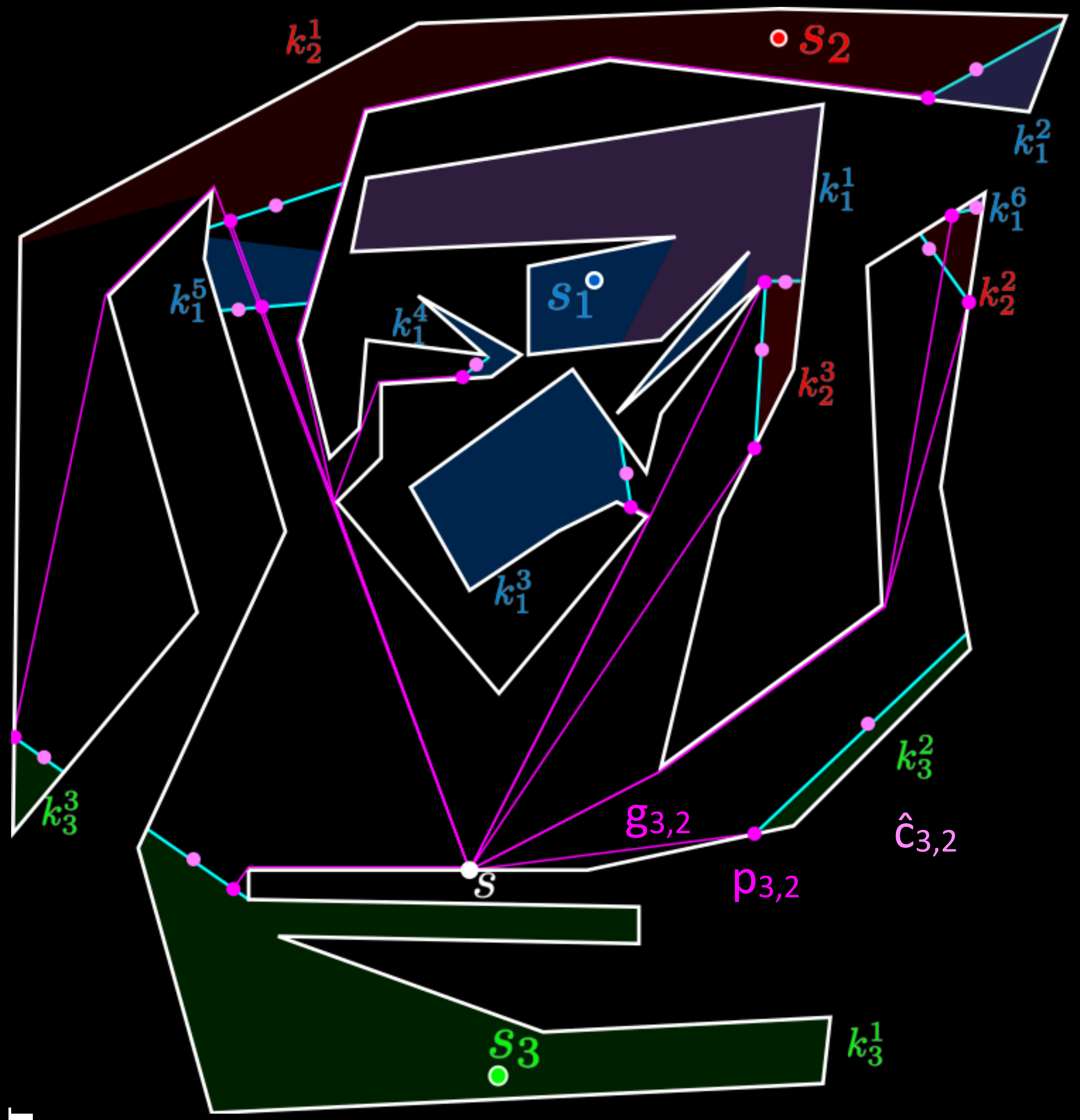

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- Create a candidate point for each connected component of the *k*-visibility region of each point in *S*.
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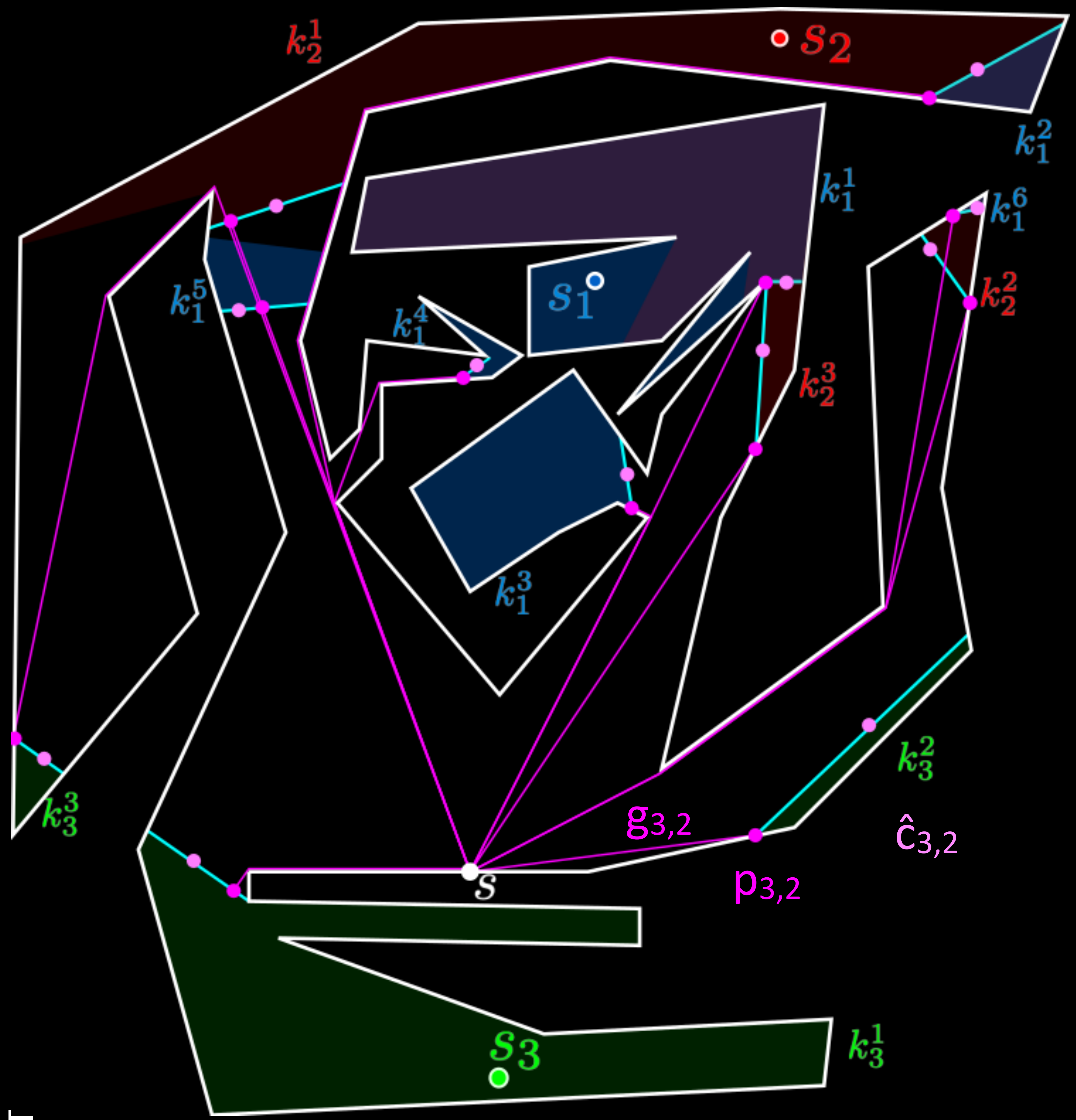

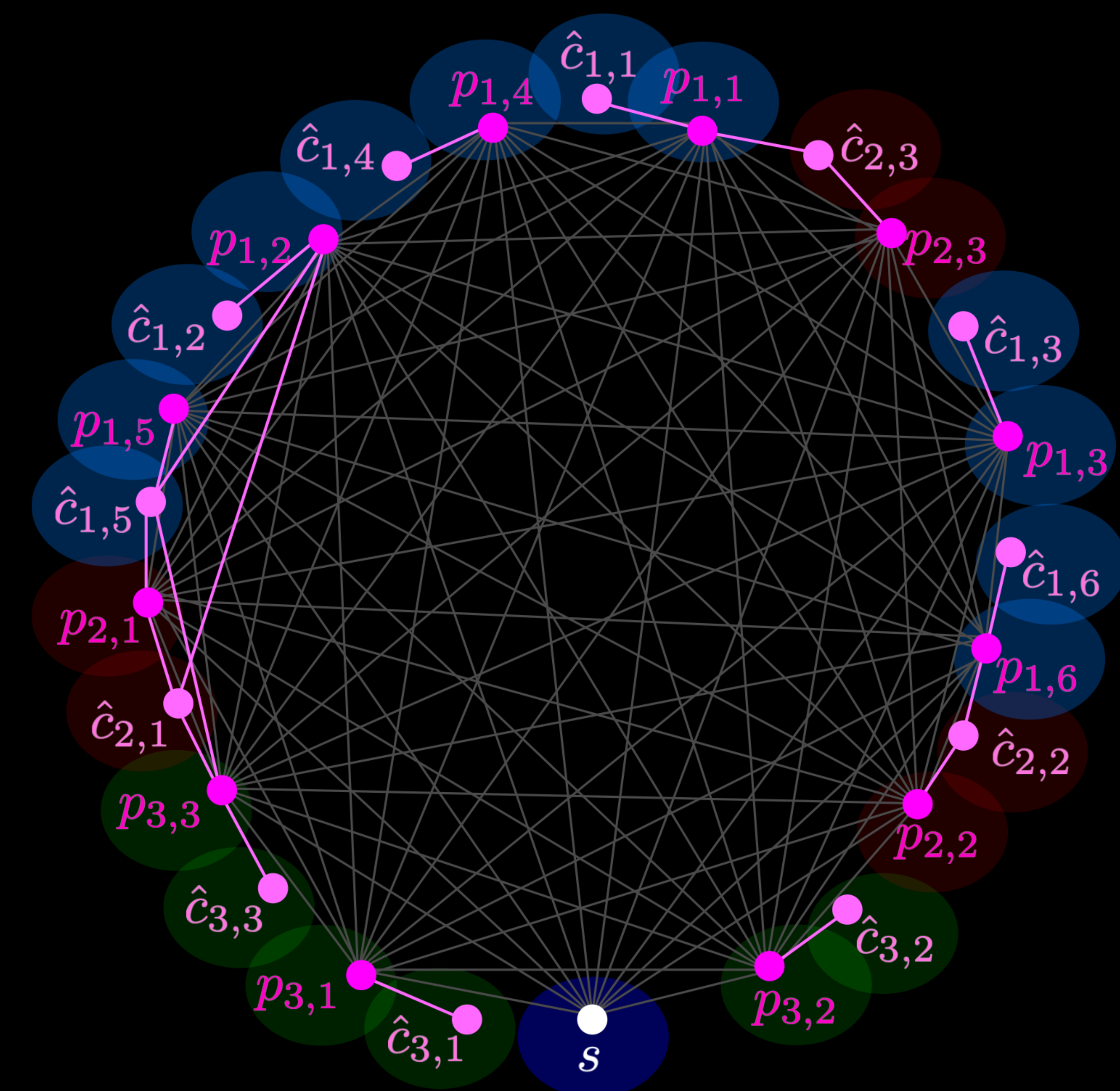

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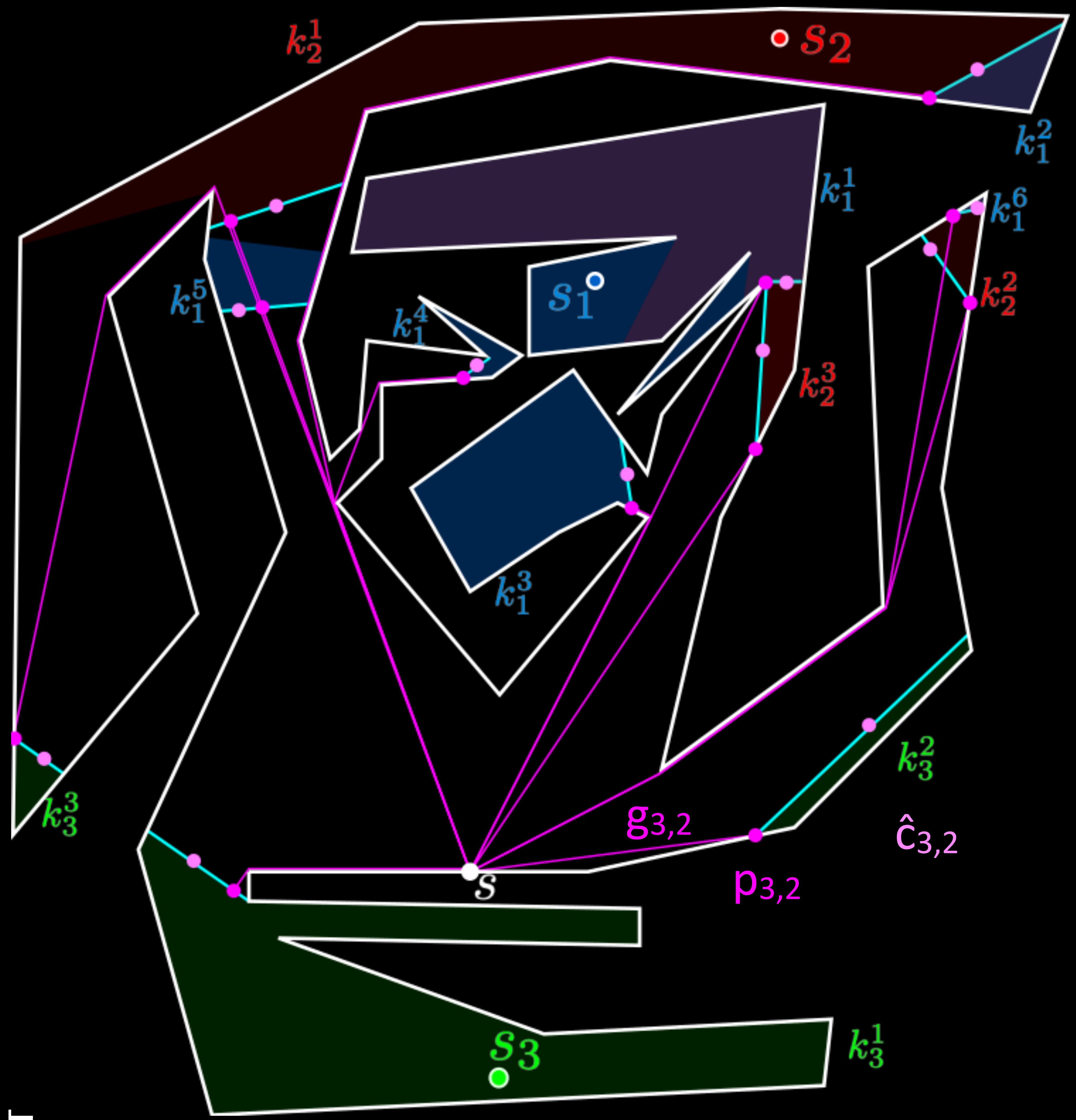

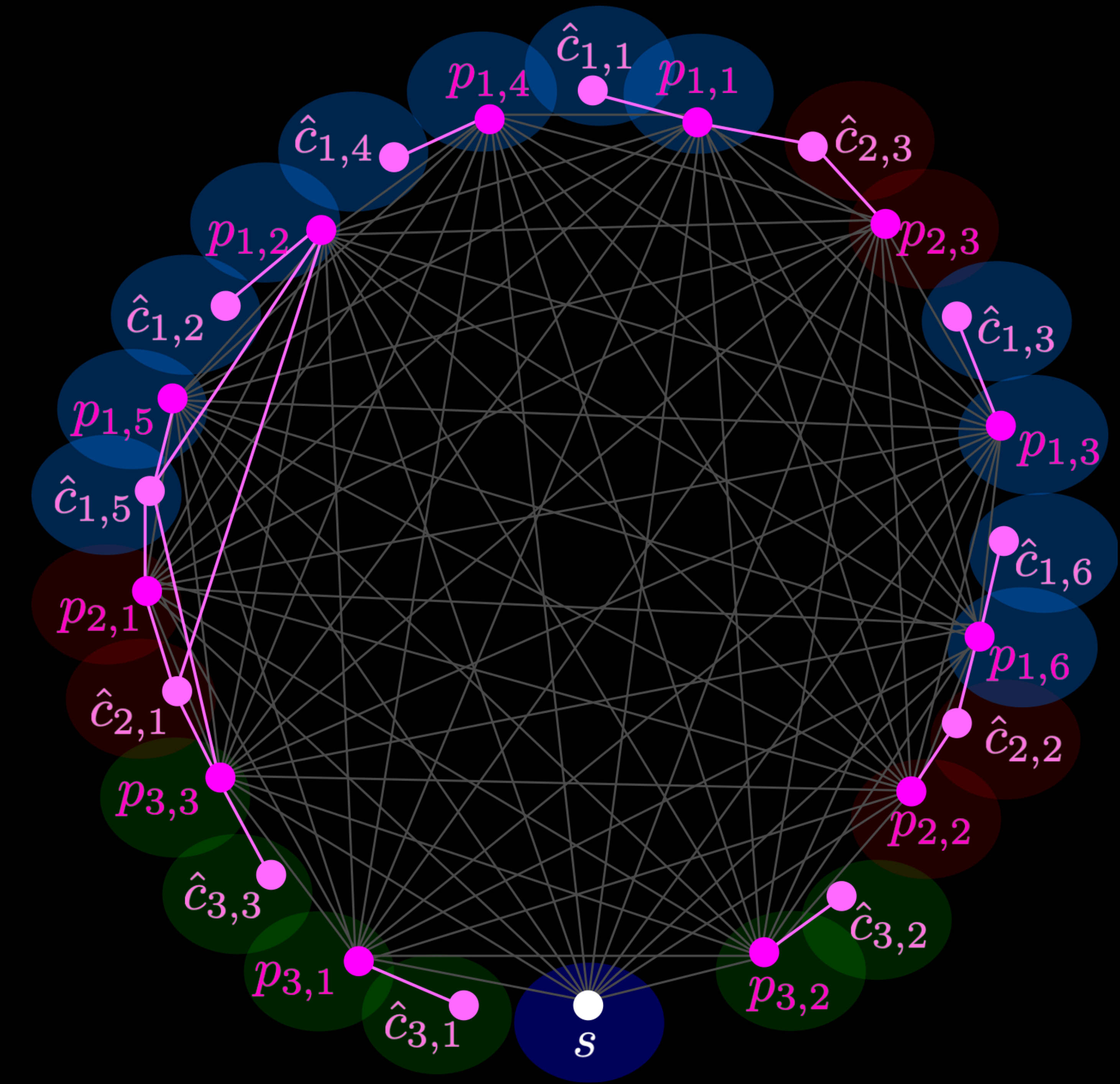

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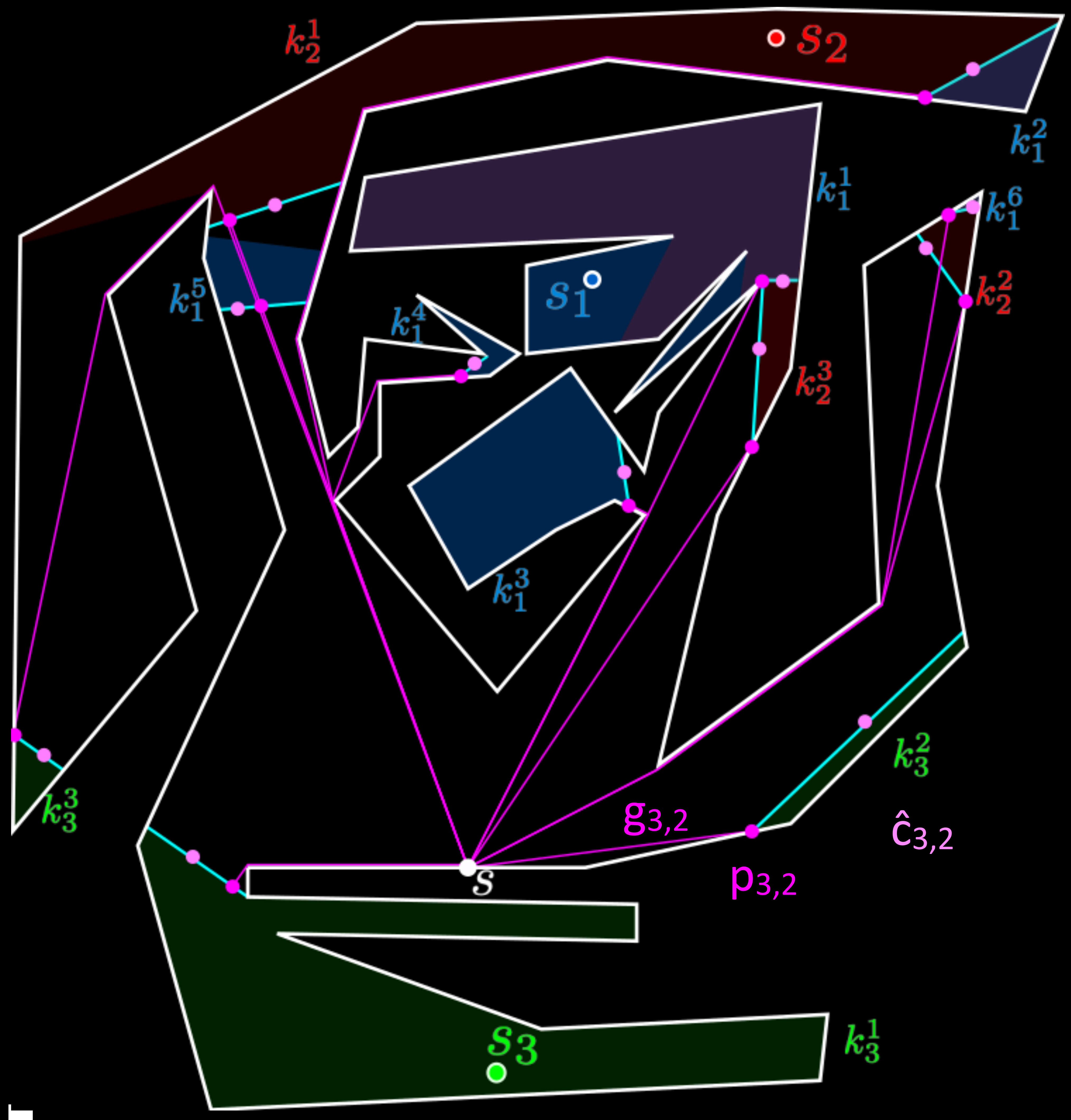

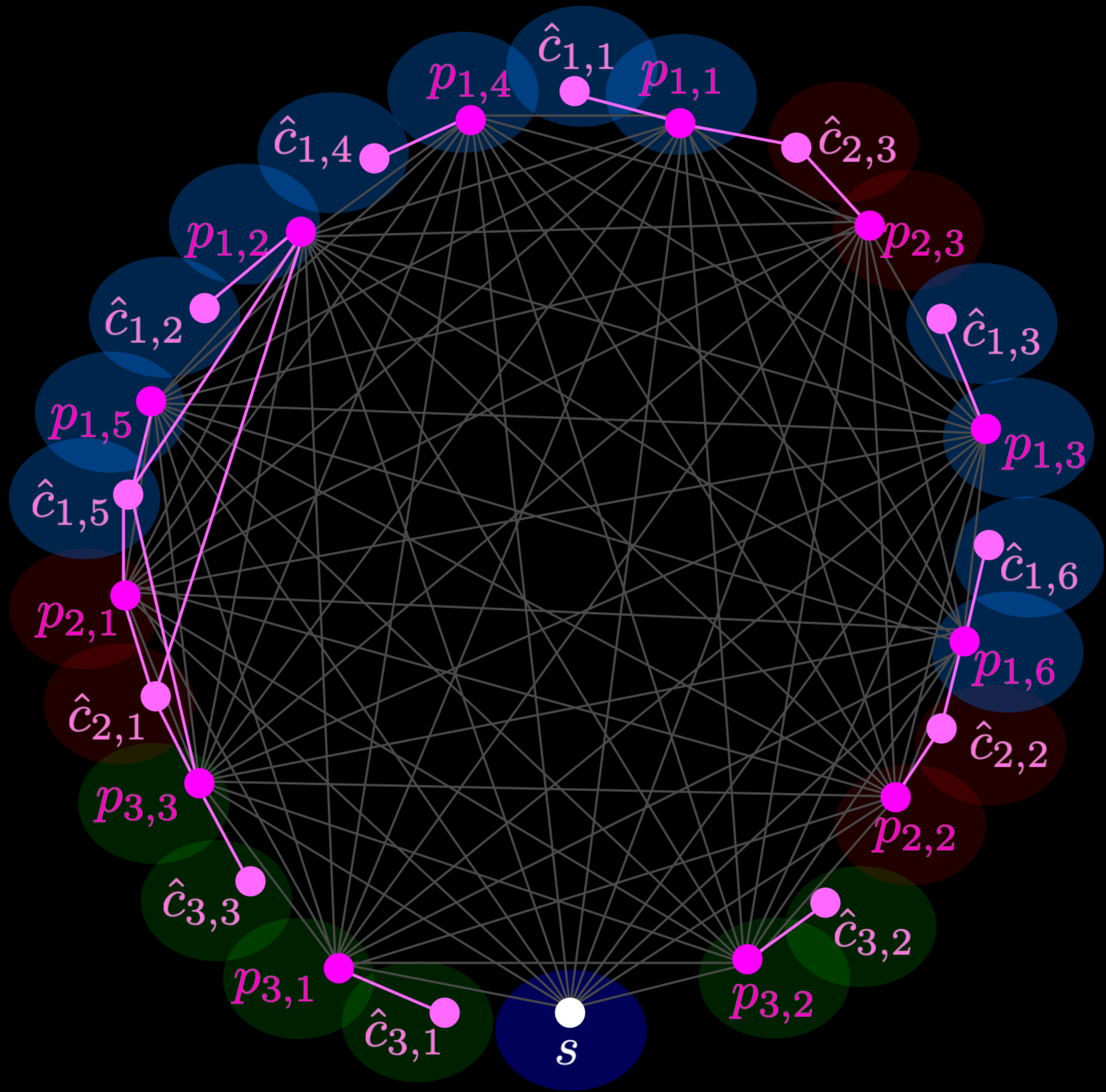

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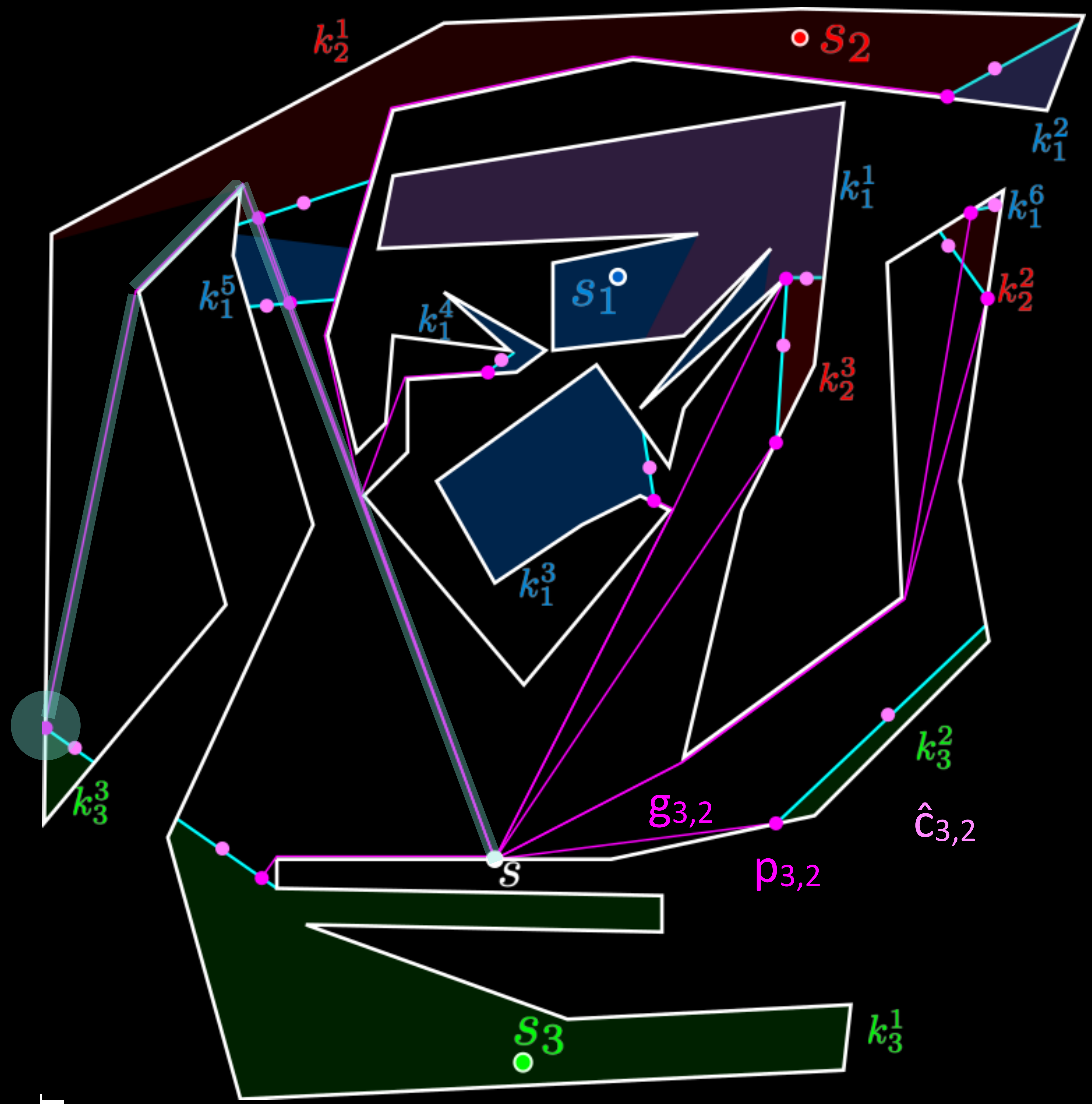
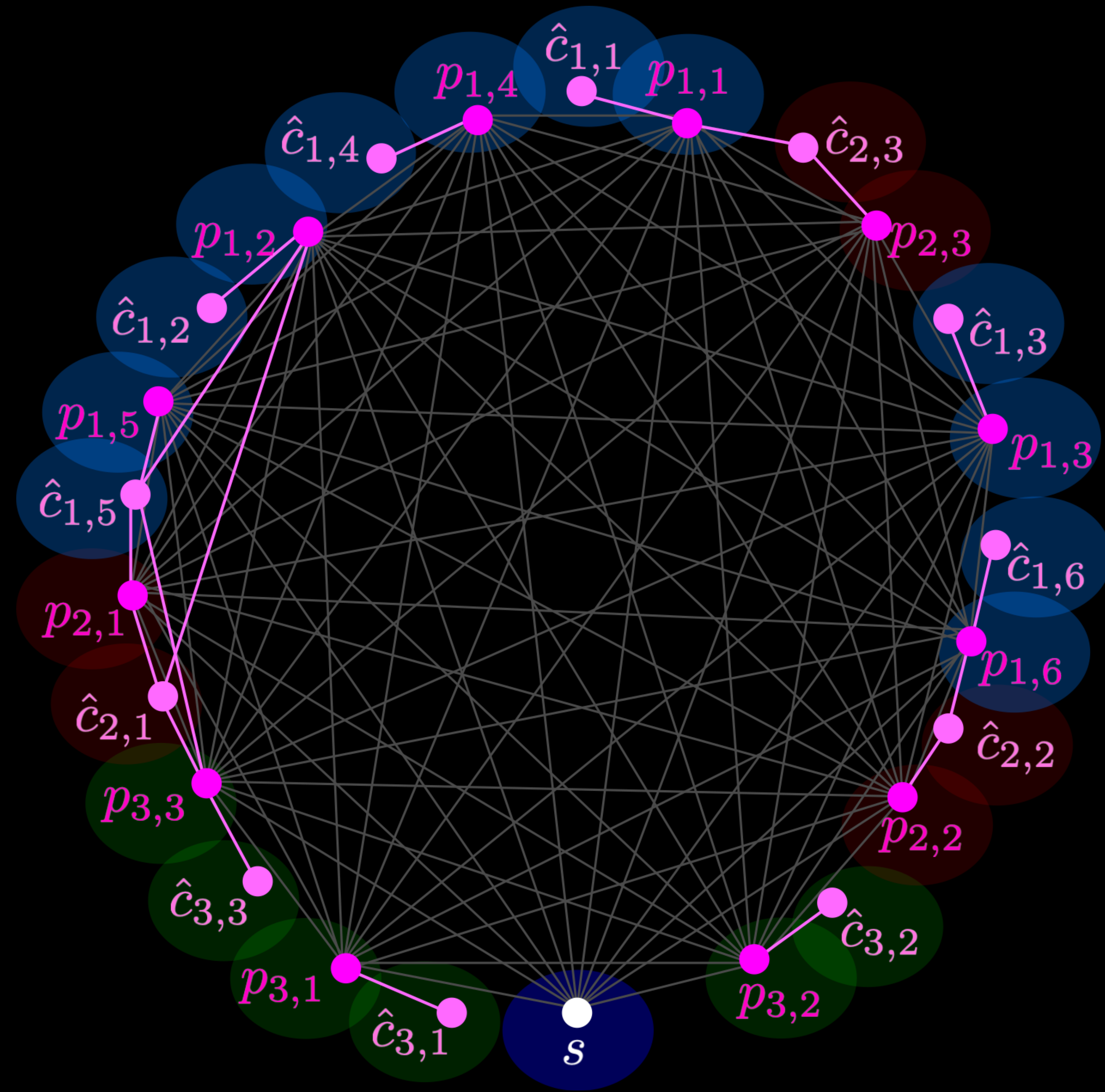
Example: When we visit k_3^3 (in point p_3^3), we also visit the cuts of k_3^3, k_2^1 and k_1^5.
Thus, we have edges from p_3^3 to $\hat{c}_3^3$, $\hat{c}_2^1$, and $\hat{c}_1^5$.





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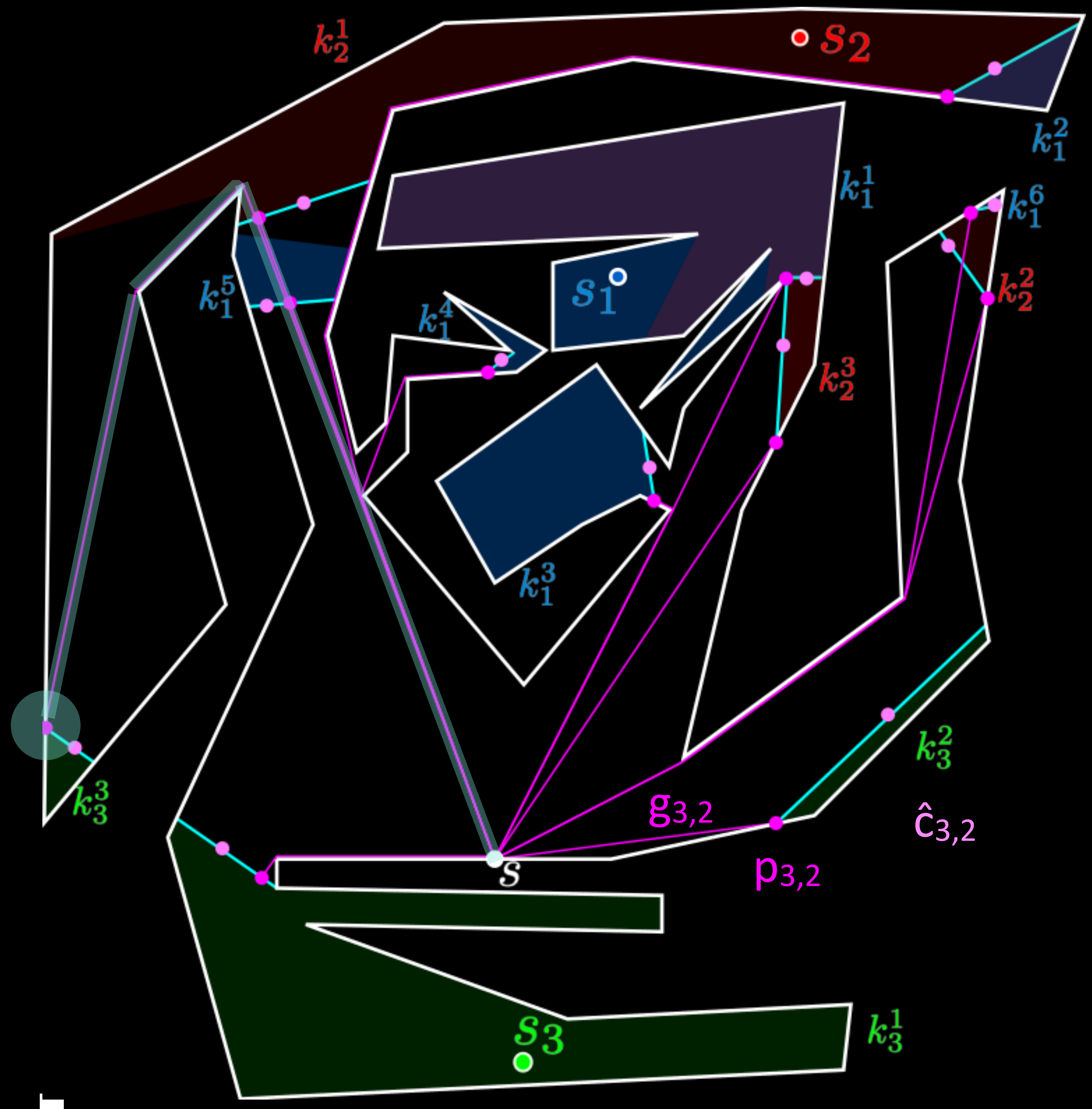

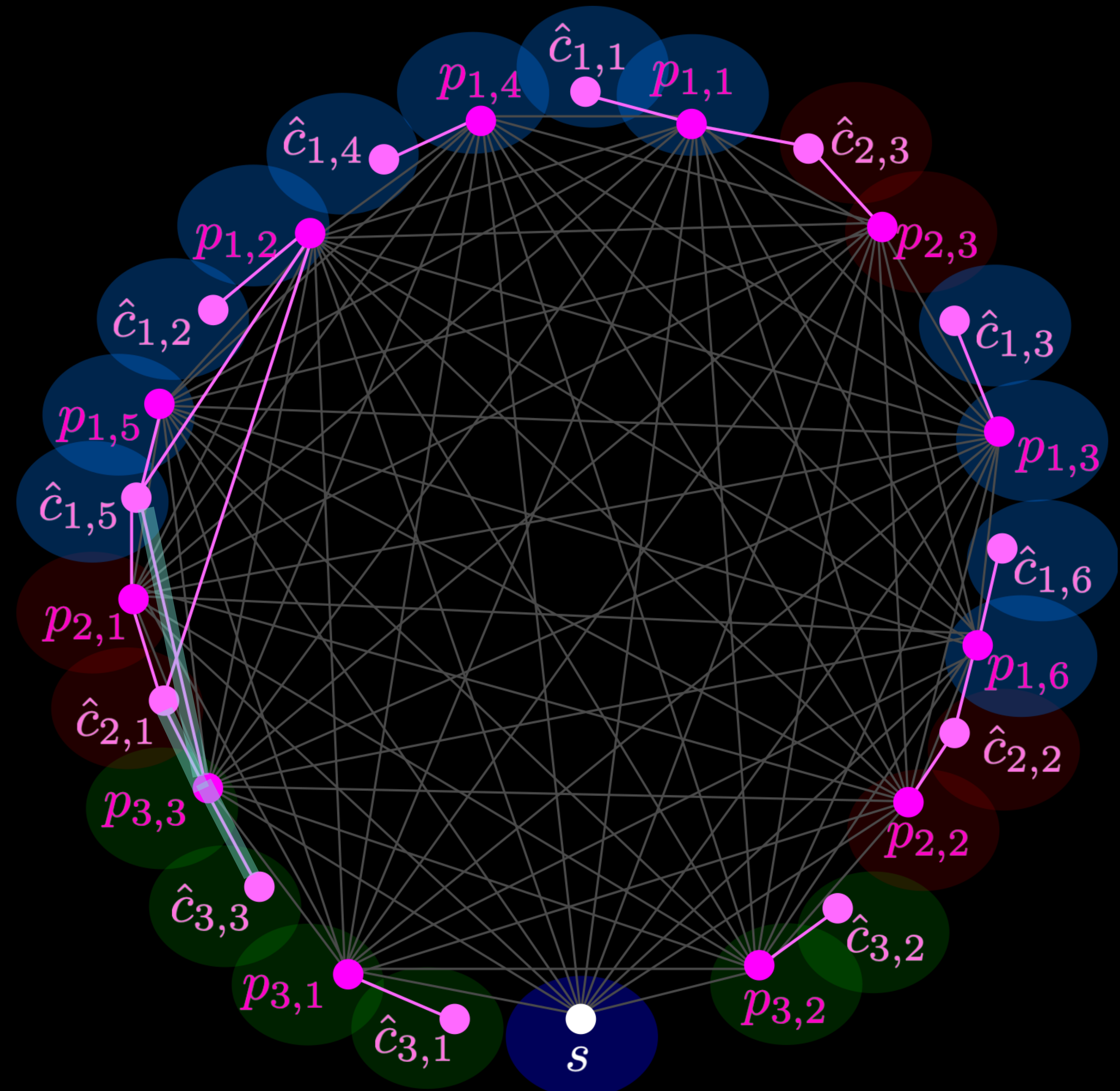

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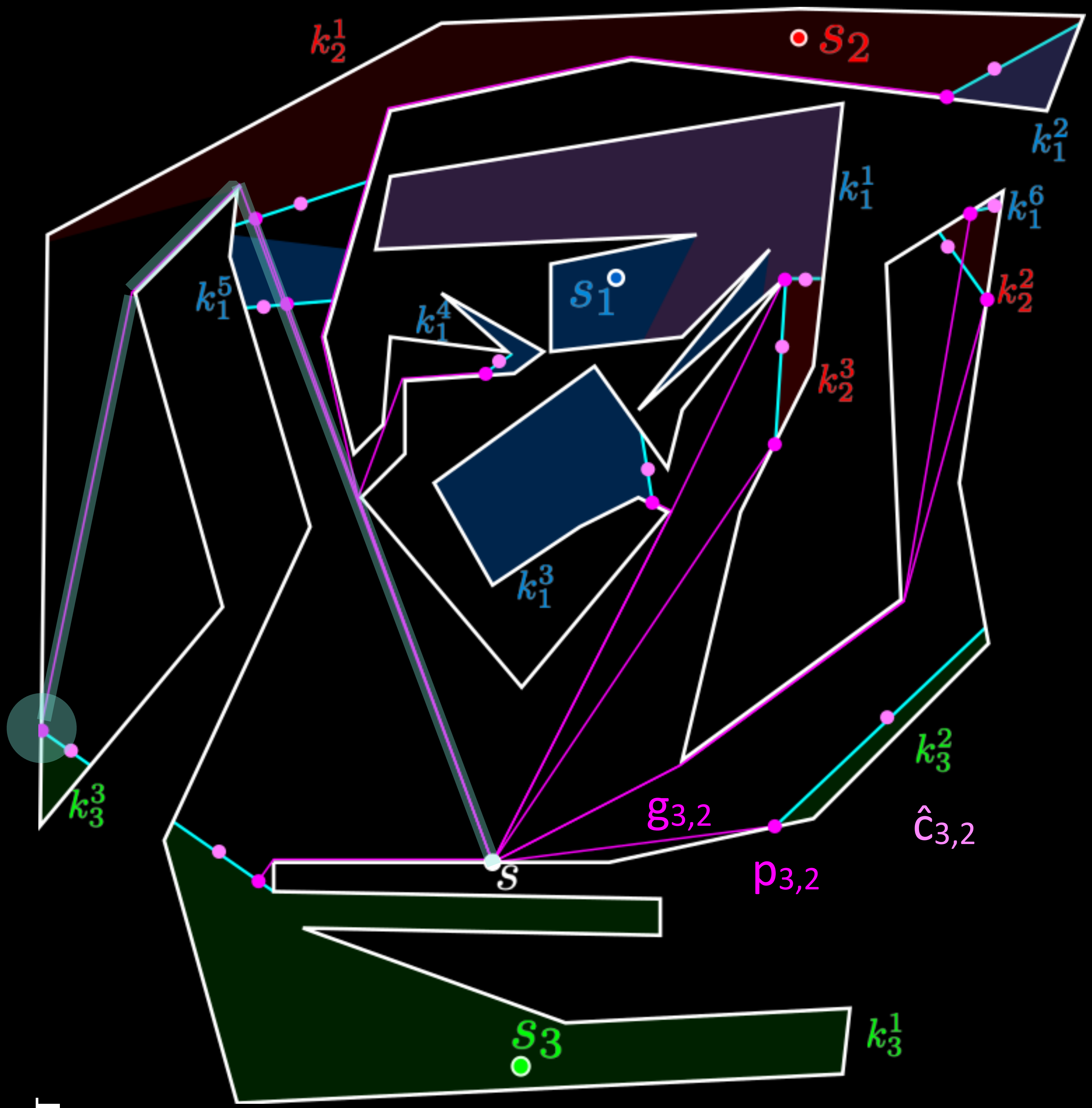

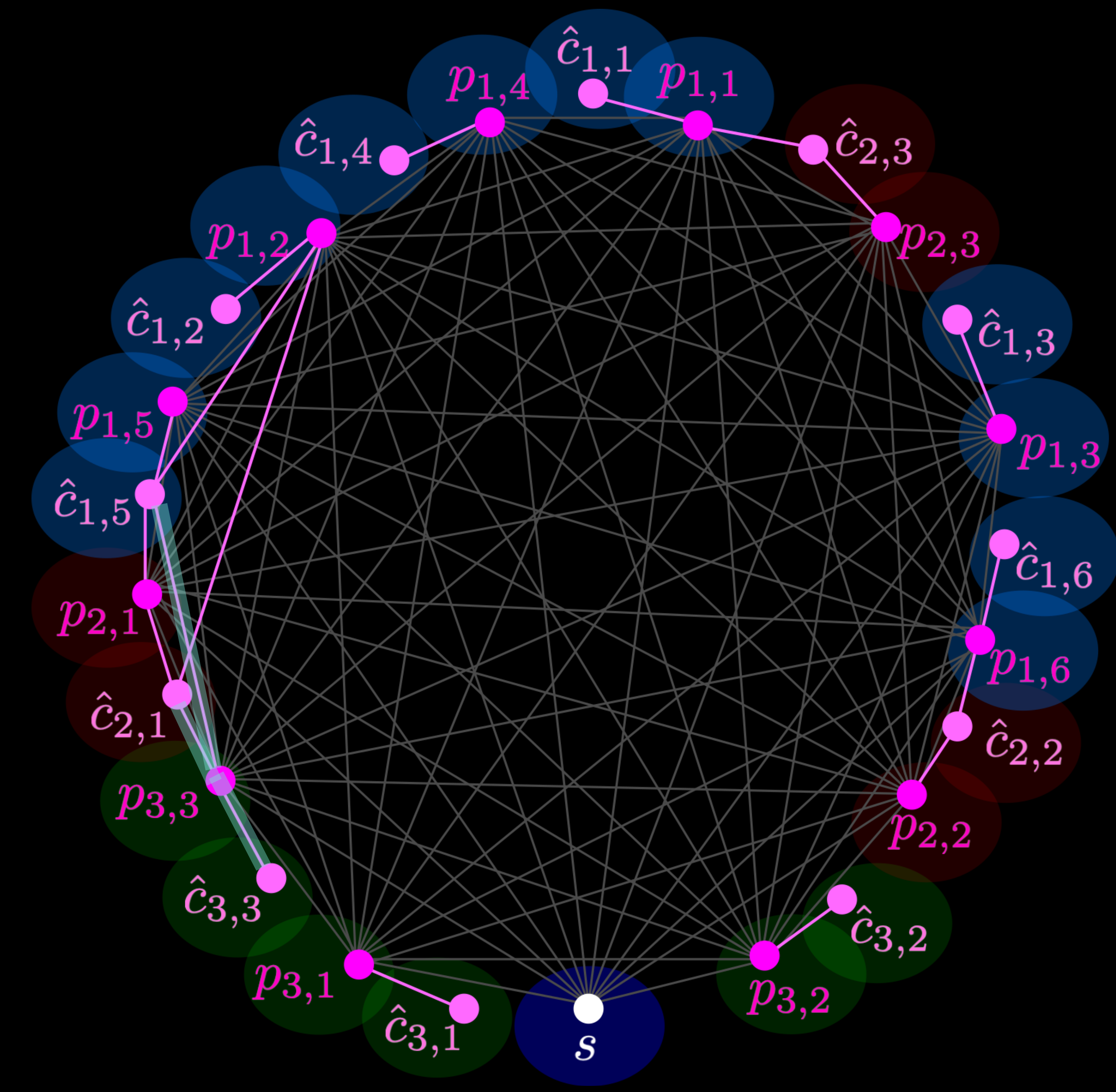

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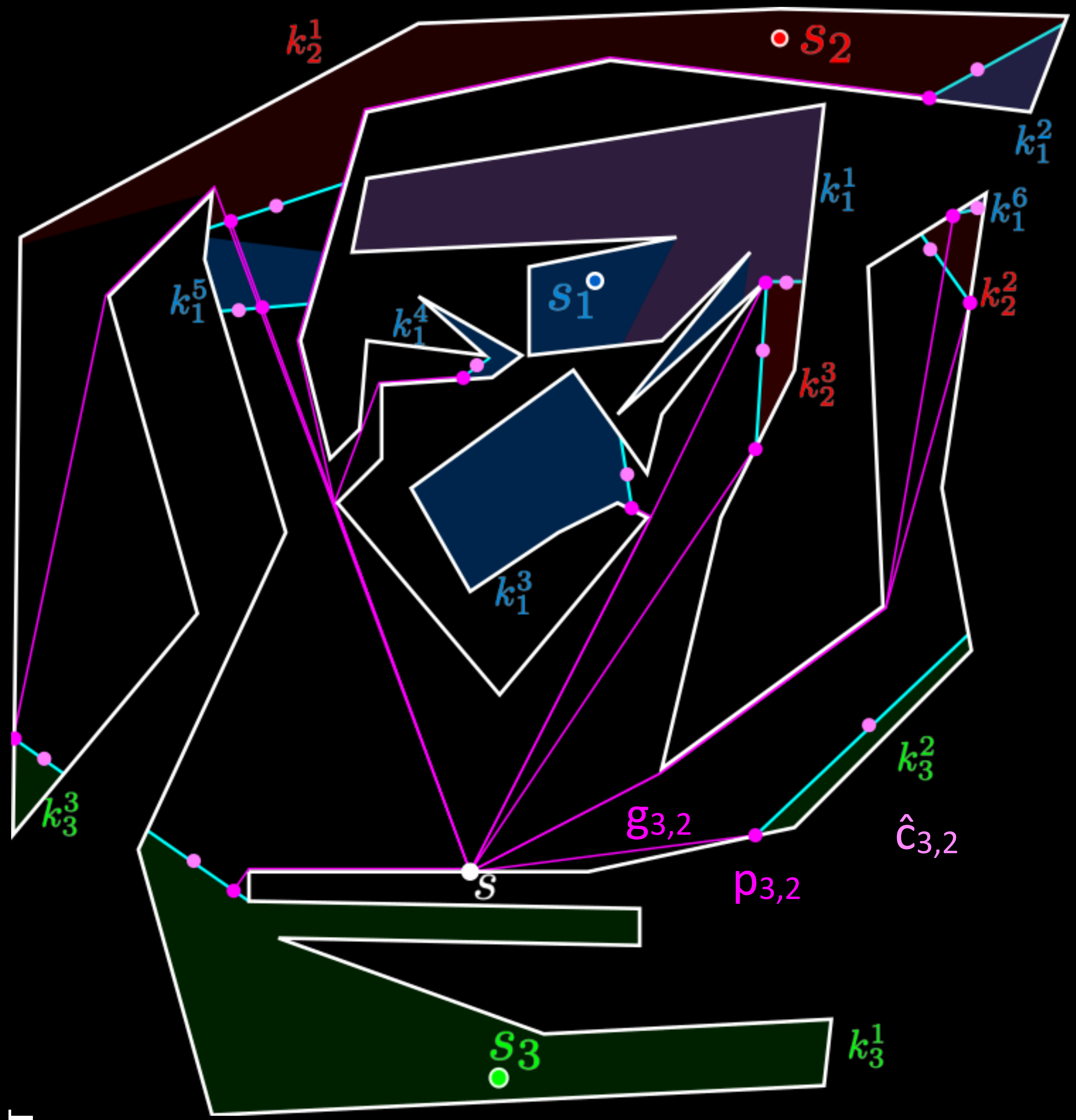

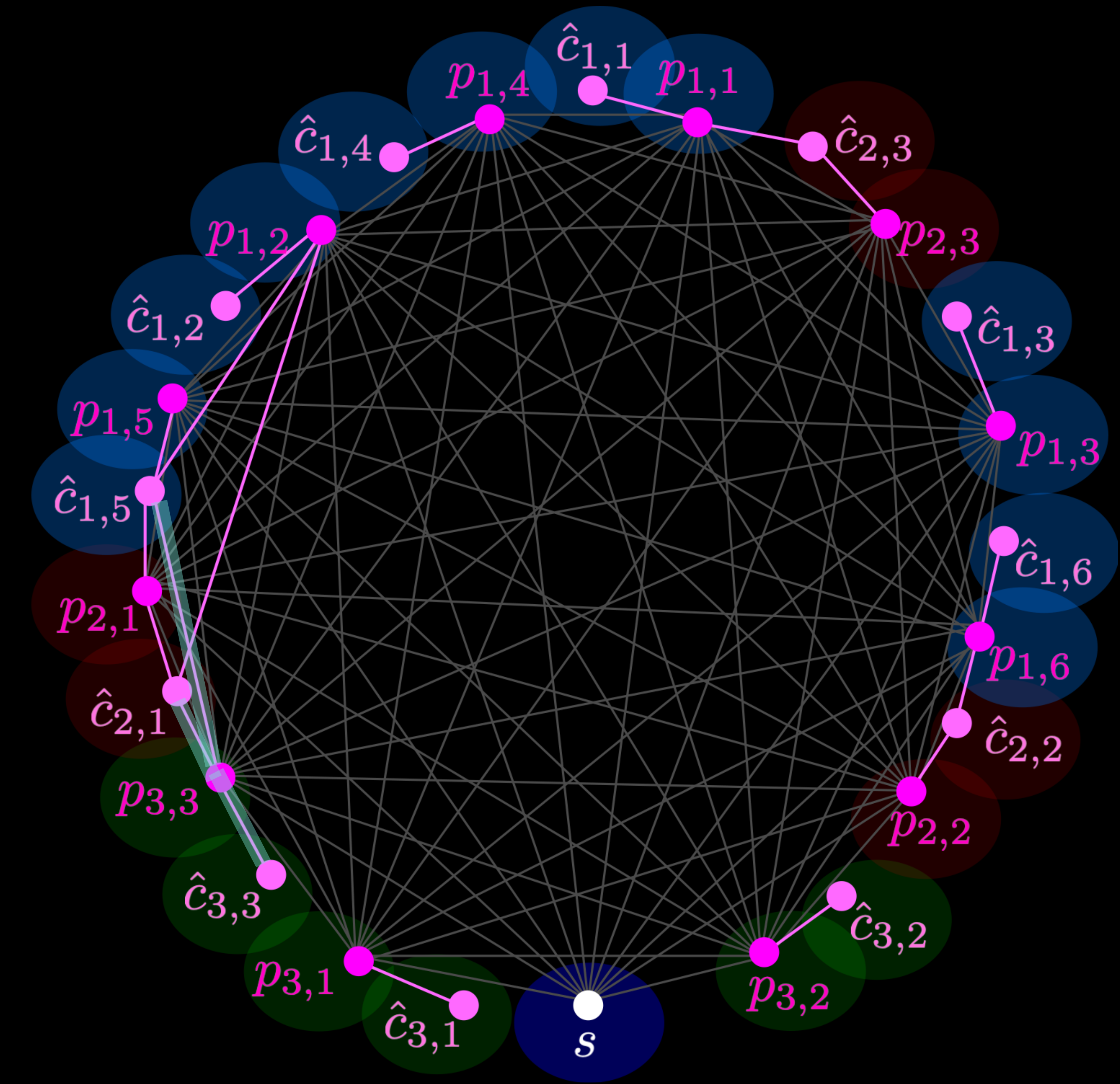

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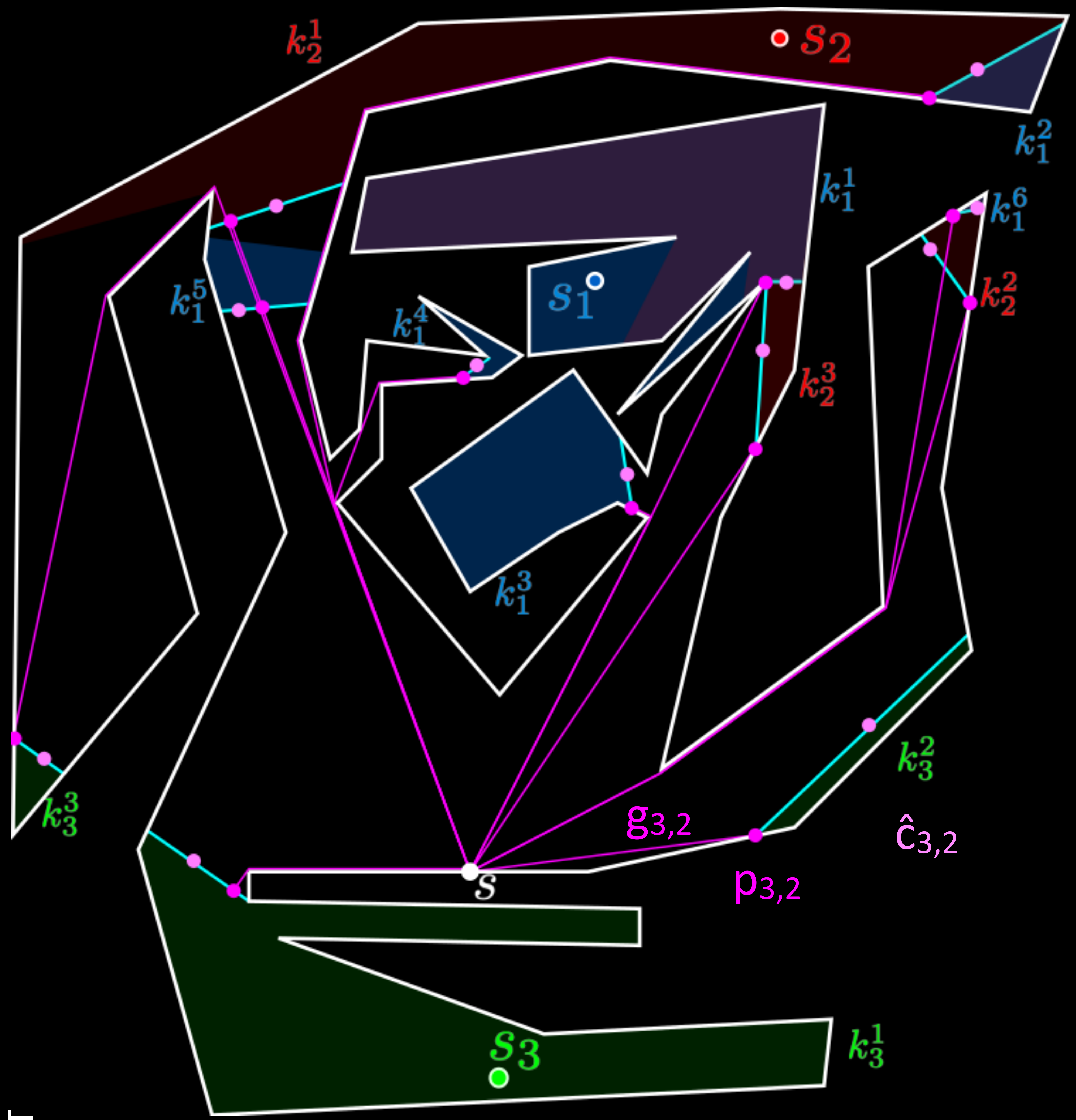

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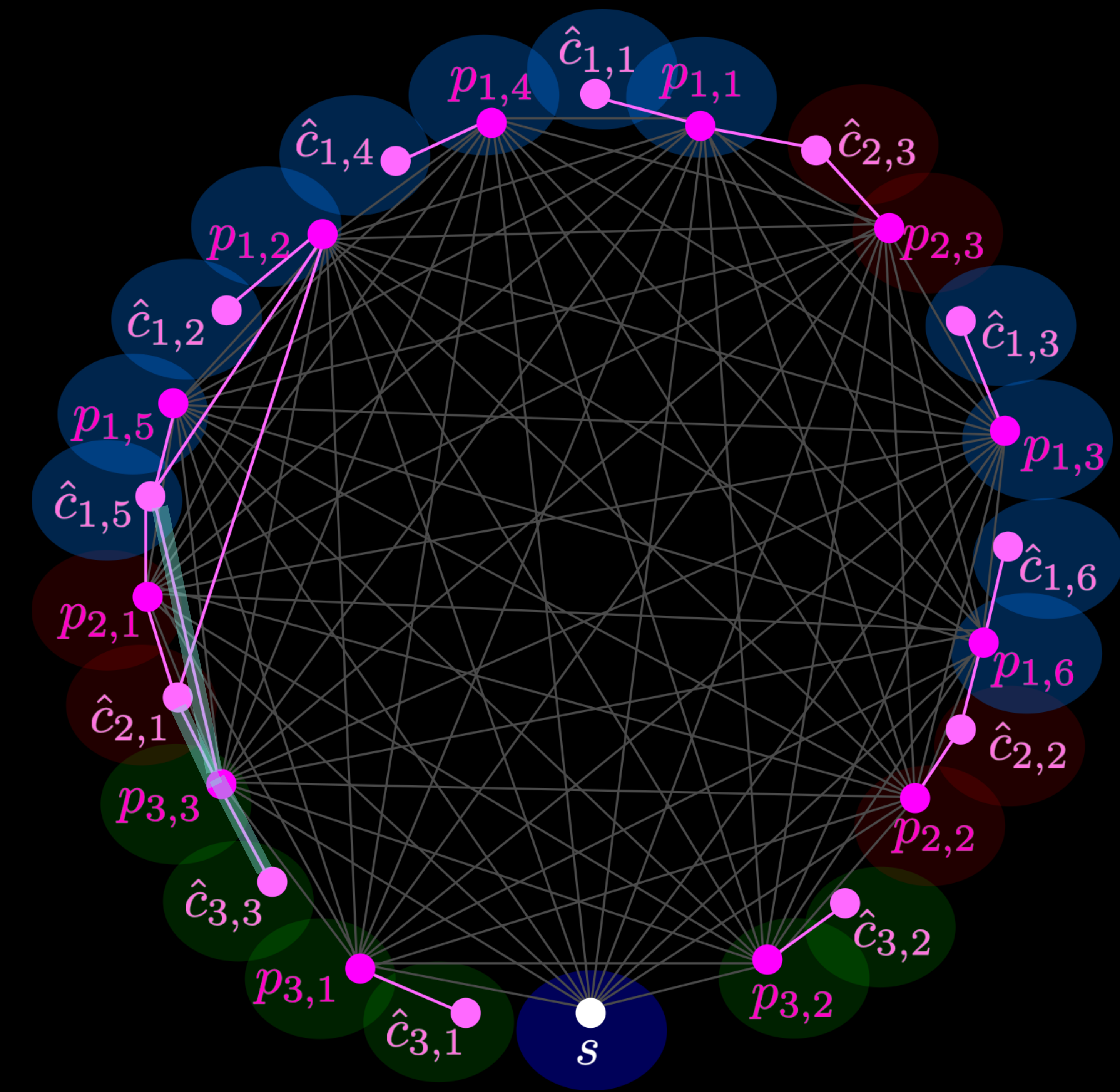

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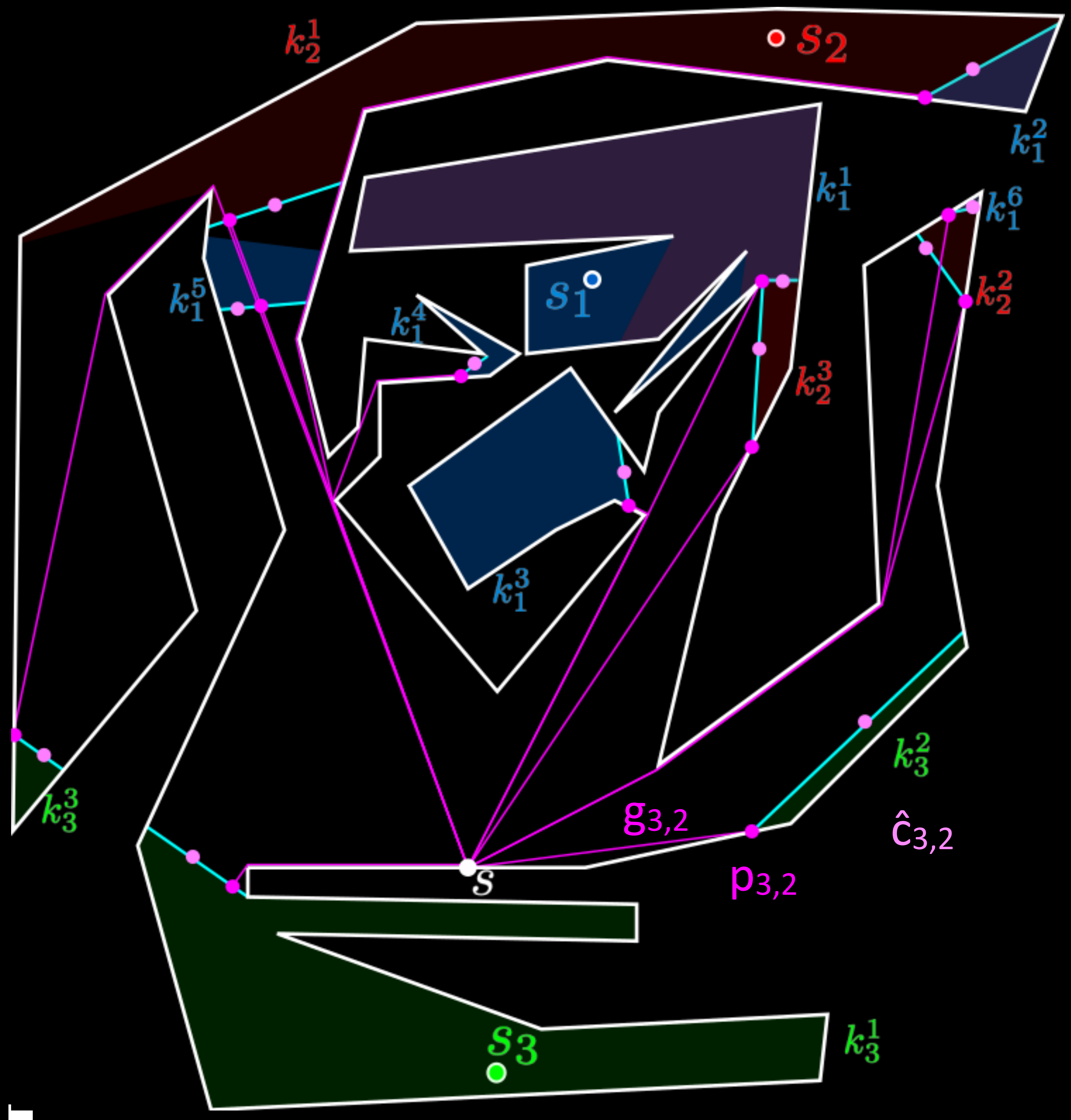

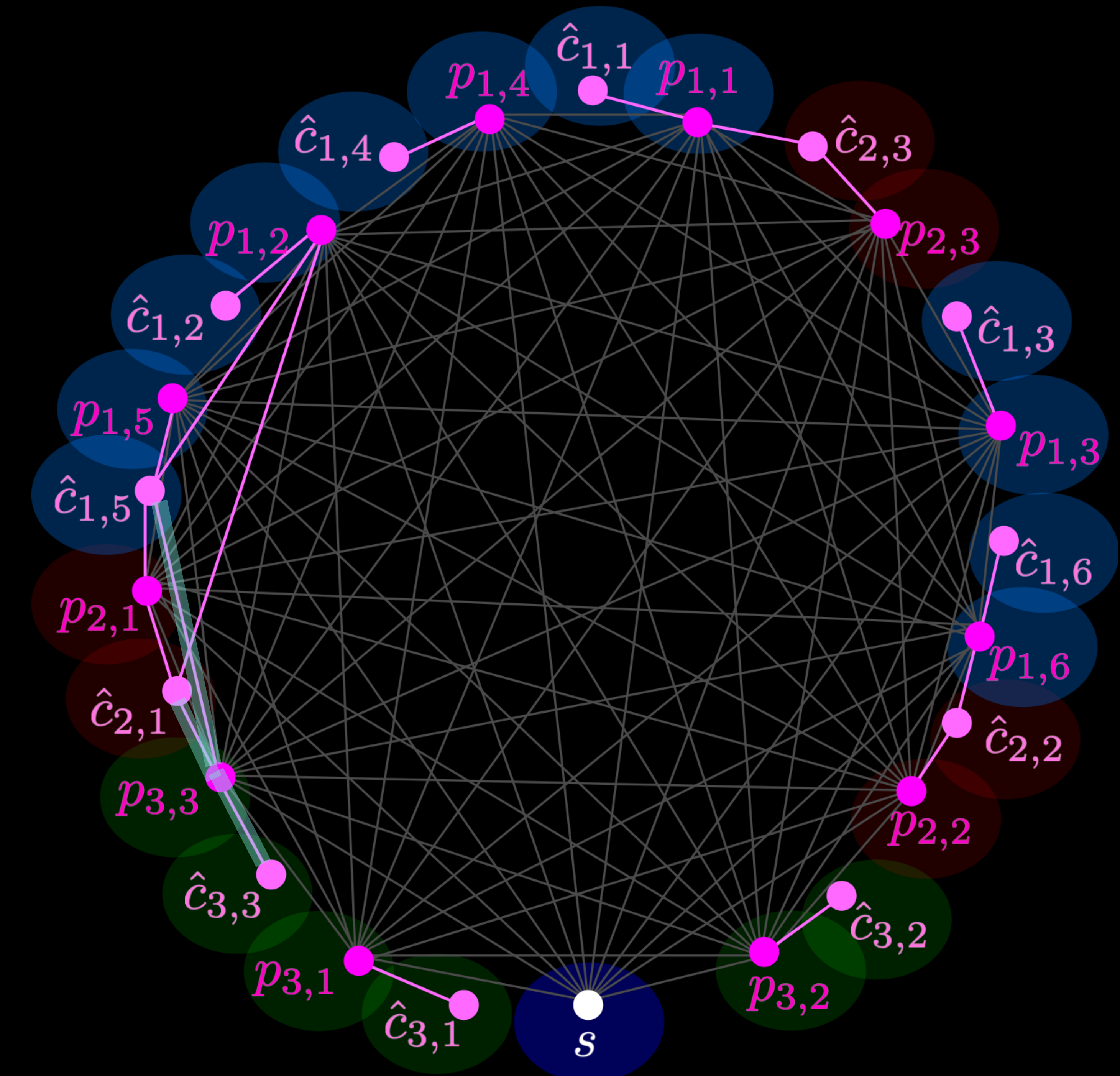

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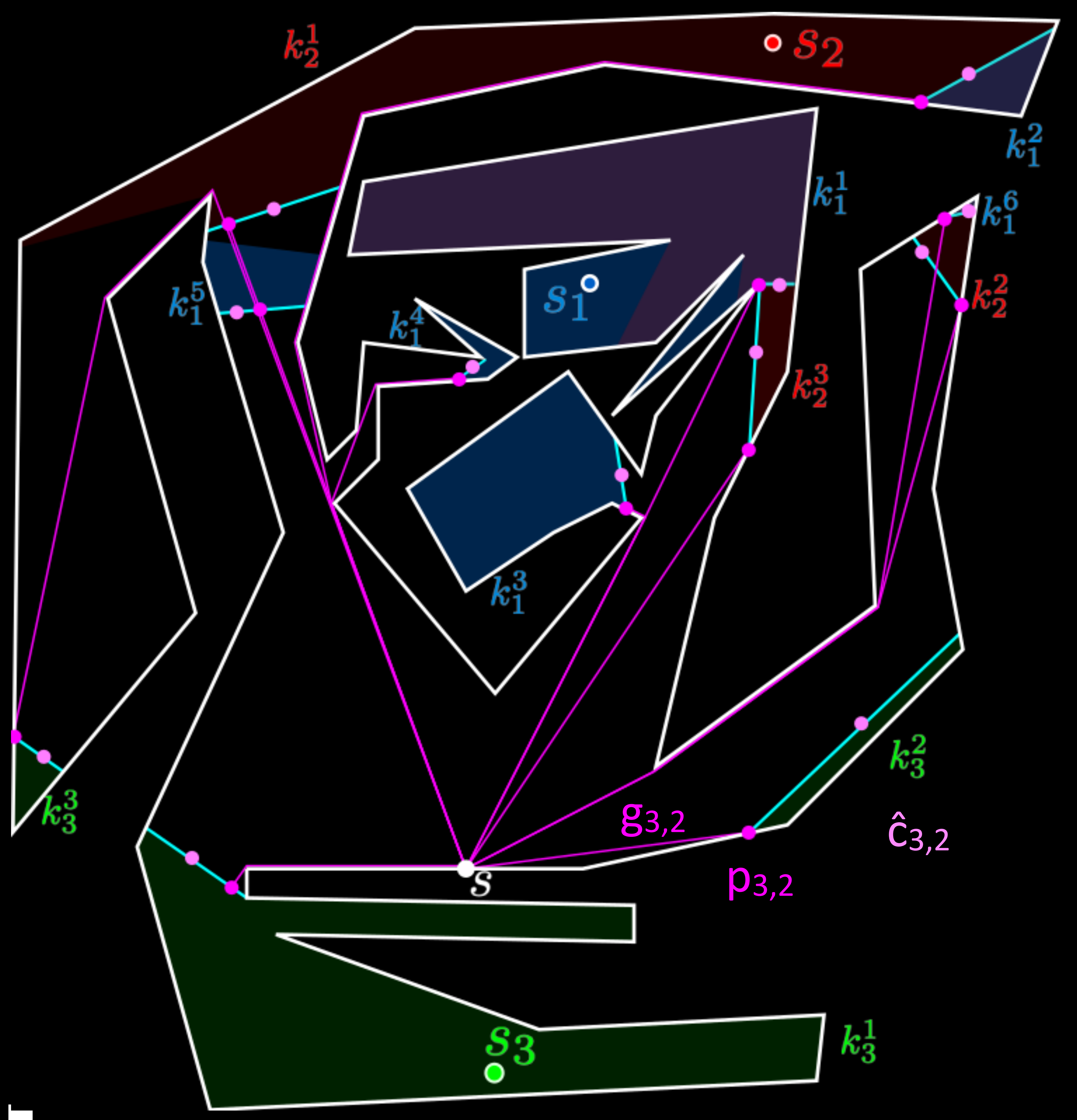

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Here:
γ_1 candidate points that belong to s_1,
γ_2 candidate points that belong to s_2,
γ_3 candidate points that belong to s_3,
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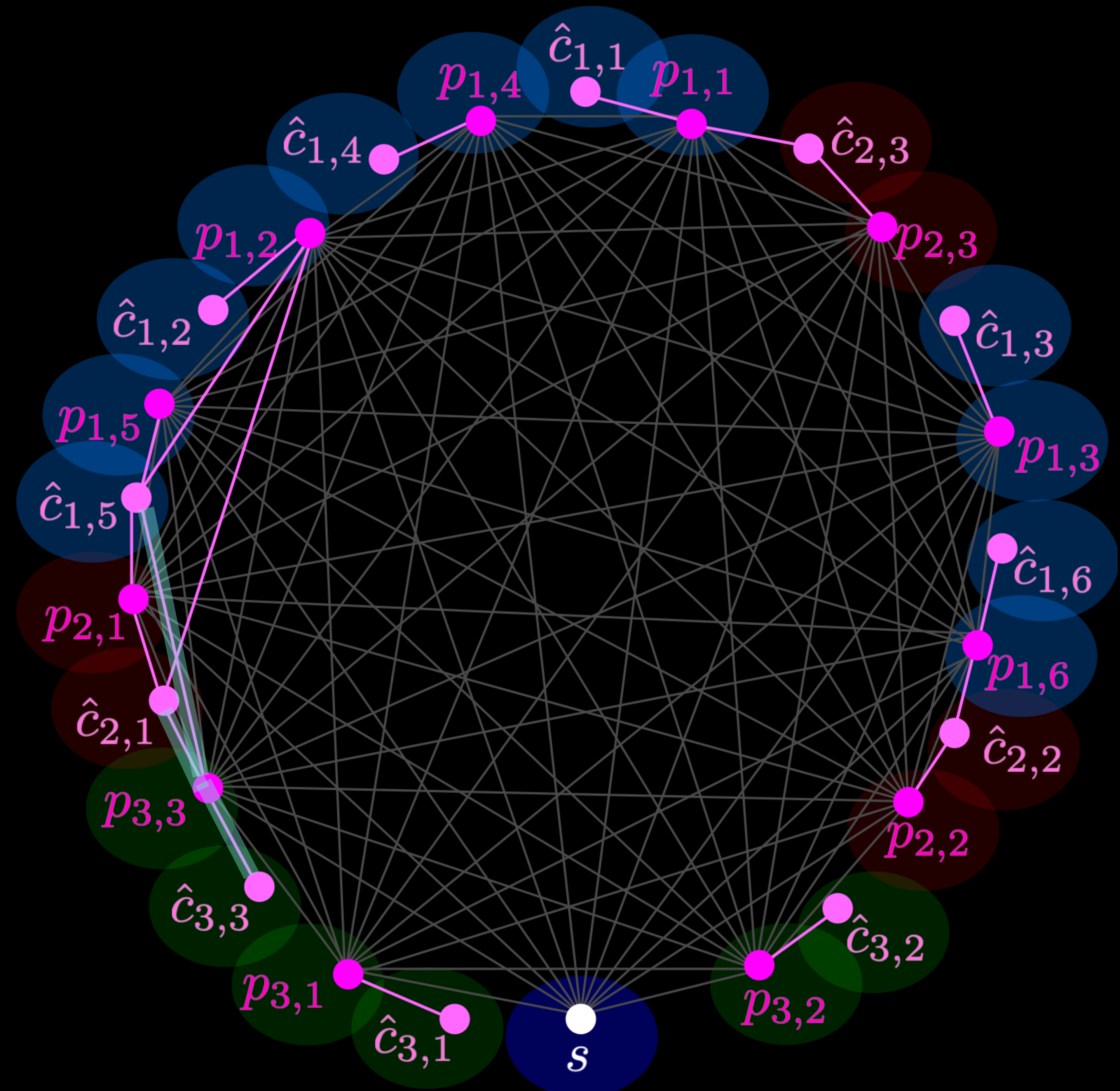

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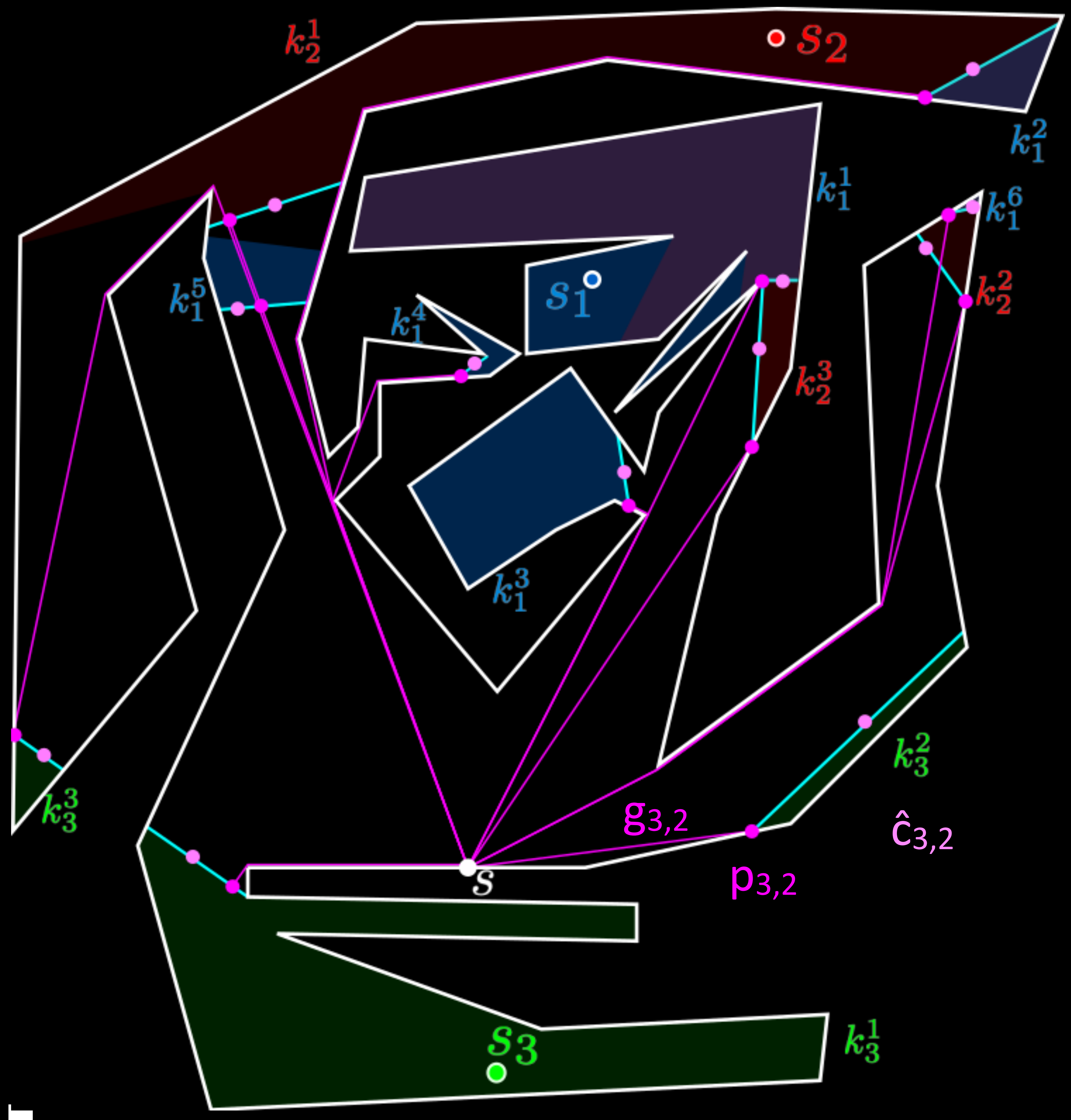

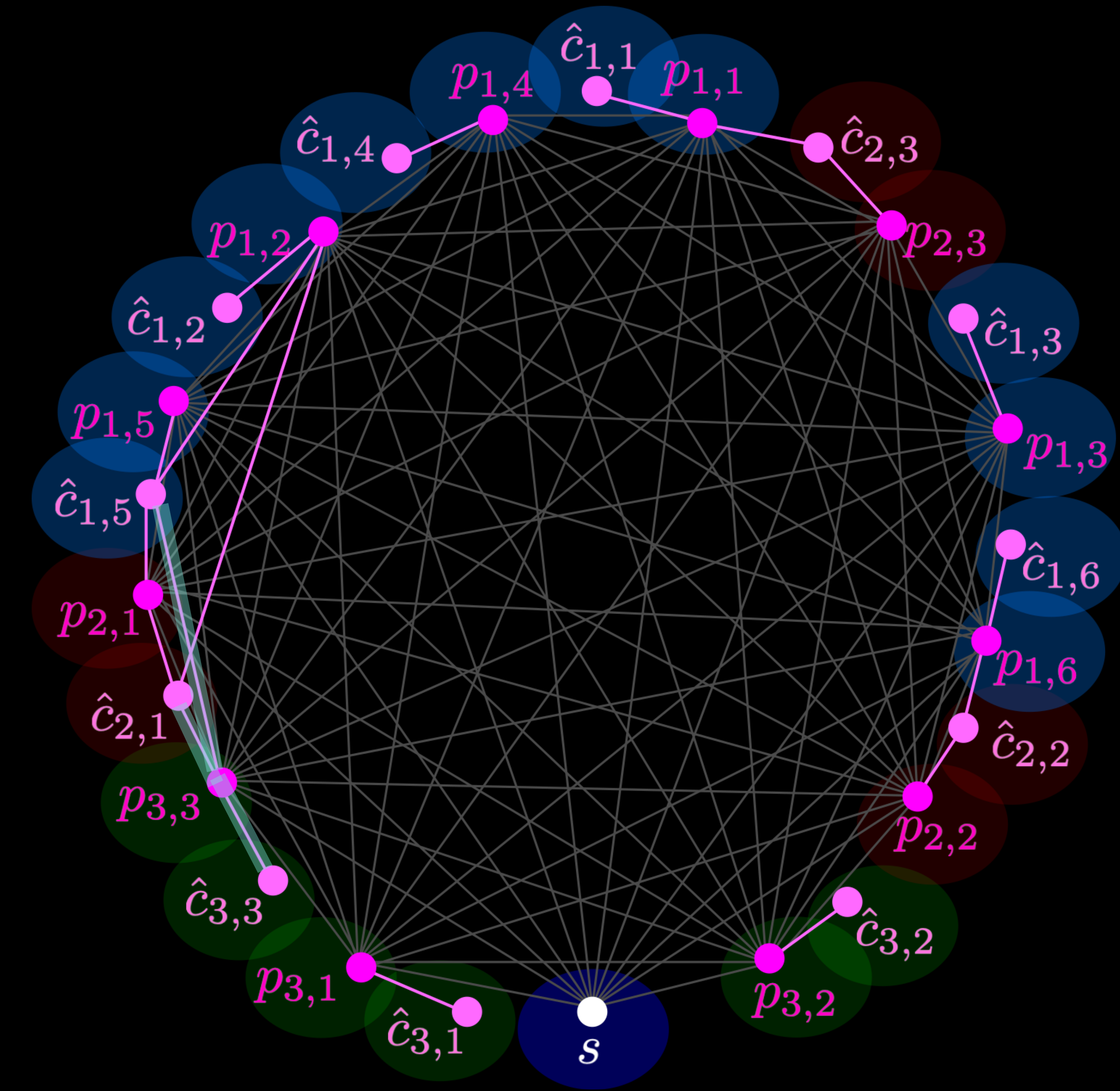

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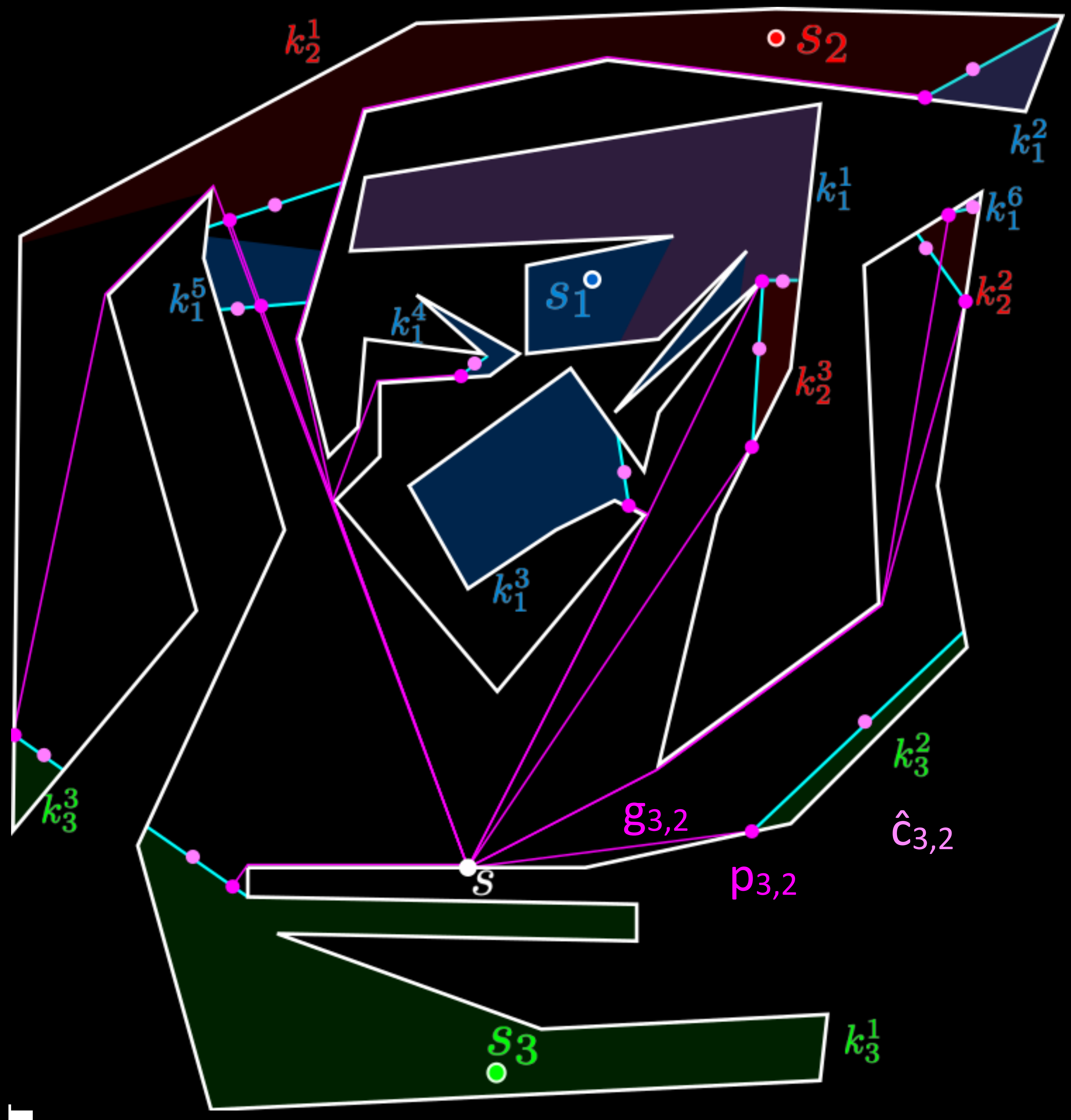

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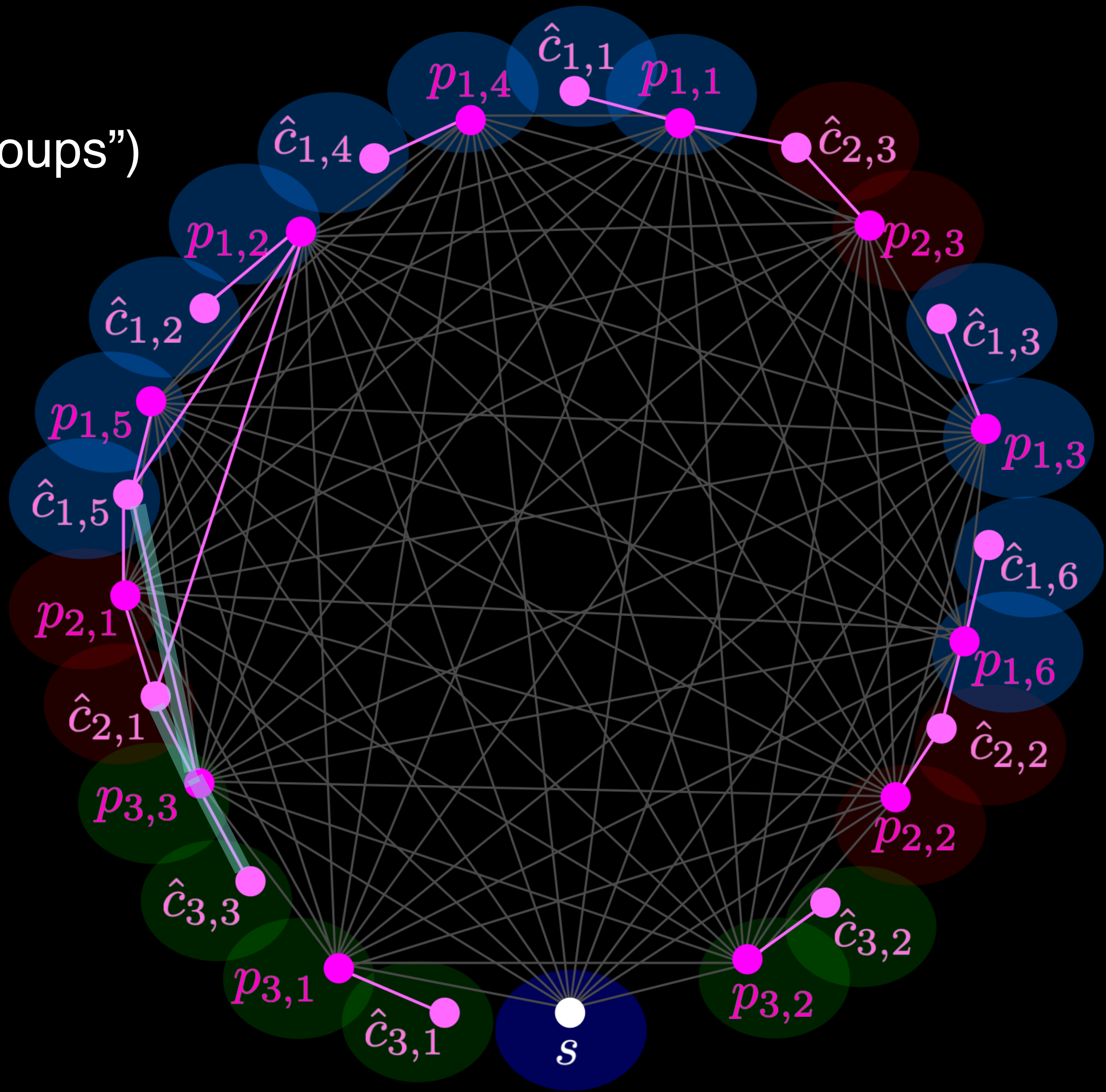

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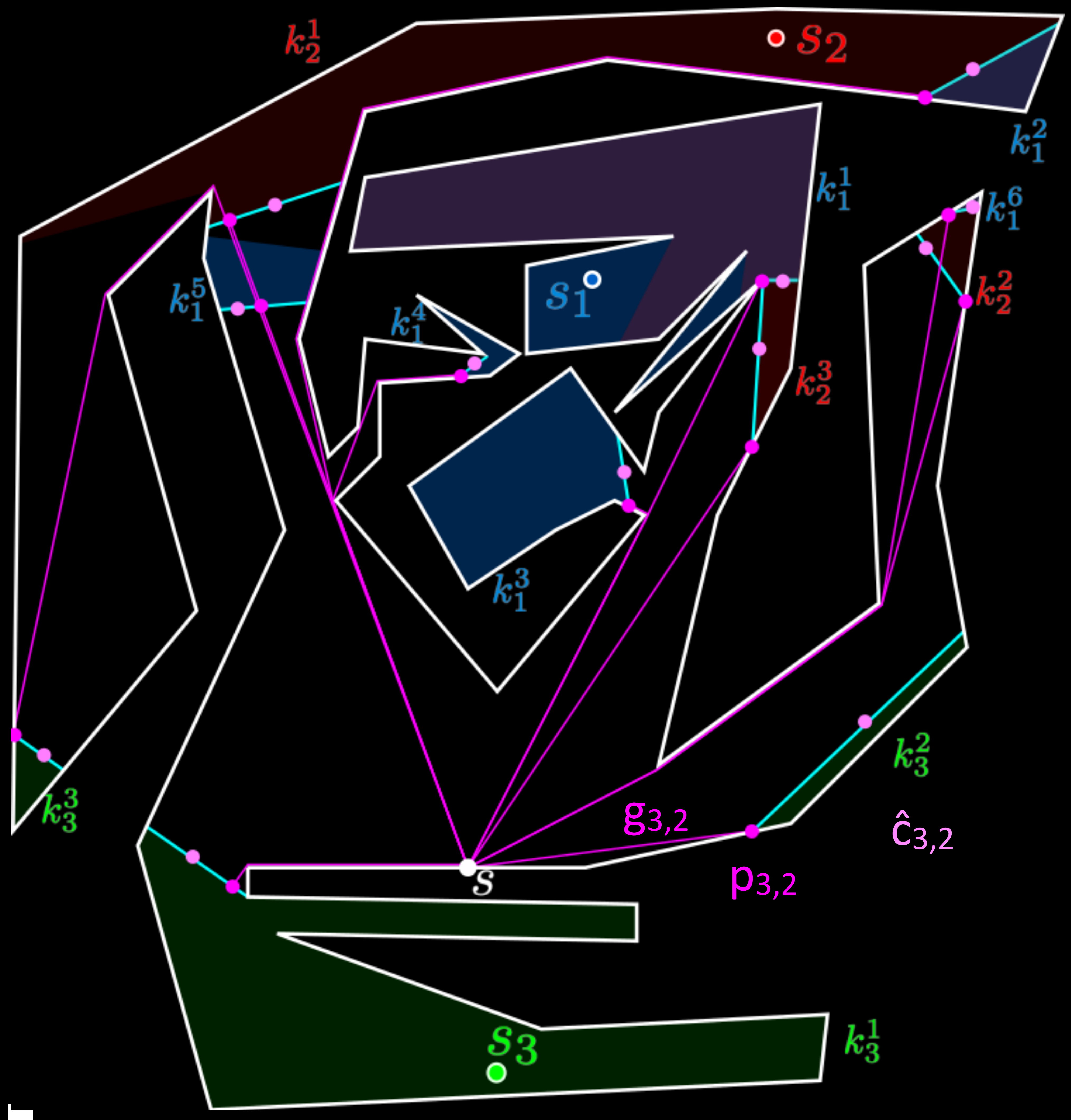

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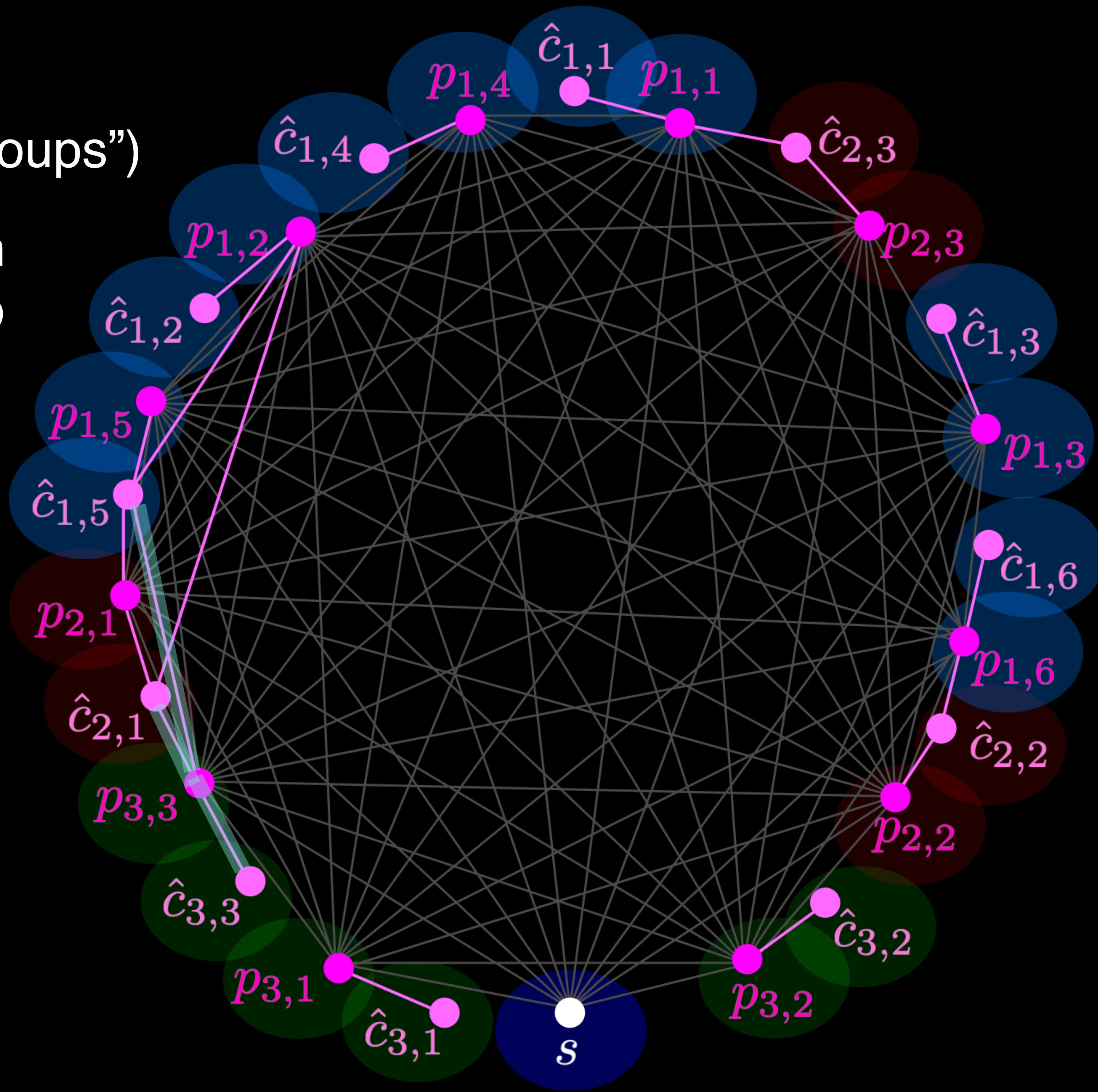

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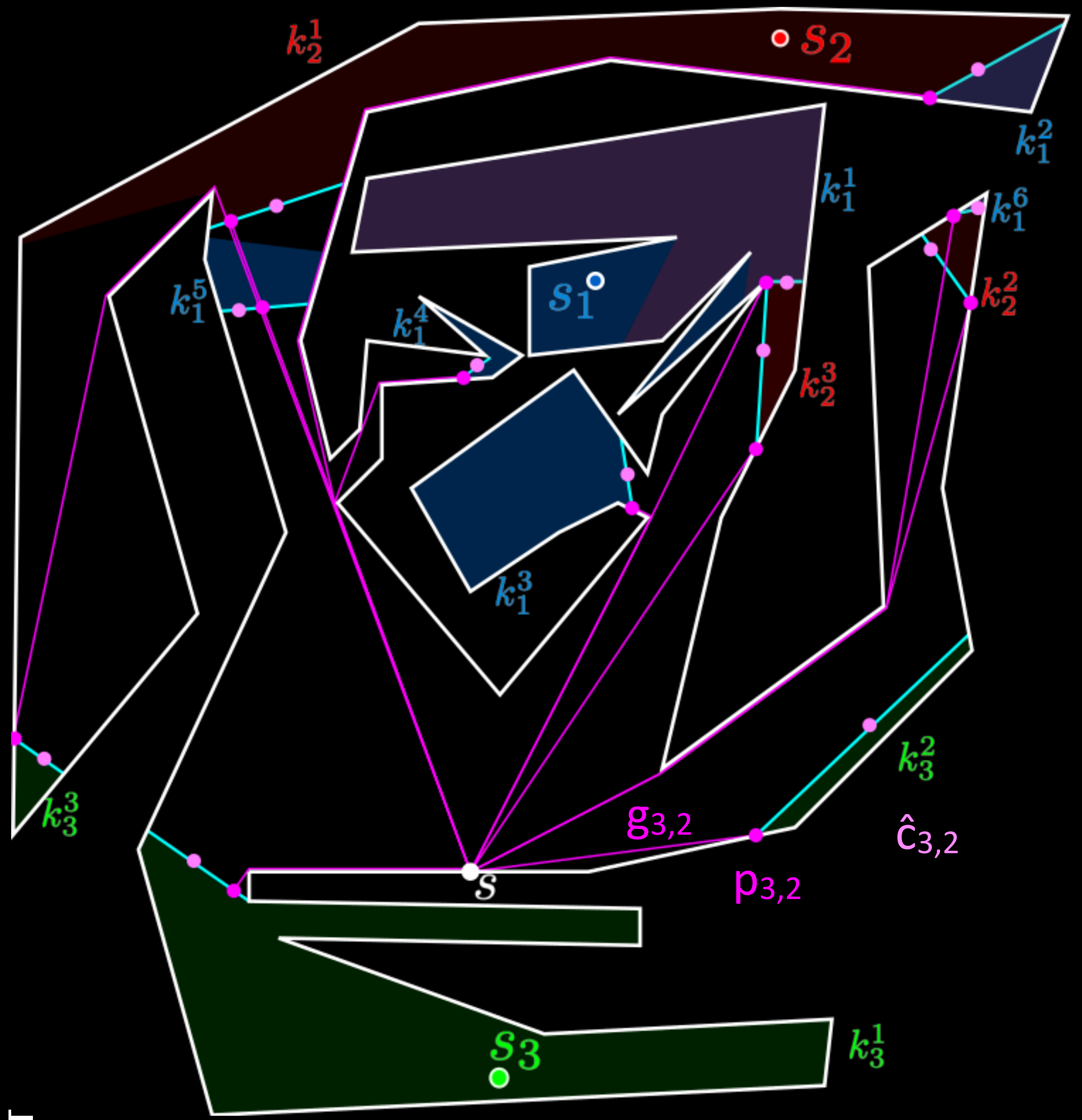

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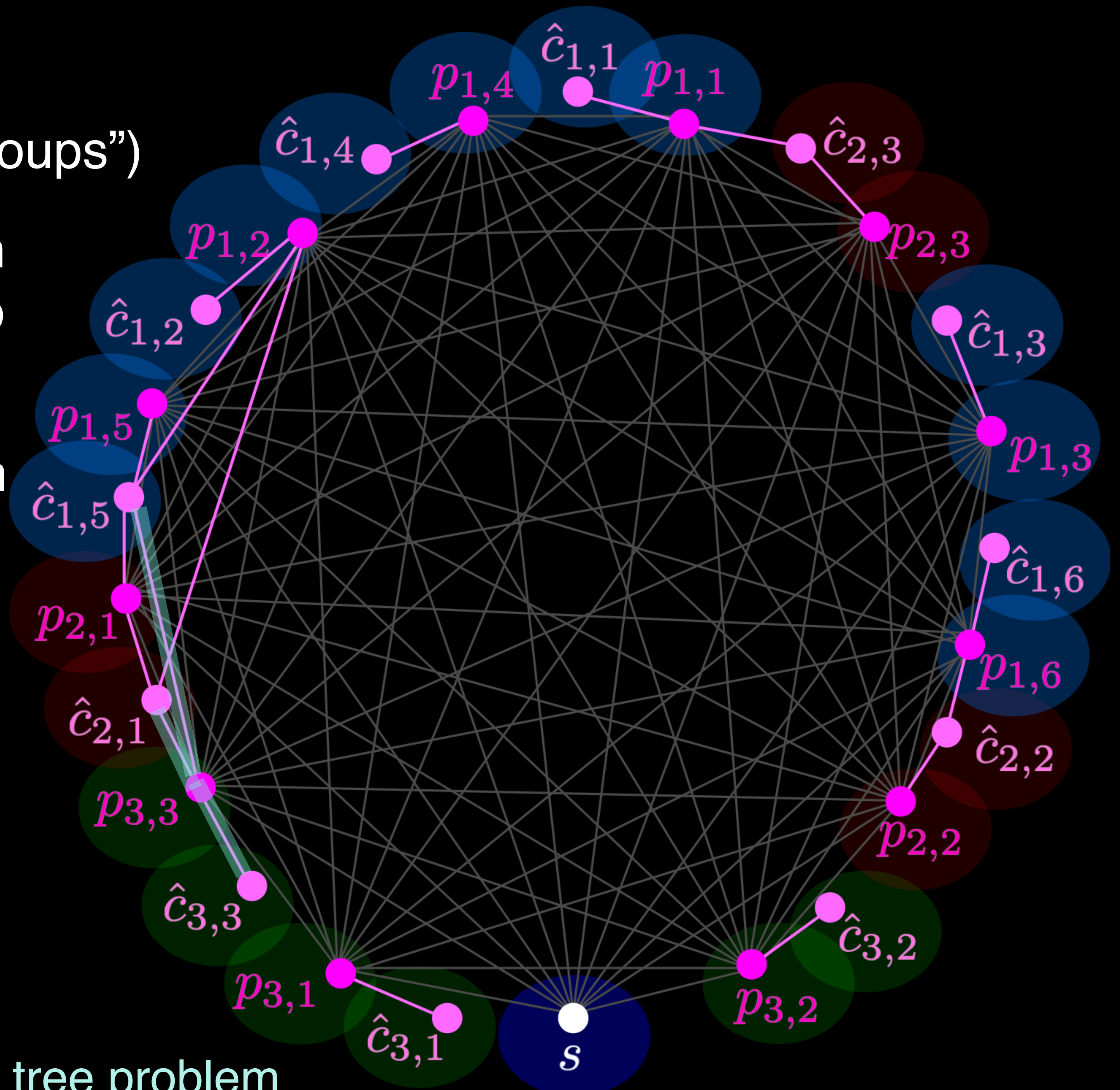

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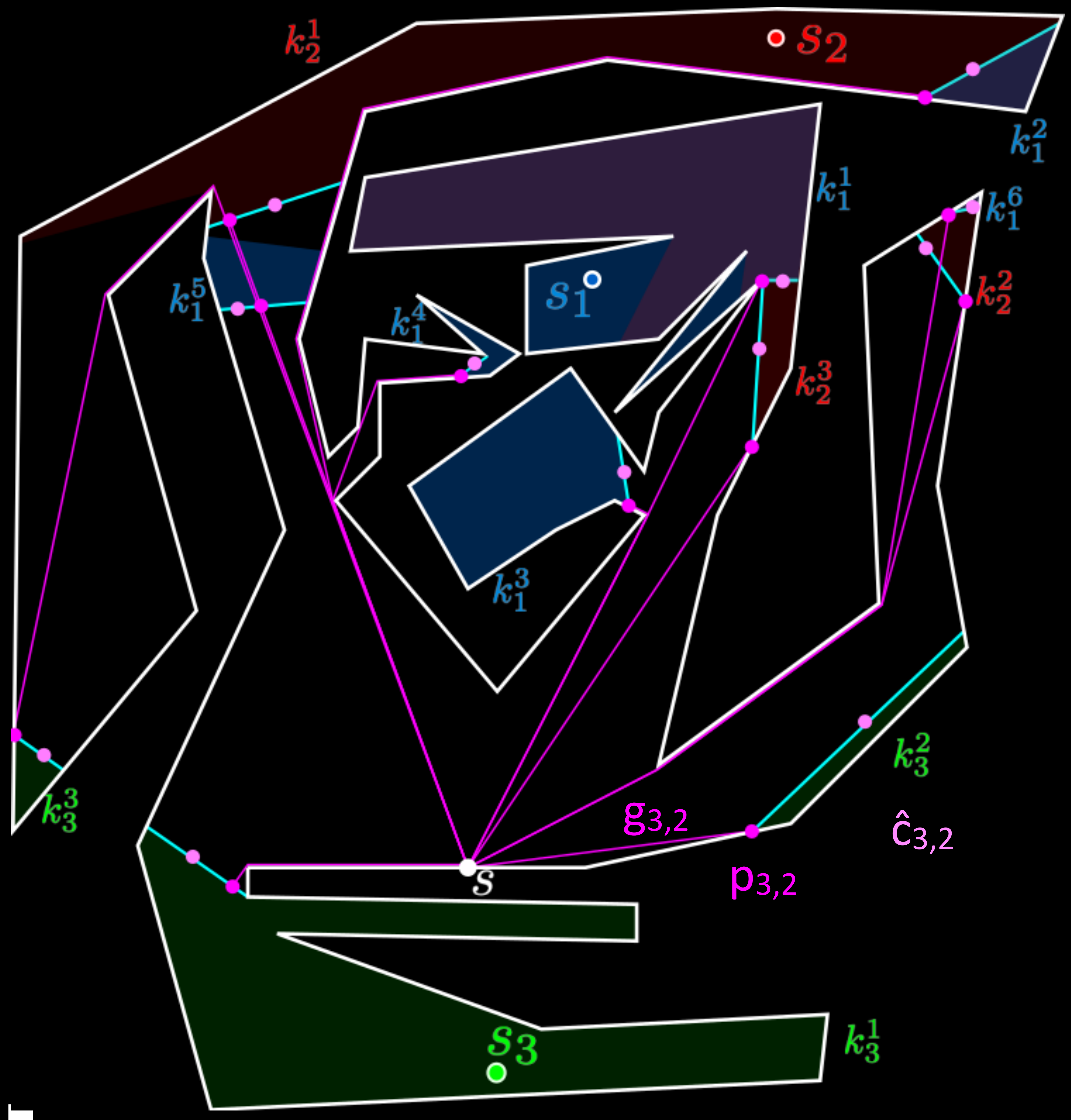

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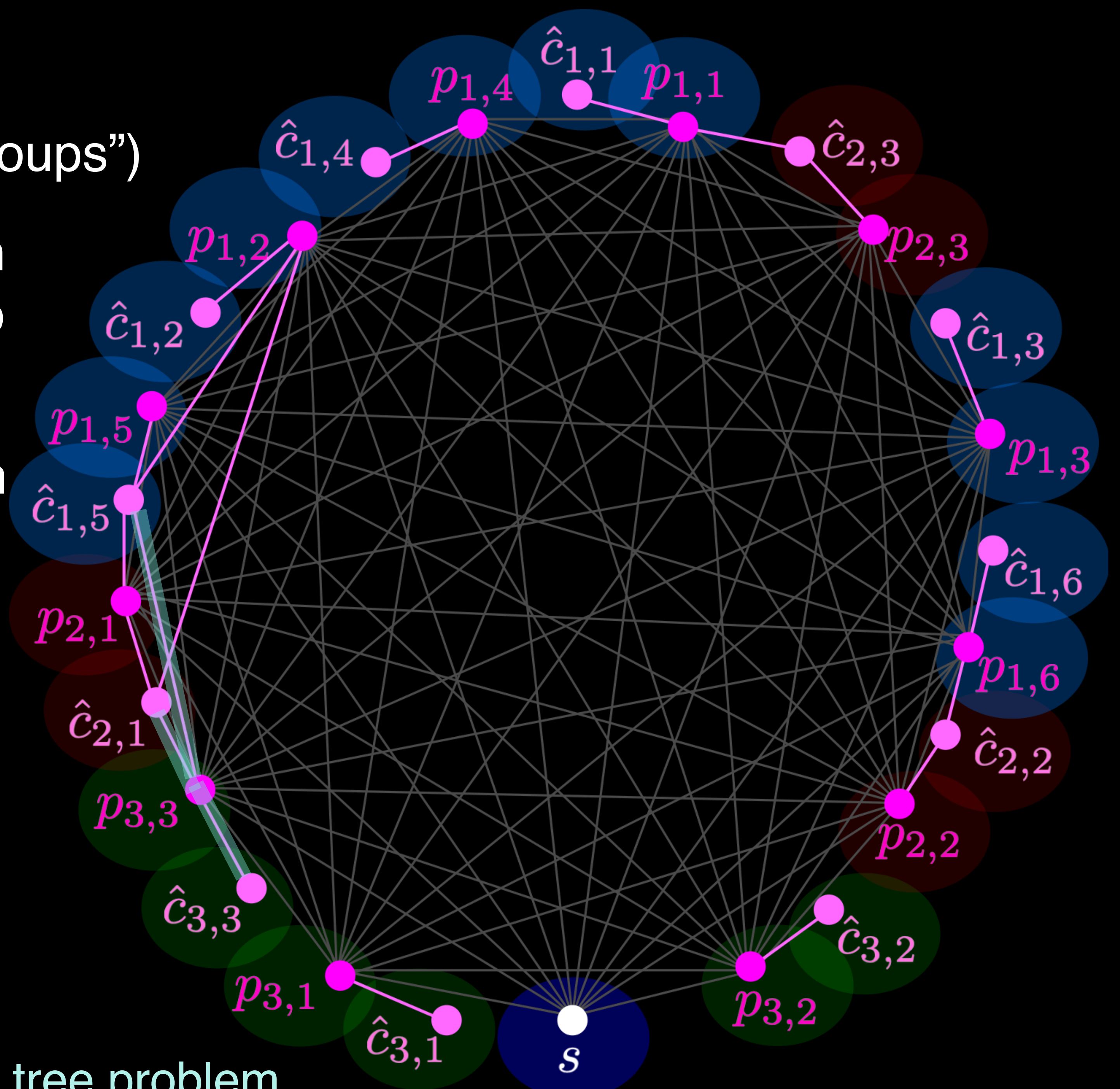

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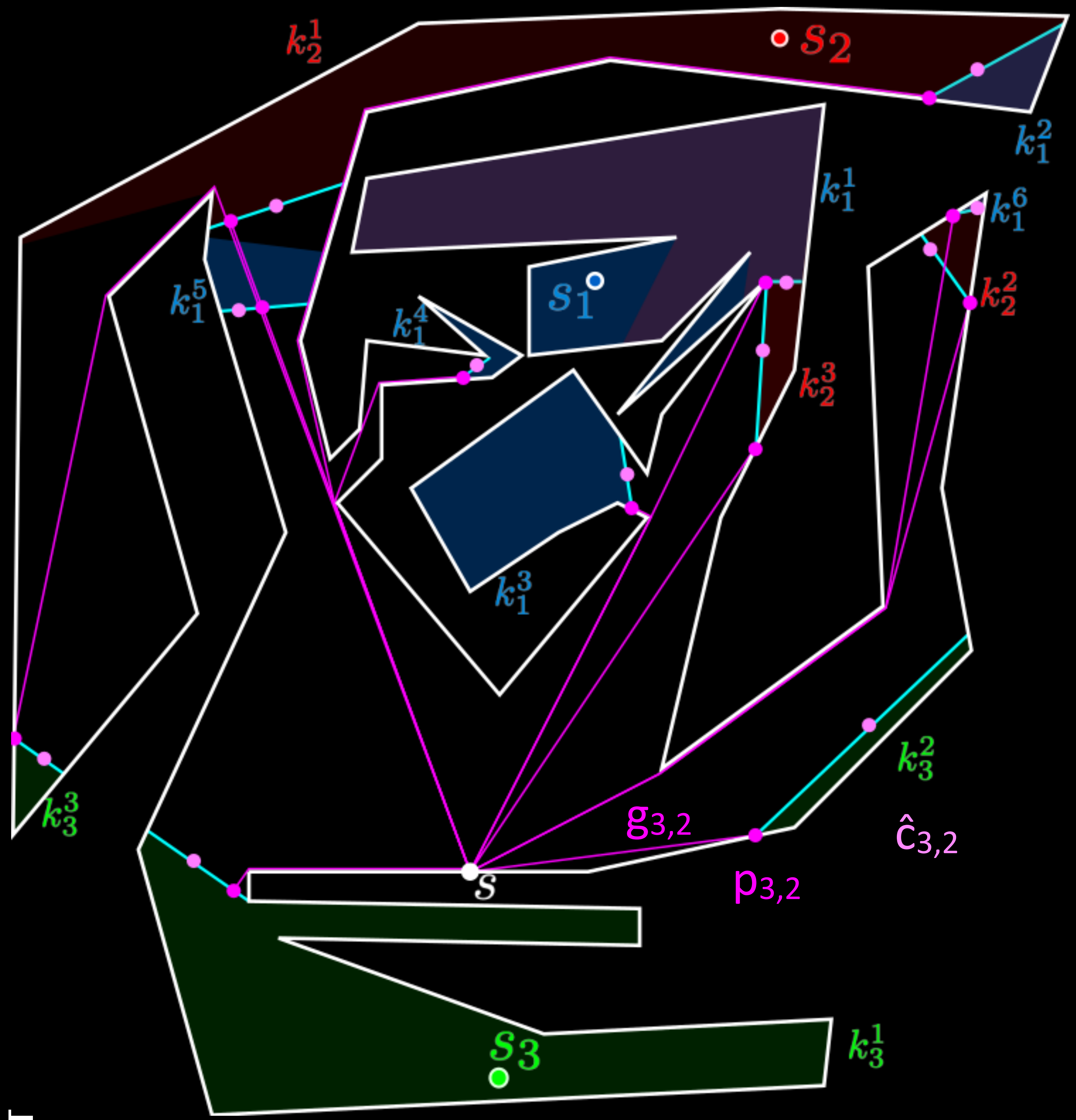

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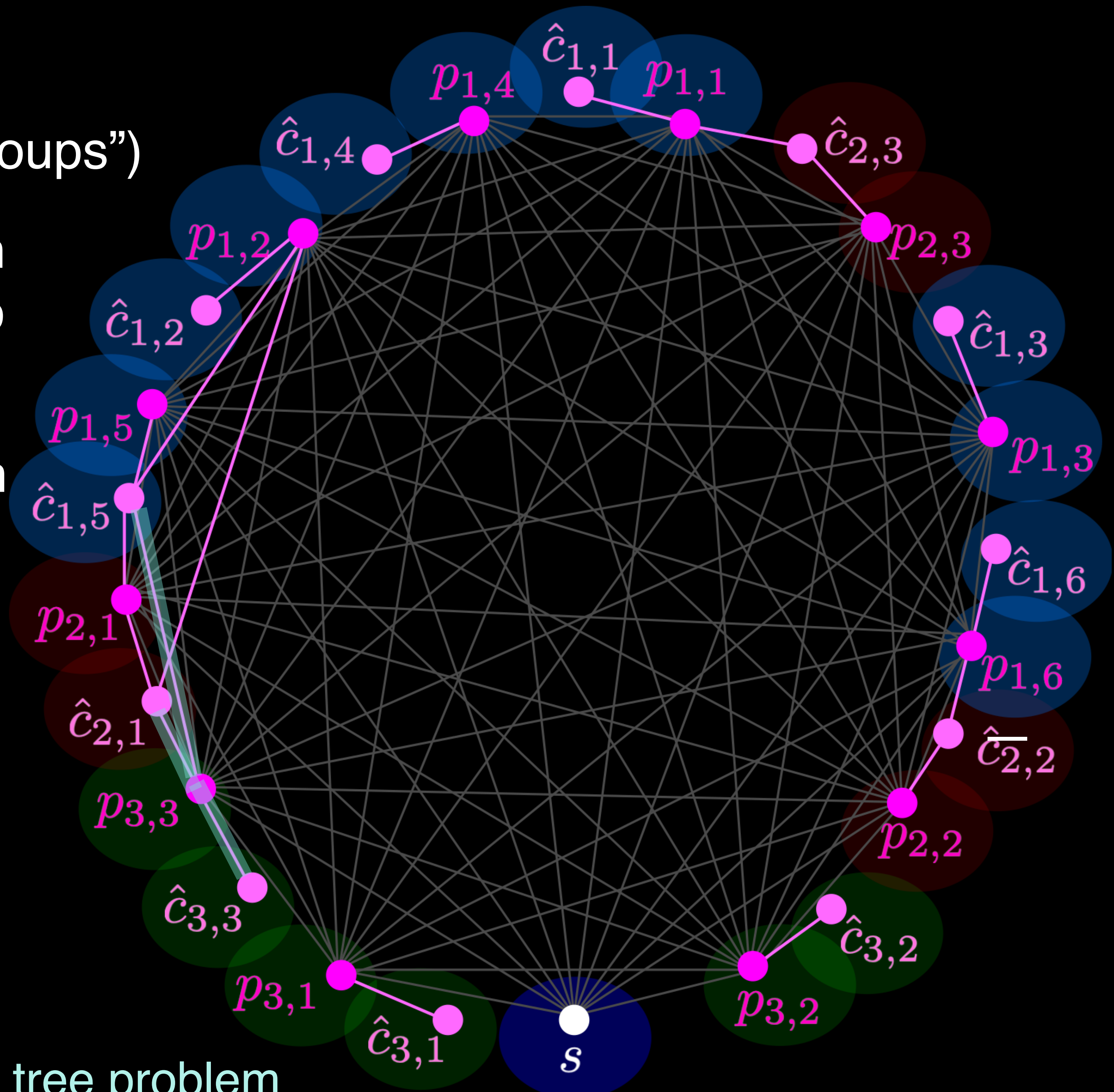

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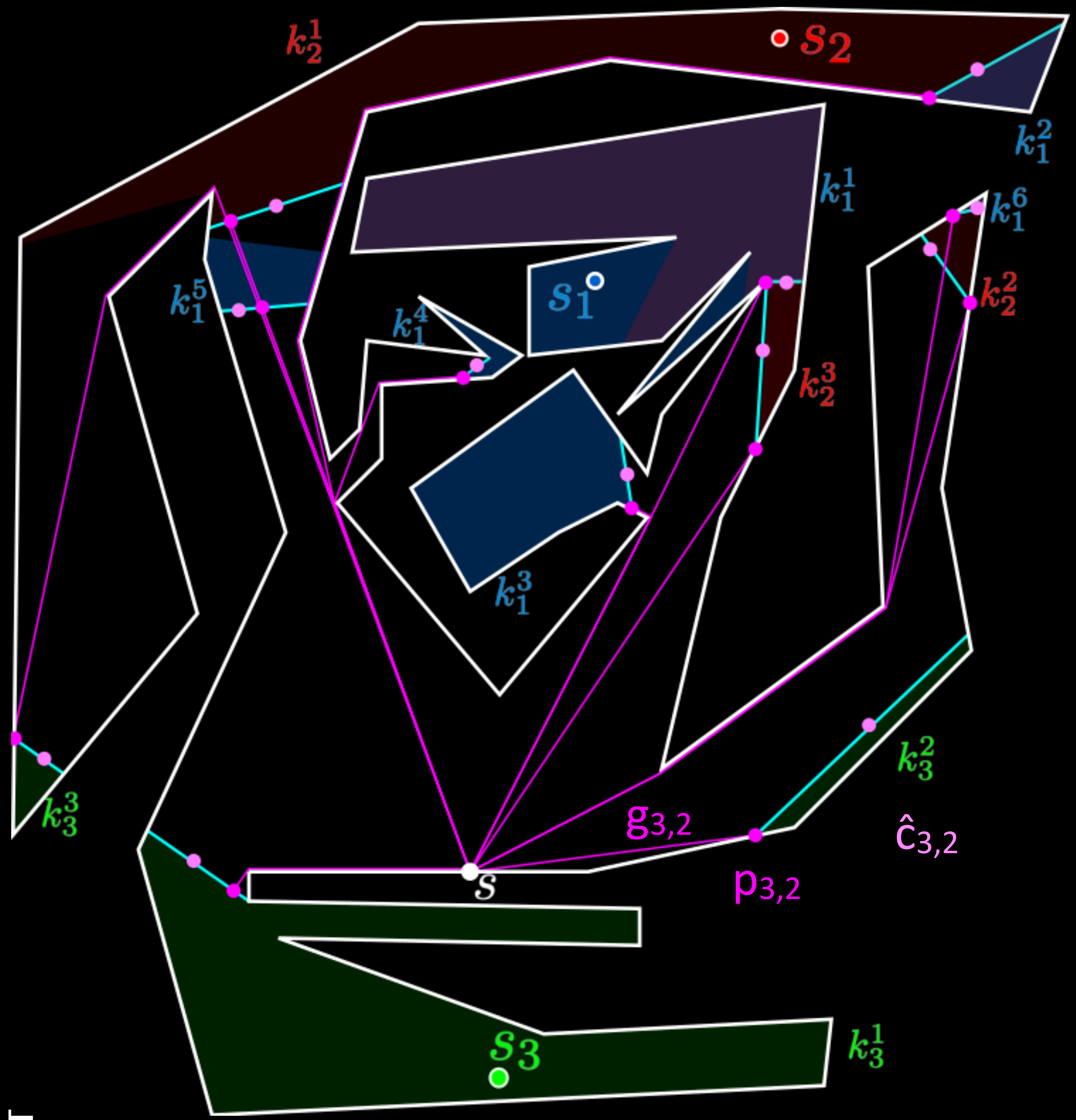

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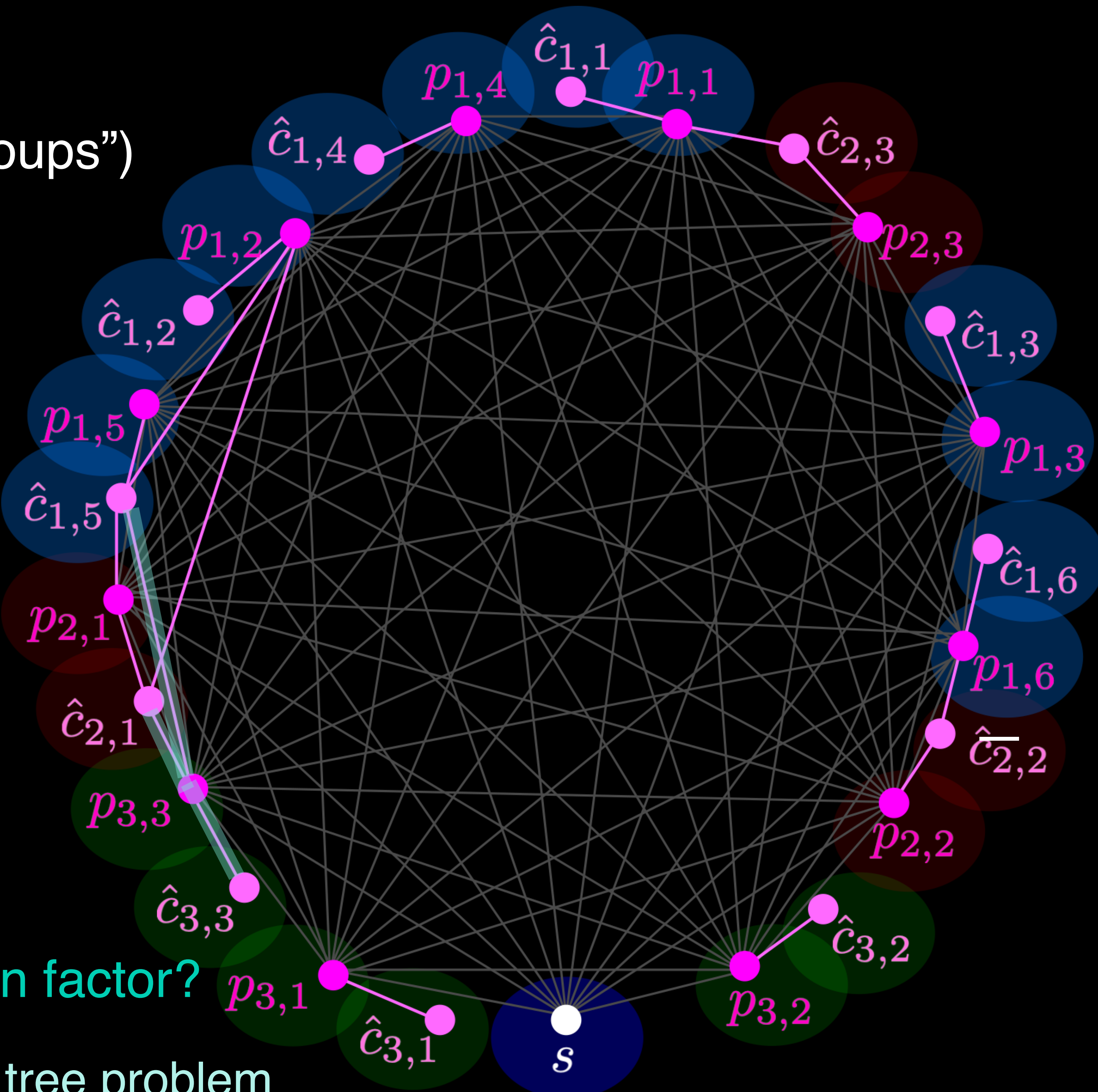

To do: why do we achieve the claimed approximation factor?

Naveen Garg, Goran Konjevod, and R. Ravi. A polylogarithmic approximation algorithm for the group Steiner tree problem

Proof idea: alter(unknown) optimal route OPT(*S*,*P*,*s*) to pass through points from *V(G)*, and new tour has length at most constant· OPT(*S*,*P*,*s*)

Proof idea: alter(unknown) optimal route OPT(S,P,s) to pass through points from $V(G)$, and new tour has length at most constant· OPT(S,P,s)

- Identify all cuts of the kVR(s_i) that OPT(S,P,s) visits—set C ($C \subseteq C^{all}$)

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- Identify all cuts of the kVR(s_i) that OPT(S,P,s) visits—set C ($C \subseteq C^{all}$)
- Let $o_{i,j}$ denote the point where OPT(S,P,s) visits $c_{i,j}$ (first time)

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- Identify all cuts of the kVR(s_i) that OPT(S,P,s) visits—set C ($C \subseteq C^{all}$)
- Let $o_{i,j}$ denote the point where OPT(S,P,s) visits $c_{i,j}$ (first time)
- Identify subset C' of essential cuts ($C' \subseteq C$)

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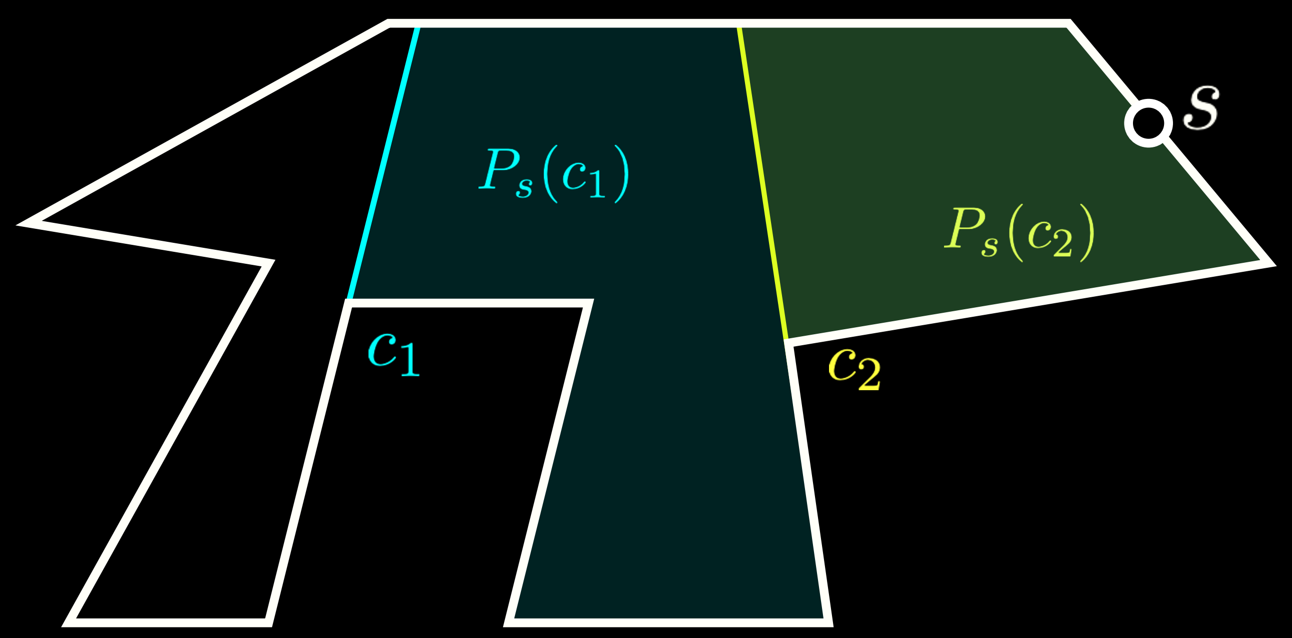

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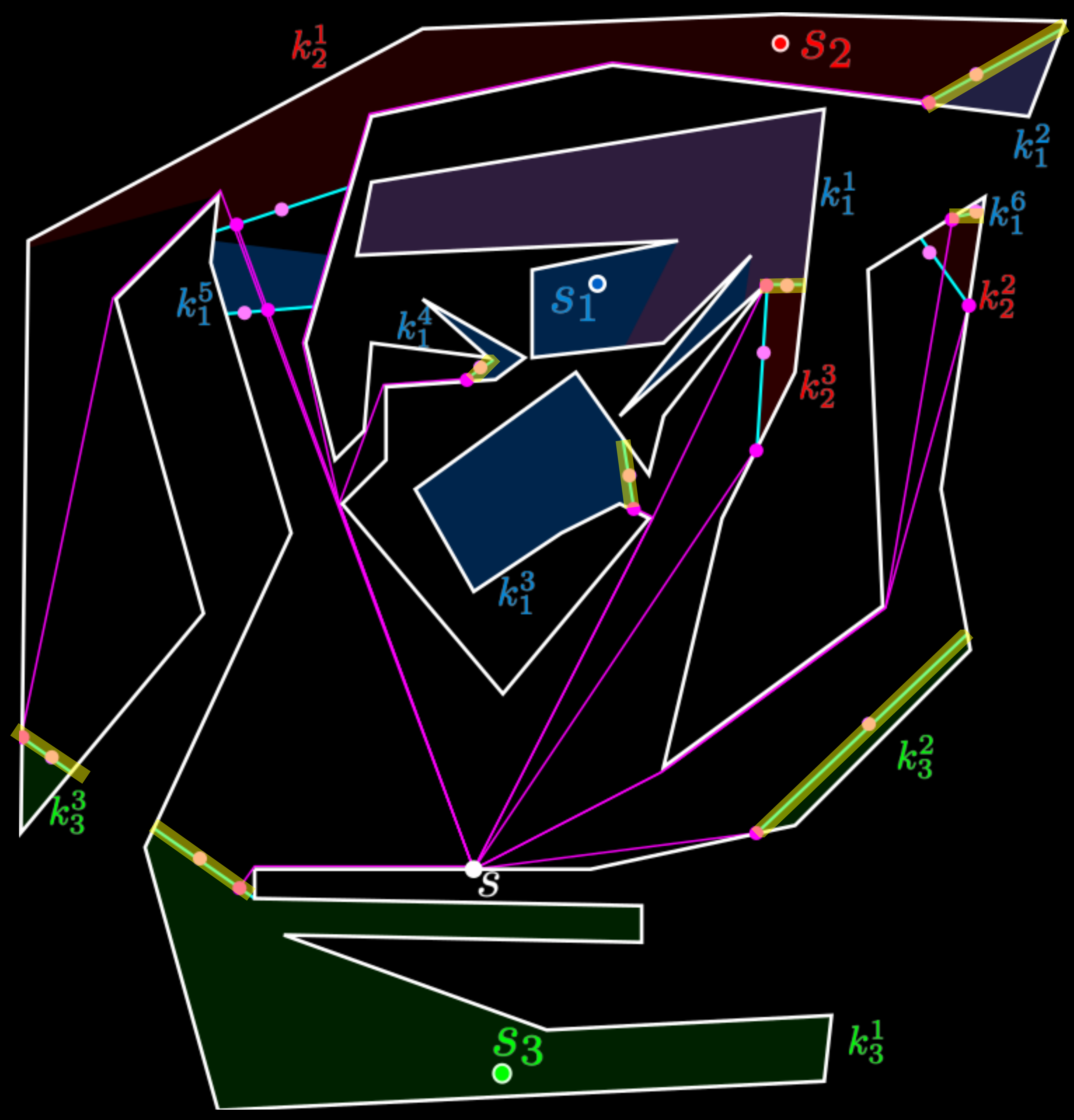

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$$\|R\| \leq \alpha_1 \cdot f(|V(G)|, |S|)\|\mathrm{OPT}_G(S,P,s)\| \leq \alpha_2 \cdot f(n|S|, |S|)\|\mathrm{CH}_P(\mathcal{P}_{C''})\| \leq \alpha_3 \cdot f(n|S|, |S|)\|\mathrm{CH}_P(\mathrm{OPT}, \mathcal{P}_{C''})\|$$
$$\leq \alpha_4 \cdot f(n|S|, |S|)\|\mathrm{OPT}(S,P,s)\|$$

with $f(N, M) = \log^2 N \log\log N \log M$

Open Problem: k-Transmitter Watchmen

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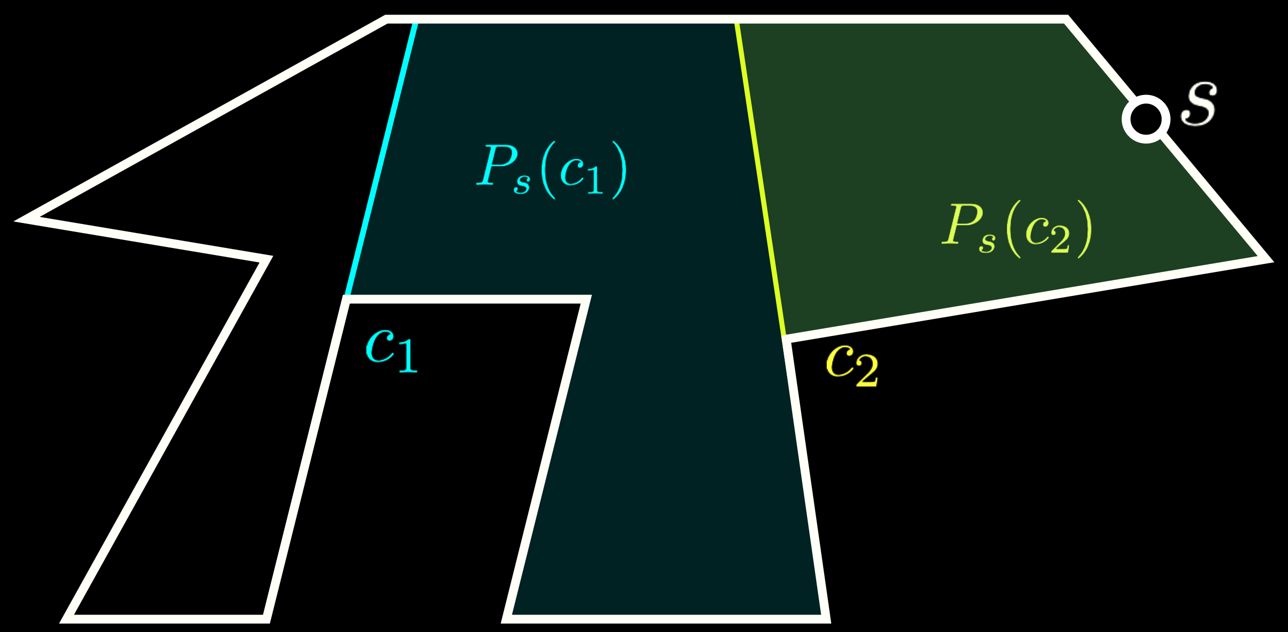

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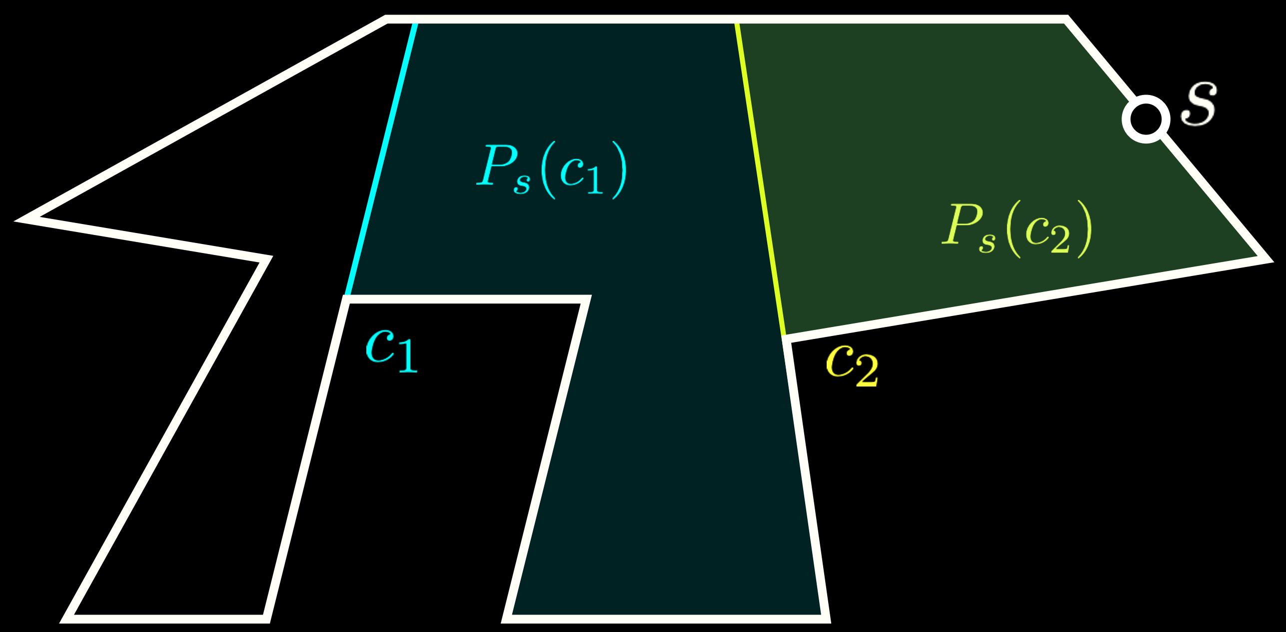

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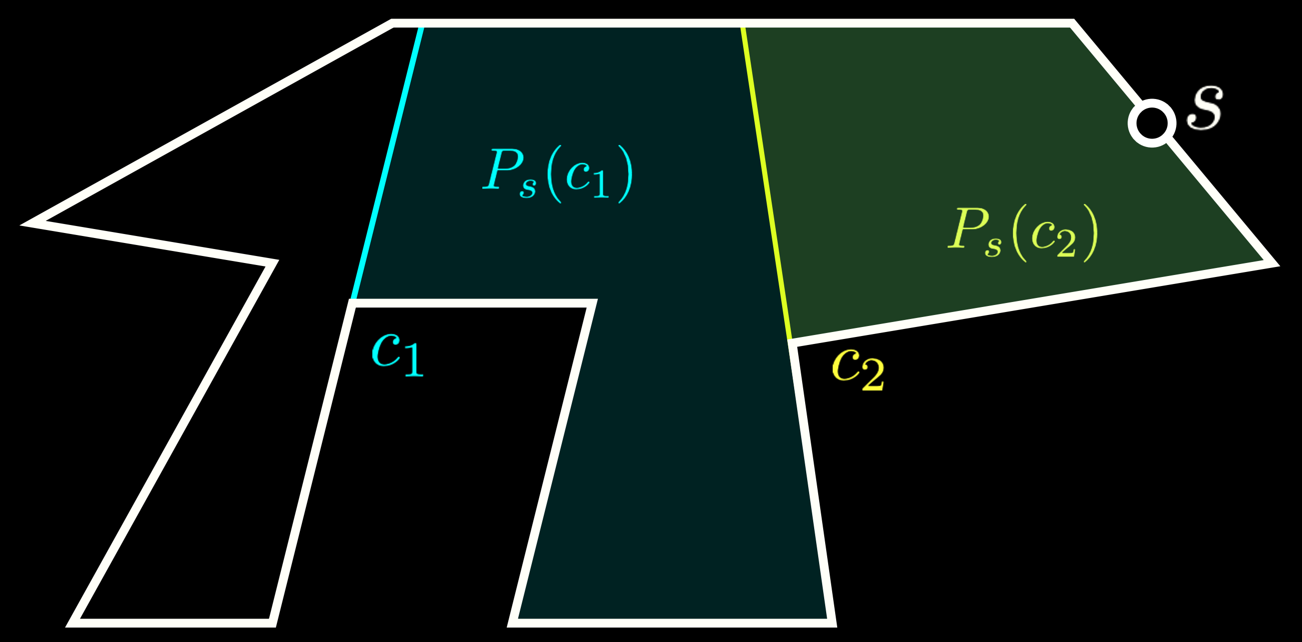

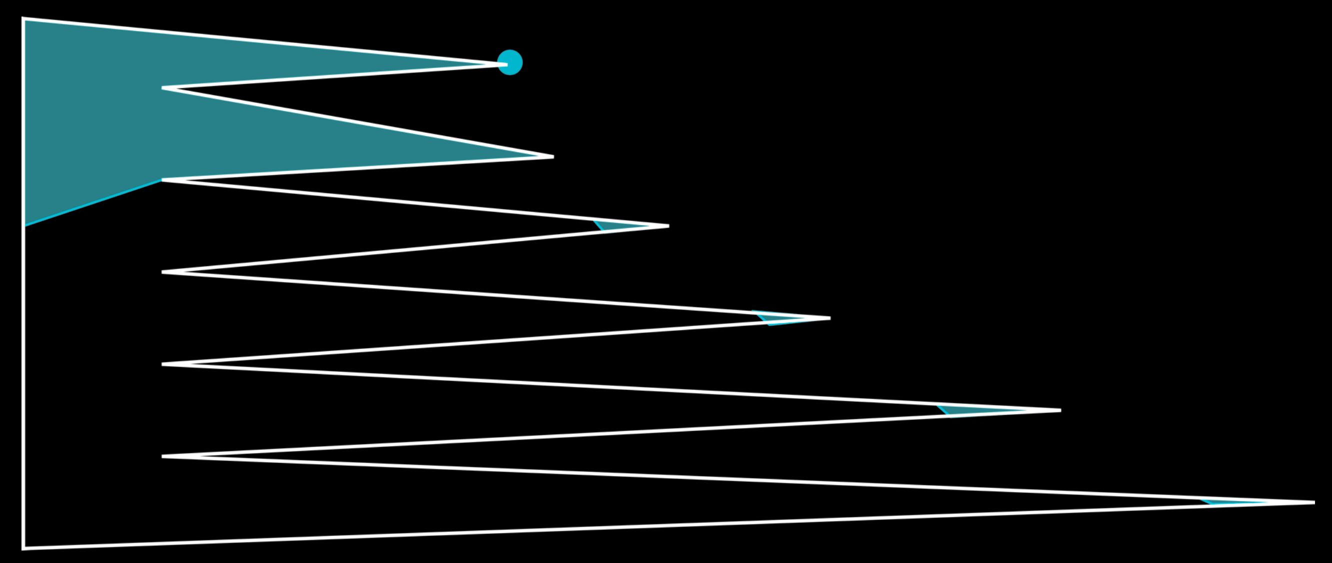

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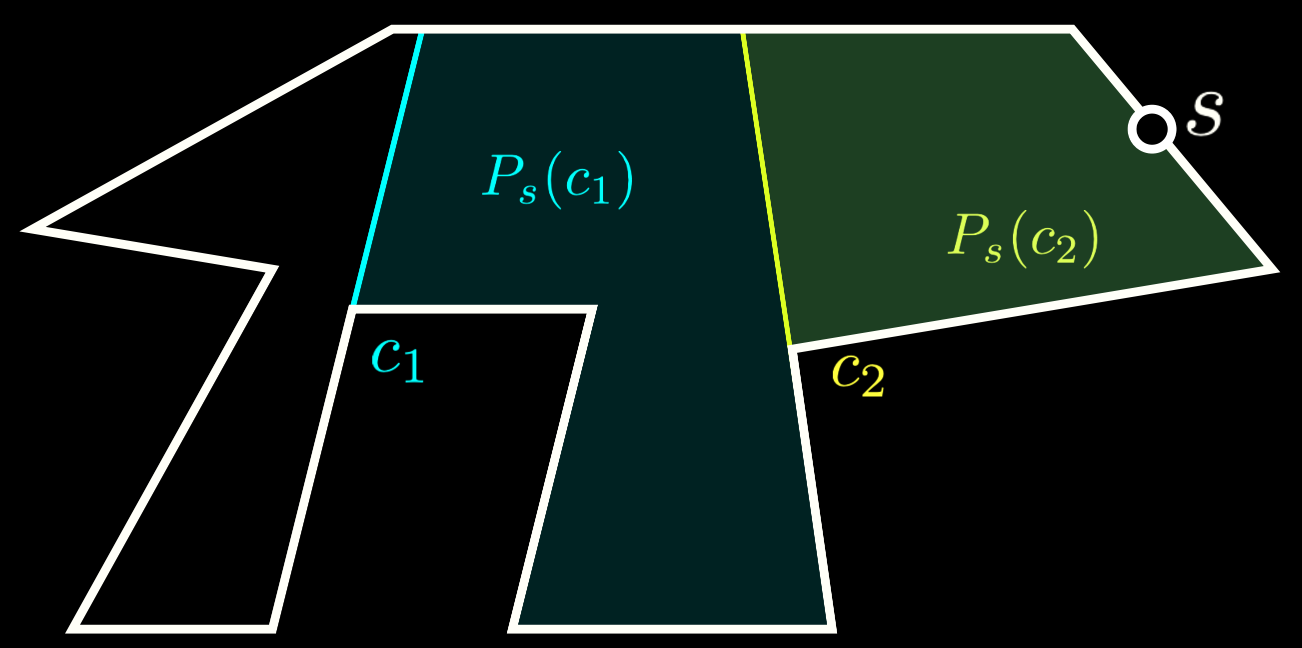

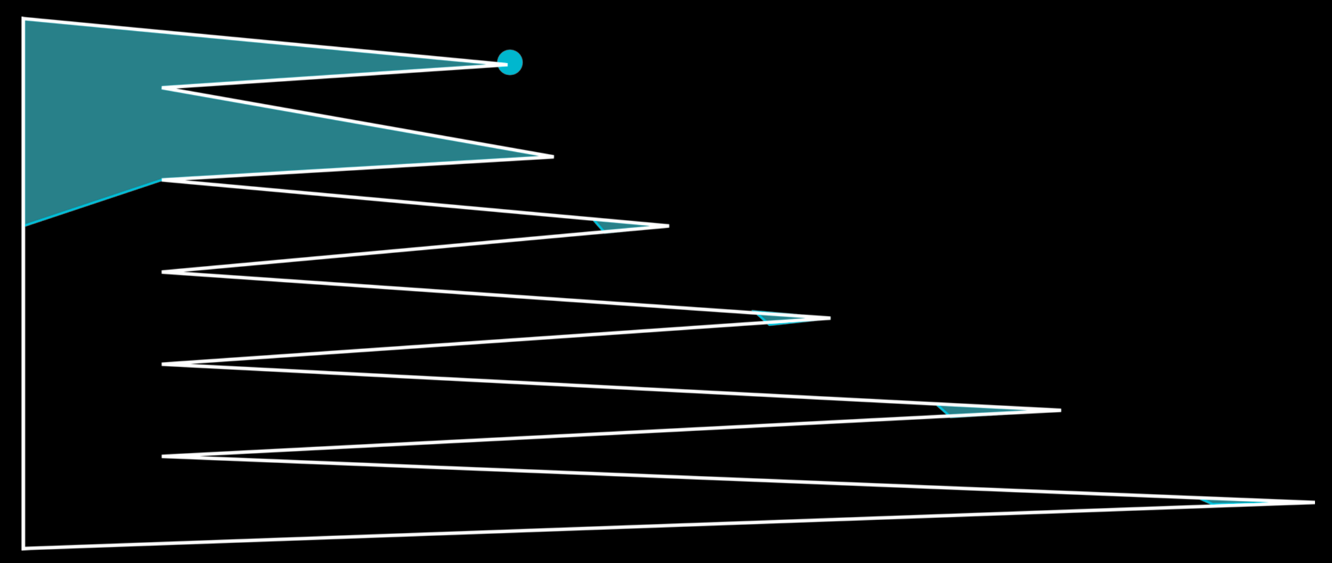

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➡ OPEN PROBLEM #2: Is there a structure like essential cuts that guarantees k-visibility of P when visited?

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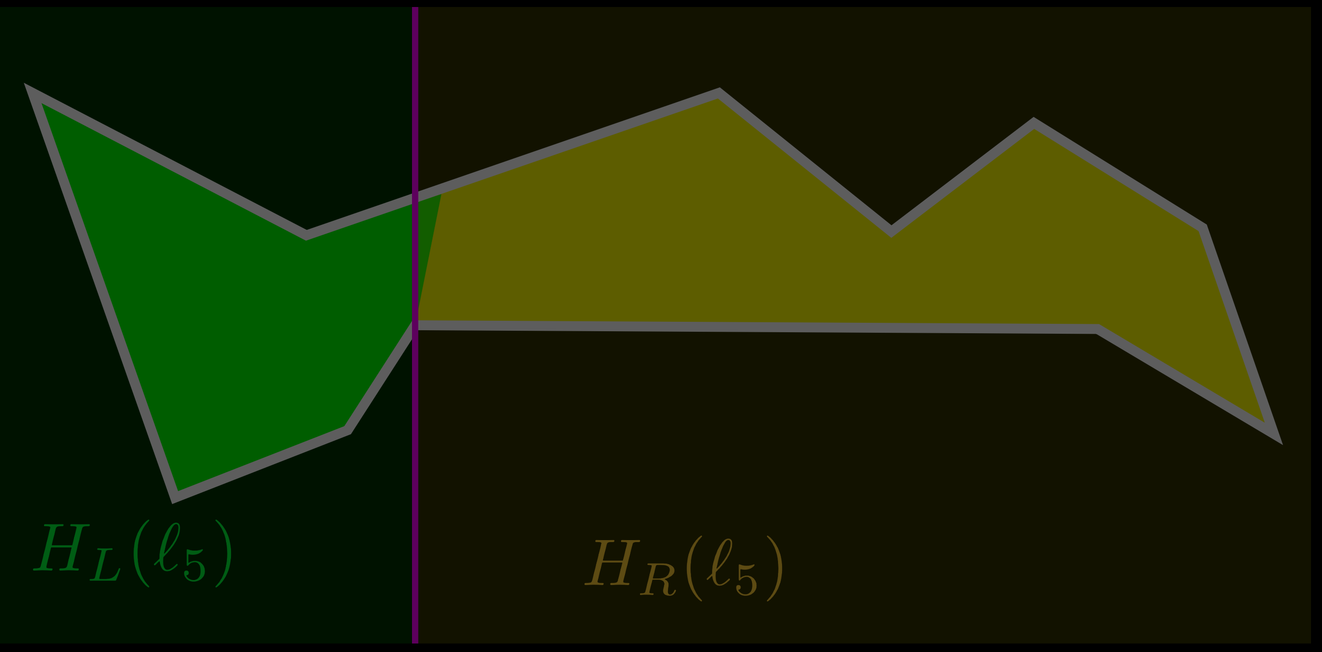
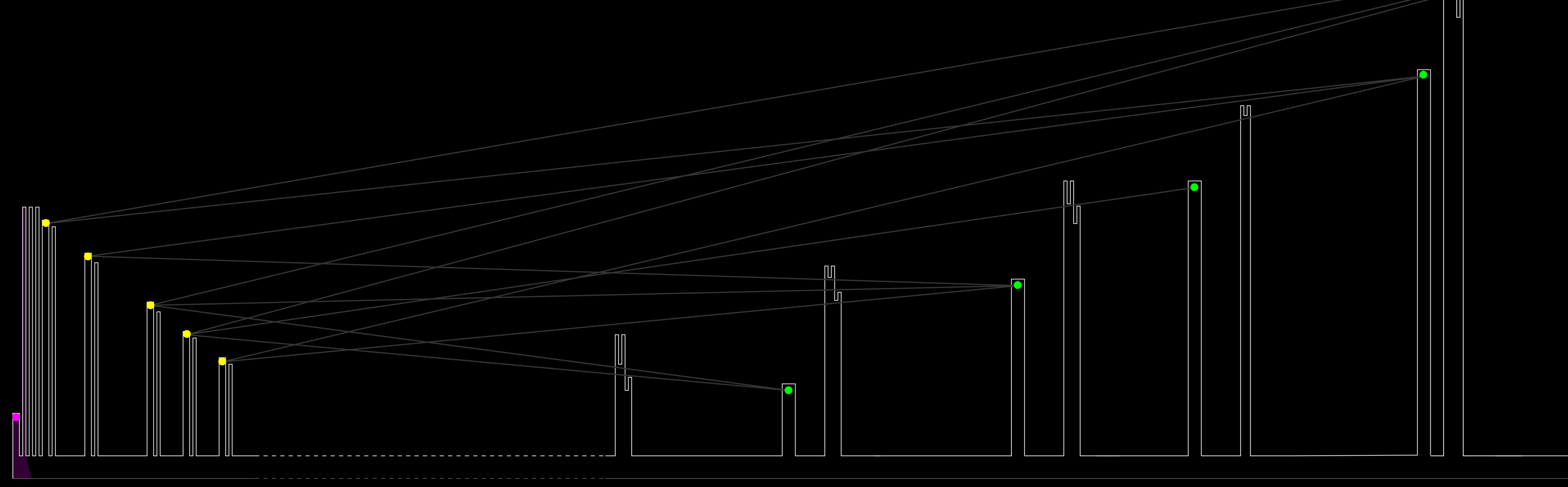
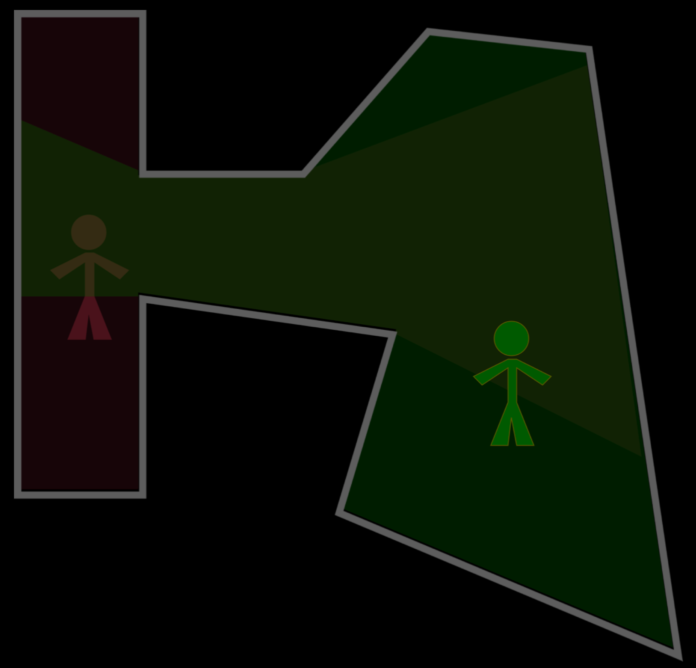
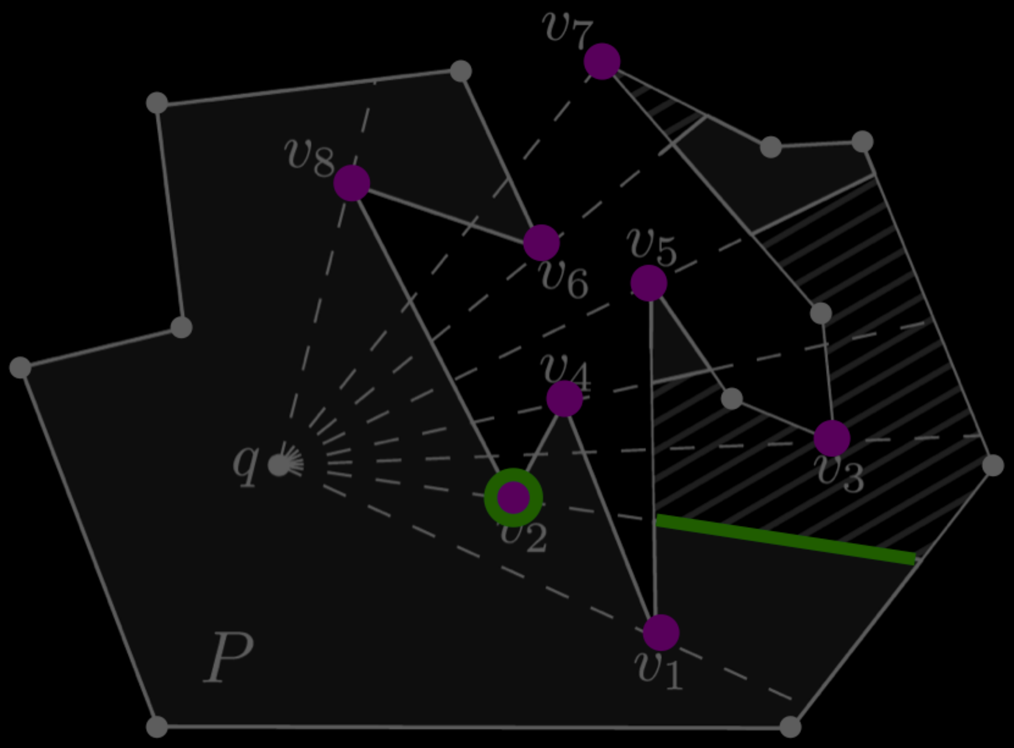
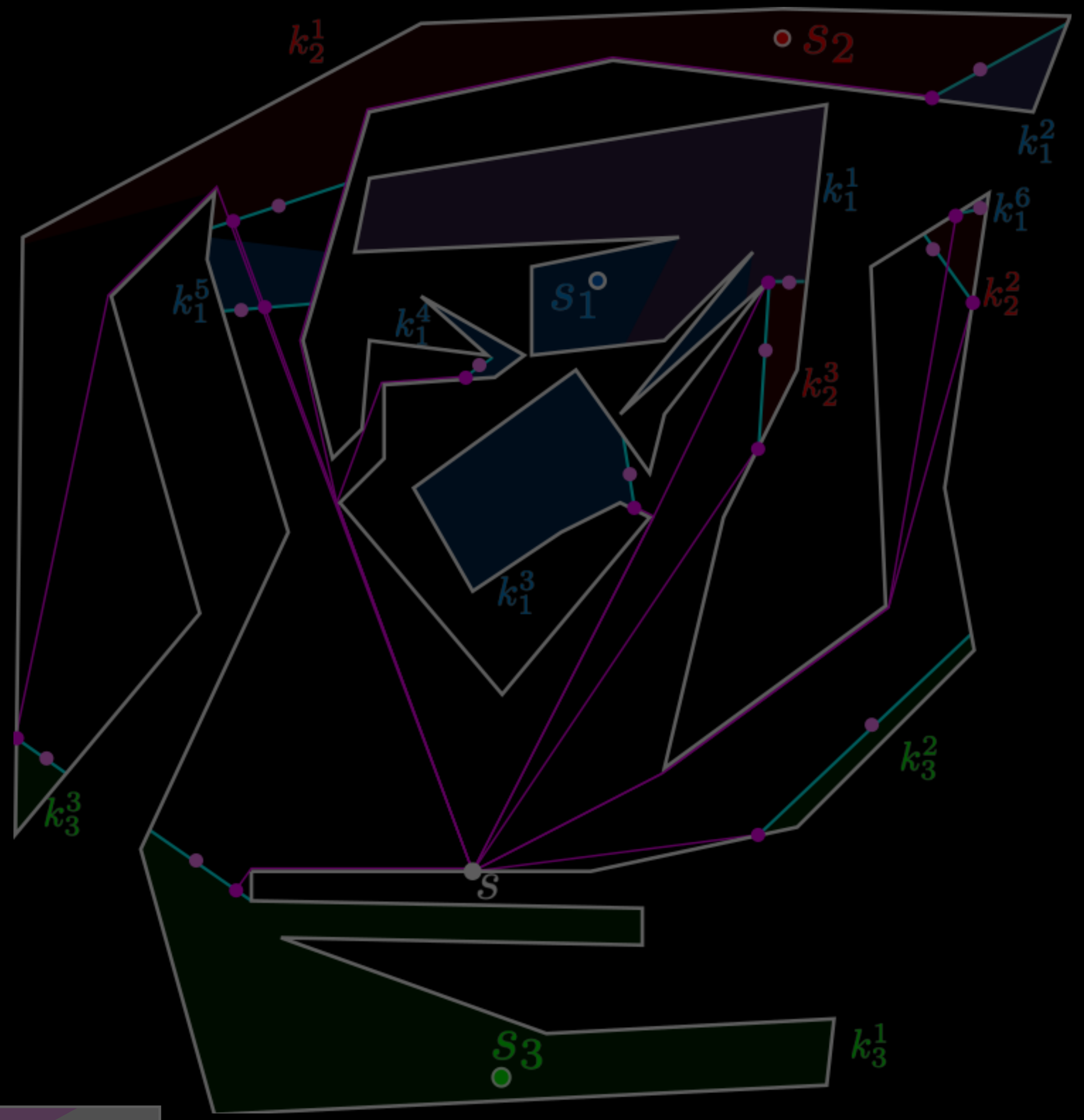
- Generally: More structural insights for *k*-transmitters











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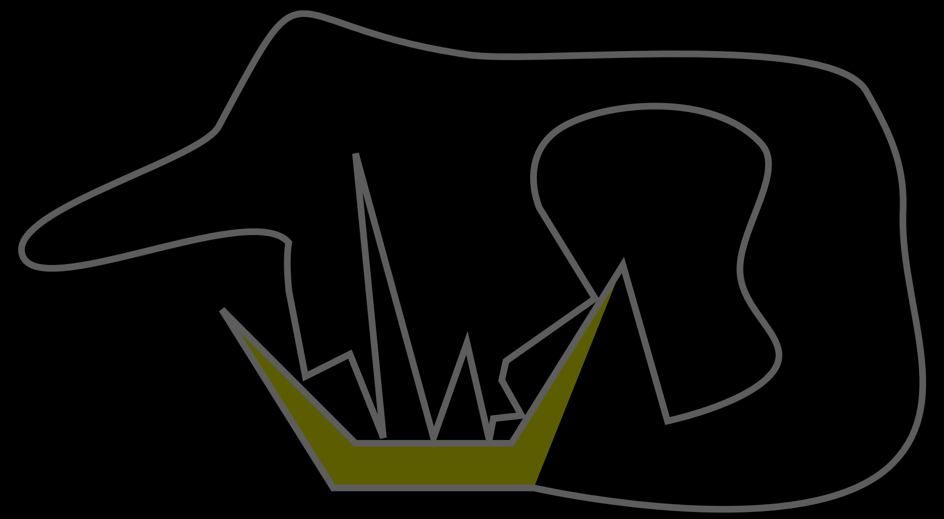

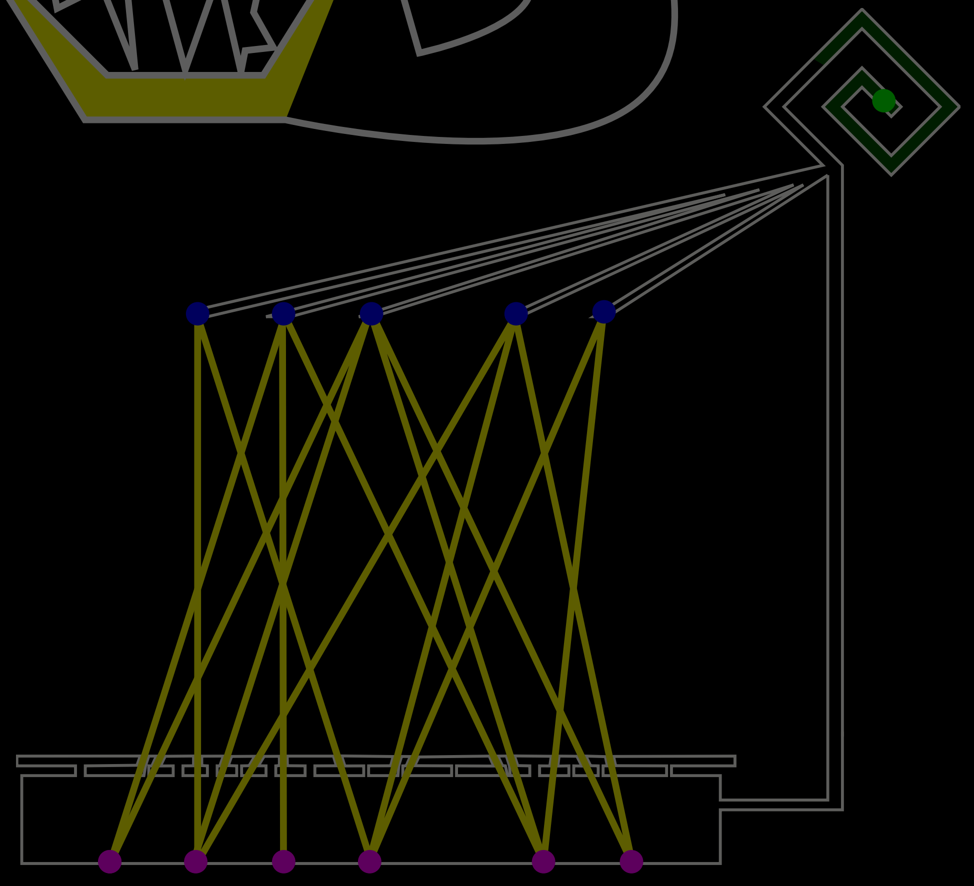

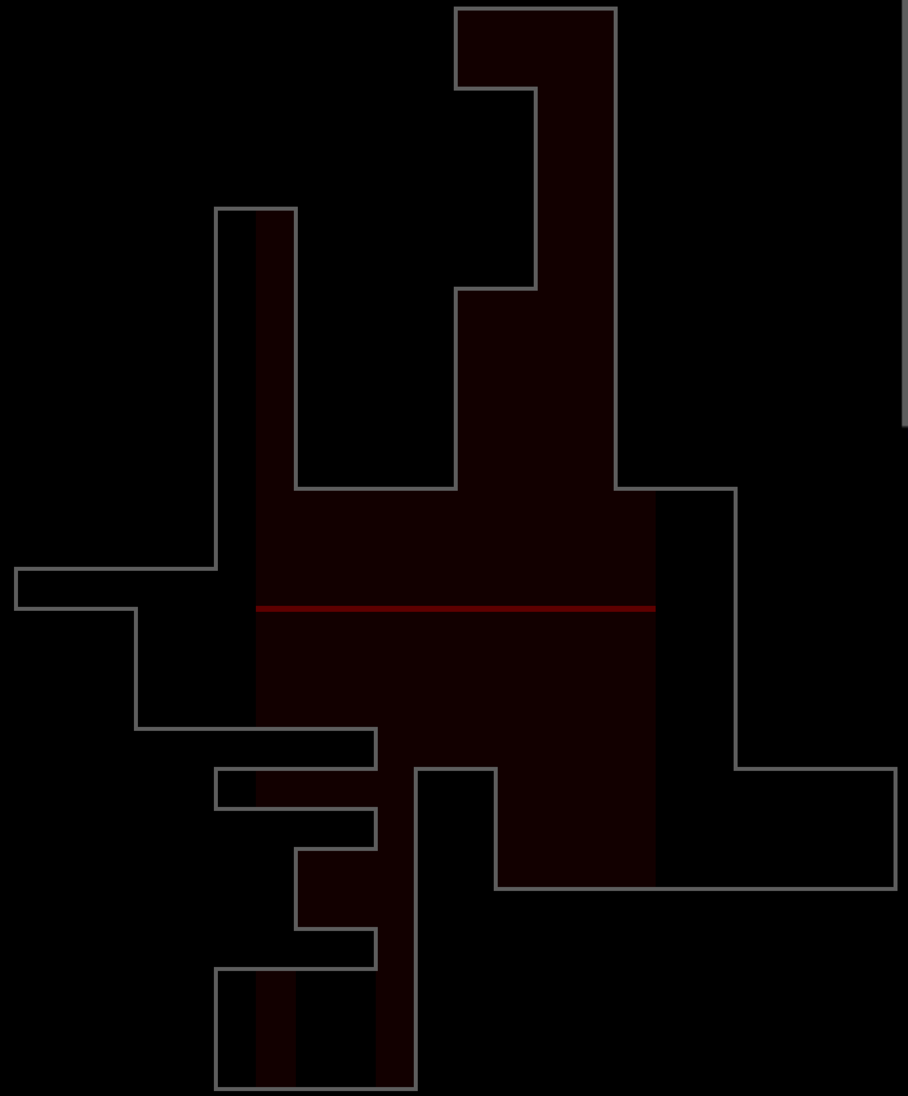

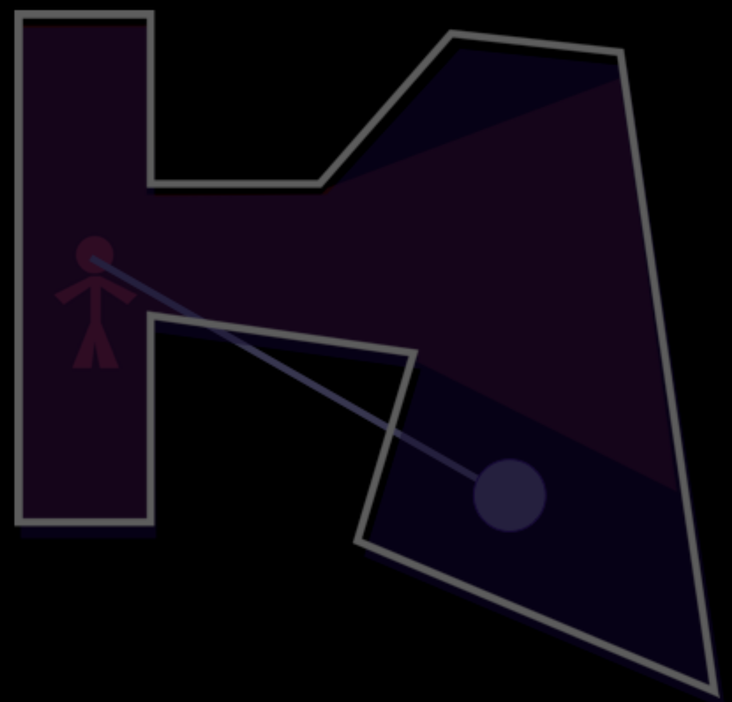

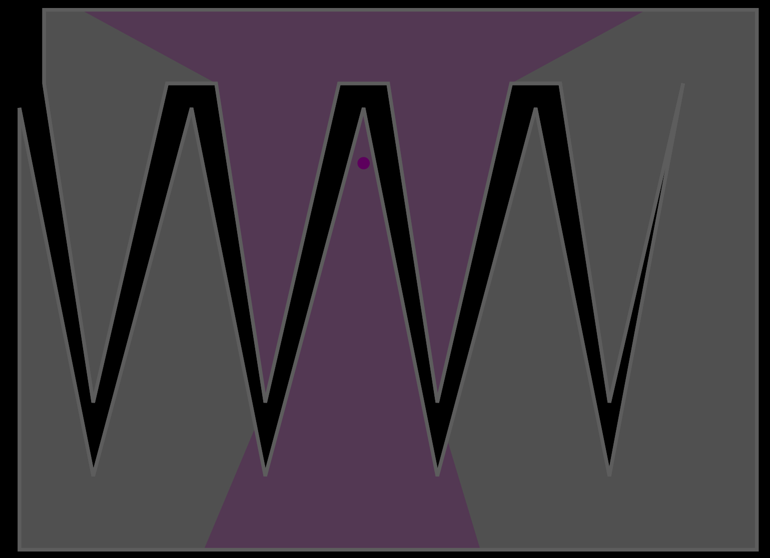

christiane.schmidt@liu.se
https://www.itn.liu.se/~chrsc91/