Stakeholder Cooperation for Improved Predictability and Lower Cost Remote Services

Joen Dahlberg, Tatiana Polishchuk, Valentin Polishchuk, **Christiane Schmidt**

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➡ **Possibilities to perturb flight schedules?** (current flight schedules consider only the single airport, ATCO might have to put a/c on hold anyhow…)

Problem Formulation

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AP1	0	0	1	1	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0
AP2	0	0	1	0	1	0	0	1	0	1	1	0	1	1	0	1	0	1	0	0	0	1	0	0	0	0	0	1	1	0	1	0	0	0	0	0
AP3	0	0	0	0	0	1	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1	0	0	0	0	0	0
AP4	0	0	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	1	0	0	0	0	1	1	1	0
AP5	0	0	1	1	0	0	0	1	0	0	1	0	0	1	1	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0

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AP1	0	0	1	1	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0
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AP3	0	0	0	0	0	1	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	1	0	0	0	1	0	1	0	0	0	0	0	0
AP4	0	0	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	1	0	0	0	0	1	1	1	0
AP5	0	0	1	1	0	0	0	1	0	0	1	0	0	1	1	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0

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AP1	0	0	1	1	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	1	0	0	0	0	0	1	0	0	1	0	0	0	0	0
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AP4	0	0	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	1	0	0	0	0	1	1	1	0
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AP4	0	0	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	1	1	0	0	0	0	1	1	1	0
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Conflict count	AP1	AP2	AP3	AP4	AP5
AP1		1058	621	366	339
AP2	1058		6473	3400	3021
AP3	621	6473		2603	2517
AP4	366	3400	2603		1449
AP5	339	3021	2517	1449	

Conflict days	AP1	AP2	AP3	AP4	AP5
AP1		341	316	278	285
AP2	341		366	363	365
AP3	316	366		362	362
AP4	278	363	362		359
AP5	285	365	362	359	

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  - ❖ Number of shifts *S*
- MAP = maximum number of airports per module

Formal problem definition:

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Flights Rescheduling and Airport-to-Module Assignment (FRAMA)

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- No conflicting airports are assigned to the same RTM
- At most MAP airports are assigned per module
- At most M modules are used

For optimisation problem: Move one constraint in objective function

Decision problem

Formal problem definition:

Flights Rescheduling and Airport-to-Module Assignment (FRAMA)

Given:

- Flight slots in a set of airports (the matrix F)
- Maximum allowable shift of a flight
- Maximum total number of allowable shifts, *S*
- Maximum number of airports per RTM, MAP
- Total number of modules, M

Find: New slots for the flights and an assignment of airports to RTMs such that

- At most *S* flights are moved
- Each flight is moved by at most Δ
- No conflicting airports are assigned to the same RTM
- At most MAP airports are assigned per module
- At most M modules are used

For optimisation problem: Move one constraint in objective function
For us: Minimize number M of used RTMs, while respecting the bounds Δ, S, MAP

Problem Complexity

Theorem: FRAMA is NP-complete, even if $\Delta = 0$ and MAP=3.

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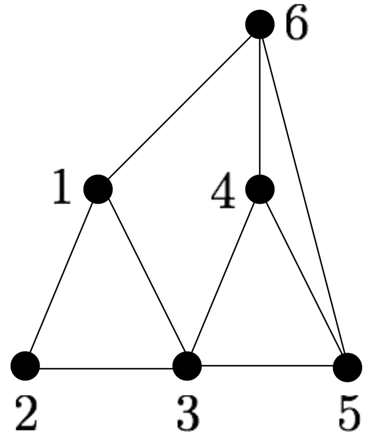

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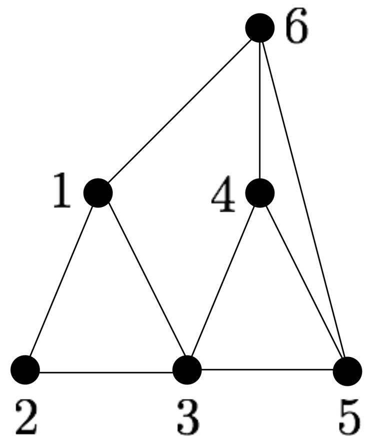

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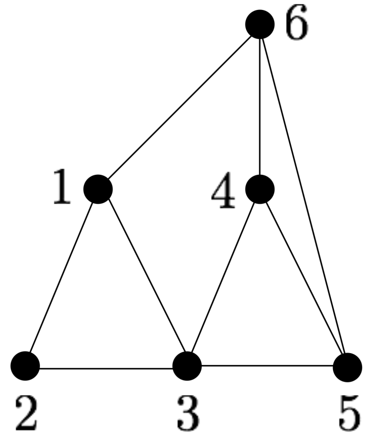

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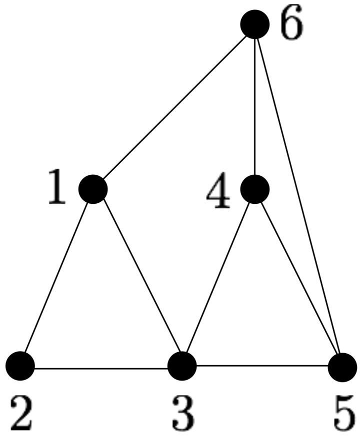

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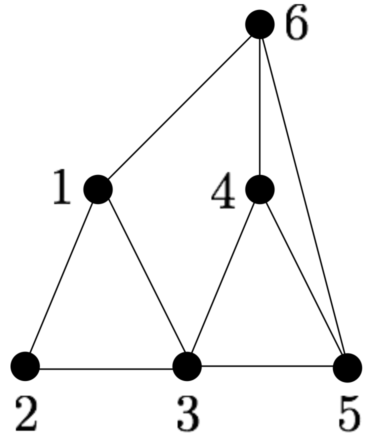

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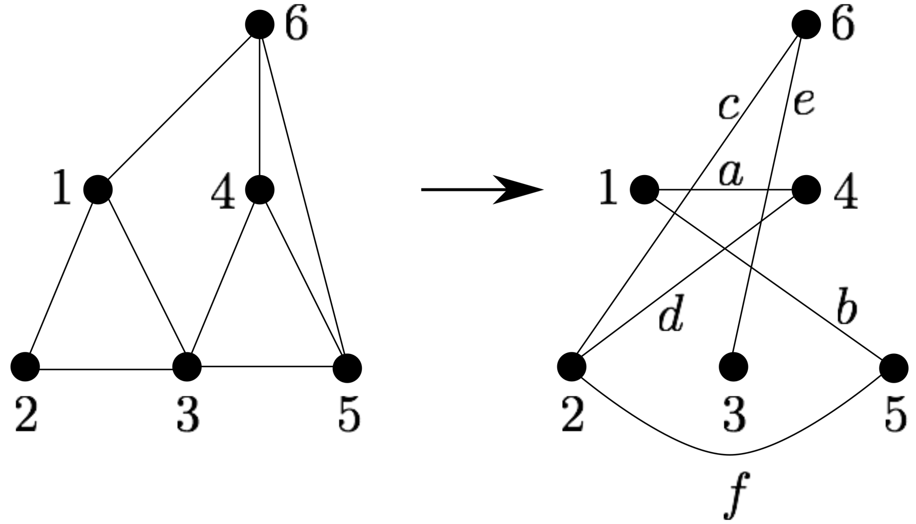

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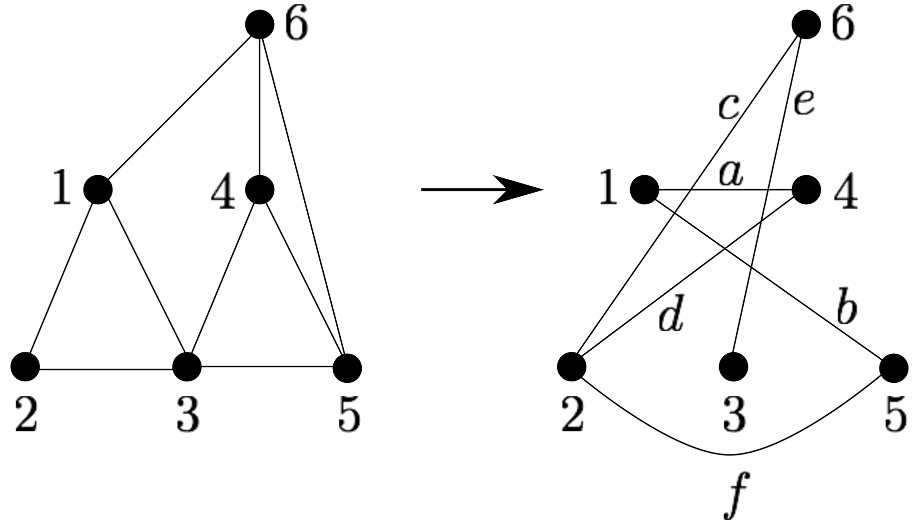

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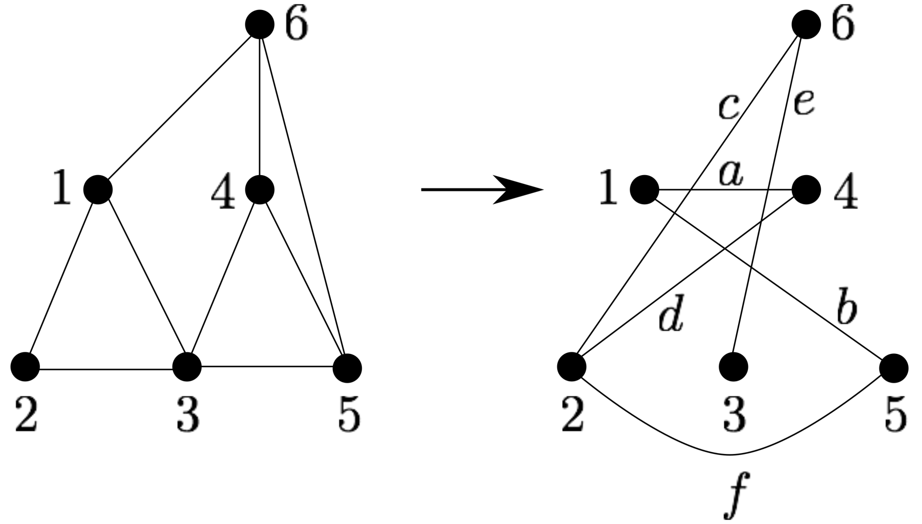

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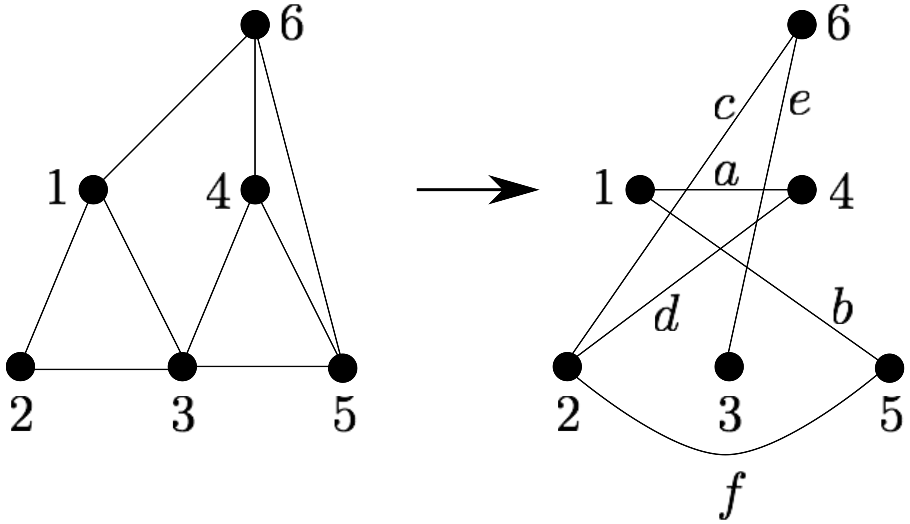

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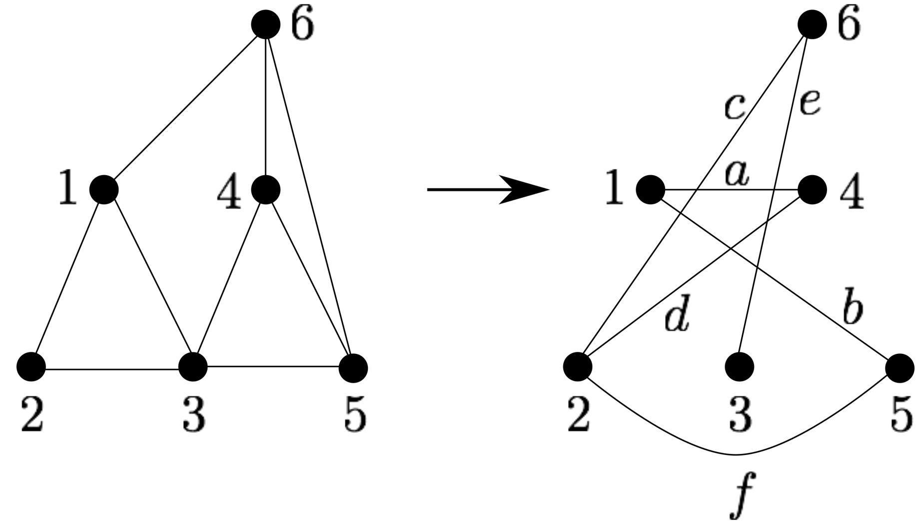

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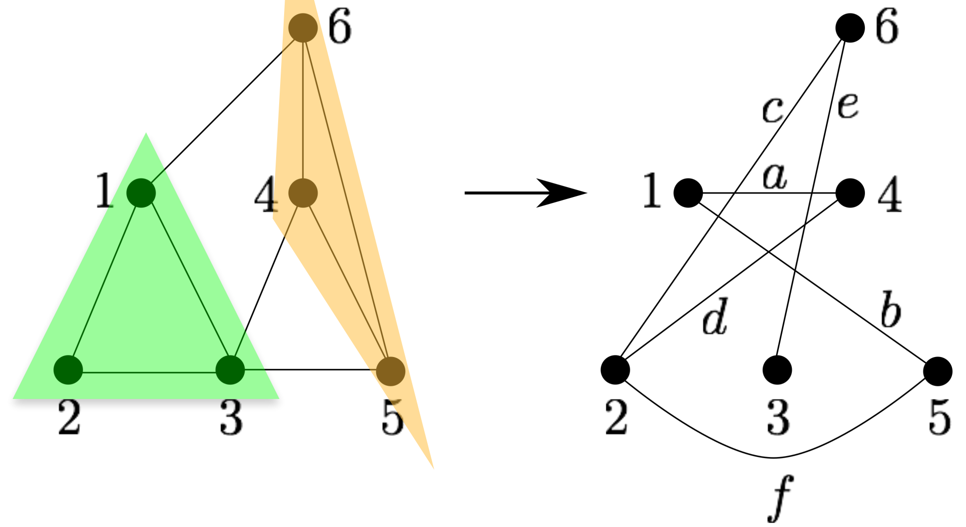

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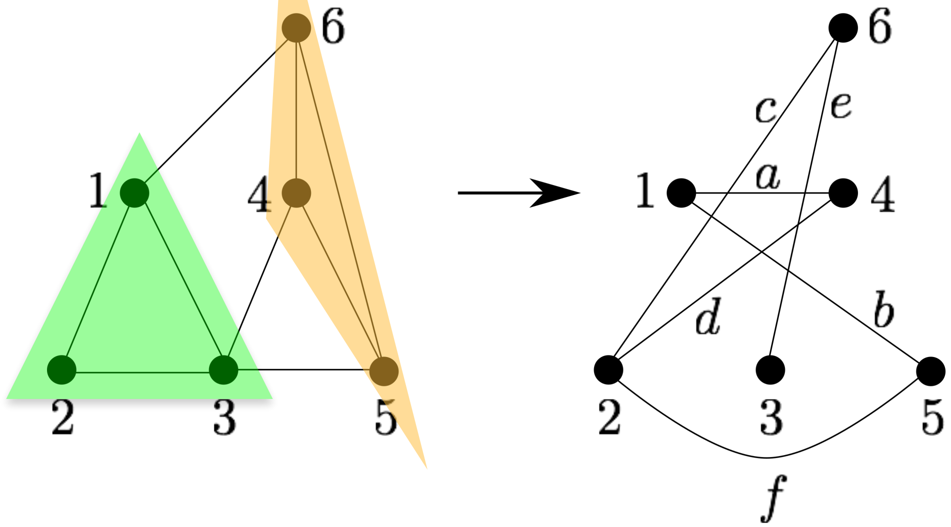

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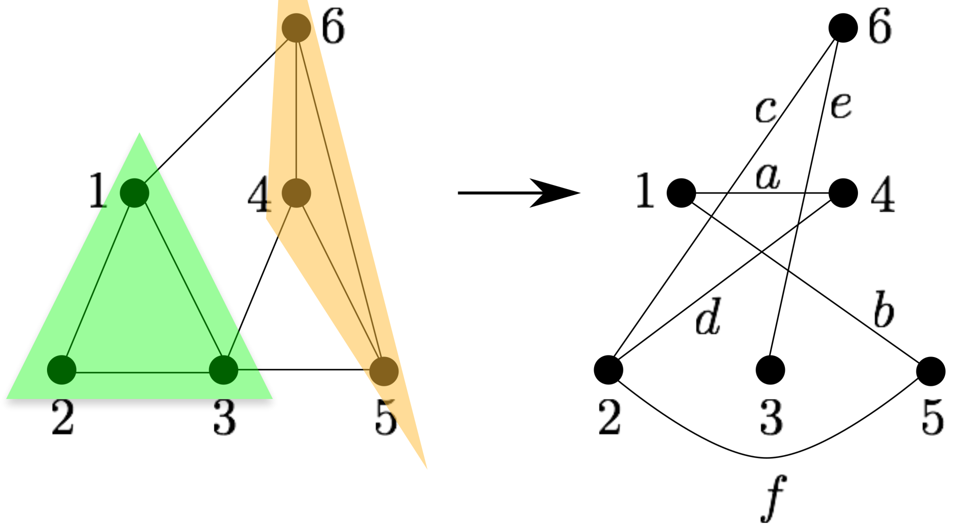

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Solution to FRAMA with Δ=0 (and, thus, *S*= 0) and MAP= 3 can be verified in polytime.

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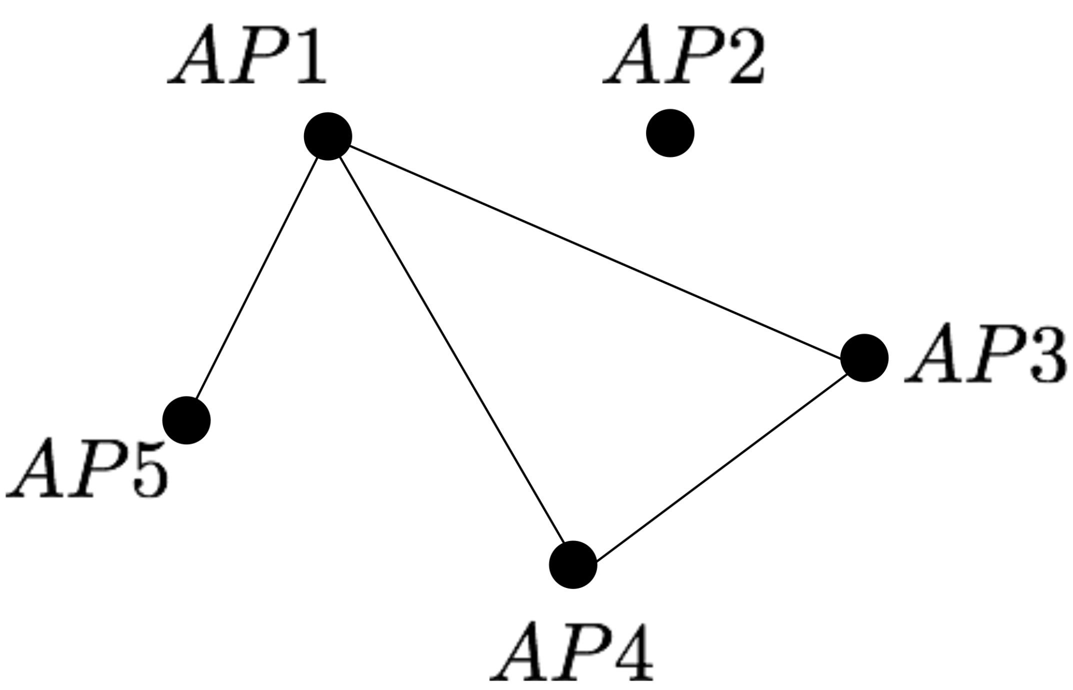

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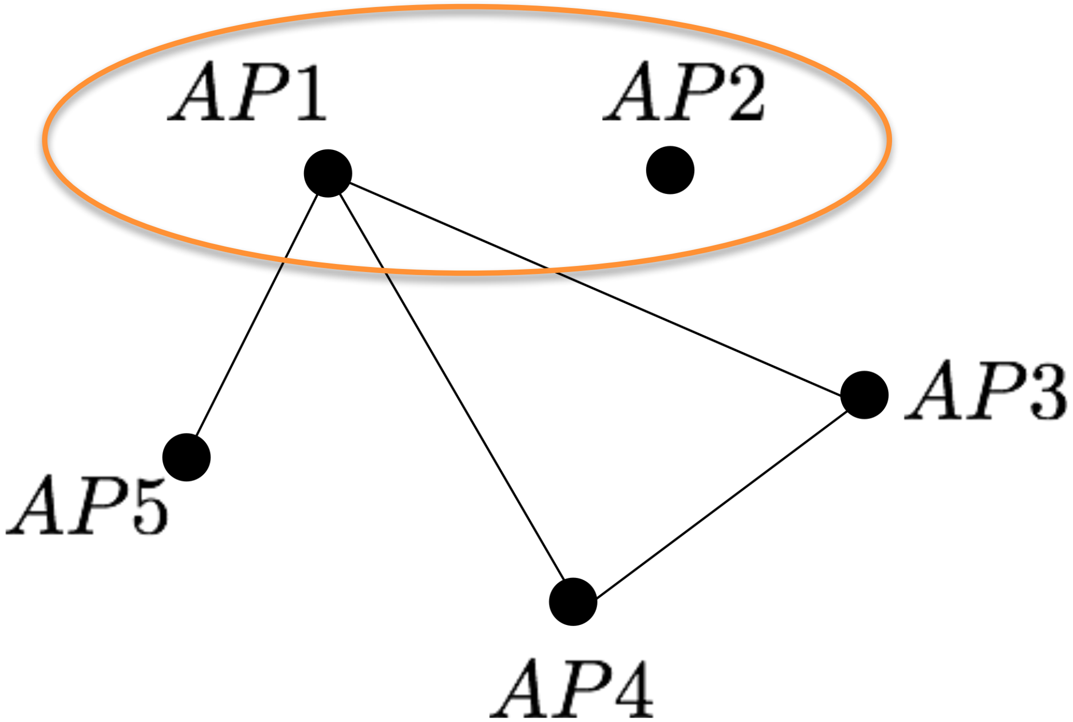

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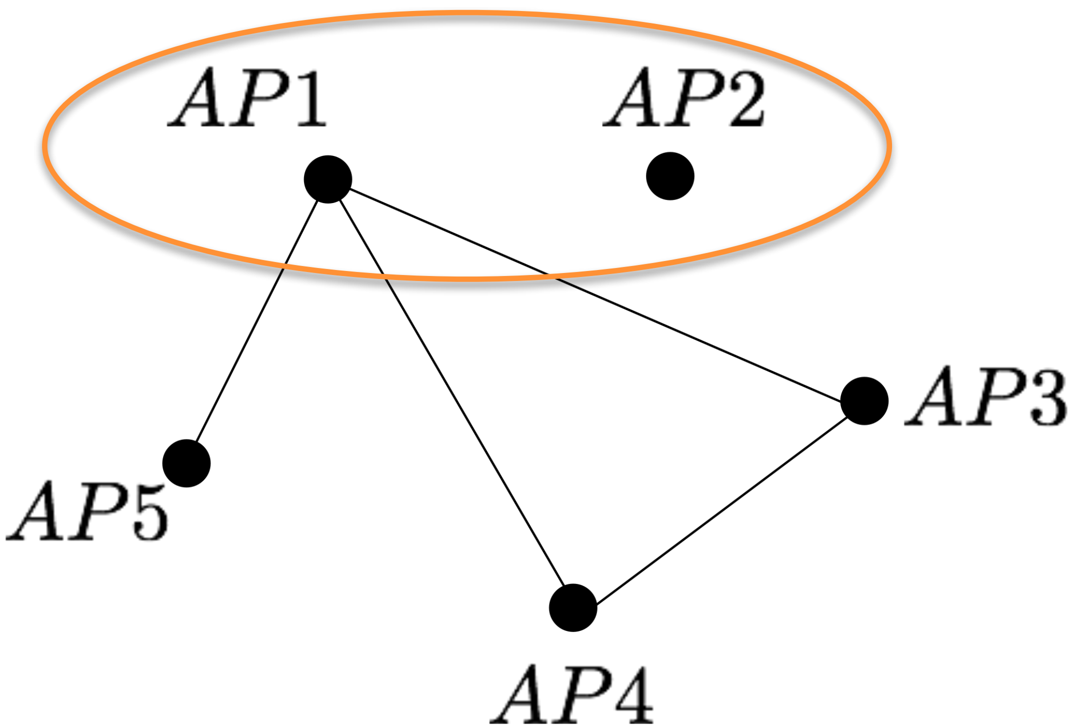

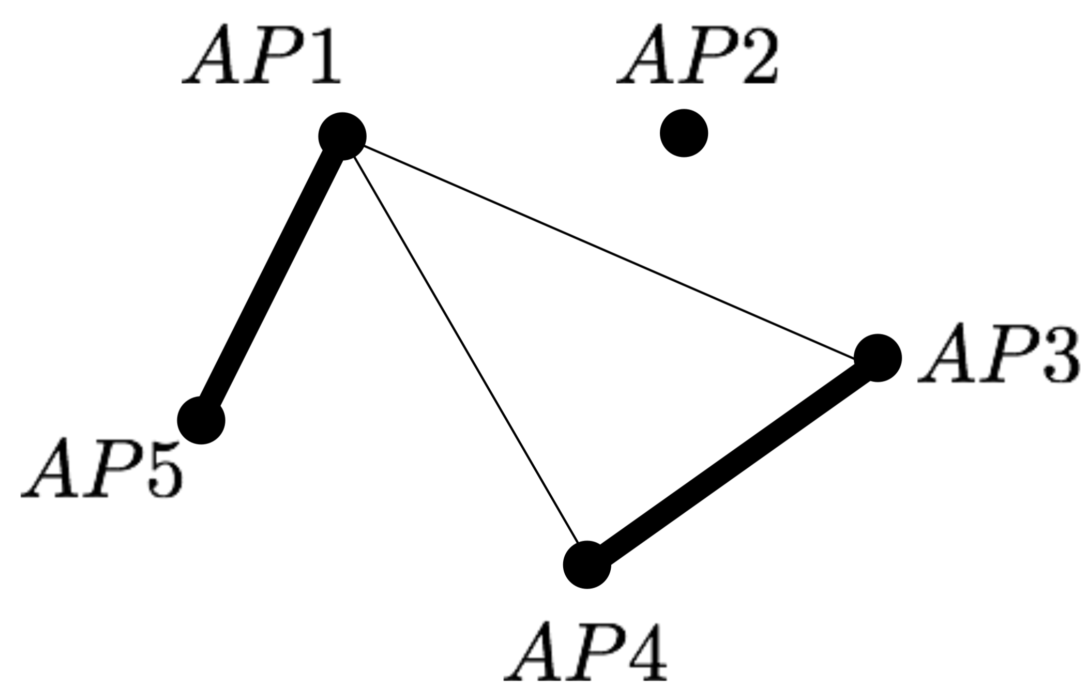

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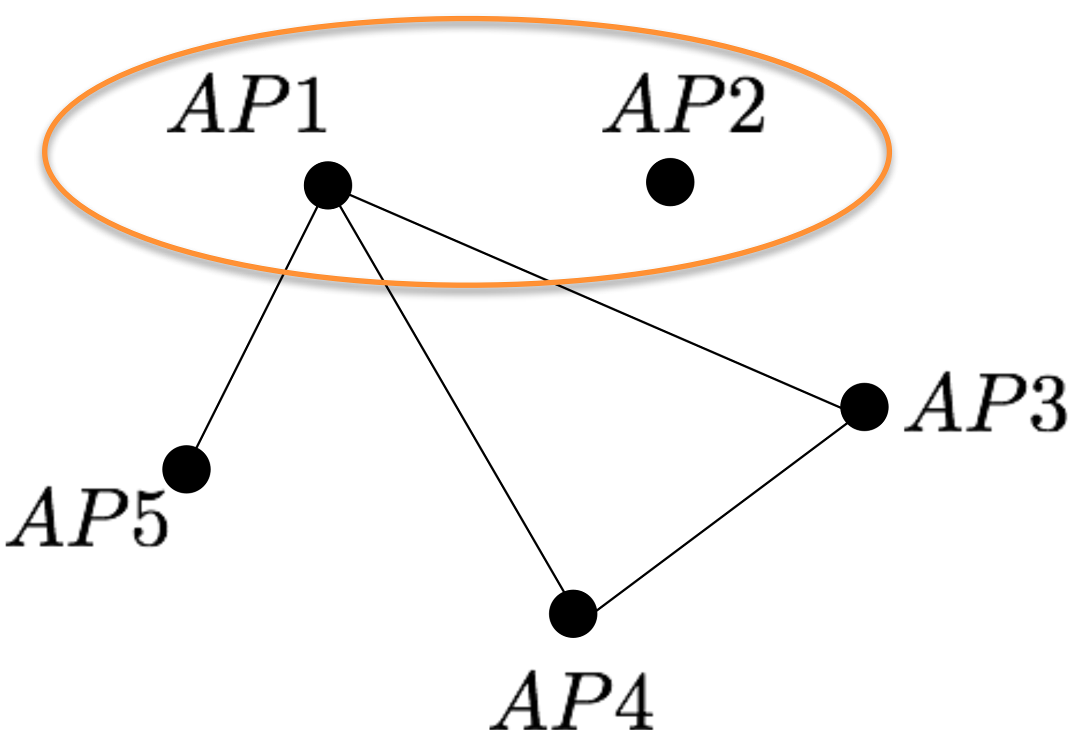

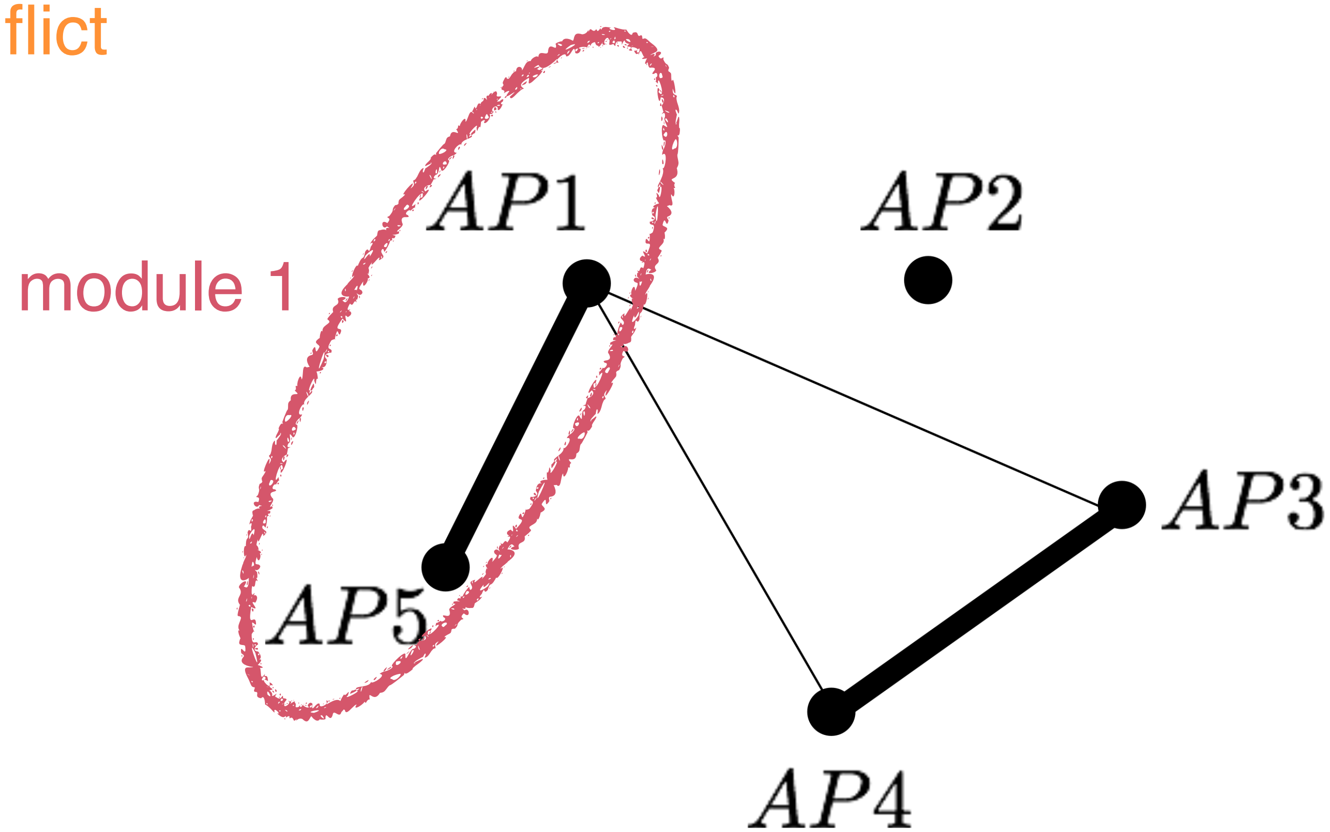

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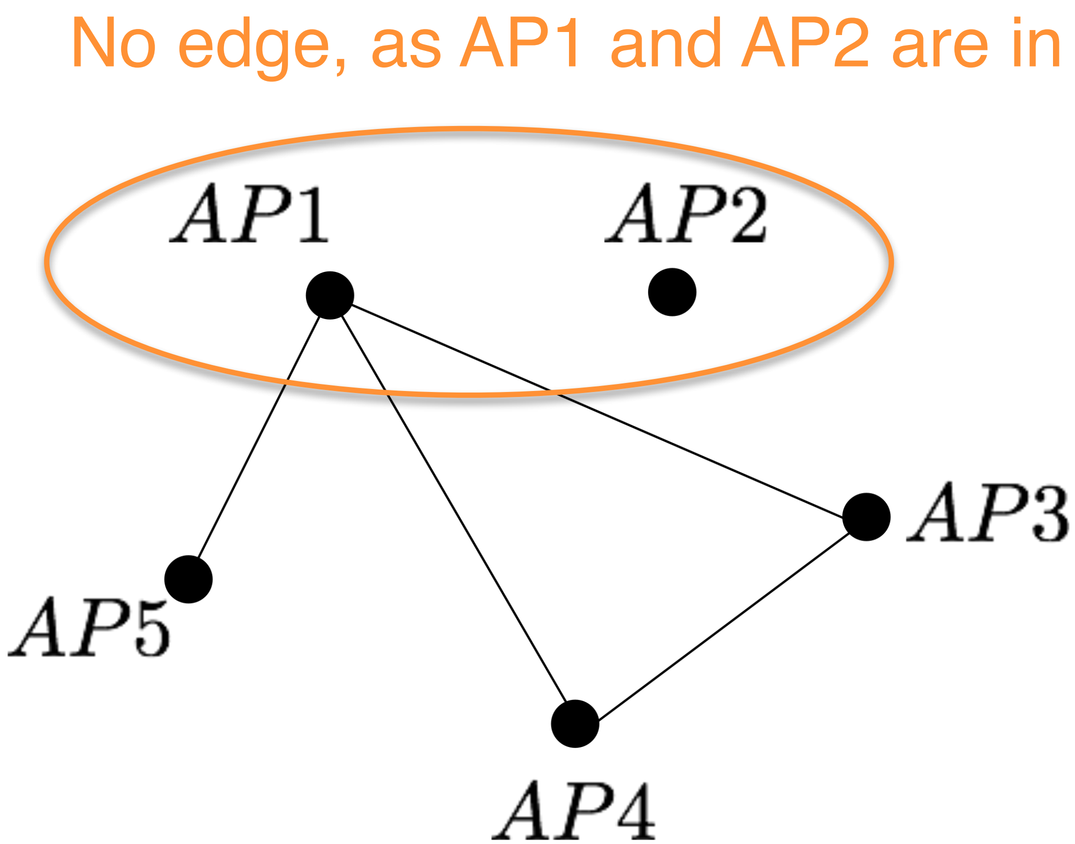

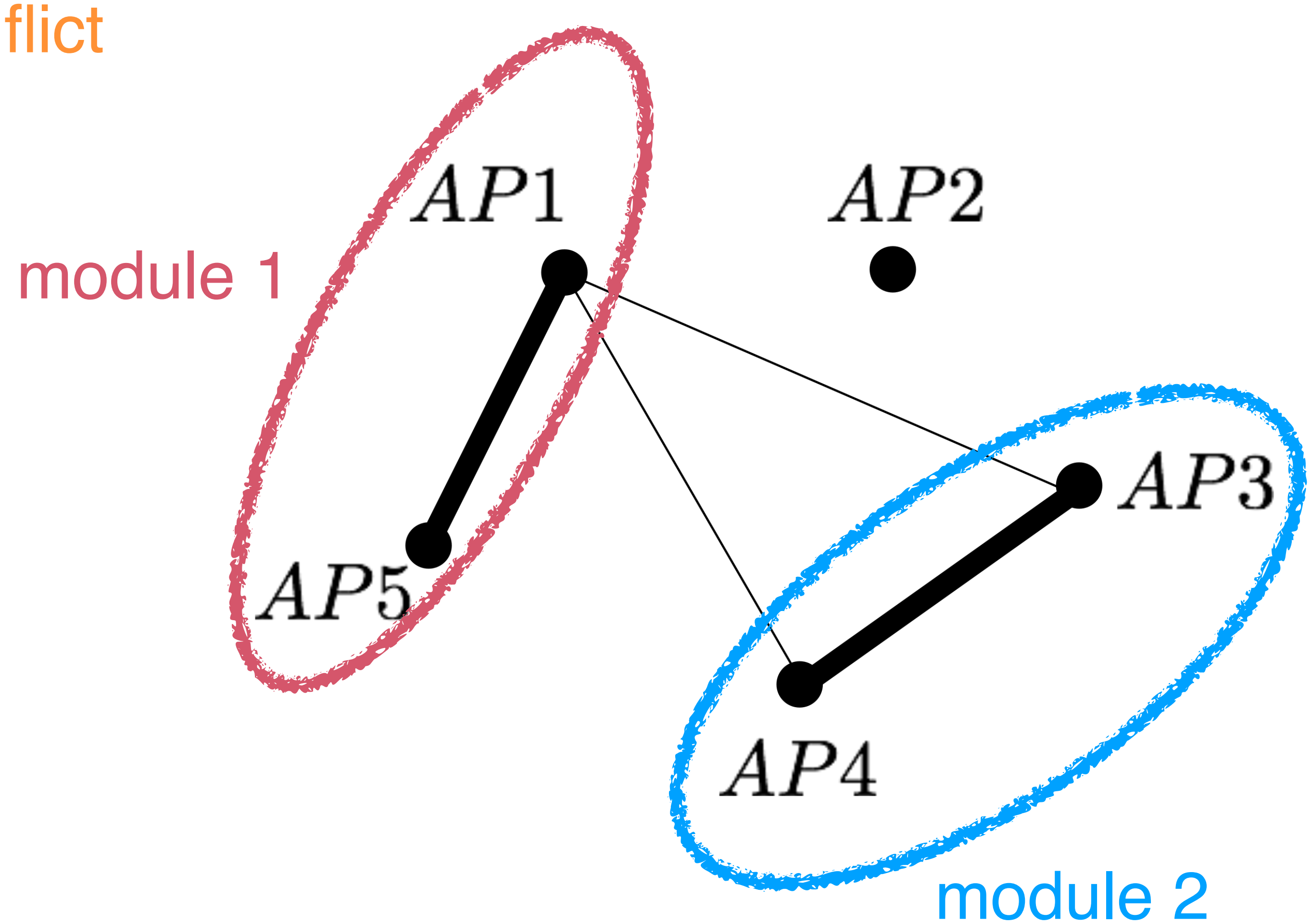

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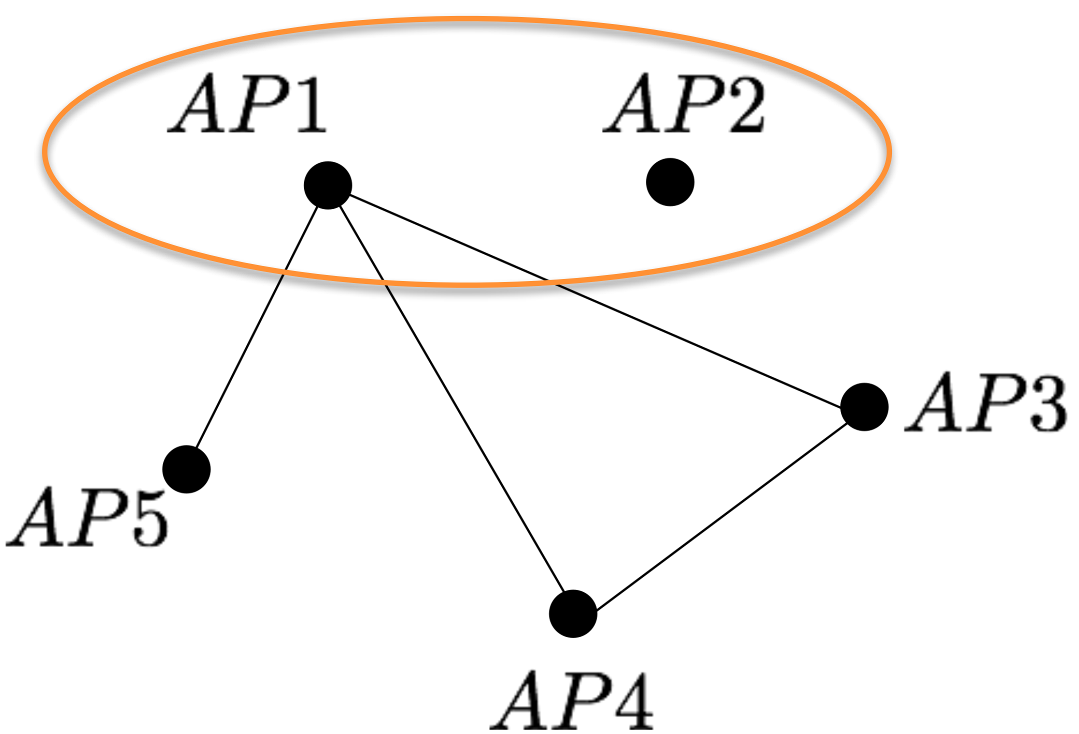

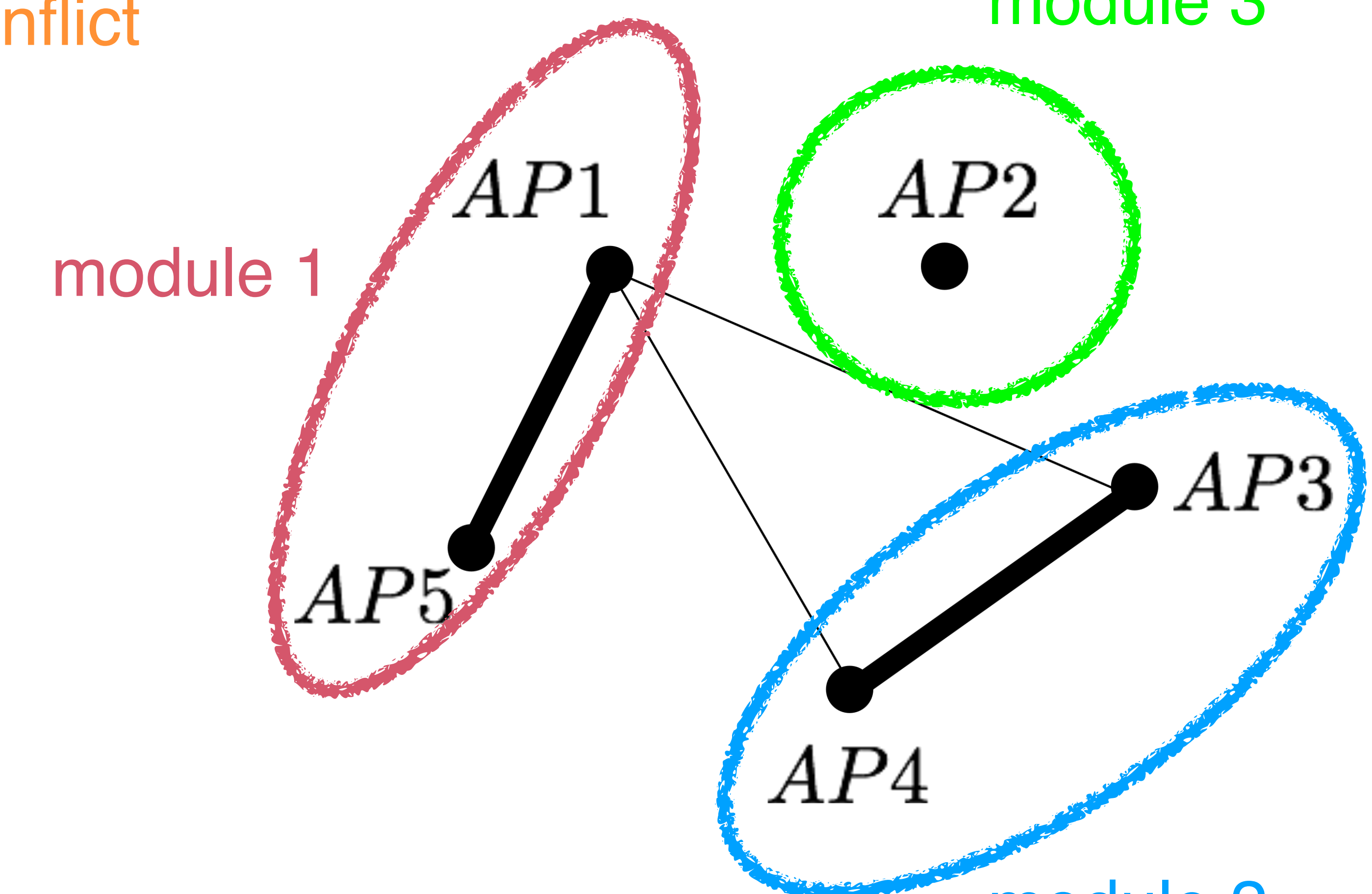

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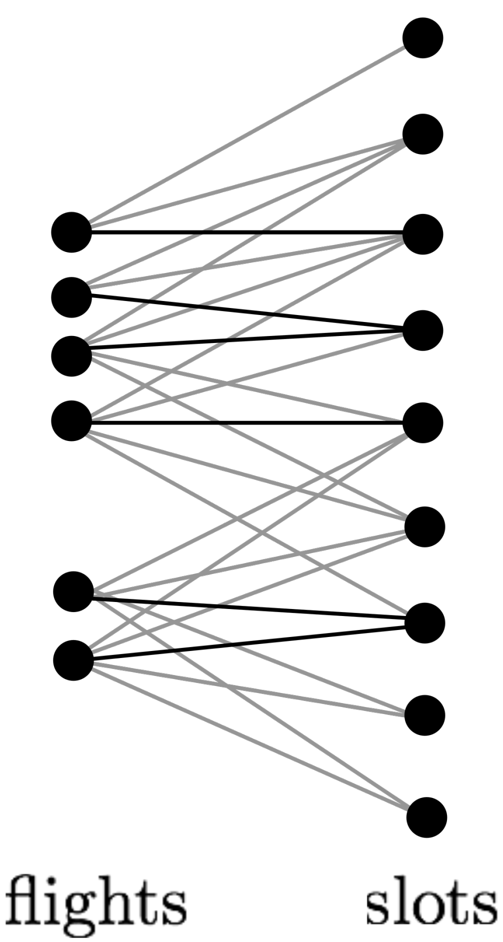

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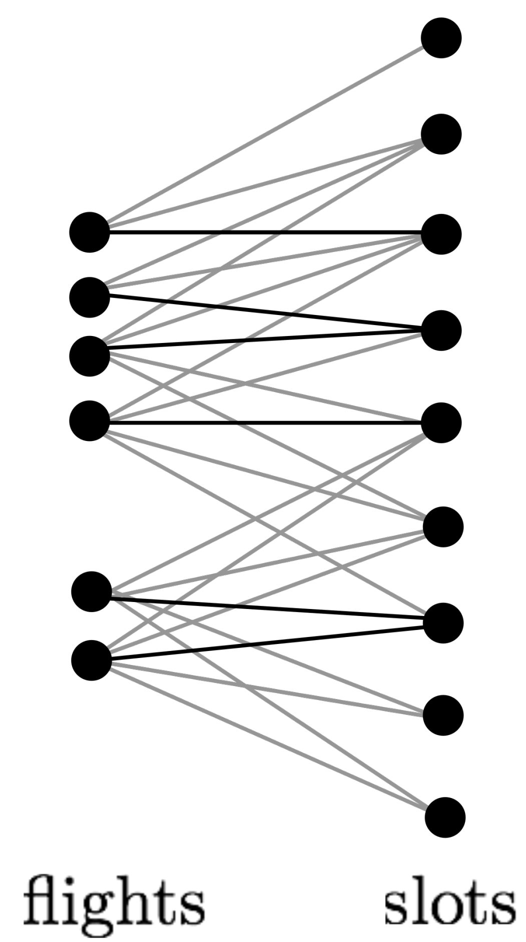

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- Flight f is connected to all slots within distance Δ/5 from its original slot
- Edge weight:
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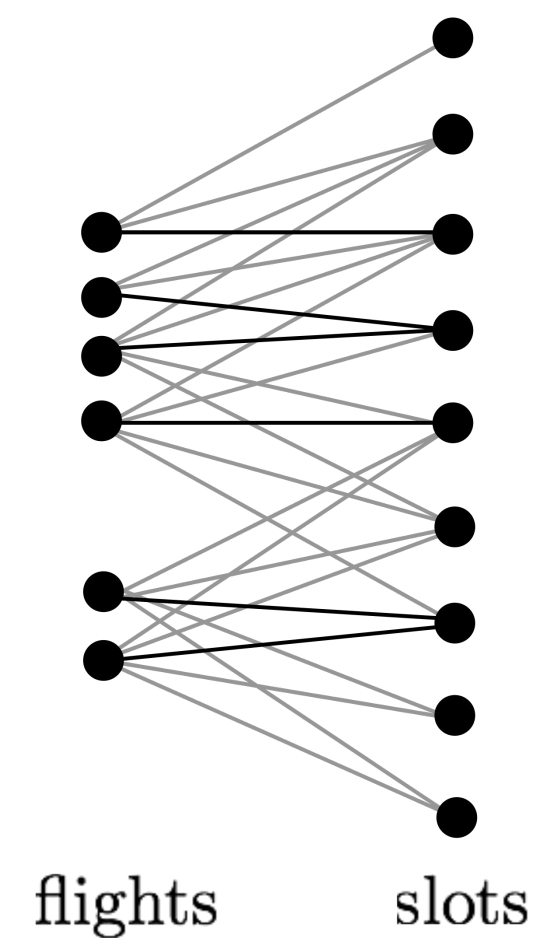

Complexity for Δ> 0 and MAP=2 unknown.

Possible heuristic:

- First remove all conflicts
- Then assign airports to RTMs

➡ Solve rescheduling and assignment problem separately

Assignment problem is trivial in the absence of conflicts (the airports are arbitrarily packed into the RTMs, with MAP airports per module)

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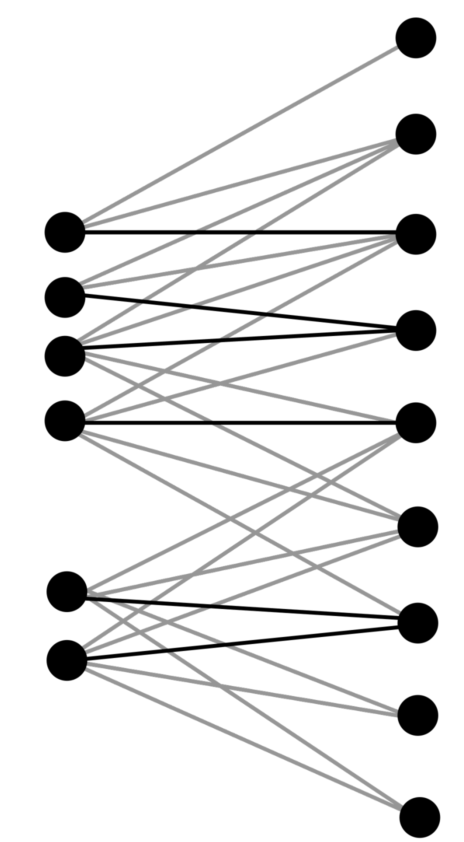

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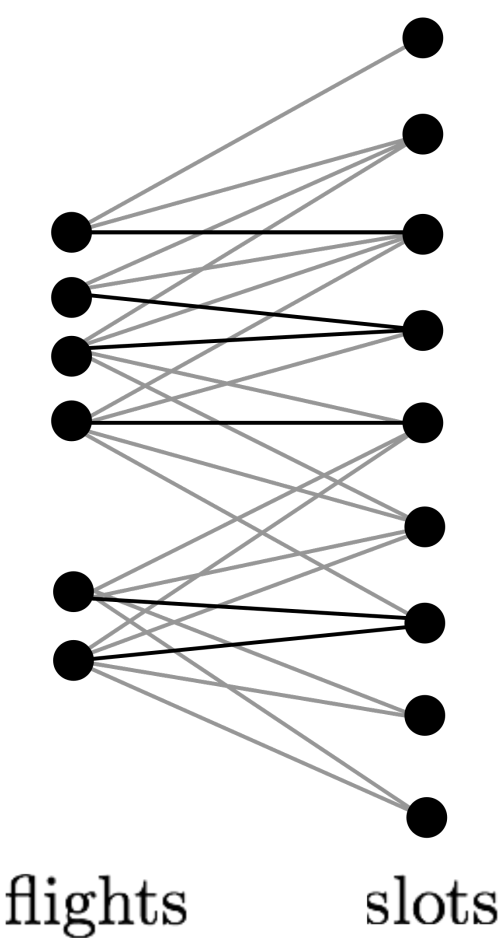

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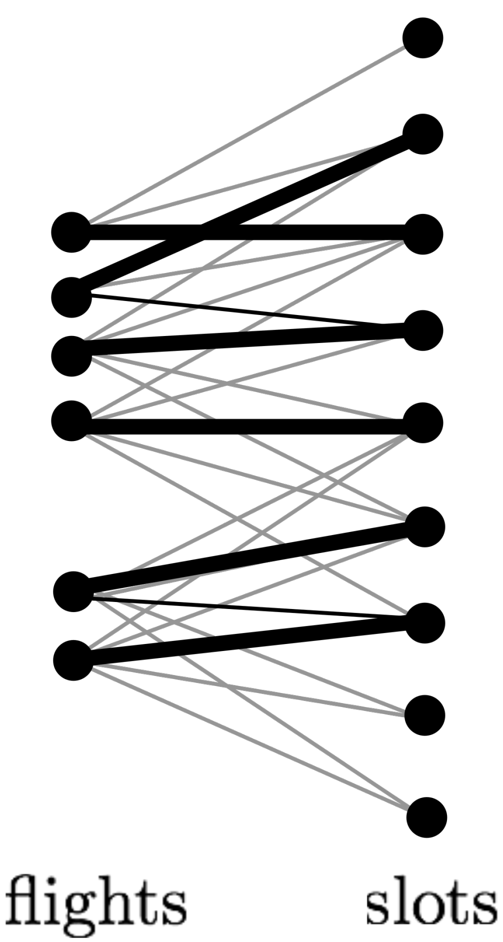

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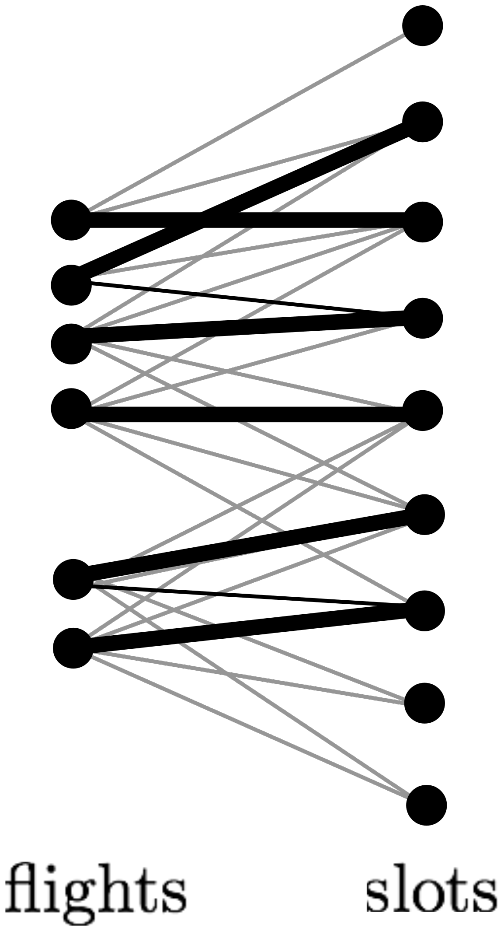

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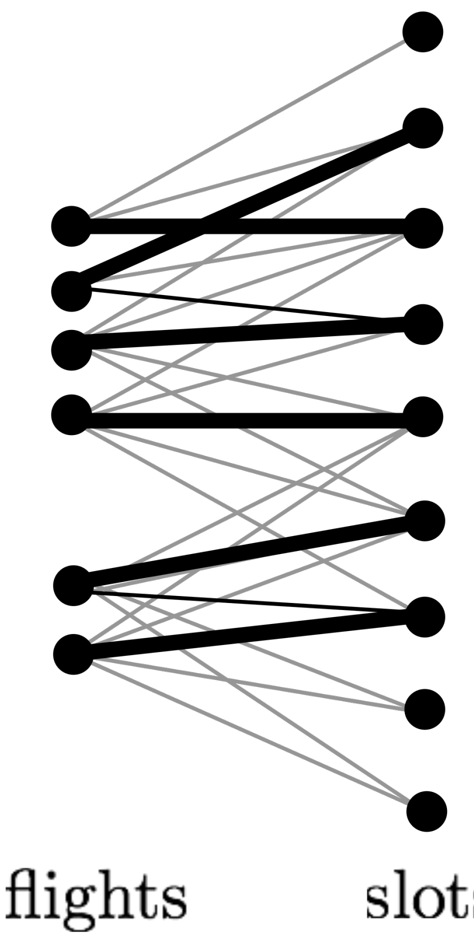
Runs in polynomial time, but may find suboptimal solutions to FRAMA (not necessary to remove all the conflicts)



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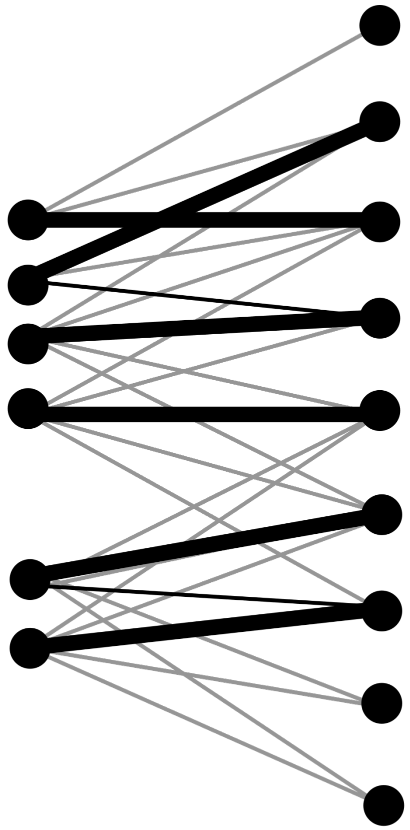
For a small number of airports: enumerate all pairs of airports



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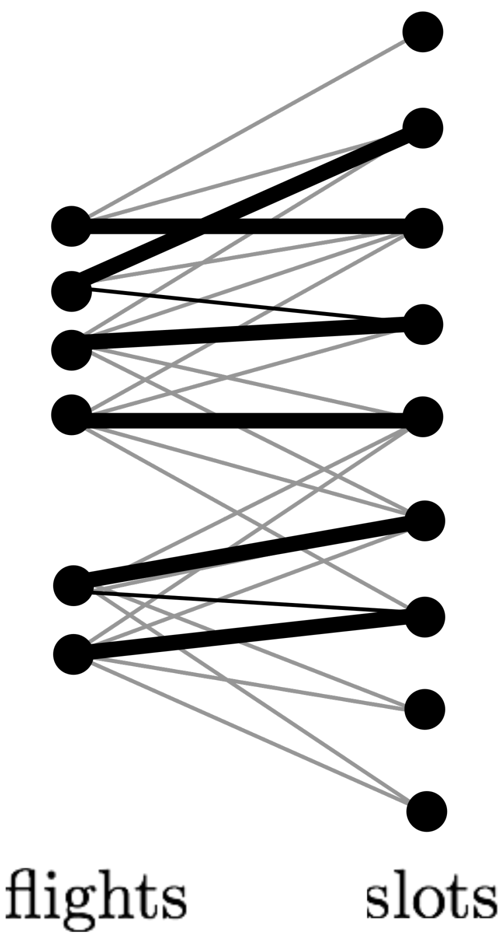

Runs in polynomial time, but may find suboptimal solutions to FRAMA (not necessary to remove all the conflicts)

For a small number of airports: enumerate all pairs of airports completely eliminate all conflicts for the given pairs (matching) with a given Δ > 0

Complexity for Δ> 0 and MAP=2 unknown.

Possible heuristic:

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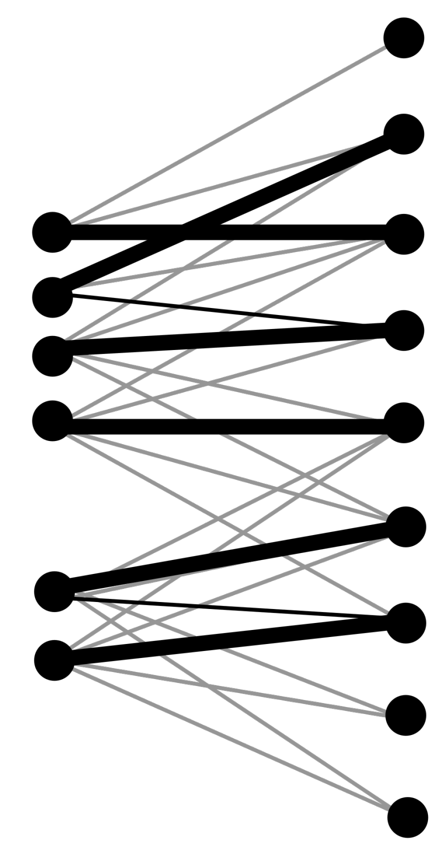
Runs in polynomial time, but may find suboptimal solutions to FRAMA (not necessary to remove all the conflicts)

For a small number of airports: enumerate all pairs of airports completely eliminate all conflicts for the given pairs (matching) with a given $\Delta > 0$ chose combination with minimum possible number of modules



IP for FRAMA

Decision variables
x_{am}: airport *a* assigned to module *m*
z_m: module *m* is used
y_{atf}: flight *f* arrives/departs at/from airport *a* in time slot *t*
w_{ab}: conflict between airport *a* and airport *b* (some *t*)

A = set of airports
M = set of modules
T = set of time slots
V_a= flights at airport *a*
p_{atf} = cost to move flight *f* at airport *a* to time slot *t*
s_{af} = scheduled time for flight *f* at airport a *i*
δ maximum shift distance for scheduled aircraft in terms of time slots: $\delta = \Delta / 5$.

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$$\min c_1 \sum_{m \in M} z_m + c_2 \sum_{a \in A} \sum_{t \in T} \sum_{f \in V_a} p_{atf} y_{atf} \tag{1}$$

$$
\begin{aligned}
s.t.\ & x_{am} && \leqslant z_m && \forall (a,m) \in A \times M && (2)\\
& \sum_{m \in M} x_{am} && = 1 && \forall a \in A && (3)\\
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& x_{am} + x_{bm} && \leqslant 2 - w_{ab} && \forall (a,b,m) \in A \times A \times M, a < b && (7)\\
& \sum_{a \in A} x_{am} && \leqslant \text{MAP} && \forall m \in M && (8)\\
& x, y, w, z && \textit{binary} && && (9)
\end{aligned}
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Some airport assigned to module *m*

$$\text{s.t. } x_{am} \leqslant z_m \quad \forall (a,m) \in A \times M \quad (2)$$

$$\sum_{m \in M} x_{am} = 1 \quad \forall a \in A \quad (3)$$

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(2) Some airport assigned to module *m* →module *m* used

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(1) c_1*#modules + c_2* sum of shifts

$$s.t.\ x_{am} \leqslant z_m \quad \forall (a,m) \in A \times M \quad (2)$$

$$\sum_{m \in M} x_{am} = 1 \quad \forall a \in A \quad (3)$$

$$\sum_{f \in V_a} y_{atf} \leqslant 1 \quad \forall (a,t) \in A \times T \quad (4)$$

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$$\sum_{a \in A} x_{am} \leqslant \text{MAP} \quad \forall m \in M \quad (8)$$

$$x, y, w, z \quad \textit{binary} \quad (9)$$

(2) Some airport assigned to module *m* →module *m* used
(3) Each airport assigned to 1 module
(4) At most 1 flight arrives/departs at airport time slot t

Decision variables
x_{am}: airport *a* assigned to module *m*
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$$\min c_1 \sum_{m \in M} z_m + c_2 \sum_{a \in A} \sum_{t \in T} \sum_{f \in V_a} p_{atf} y_{atf} \quad (1)$$

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$$s.t. \quad x_{am} \leqslant z_m \quad \forall (a,m) \in A \times M \quad (2)$$

(2) Some airport assigned to module *m* →module *m* used

$$\sum_{m \in M} x_{am} = 1 \quad \forall a \in A \quad (3)$$

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$$\sum_{f \in V_a} y_{atf} \leqslant 1 \quad \forall (a,t) \in A \times T \quad (4)$$

(4) At most 1 flight arrives/departs at airport time slot t

$$\sum_{t=\max(1, s_{af}-\delta)}^{\min(|T|, s_{af}+\delta)} y_{atf} = 1 \quad \forall (a,f) \in A \times V_a \quad (5)$$

(5) Each flight ±δ from scheduled time

$$\sum_{f \in V_a} y_{atf} + \sum_{f \in V_b} y_{btf} \leqslant 1 + w_{ab} \forall (a,b,t) \in A \times A \times T, a < b \quad (6)$$

$$x_{am} + x_{bm} \leqslant 2 - w_{ab} \forall (a,b,m) \in A \times A \times M, a < b \quad (7)$$

$$\sum_{a \in A} x_{am} \leqslant \text{MAP} \quad \forall m \in M \quad (8)$$

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Decision variables

x_{am}: airport *a* assigned to module *m*

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min # shifts: $p_{atf}=1$ if $t \neq s_{af}$; $p_{atf}=0$ if $t=s_{af}$

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δ maximum shift distance for scheduled aircraft in terms of time slots: $\delta = \Delta/5$.

$$\min c_1 \sum_{m \in M} z_m + c_2 \sum_{a \in A} \sum_{t \in T} \sum_{f \in V_a} p_{atf} y_{atf} \quad (1)$$

(1) c_1*#modules + c_2* sum of shifts

$$s.t. \quad x_{am} \leqslant z_m \quad \forall (a,m) \in A \times M \quad (2)$$

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(4) At most 1 flight arrives/departs at airport time slot t

$$\sum_{t=\max(1, s_{af}-\delta)}^{\min(|T|, s_{af}+\delta)} y_{atf} = 1 \quad \forall (a,f) \in A \times V_a \quad (5)$$

(5) Each flight $\pm\delta$ from scheduled time

$$\sum_{f \in V_a} y_{atf} + \sum_{f \in V_b} y_{btf} \leqslant 1 + w_{ab} \forall (a,b,t) \in A \times A \times T, a < b \quad (6)$$

(6) Two a/c at same slot at airports *a* and *b*

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(7) If ∃ conflict → airports not same module

$$\sum_{a\in A} x_{am} \leqslant \text{MAP} \quad \forall m\in M \quad (8)$$

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(2) Some airport assigned to module *m* → module *m* used
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(4) At most 1 flight arrives/departs at airport time slot t
(5) Each flight ±δ from scheduled time
(6) Two a/c at same slot at airports *a* and *b* → two airports in conflict
(7) If ∃ conflict → airports not same module
(8) Max MAP airports to each module

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IP formulation of FRAMA optimises c_1*M + c_2*S (could move one in constraint)

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IP formulation of FRAMA optimises c_1*M + c_2*S (could move one in constraint)
We choose c_1 and c_2 such that minimizing the modules is the primary objective: $c_1 >> c_2$

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➡ IP formulation solves FRAMA!

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Experimental Study

Additional airports considered for remote operation in Sweden:

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- **Airport 1 (AP1):** Small airport with low traffic, few scheduled flights per hour, non-regular helicopter traffic, sometimes special testing activities.
- **Airport 2 (AP2):** Low to medium-sized airport, multiple scheduled flights per hour, regular special traffic flights (usually open 24/7, with exceptions).
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One optimization problem for each pair (Δ, MAP)

Original Traffic

MAP=5

δ	# of modules	# of shifts $= S$	maximum shift (in mins) $= \Delta$
0	5	0	-
1	2	32	5
2	2	27	10
3	2	26	15
4	2	26	-
5	2	26	-
6	2	26	-
7	1	118	35
8	1	108	40
9	1	99	45
10	1	91	50
11	1	85	55
12	1	83	60
13	1	81	65
14	1	79	70
15	1	78	75
16	1	75	80
17	1	75	85
18	1	75	90
19	1	74	95
20	1	74	100
21	1	73	105

We have 12 x 24 = 288 slots for flight movements

➡ with sufficiently large shifts 233 flight movements in single module

Original Traffic

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MAP=5

δ	# of modules	# of shifts $= S$	maximum shift (in mins) $= \Delta$
0	5	0	-
1	2	32	5
2	2	27	10
3	2	26	15
4	2	26	-
5	2	26	-
6	2	26	-
7	1	118	35
8	1	108	40
9	1	99	45
10	1	91	50
11	1	85	55
12	1	83	60
13	1	81	65
14	1	79	70
15	1	78	75
16	1	75	80
17	1	75	85
18	1	75	90
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No rescheduling allowed: need 5 RTMs

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9	1	99	45
10	1	91	50
11	1	85	55
12	1	83	60
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No rescheduling allowed: need 5 RTMs
Reschedule at most ±5 minutes: 2 RTMs

We have 12 x 24 = 288 slots for flight movements

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0	5	0	-
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5	2	26	-
6	2	26	-
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8	1	108	40
9	1	99	45
10	1	91	50
11	1	85	55
12	1	83	60
13	1	81	65
14	1	79	70
15	1	78	75
16	1	75	80
17	1	75	85
18	1	75	90
19	1	74	95
20	1	74	100
21	1	73	105

No rescheduling allowed: need 5 RTMs

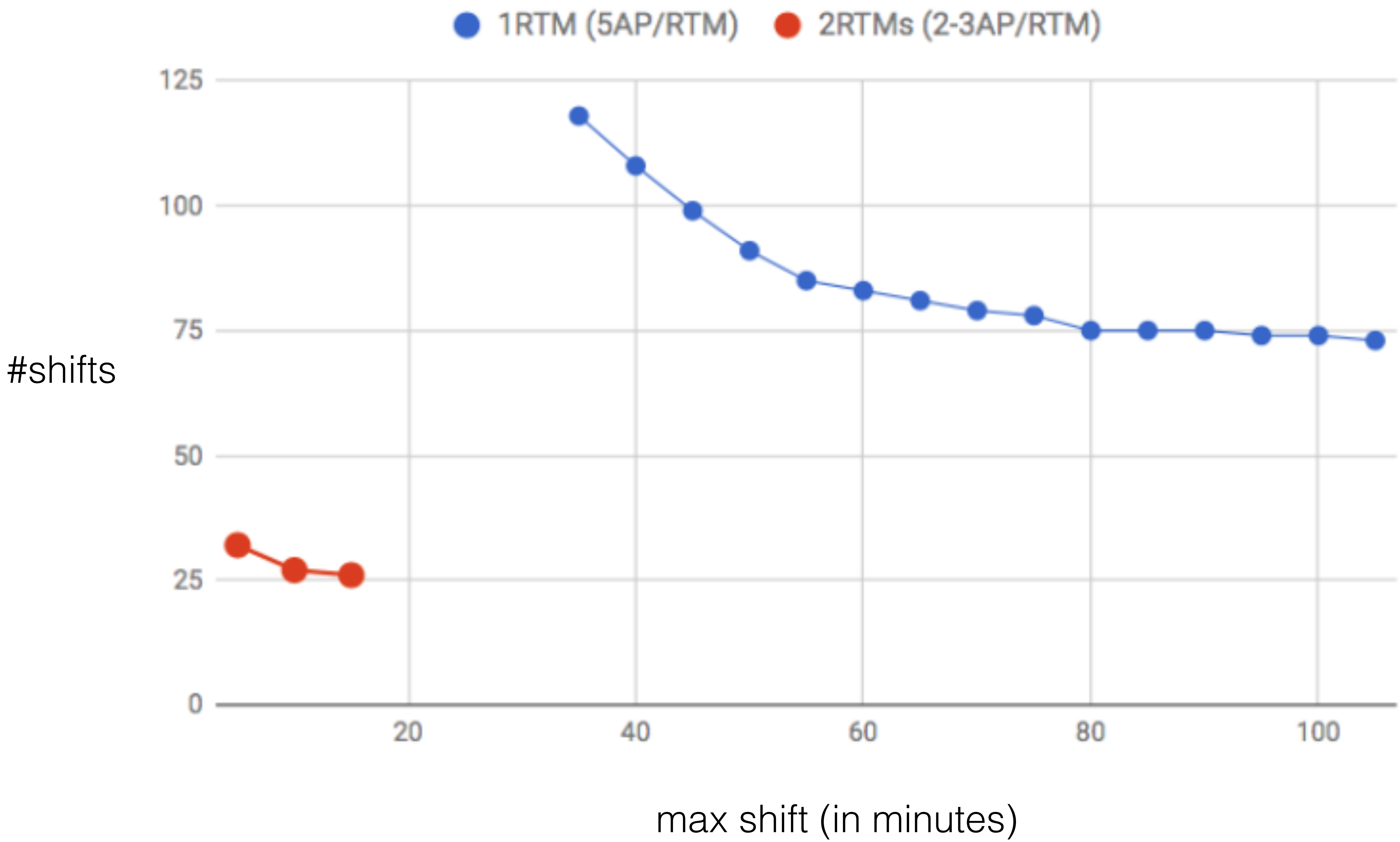
Reschedule at most ±5 minutes: 2 RTMs

For 1 RTM: we need to reschedule by ±35 mins

We have 12 x 24 = 288 slots for flight movements

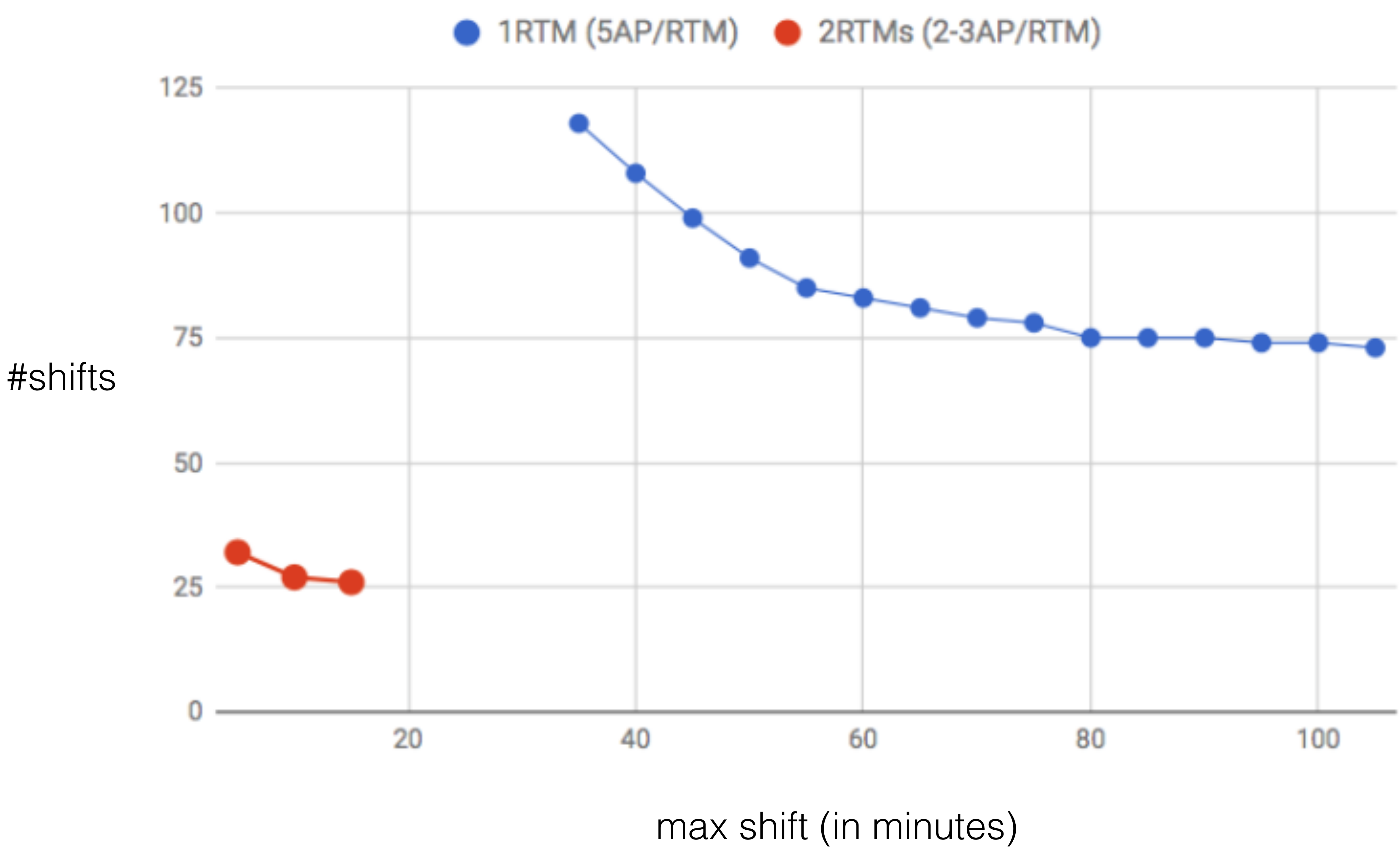
➡ with sufficiently large shifts 233 flight movements in single module

Original Traffic



Original Traffic

Shows tradeoffs: more shifts — larger shifts (more minutes) — more APs/module



Original Traffic

MAP=4

δ	M	S
0	5	0
1	2	32
2	2	27
3	2	26

MAP=3

δ	M	S
0	5	0
1	2	32

MAP=2

δ	# of modules	# of shifts
0	5	0
1	3	7

All 286 movements

In case of a self-induced conflict: model shifts either of them

All 286 movements

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➡ we start with possible more than one flight movement per time slot and airport

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MAP=5

δ	M	S
0	infeasible	infeasible
1	infeasible	infeasible
2	2	103
3	2	80
4	2	79
36	2	79
37	1	158
38	1	154

All 286 movements

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δ	M	S
0	infeasible	infeasible
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3	2	80
4	2	79
36	2	79
37	1	158
38	1	154

For 233 movs 2 RTMs were enough for δ=1, now δ=2

All 286 movements

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For 233 movs 2 RTMs were enough for δ=1, now δ=2

For 233 movs 1RTM was enough for δ=7, now δ=37

All 286 movements

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δ	M	S
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2	2	103
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37	1	158
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For 233 movs 2 RTMs were enough for δ=1, now δ=2

For 233 movs 1RTM was enough for δ=7, now δ=37

MAP=4

δ	M	S
0	infeasible	infeasible
1	infeasible	infeasible
2	2	103
3	2	80
4	2	79
288	2	79

All 286 movements

In case of a self-induced conflict: model shifts either of them

➡ we start with possible more than one flight movement per time slot and airport

➡ δ=0 infeasible by definition

MAP=5

δ	M	S
0	infeasible	infeasible
1	infeasible	infeasible
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3	2	80
4	2	79
36	2	79
37	1	158
38	1	154

For 233 movs 2 RTMs were enough for δ=1, now δ=2

For 233 movs 1RTM was enough for δ=7, now δ=37

MAP=4

δ	M	S
0	infeasible	infeasible
1	infeasible	infeasible
2	2	103
3	2	80
4	2	79
288	2	79

MAP=3

δ	M	S
0	infeasible	infeasible
1	infeasible	infeasible
2	2	103
3	2	80
4	2	79
288	2	79

All 286 movements

In case of a self-induced conflict: model shifts either of them
➡ we start with possible more than one flight movement per time slot and airport
➡ δ=0 infeasible by definition

MAP=5

δ	M	S
0	infeasible	infeasible
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For 233 movs 1RTM was enough for δ=7, now δ=37

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δ	M	S
0	infeasible	infeasible
1	infeasible	infeasible
2	2	103
3	2	80
4	2	79
288	2	79

MAP=3

δ	M	S
0	infeasible	infeasible
1	infeasible	infeasible
2	2	103
3	2	80
4	2	79
288	2	79

MAP=2

δ	M	S
0	infeasible	infeasible
1	infeasible	infeasible
2	3	61
3	3	61
4	3	60
288	3	60

Computation times: Solve in two steps

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We solve two optimisation with $c_2 = 0$ and $c_1 = 0$ respectively and fix the $\sum z_k$ to the optimal number of modules used when solving the second optimization problem.

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δ	# of modules	# of shifts $= S$	computation time in sec
0	infeasible	-	-
1	infeasible	-	-
2	2	103	1,40
3	2	80	1,26
4	2	79	1,79
36	2	79	7,97
37	1	158	8,42
38	1	154	9,34
39	1	151	40,84
40	1	149	46,61
41	1	147	45,12
42	1	144	38,10
43	1	141	40,20
44	1	139	43,57
45	1	137	9,24
46	1	136	106,31
47	1	135	148,79
48	1	134	100,03
49	1	133	94,08
50	1	132	479,12
51	1	130	433,79
52	1	128	348,83
53	1	126	11,65
288	1	126	46,49

Computation times: Solve in two steps

We solve two optimisation with $c_2 = 0$ and $c_1 = 0$ respectively and fix the $\sum z_k$ to the optimal number of modules used when solving the second optimization problem.

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288	1	126	46,49

MAP=4

δ	# of modules	# of shifts $= S$	computation time in sec
0	infeasible	-	-
1	infeasible	-	-
2	2	103	1,31
3	2	80	1,06
4	2	79	1,22
288	2	79	60,92

Computation times: Solve in two steps

We solve two optimisation with $c_2 = 0$ and $c_1 = 0$ respectively and fix the $\sum z_k$ to the optimal number of modules used when solving the second optimization problem.

MAP=5

δ	# of modules	# of shifts $= S$	computation time in sec
0	infeasible	-	-
1	infeasible	-	-
2	2	103	1,40
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4	2	79	1,79
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288	1	126	46,49

MAP=4

δ	# of modules	# of shifts $= S$	computation time in sec
0	infeasible	-	-
1	infeasible	-	-
2	2	103	1,31
3	2	80	1,06
4	2	79	1,22
288	2	79	60,92

MAP=3

δ	# of modules	# of shifts $= S$	computation time in sec
0	infeasible	-	-
1	infeasible	-	-
2	2	103	1,36
3	2	80	1,28
4	2	79	1,09
288	2	79	51,79

Computation times: Solve in two steps

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We solve two optimisation with $c_2 = 0$ and $c_1 = 0$ respectively and fix the $\sum z_k$ to the optimal number of modules used when solving the second optimization problem.

MAP=5

δ	# of modules	# of shifts $= S$	computation time in sec
0	infeasible	-	-
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2	2	103	1,40
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288	1	126	46,49

MAP=4

δ	# of modules	# of shifts $= S$	computation time in sec
0	infeasible	-	-
1	infeasible	-	-
2	2	103	1,31
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MAP=3

δ	# of modules	# of shifts $= S$	computation time in sec
0	infeasible	-	-
1	infeasible	-	-
2	2	103	1,36
3	2	80	1,28
4	2	79	1,09
288	2	79	51,79

MAP=2

δ	# of modules	# of shifts $= S$	computation time in sec
0	infeasible	-	-
1	infeasible	-	-
2	3	61	0,55
3	3	61	1,09
4	3	60	0,98
288	3	60	100,30

Increased Traffic Volume

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Duplicate each of the original flight movements

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Duplicate each of the original flight movements
Shift randomly by plus/minus one hour

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Duplicate each of the original flight movements
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Shift again, randomly, by plus/minus 15 minutes

Increased Traffic Volume

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If two flight movements end up in the same slot, one of the movements is deleted

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“2x” data created from all data of the year 2016

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“2x” data created from all data of the year 2016
➡ shifted duplicates of flights from October 18, 2016 and October 20, 2016 may now happen on October 19, 2016

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➡ Not exactly twice the number of movements

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Duplicate each of the original flight movements
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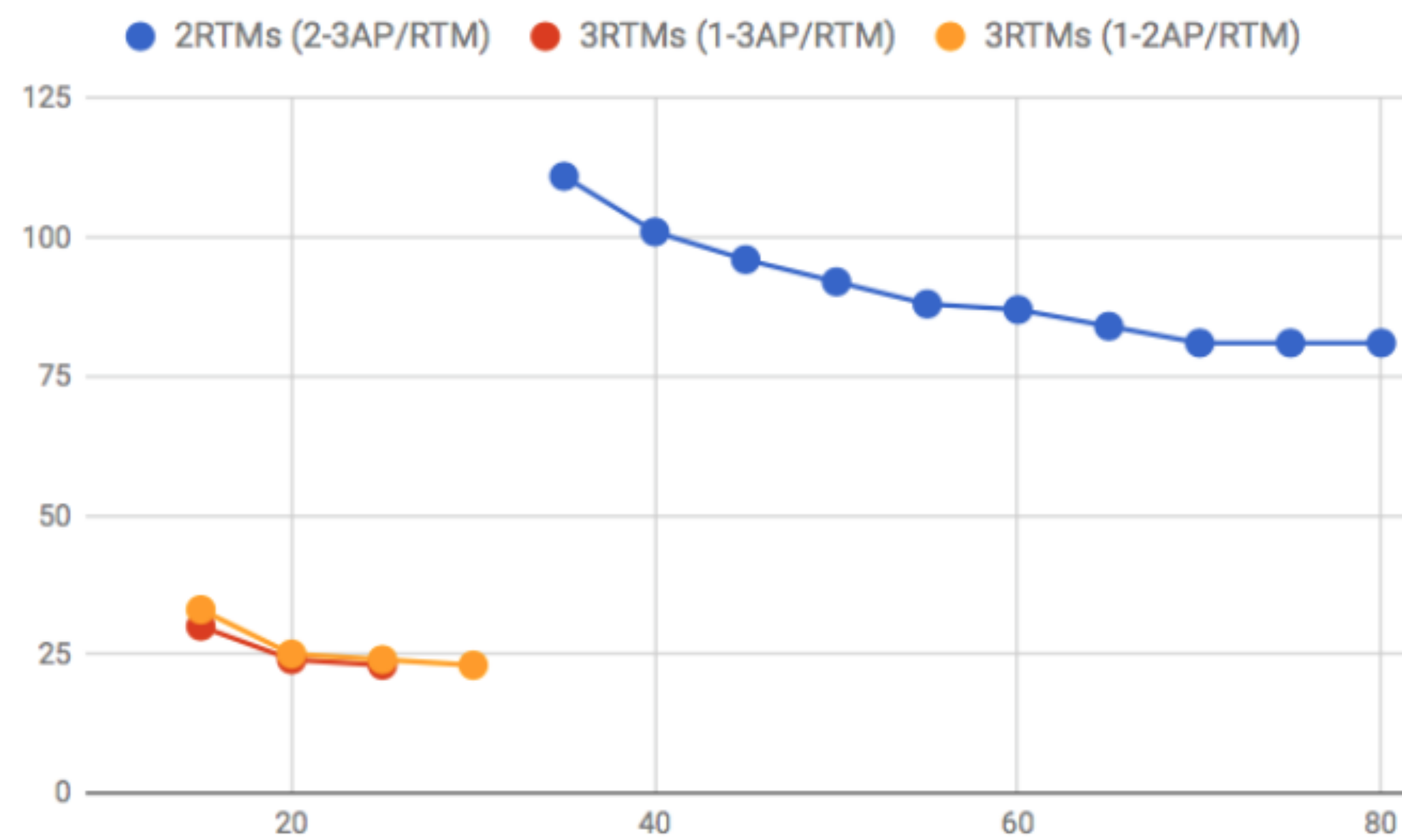
If two flight movements end up in the same slot, one of the movements is deleted
“2x” data created from all data of the year 2016
➡ shifted duplicates of flights from October 18, 2016 and October 20, 2016 may now happen on October 19, 2016
➡ Not exactly twice the number of movements
- October 19: data set has 416 flight movements (after deleting double movements in time slots) out of 575 flight movements (all of the movements from 2016 that the duplication and shifting process produces)

Increased Traffic Volume

δ	# of modules	S	Δ	S for 3RTMs (1-3AP/RTM)	S for 3RTMs (1-2AP/RTM)
0	5	0	-	-	-
1	3	30	5	30	33
2	3	24	10	24	25
3	3	23	15	23	24
4	3	23	20	-	23
5	2	111	25	-	-
6	2	101	30	-	-
7	2	96	35	-	-
8	2	92	40	-	-
9	2	88	45	-	-
10	2	87	50	-	-
11	2	84	55	-	-
12	2	81	60	-	-
13	2	81	65	-	-
14	2	81	70	-	-
15	2	81	75	-	-
16	2	80	80	-	-



Increased Traffic Volume

δ	# of modules	S	Δ	S for 3RTMs (1-3AP/RTM)	S for 3RTMs (1-2AP/RTM)
0	5	0	-	-	-
1	3	30	5	30	33
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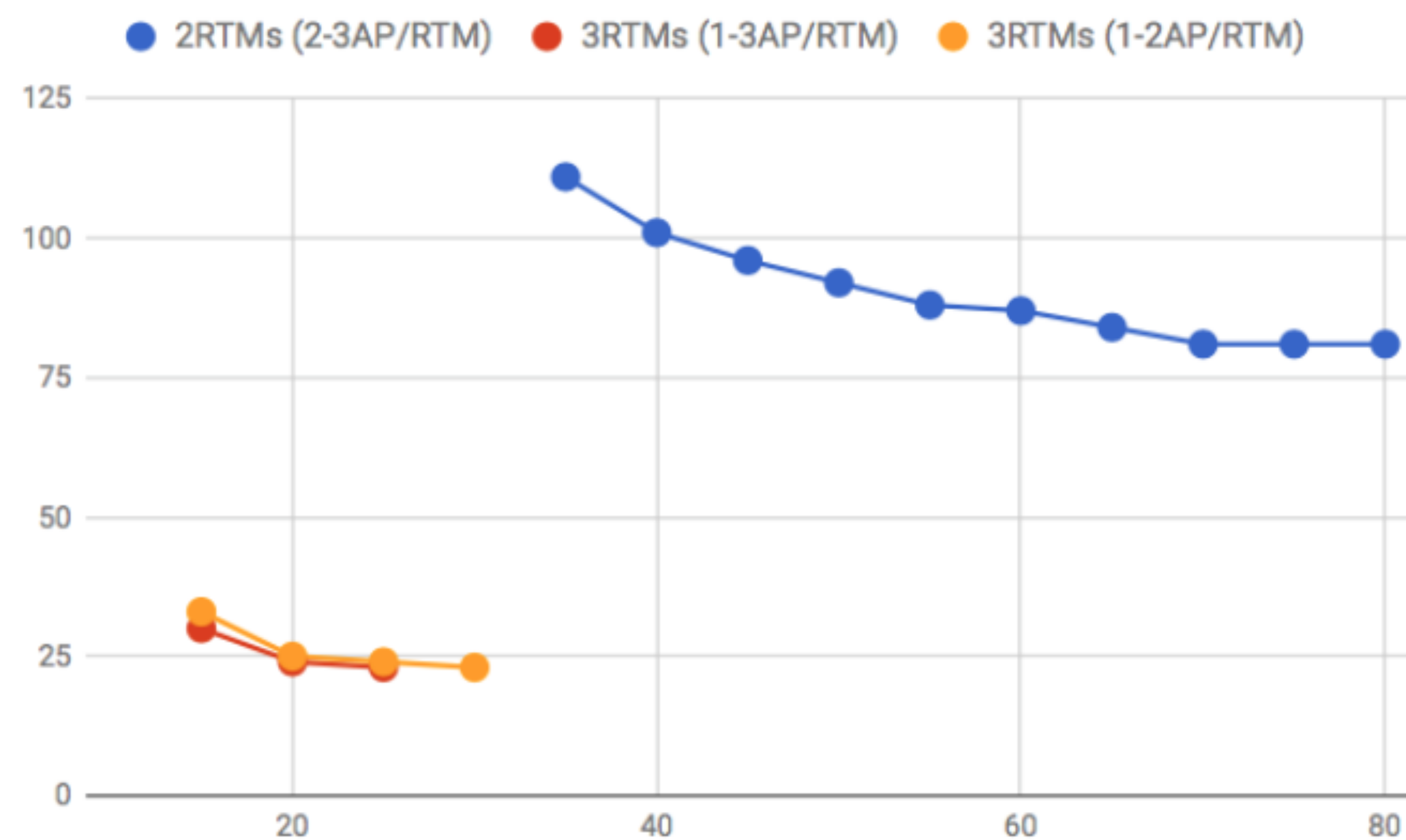
4	3	23	20	-	23
5	2	111	25	-	-
6	2	101	30	-	-
7	2	96	35	-	-
8	2	92	40	-	-
9	2	88	45	-	-
10	2	87	50	-	-
11	2	84	55	-	-
12	2	81	60	-	-
13	2	81	65	-	-
14	2	81	70	-	-
15	2	81	75	-	-
16	2	80	80	-	-



For MAP=2 we get the optimum of 3RTMs for δ=1
33 shifts ↔ 7 shifts for original traffic

Increased Traffic Volume

Same tradeoffs: more shifts — larger shifts (more minutes) — more APs/module

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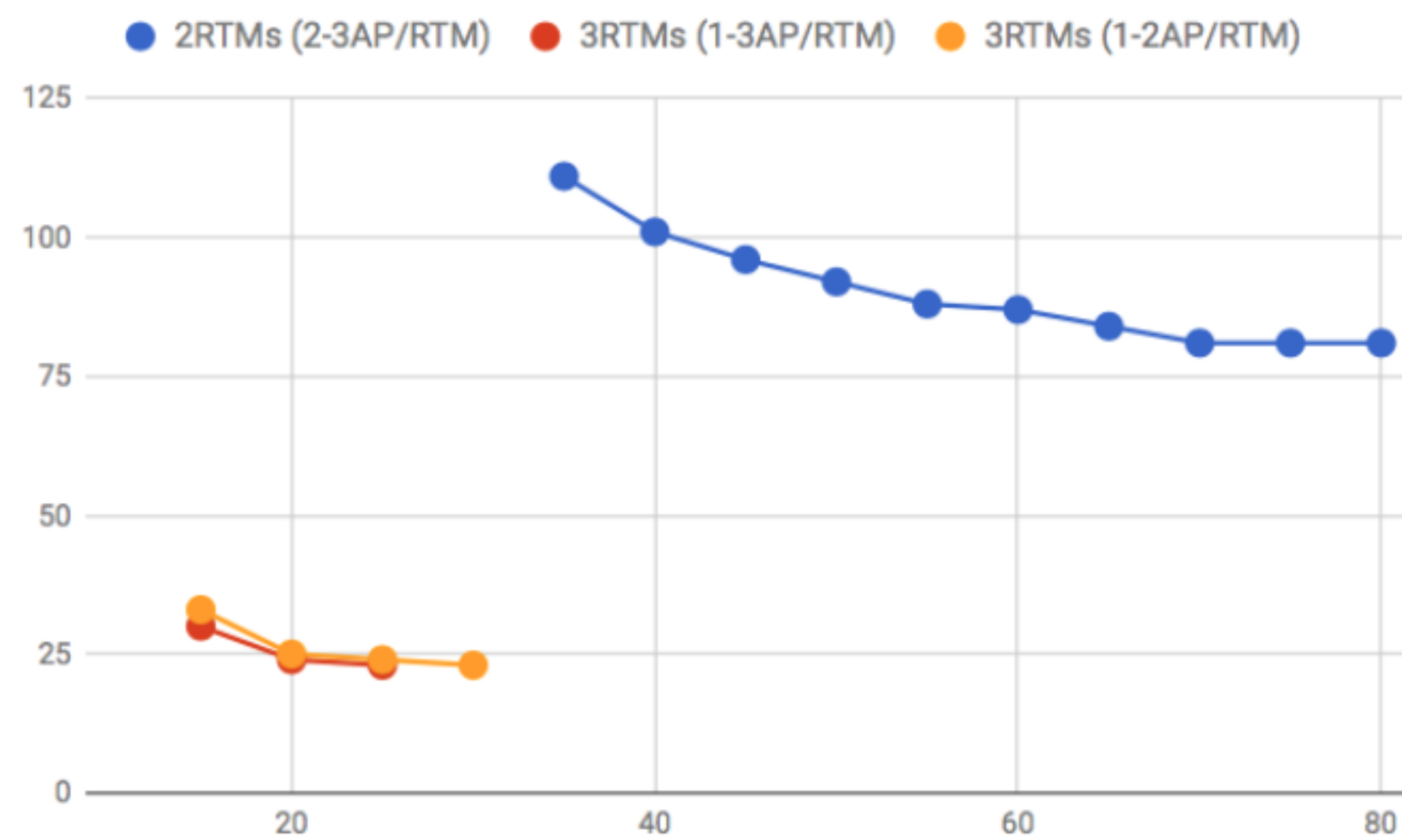
1	3	30	5	30	33
2	3	24	10	24	25
3	3	23	15	23	24
4	3	23	20	-	23
5	2	111	25	-	-
6	2	101	30	-	-
7	2	96	35	-	-
8	2	92	40	-	-
9	2	88	45	-	-
10	2	87	50	-	-
11	2	84	55	-	-
12	2	81	60	-	-
13	2	81	65	-	-
14	2	81	70	-	-
15	2	81	75	-	-
16	2	80	80	-	-



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Conclusion/Future Work

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Future Work

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- Optimization problem for remote towers (FRAMA):

Future Work

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  - Shifts flights to other, nearby, slots

Future Work

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➡ Show applicability of our approach

Future Work

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➡ Show applicability of our approach

➡ Tradeoffs: more shifts — larger shifts (more minutes) — more APs/module

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➡ Tradeoffs: more shifts — larger shifts (more minutes) — more APs/module

➡ Minor shifts (few minutes) can significantly reduce necessary number of modules

Future Work

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Conclusion

- Optimization problem for remote towers (FRAMA):
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Future Work

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Future Work

- Our conflict definition may be too conservative/precautionary

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➡ Continues discussion with operations

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➡ Cooperation between airlines, airport owners and ANSPs may help in reduction of RTC operation costs

Future Work

- Our conflict definition may be too conservative/precautionary
- They cannot be discarded, and will influence staff planning

➡ Continues discussion with operations
➡ Possibly: distinguish arrival/departures

Conclusion

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➡ Show applicability of our approach
➡ Tradeoffs: more shifts — larger shifts (more minutes) — more APs/module
➡ Minor shifts (few minutes) can significantly reduce necessary number of modules
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Future Work

- Our conflict definition may be too conservative/precautionary
- They cannot be discarded, and will influence staff planning

➡ Continues discussion with operations
➡ Possibly: distinguish arrival/departures
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Conclusion

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