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# A Framework for Integrated Terminal Airspace Design

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- Traditionally:
  - 1. Routes, 2. Sectors
  - 1. Sectors, 2. Routes
- Why?
  - Computational limits
  - Historical reasons
- Here: two unified approaches to airspace design
  ➡ Simultaneous design of paths and sectors

(I) **MIP-based approach**

- ❖ Combines two of our prior MIPs: one for TMA sectorization and one for STARs in the TMA
- ❖ Integrates constraints on the interaction between sector boundary and arrival routes

(II) **Voronoi-based approach**

- ❖ Based on Voronoi diagram of “hotspots” of controller attention
- ❖ Can be used for any route design
- ❖ Idea: **Computation of best possible routes more important than to optimise sector boundaries**
  - ❖ Routes determine how fast and with how much fuel aircraft can reach and leave the runway, and good design supports controllers to maintain safe separation.
  - ❖ Sectors should guarantee that
    - Points of increased controller interest are not too close to sector boundary
    - Taskload of the different controllers is balanced
  - ➡ Important: sector boundary as far away from “hotspots” as possible
  - ➡ Exact location of remaining sector boundary not as important as exact run of routes.
- ➡ Goal: sectors that separated hotspots of routes as much as possible while balancing controller taskload

# Identification of Hotspots

Goal: define the potential conflict points, the **hotspots**, of any route design
➡ Define important part of the interaction between routes and sectors

Two-step process in interviews with ATCOs:

1. ATCOs identified hotspots for different SID and STAR combinations.
2. Discussed which type of hotspots any kind of design will induce (step to a general route-hotspot relation)

Hotspots $\mathcal{H}$:

- **Runway**
- **Entry and exit points with high traffic load**
- **Intersection points of SIDs and STARs**

Second round: assign a weight $\omega_\eta$ to each hotspot $\eta \in \mathcal{H}$.

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# Review Grid-based IP formulation for STARs

# Review Grid-based IP formulation for STARs

- Square grid in the TMA
- Snap locations of the entry points and the runway onto the grid

- $\mathcal{EP}$: set of (snapped) entry points
- R: runway

- G = (V,E):
  - Every grid node connected to its 8 neighbors
- $\ell_{i,j}$ length of an edge (i, j)

1. **No more than two routes merge at a point:** in-degree ≤ 2
2. **Merge point separation:** distance threshold L
3. **No sharp turns:** angle threshold $\alpha$, minimum edge length L
4. **Obstacle avoidance**
5. **STAR–SID separation:**
   STAR–SID crossings far from the runway,
   where arriving and departing planes sufficiently separated **vertically**
   (difference of descend and climb slopes)

# Review Grid-based IP formulation for STARs

$x_e$ decision variables: edge e participates in the STAR.

$f_e$ flow variables: gives the flow on edge e = (i, j) (i.e., from i to j )

$$\sum_{k:(k,i)\in E} f_{ki} - \sum_{j:(i,j)\in E} f_{ij} = \begin{cases} \sum_{k\in\mathcal{EP}} \varkappa_k & i = R \\ -\varkappa_i & i \in \mathcal{EP} \\ 0 & i \in V \setminus \{\mathcal{EP} \cup R\} \end{cases} \qquad (1)$$

Flow from all entry points reaches runway
Flow of one leaves each entry point
Flow conservation

$$x_e \geq \frac{f_e}{|\mathcal{EP}|} \qquad \forall e \in E \qquad (2)$$

Edges with positive flow are in STAR

$$f_e \geq 0 \qquad \forall e \in E \qquad (3)$$

Flow non-negative

$$x_e \in \{0,1\} \qquad \forall e \in E \qquad (4)$$

Edge decision variables are binary

$$\sum_{k:(k,i)\in E} x_{ki} \leq 2 \qquad \forall i \in V \setminus \{\mathcal{EP} \cup R\} \qquad (5)$$

$$\sum_{j:(i,j)\in E} x_{ij} \leq 1 \qquad \forall i \in V \setminus \{\mathcal{EP} \cup R\} \qquad (6)$$

$$\sum_{k:(k,R)\in E} x_{kR} = 1 \qquad (7)$$

$$\sum_{j:(R,j)\in E} x_{Rj} \leq 0 \qquad (8)$$

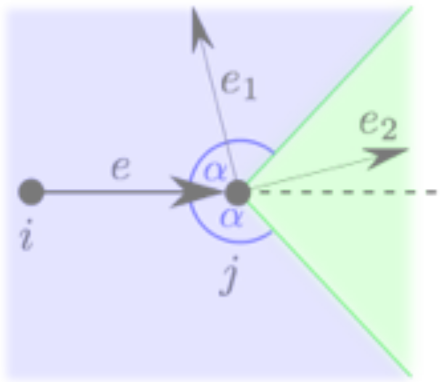
$$\sum_{k:(k,i)\in E} x_{ki} \leq 0 \qquad \forall i \in \mathcal{EP} \qquad (9)$$

$$\sum_{j:(i,j)\in E} x_{ij} = 1 \qquad \forall i \in \mathcal{EP} \qquad (10)$$

Degree constraints:
outdegree of every vertex at most 1,
maximum indegree is 2.
Runway only one ingoing, entry points only one outgoing edge.

$$a_e x_e + \sum_{f\in A_e} x_f \leq a_e \qquad \forall e \in E \qquad (11)$$

$a_e = |A_e|$




If an edge $x_e$ the angle to the consecutive segment of a route is never

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**Objective functions:**

$$\min \sum_{e \in E} \ell_e f_e \quad \text{demand-weighted paths length} \qquad (1)$$

$$\min \sum_{e \in E} \ell_e x_e \quad \text{tree weight} \qquad (2)$$

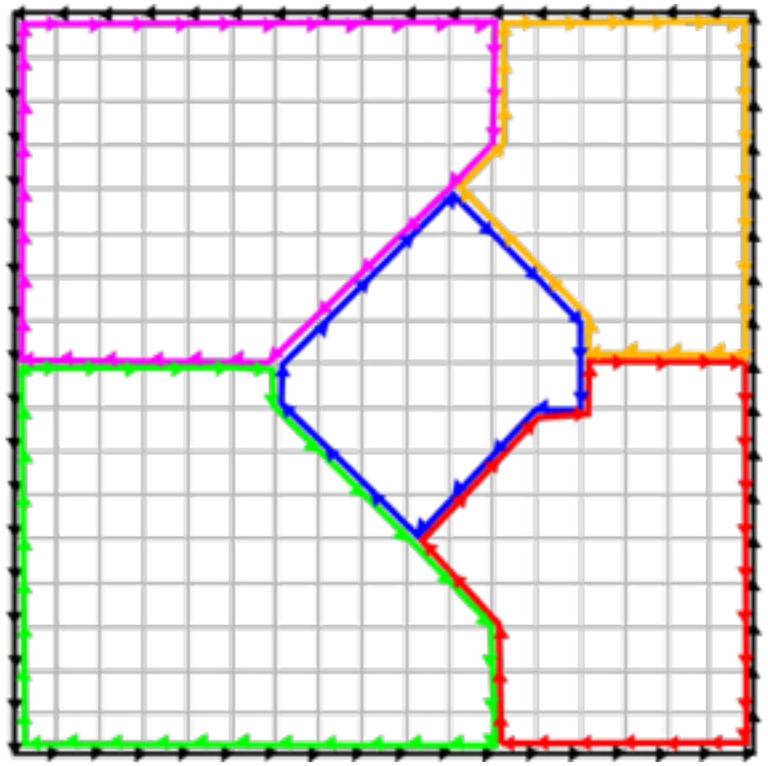
# Review Grid-based IP formulation for Sectorization

**Sectorization Problem:**
Given: The coordinates of the TMA, defining a polygon P , the number of sectors |S|, and a set C of constraints on the resulting sectors.
Find: A sectorization of P with k = |S| , fulfilling C.

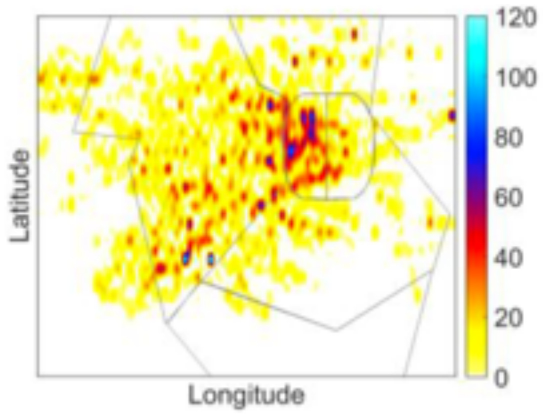
Possible constraints for sectorization:
**(a) Balanced taskload**
**(b) Connected sectors**
**(c) Nice shape** (smooth boundary and an easily memorable shape)
**(d) Convex sectors** ((straight-line) flight cannot enter and leave a convex sector multiple times)
**(e) Interior conflict points** ( Points that require increased attention from ATCOs should lie in the sector's interior.)

- Square grid in the TMA
- $G_2 = (V_2, E_2)$:
  - Every grid node connected to its 8 neighbors
- $N(i)$ = set of neighbors of i (including i)
- $\ell_{i,j}$ length of an edge (i, j)

Main idea: use an artificial sector, $S_0$, that encompasses the complete boundary of P, using all counterclockwise (ccw) edges.



Taskload?
We use heat maps of the density of weighted clicks as an input.
BUT: we do not depend on specific maps.



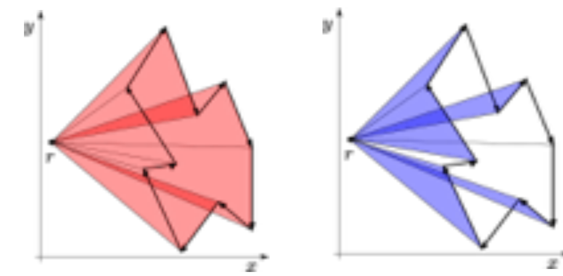

[E. Zohrevandi, V. Polishchuk, J. Lundberg, Å. Svensson, J. Johansson, and B. Josefsson. Modeling and analysis of controller's taskload in different predictability conditions, 2016]

**Many concepts to assign sectors correct area and taskload, and to enforce convex sectors**
**→ we do not go into all details (see paper for detailed description)**

# Review Grid-based IP formulation for Sectorization

$$
\begin{aligned}
y_{i,j,0} &= 1 && \forall (i,j) \in S_0 \\
\sum_{s\in\mathcal{S}^*} y_{i,j,s} - \sum_{s\in\mathcal{S}^*} y_{j,i,s} &= 0 && \forall (i,j) \in E_2 \\
y_{i,j,s} + y_{j,i,s} &\le 1 && \forall (i,j) \in E_2, \forall s \in \mathcal{S}^* \\
\sum_{s\in\mathcal{S}^*} y_{i,j,s} &\le 1 && \forall (i,j) \in E_2 \\
\sum_{(i,j)\in E_2} y_{i,j,s} &\ge 3 && \forall s \in \mathcal{S}^* \\
y_{i,j,s} &\in \{0,1\} && \forall (i,j) \in E_2, \forall s \in \mathcal{S}^* \\
\sum_{l\in V_2:(l,i)\in E_2} y_{l,i,s} - \sum_{j\in V_2:(i,j)\in E_2} y_{i,j,s} &= 0 && \forall i \in V_2, \forall s \in \mathcal{S}^* \\
\sum_{l\in V_2:(l,i)\in E_2} y_{l,i,s} &\le 1 && \forall i \in V_2, \forall s \in \mathcal{S}^*
\end{aligned}
$$

$\Longrightarrow$ Union of the |S| sectors completely covers the TMA.

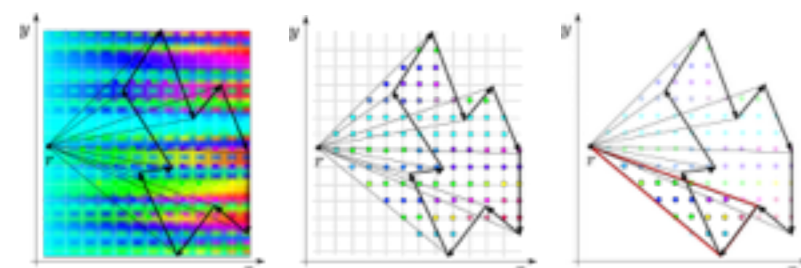


$$
\begin{aligned}
\sum_{(i,j)\in E_2} b_{i,j}\, y_{i,j,s} - a_s &= 0 && \forall s \in \mathcal{S}^* \\
\sum_{s\in\mathcal{S}} a_s &= a_0 \\
a_s &\ge a_{LB} && \forall s \in \mathcal{S}
\end{aligned}
$$

Assign sectors correct area (and balance it)

$$
\begin{aligned}
\sum_{(i,j)\in E_2} h_{i,j}\, y_{i,j,s} - t_s &= 0 && \forall s \in \mathcal{S}^* \\
t_s &\ge t_{LB} && \forall s \in \mathcal{S}
\end{aligned}
$$

Assign sectors correct taskload and balance it



$$
q^s_{j,m} = \frac{1}{2}\left( \sum_{i:(i,j)\in E_2} p_{i,j,m}\, y_{i,j,s} - \sum_{l:(j,l)\in E_2} p_{j,l,m}\, y_{j,l,s} \right) \quad \forall s \in \mathcal{S},\ \forall j \in V_2,\ \forall m \in \mathcal{M}
$$

$$
\begin{aligned}
qabs^s_{j,m} &\ge q^s_{j,m} \quad \forall s \in \mathcal{S}, \forall j \in V_2,\ \forall m \in \mathcal{M} \\
qabs^s_{j,m} &\ge -q^s_{j,m} \quad \forall s \in \mathcal{S}, \forall j \in V_2,\ \forall m \in \mathcal{M}
\end{aligned}
$$

$$
\begin{aligned}
z^s_{i,j,m} &\ge 0 \quad \forall i,j \in V_2\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M} \\
z^s_{i,j,m} &\le qabs^s_{j,m} \quad \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M} \\
z^s_{i,j,m} &\le y_{i,j,s} \quad \forall i,j \in V_2\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}
\end{aligned}
$$

$$
z^s_{i,j,m} \ge y_{i,j,s} - 1 + qabs^s_{j,m} \forall i,j \in V_2\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}
$$

$$
\sum_{i\in V_2}\sum_{j\in V_2} z^s_{i,j,m} = 2\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}
$$

All sectors convex

**Objective Function:**

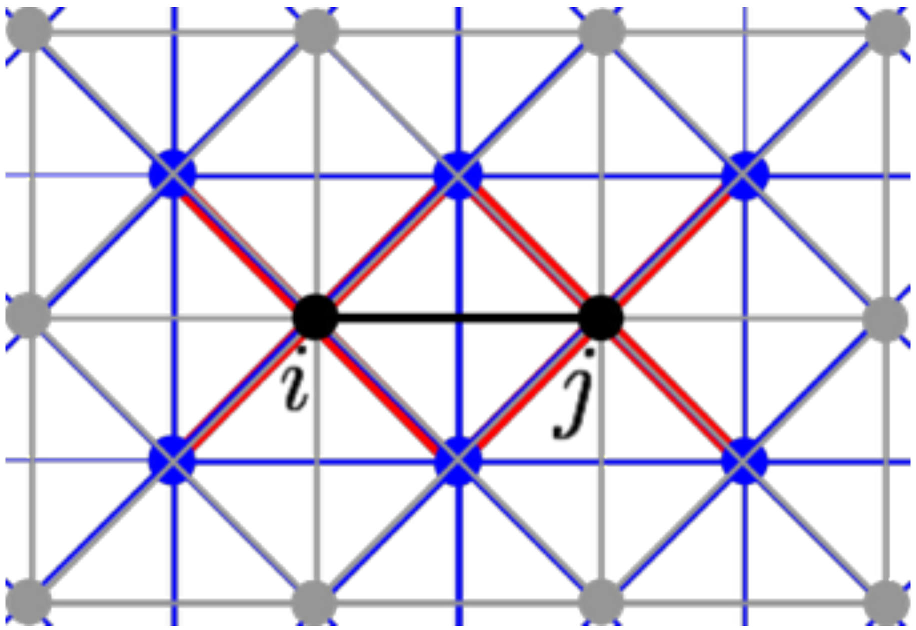
$$
\min \sum_{s\in\mathcal{S}} \sum_{(i,j)\in E} (\gamma \ell_{i,j} + (1-\gamma) w_{i,j})\, y_{i,j,s}, \quad 0 \le \gamma < 1
$$

$$
w_{i,j} = h_i + h_j
$$

$$
w_{i,j} = \sum h_k + \sum h_l
$$

# The Combined MIP

- Compute sectors and routes simultaneously
  ➡ Variables for selecting routes ($x_e$ and $f_e$) and for selecting boundary edges ($y_{i,j,s}$)
- Interaction (possibly achieve only close to orthogonal intersections)




Grid for route edge selection

Grid for sector boundary edge selection

If edge (i,j) is used for sector boundary
➡ These edges are forbidden for routes
(can be defined depending on goal)

- Route vertices of different degree induce heat values at their location
- These get split by the sectors

➡ Constraint that properly assign these heat values.

- **Computationally expensive to solve!!**

# The Voronoi-based Approach

REMINDER

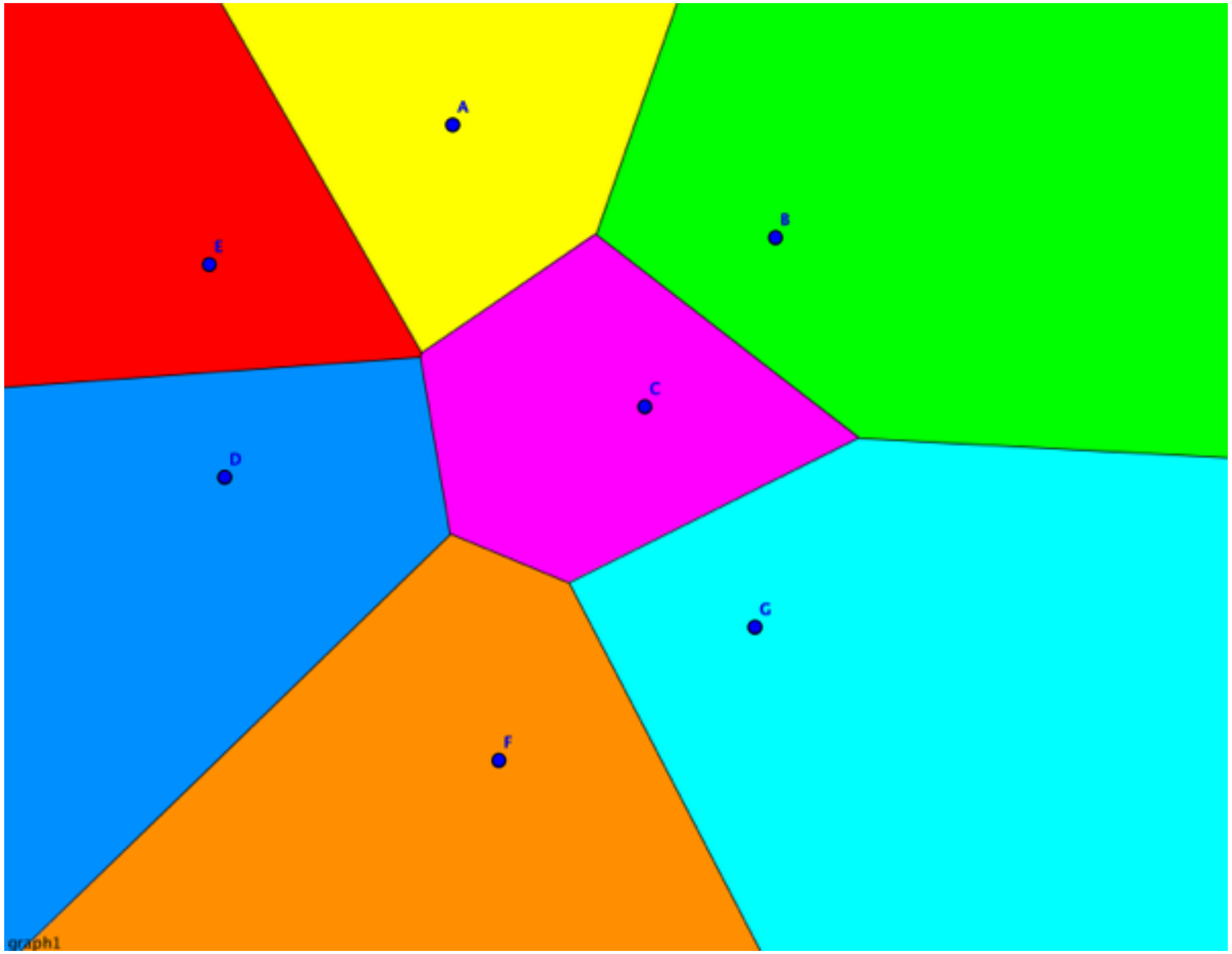
❖ Idea: **Computation of best possible routes more important than to optimise sector boundaries**

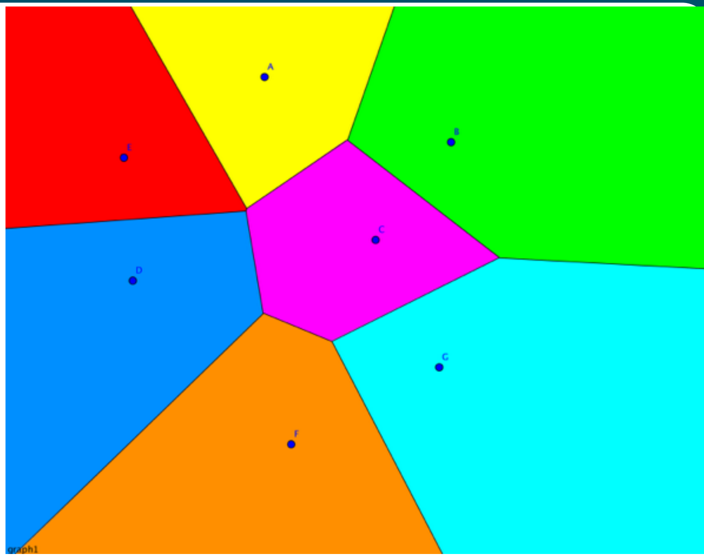
- ❖ Routes determine how fast and with how much fuel aircraft can reach and leave the runway, and good design supports controllers to maintain safe separation.
- ❖ Sectors should guarantee that
  - Points of increased controller interest are not too close to sector boundary
  - Taskload of the different controllers is balanced
- ➡ Important: sector boundary as far away from “hotspots” as possible
- ➡ Exact location of remaining sector boundary not as important as exact run of routes.

➡ Goal: sectors that separated hotspots of routes as much as possible while balancing controller taskload

- Also nice to have: simple shape and convex sectors
- Convexity defined:
  - Geometrically (for any point of pairs in the sector the straight line connection is fully contained in the sector as well)
  - Trajectory-based (no route enters the same sector more than once)

What is a Voronoi diagram?
Given: Set of sites S (points in the plane)
Voronoi cell of a site s: points in the plane that are closer to s than to any other site
**Voronoi diagram Vor(S)**: the collection of boundaries, that is, the points that do not have a unique nearest site.





Natural choice for sites: the hotspots (potential conflict points) we want to separate

➡ Edges of the Voronoi diagram as far away from the sites=hotspots as possible

➡ Choose subset of the edges as sector boundary guarantees that sector boundary is as far away from hotspots as possible

When we compute routes, they automatically define our set of hotspots $\mathcal{H}$

➡ Directly implies resulting Voronoi diagram of hotspots

- Each Voronoi cell is geometrically convex

- Now: **merge** Voronoi cells into sectors such that:
  - ✦ We obtain **k sectors**
  - ✦ Each sector is **connected**
  - ✦ We **balance** either the sector **area or taskload**
  - ✦ Nice to have: resulting sectors are **trajectory-based convex**

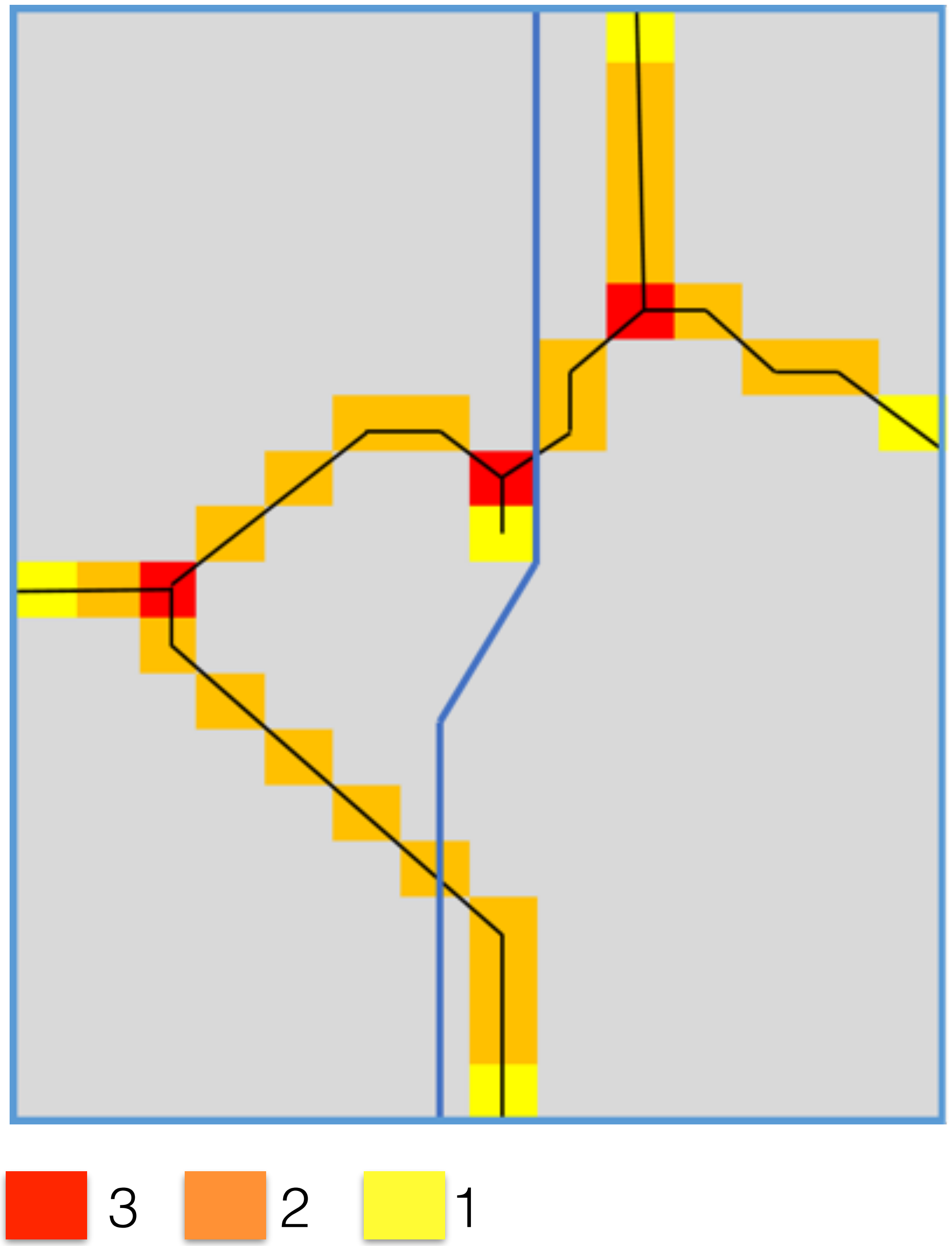
## Experimental Study: Arlanda Airport

MIP-based Approach

MIP is computationally expensive.
Here, we only want to give an example to show the general applicability.

- 2 sectors
- Only STAR
- Simplified heat value assignment: $h_q = \delta(q)$ $\forall q \in V_2$ (and do not set higher heat values for entry points and runway beforehand)
- Do not require sectors to be convex
- Balance taskload ($c_2$=1)

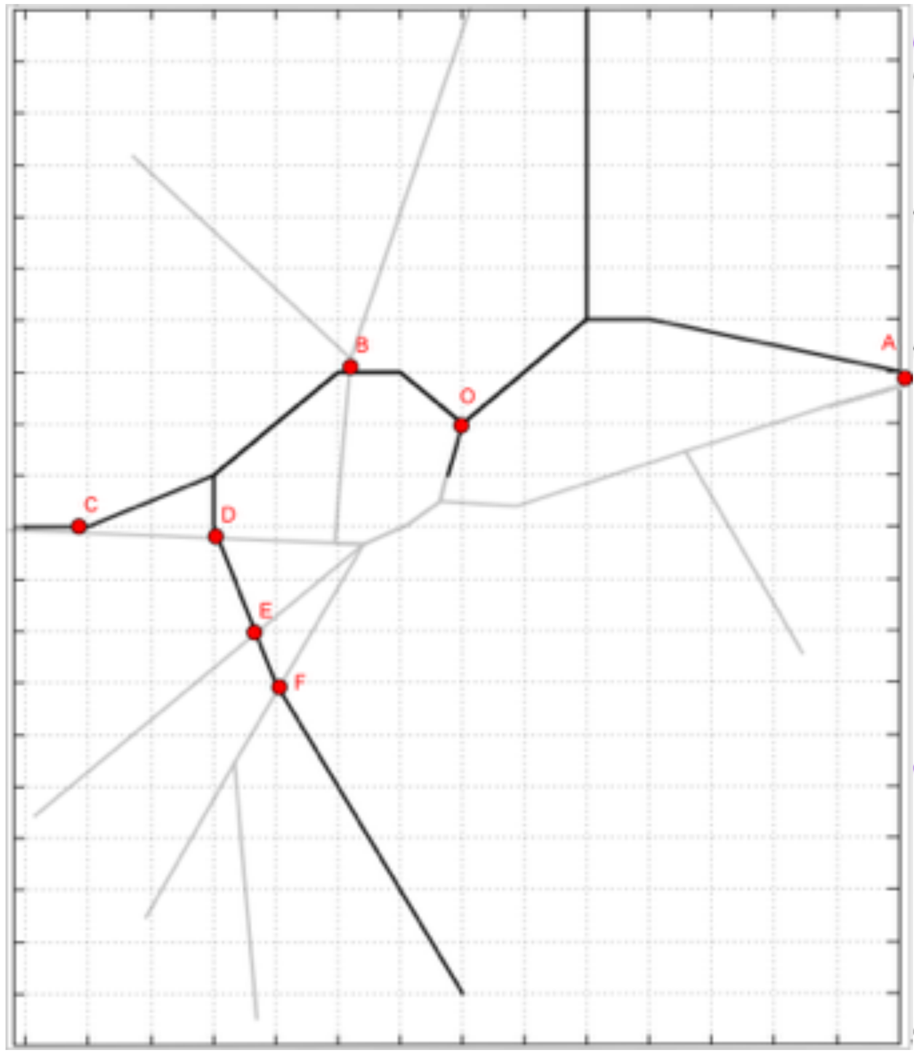
Both sectors are trajectory-based convex.



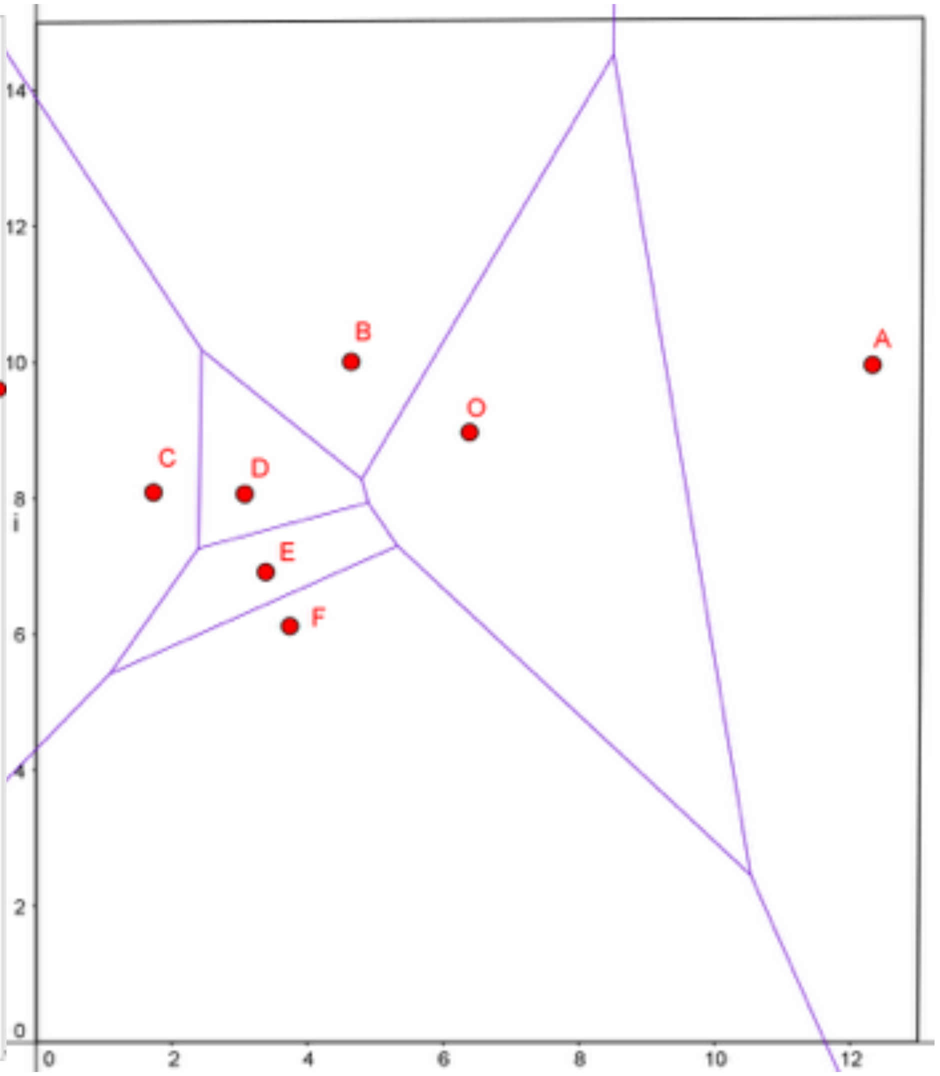
# Voronoi-based Approach

# Example: Routes created by routes MIP


# Hotspot identification

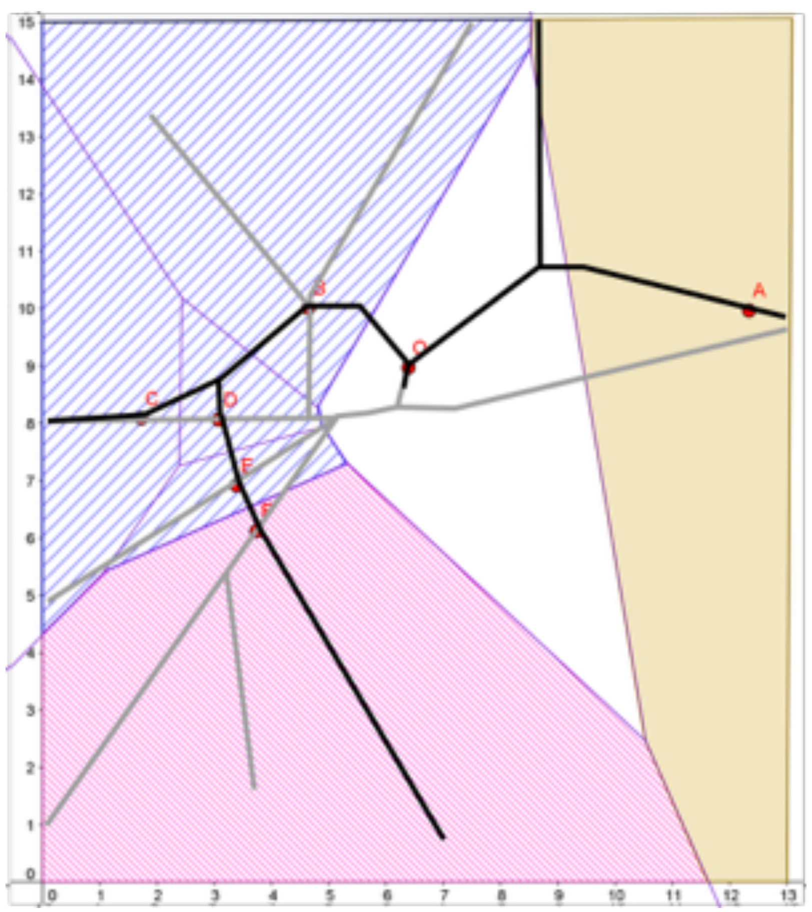


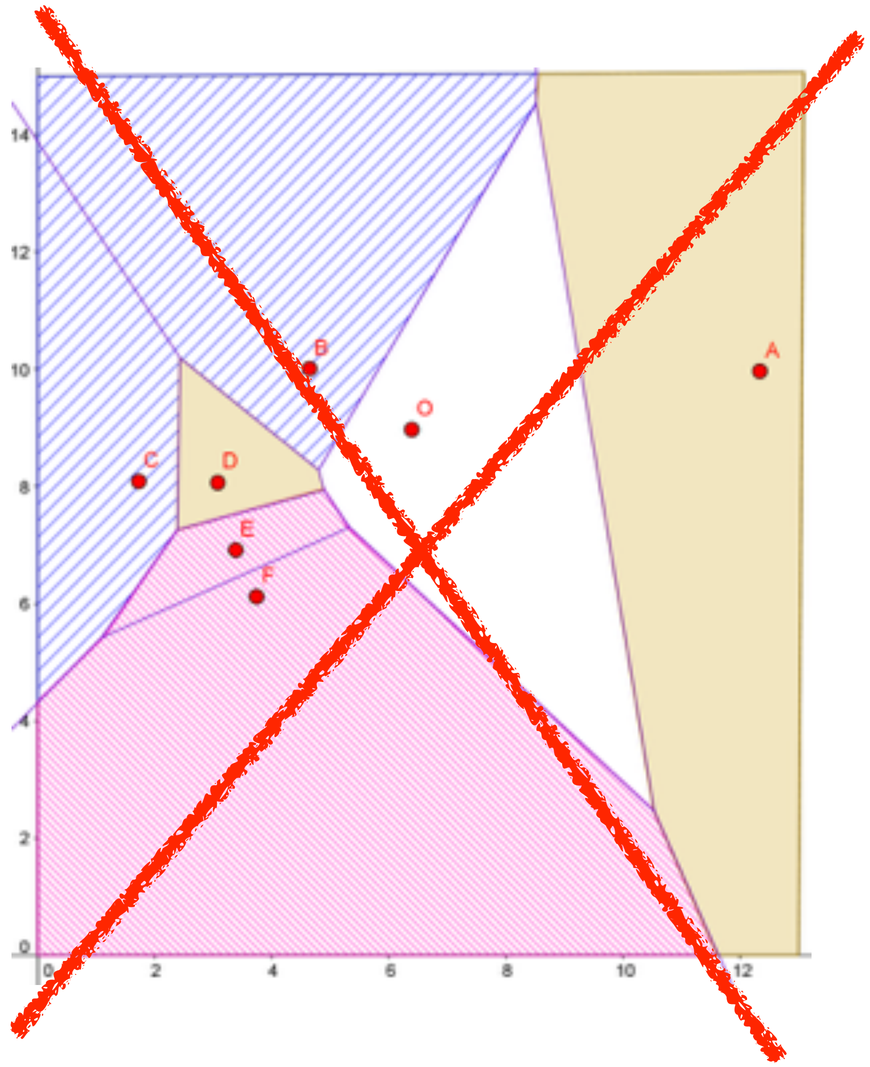
# Voronoi diagram of hotspots

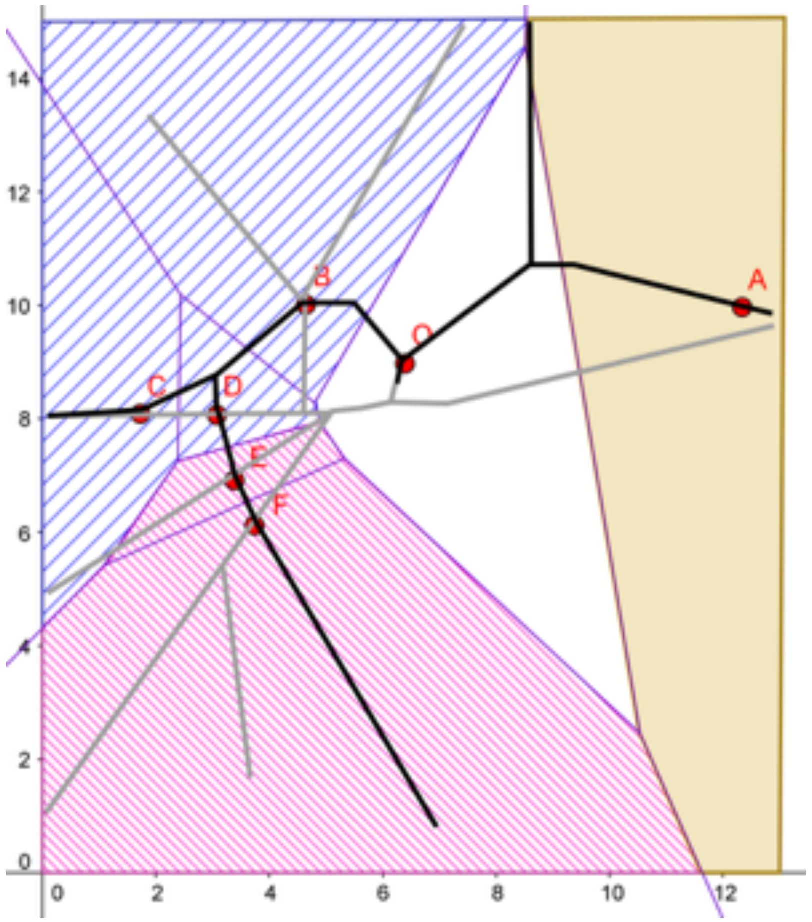


TODO: merge cells into 4 sectors (runway=O separate sector)

# (a) Balance area

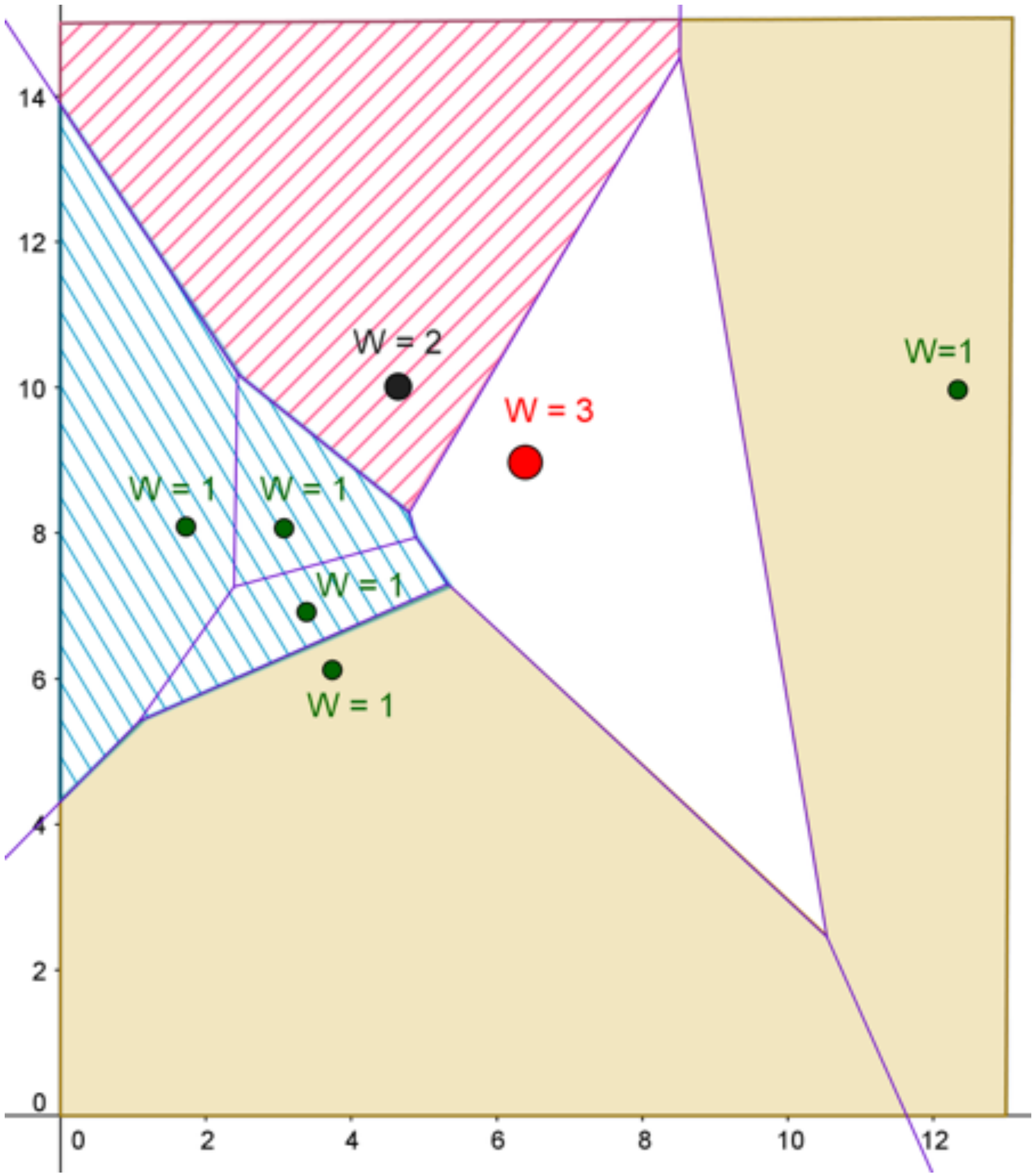


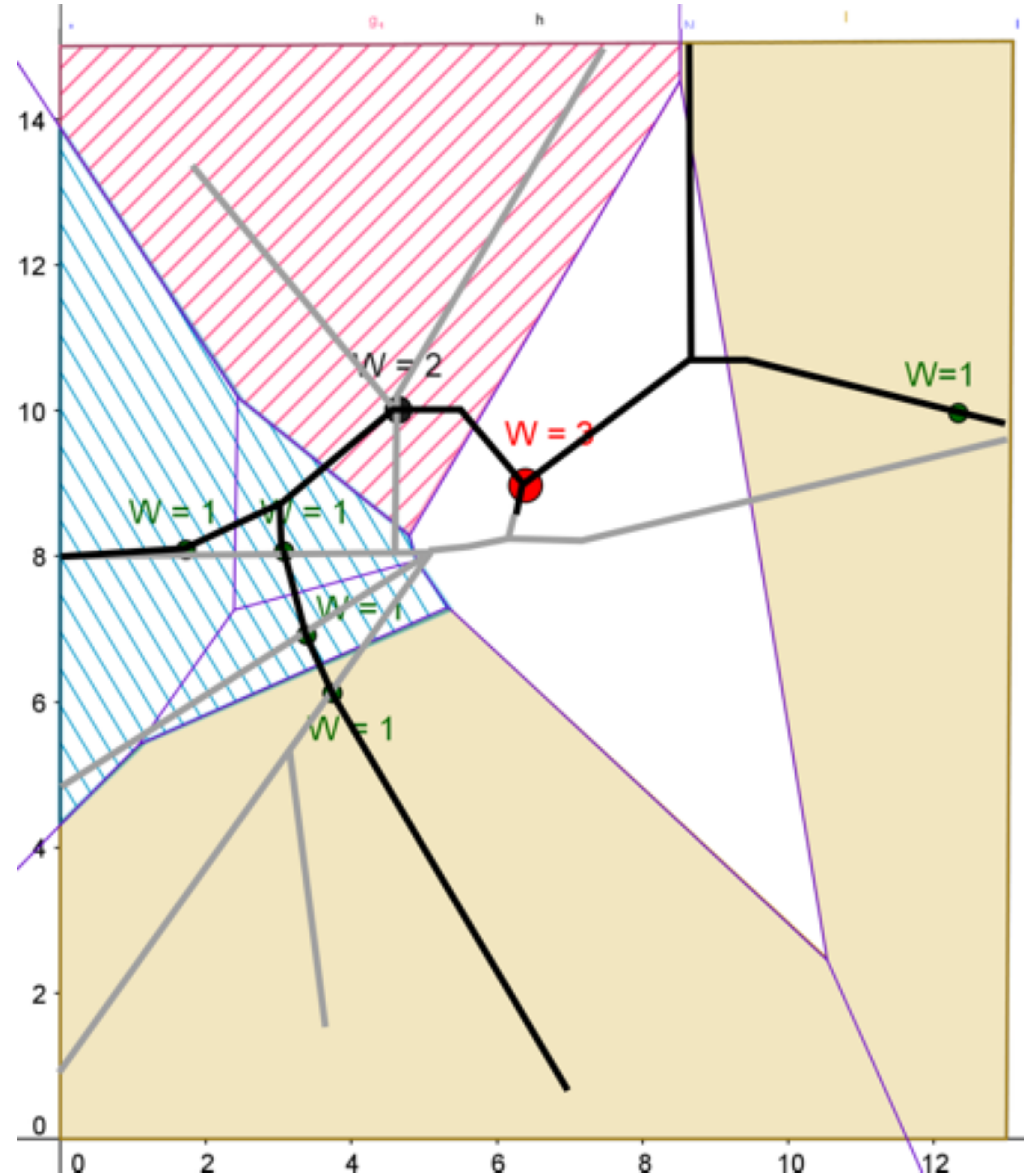




For both: all sectors are trajectory-based convex.

# (b) Balance taskload ➜ Weight assigned to hotspots




All sectors are trajectory-based convex.

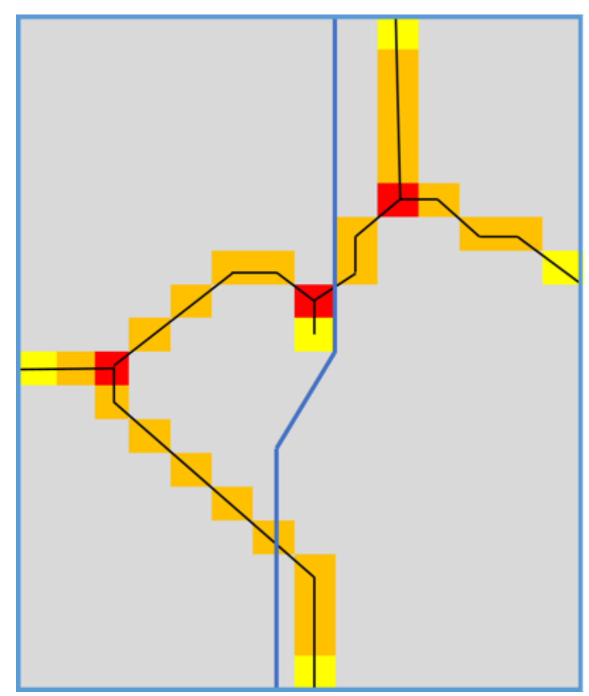
# Conclusion/Outlook

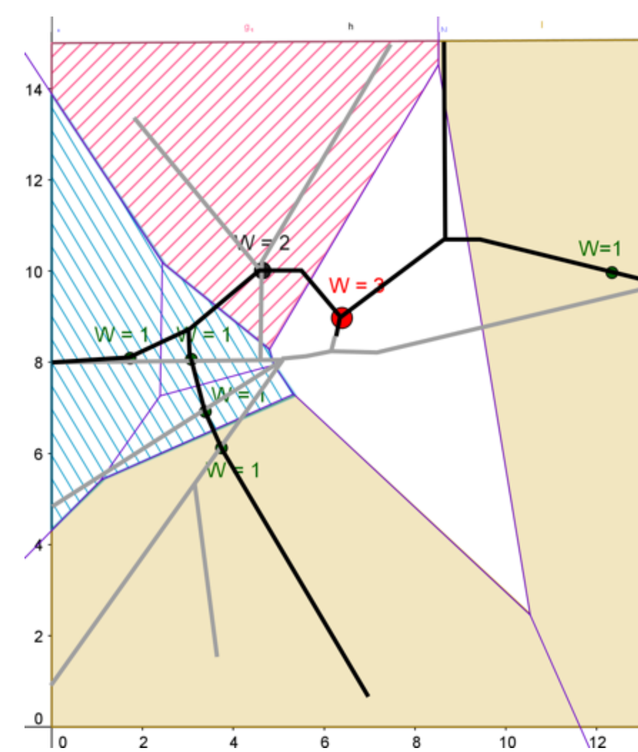
## Conclusion

- Two approaches for simultaneous design of paths and sectors
  (I)MIP-based: powerful but computationally expensive
  (II)Voronoi-based: geometrically separate hotspots by sector boundary
- Showed applicability with first experiments on Stockholm TMA
- Move from complicated sectors+simple routes to detailed routes+simple sectors

## Outlook

- Try Voronoi-based approach with other STARs (and SIDs)
- Define disks of different size around the hotspots depending on weight ➜ More intense hotspots guaranteed further away from sector boundary
- Include SIDs in MIP-based approach
- Extended experiments





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**Thank you.**