# Computational Complexity and Bounds for Norinori and LITS

Michael Biro, **Christiane Schmidt**

SWARTHMORE

LINKÖPING UNIVERSITY

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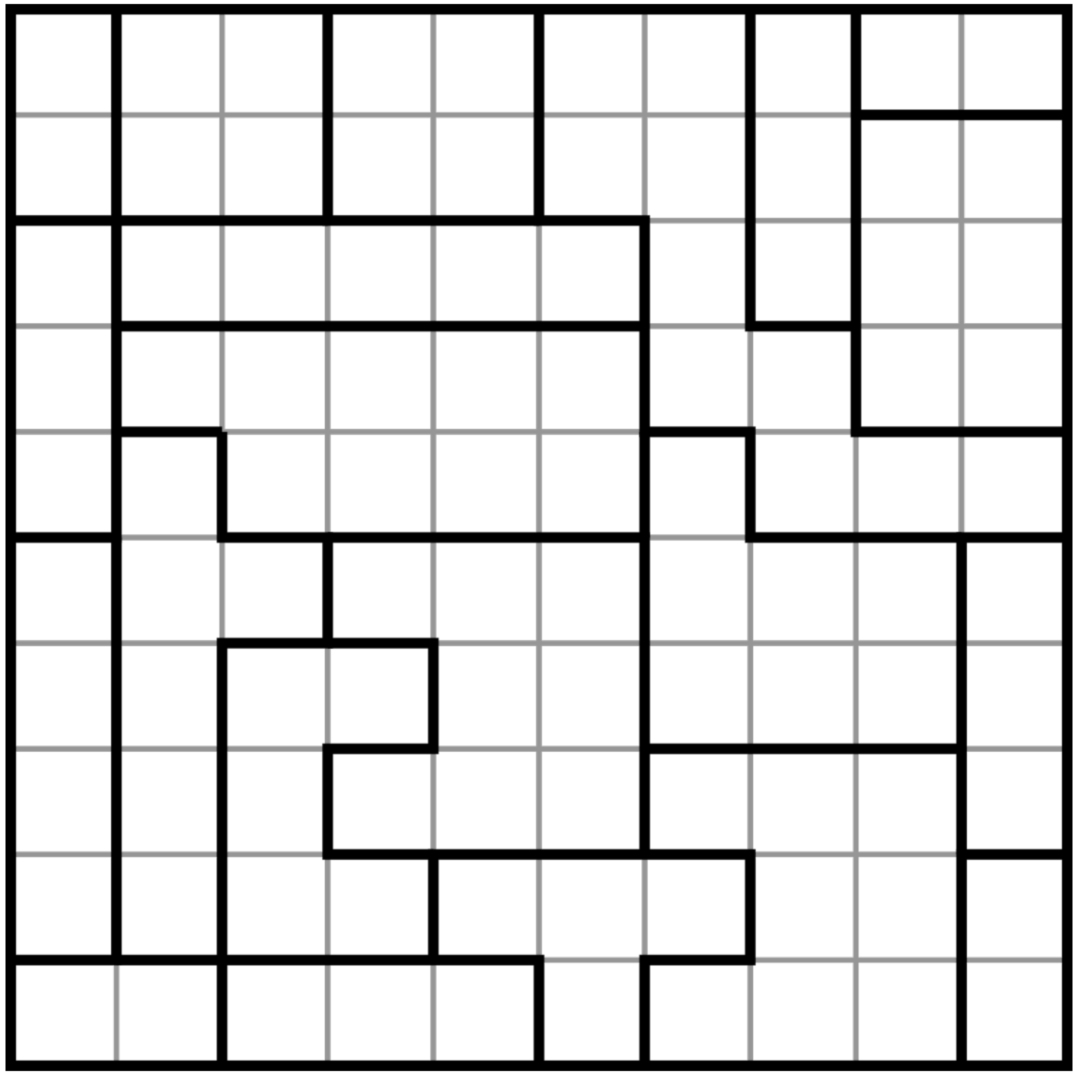

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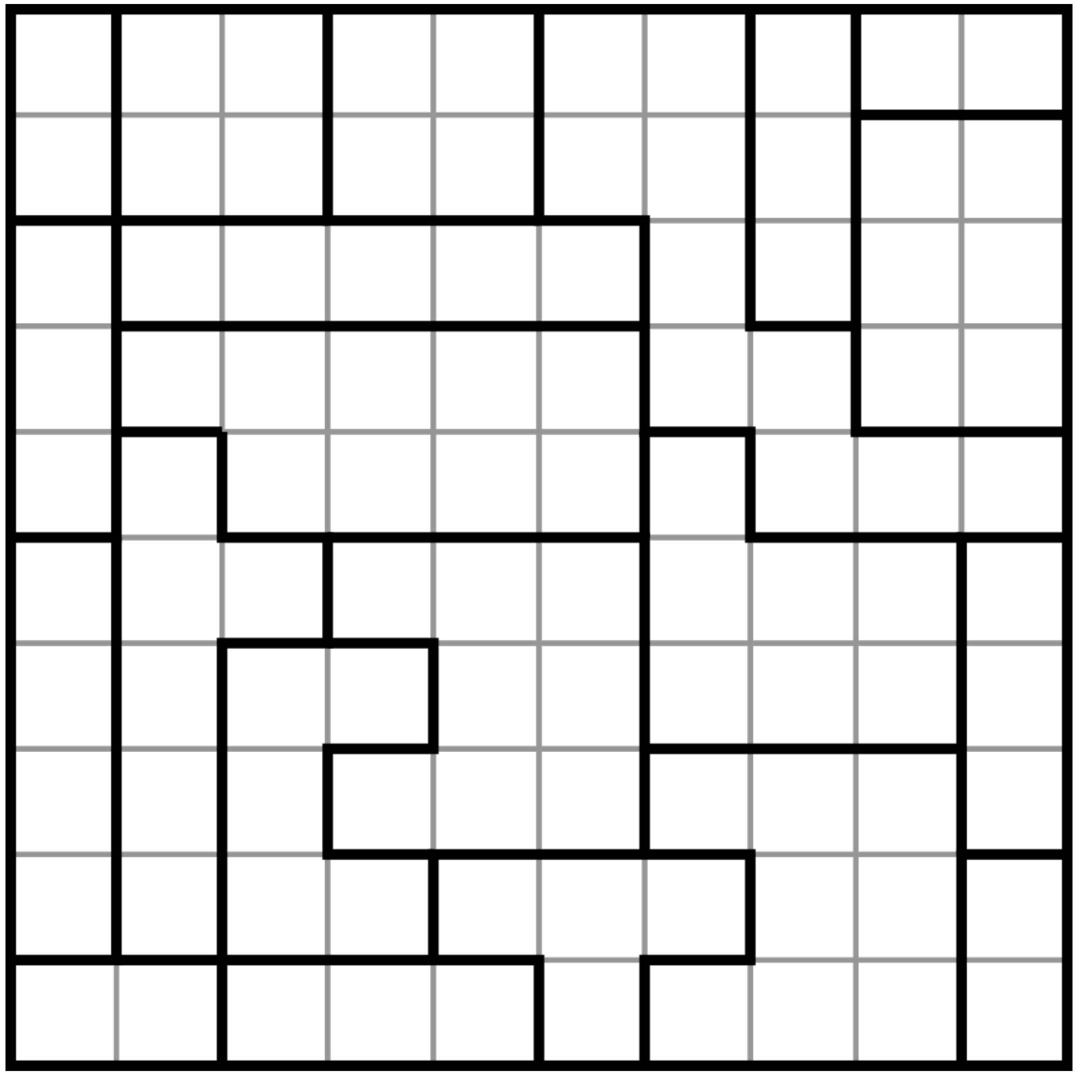

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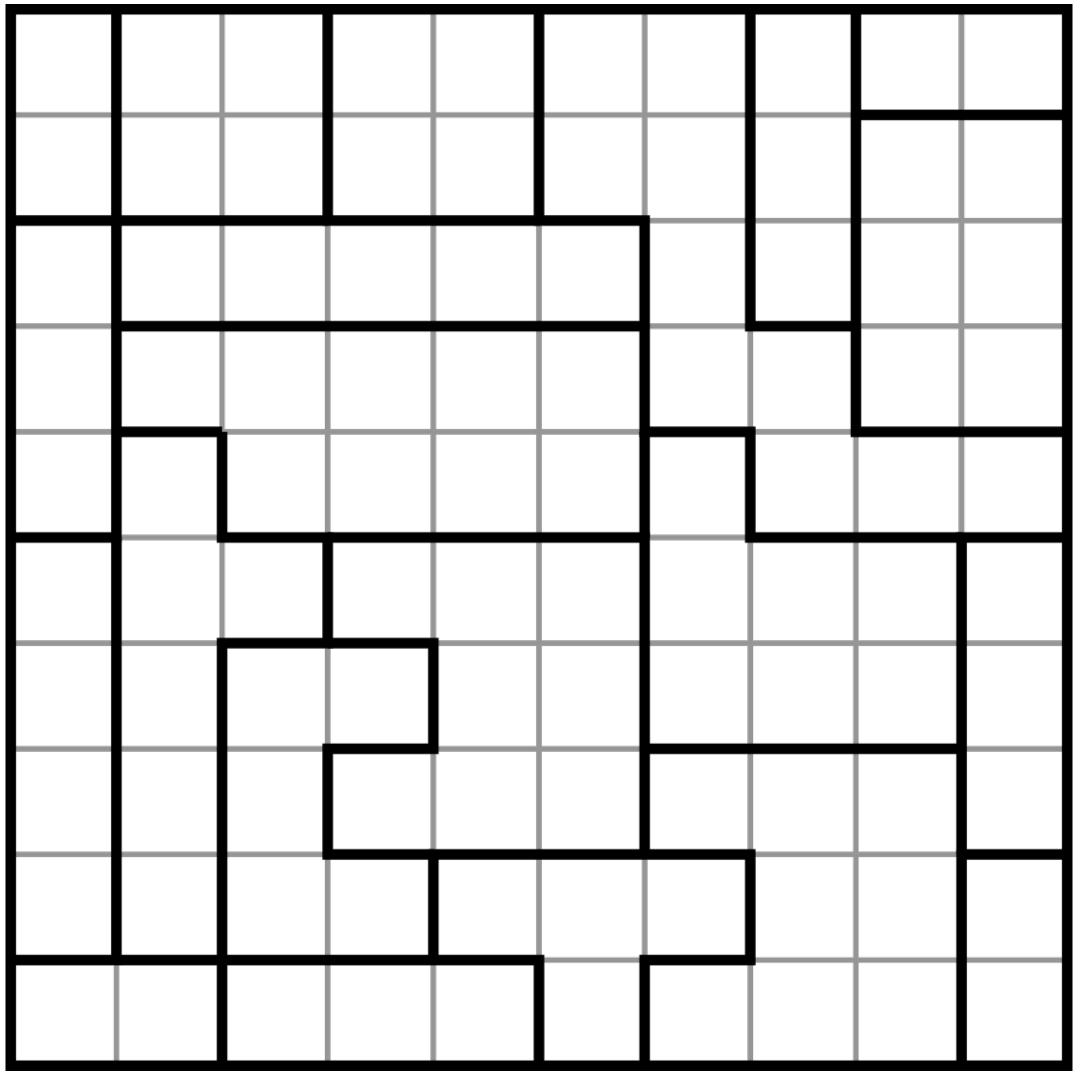

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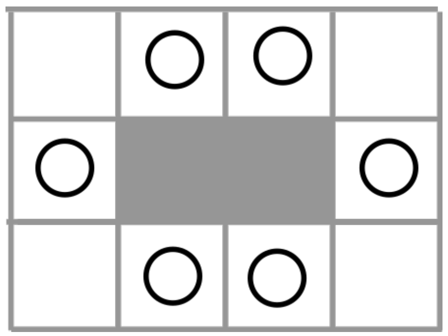

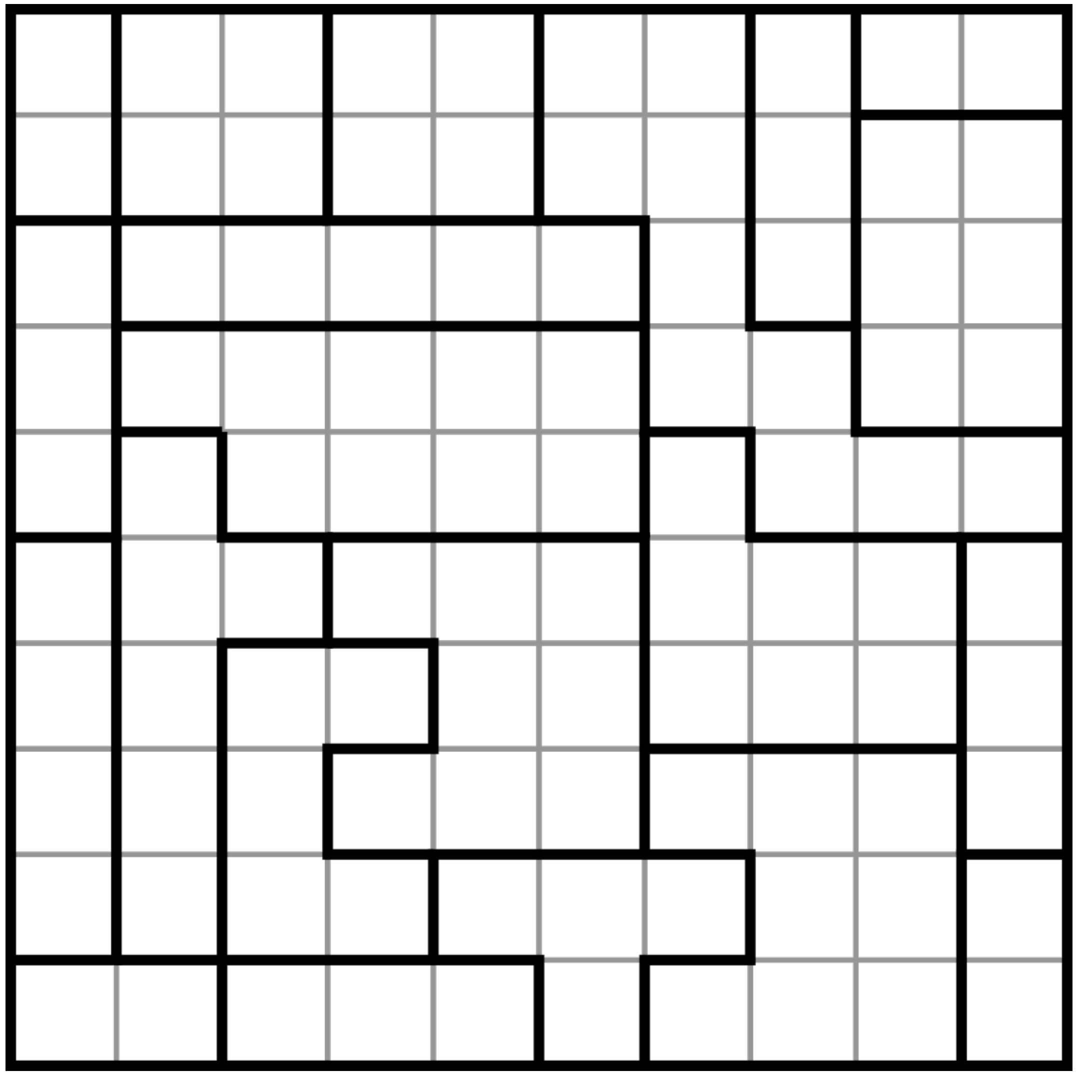

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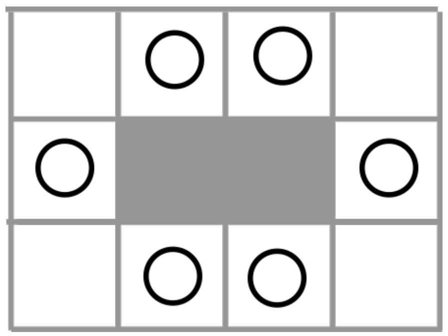

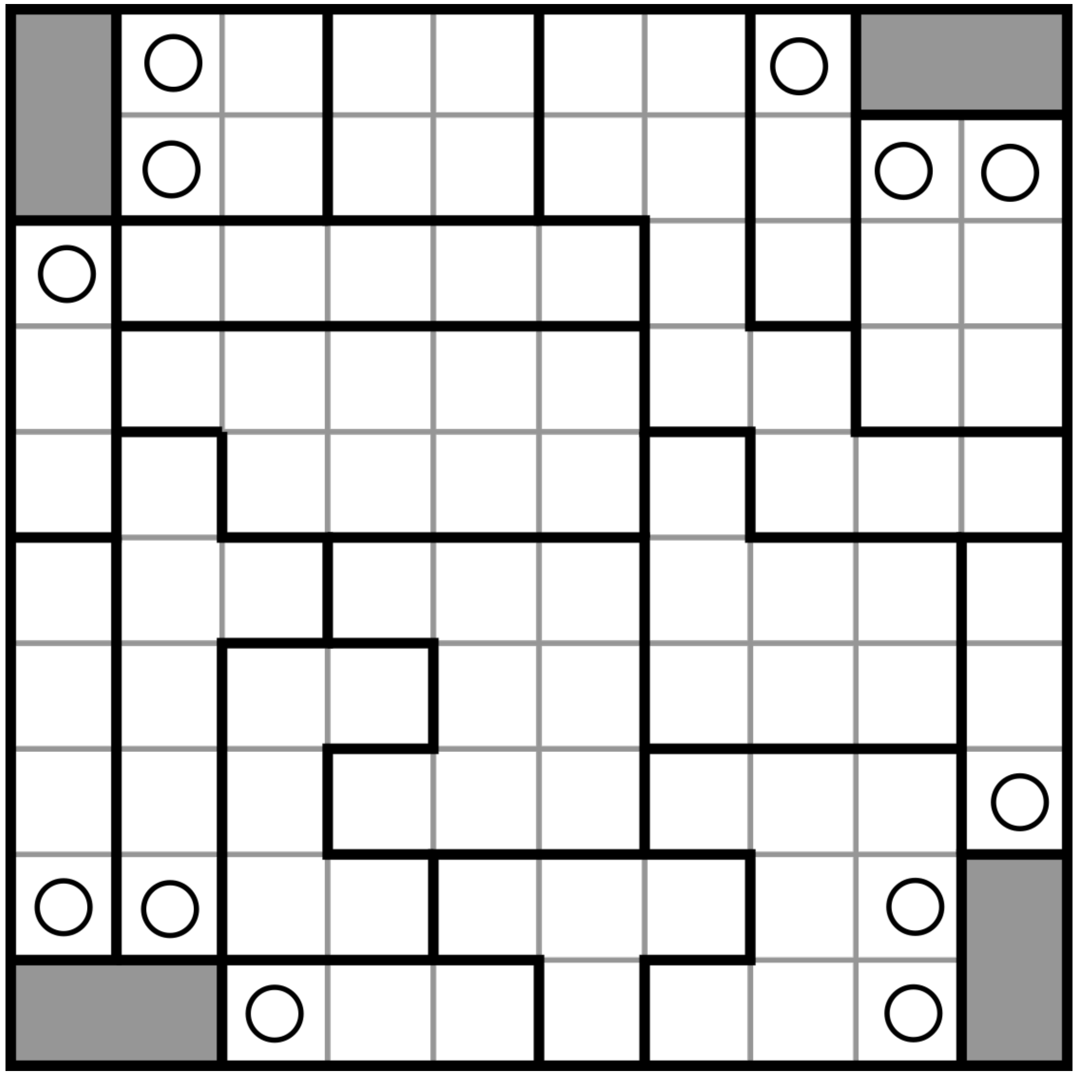

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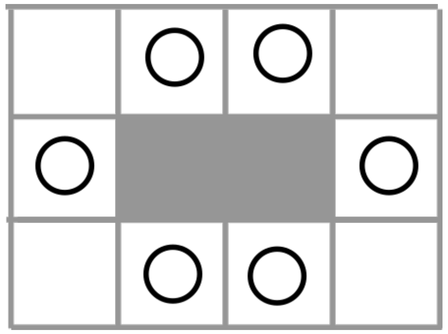

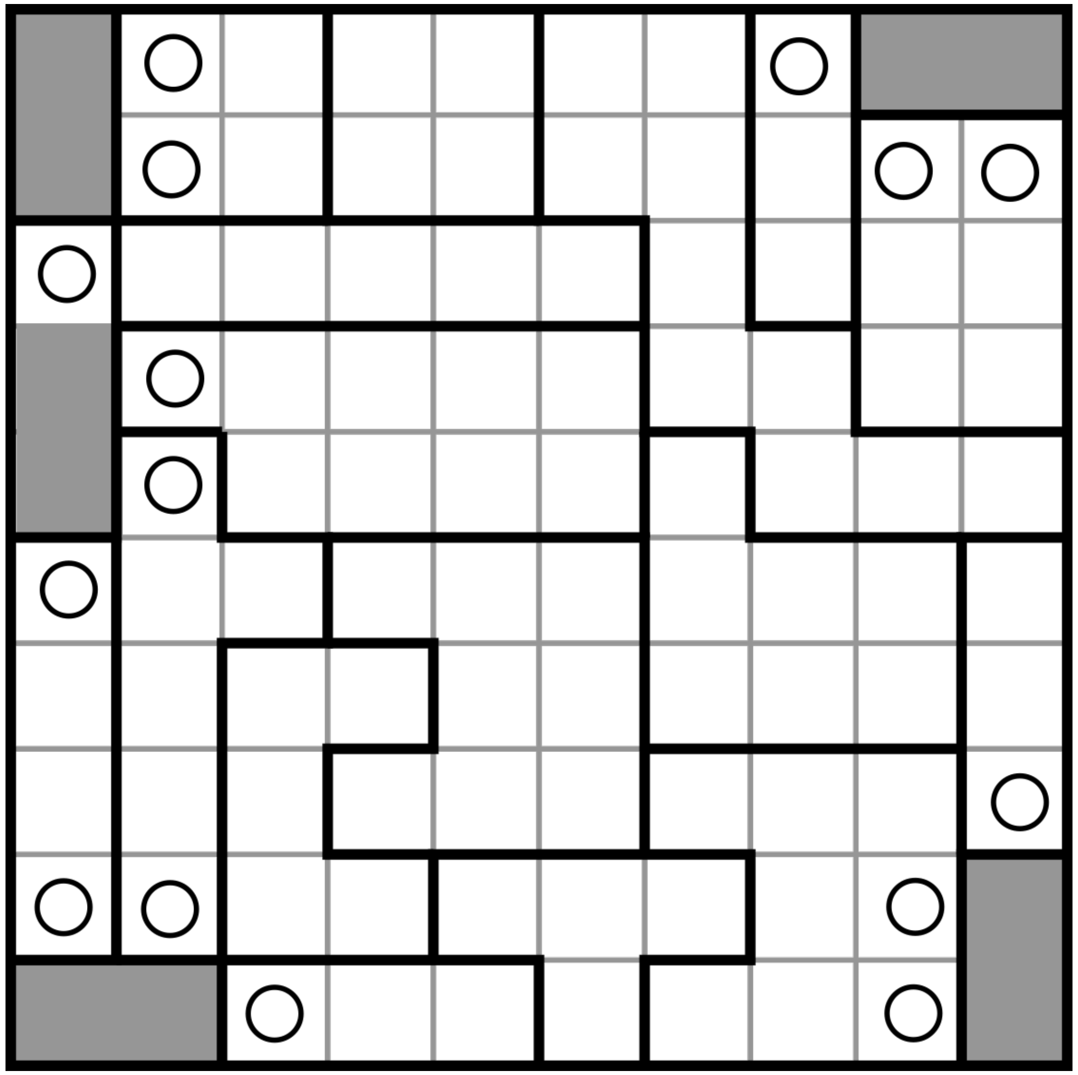

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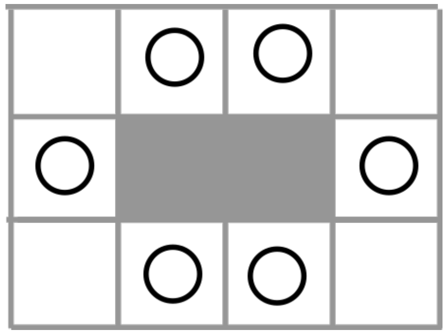

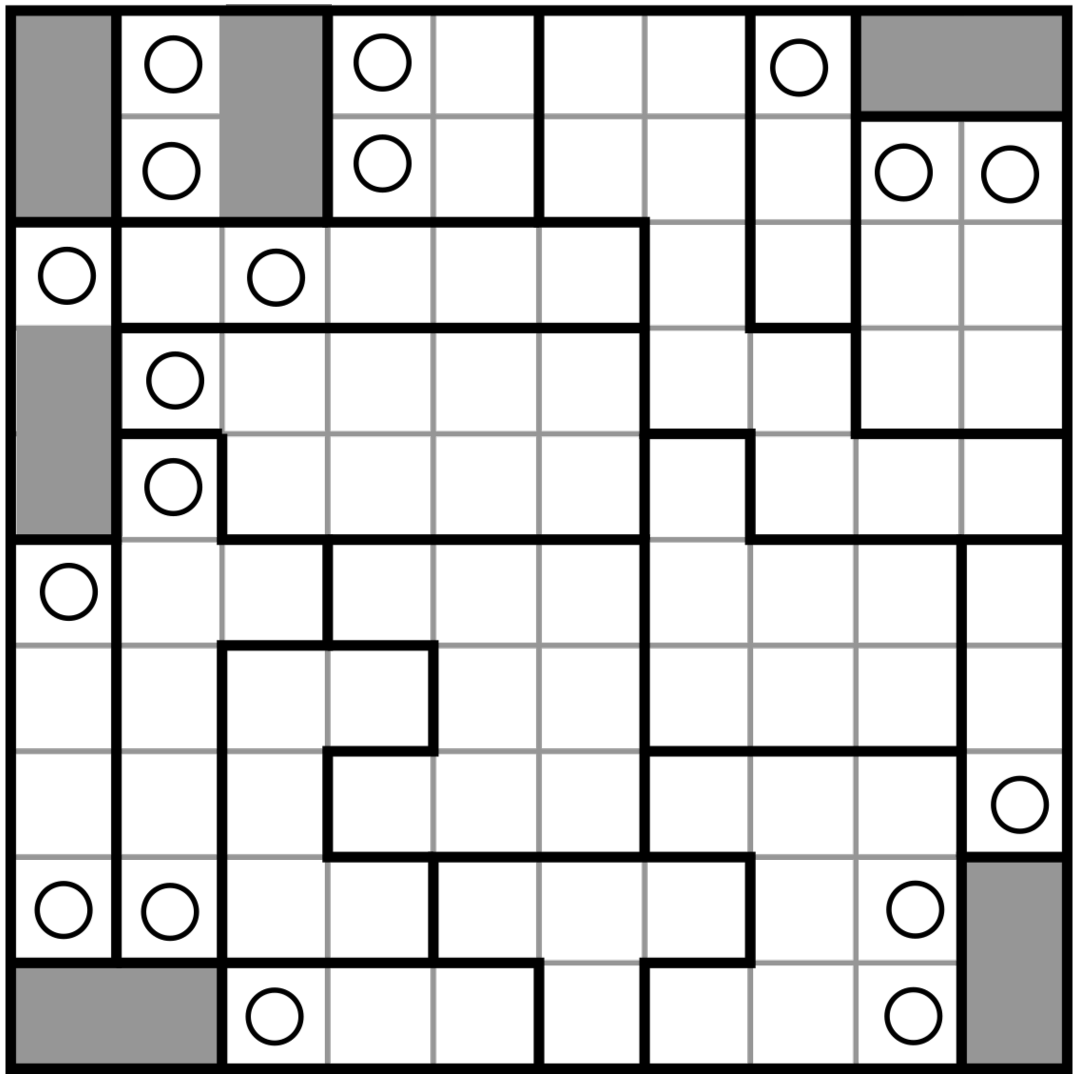

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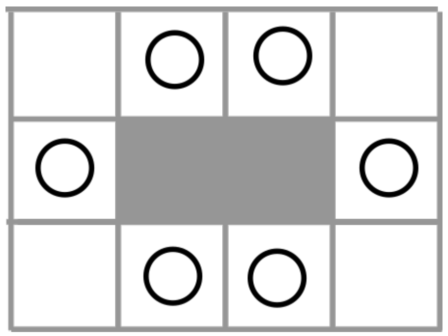

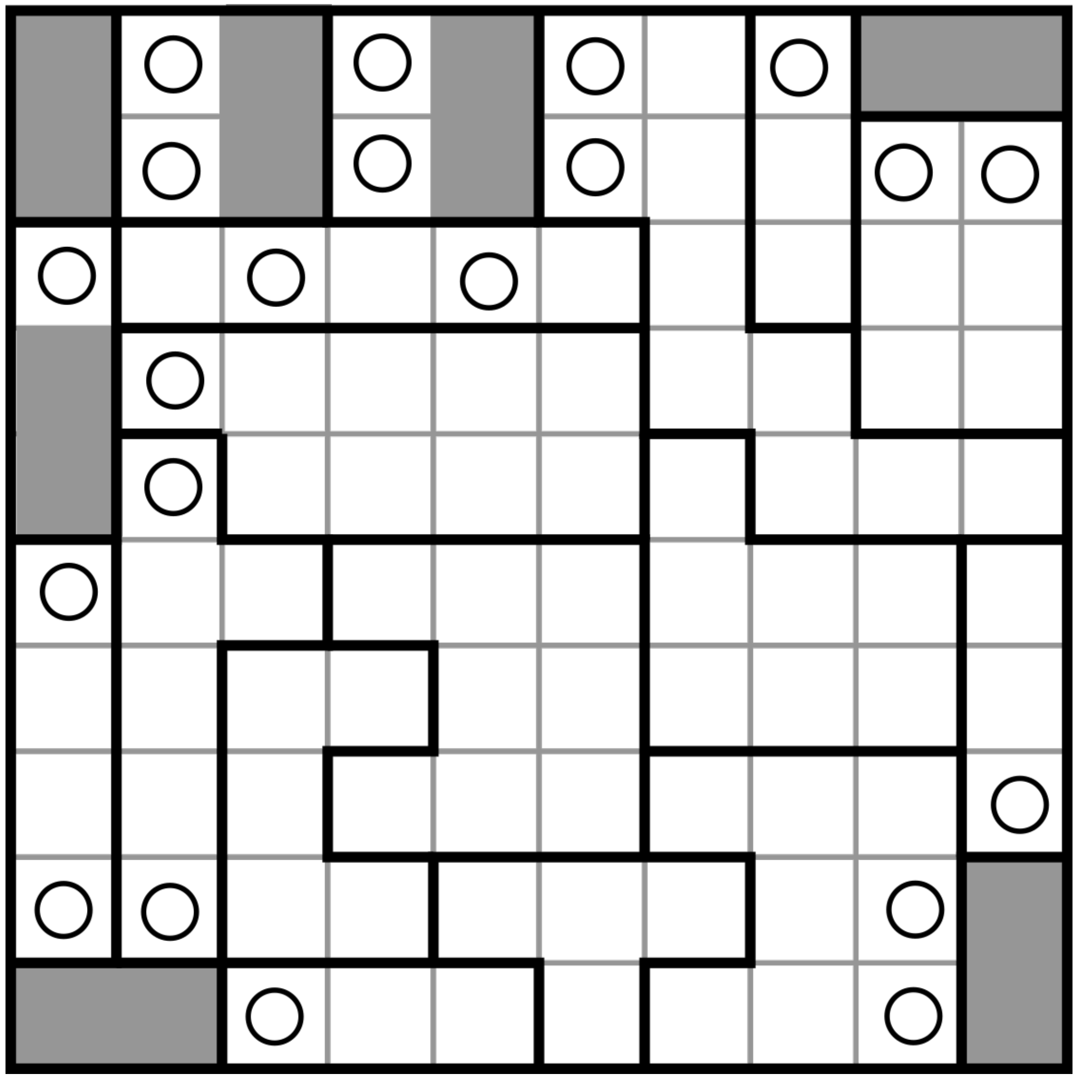

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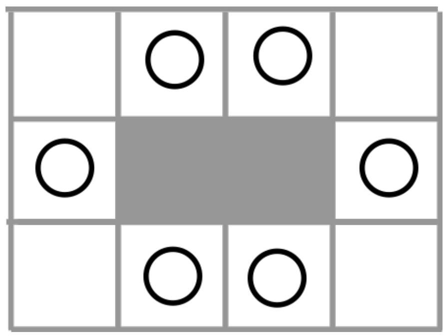

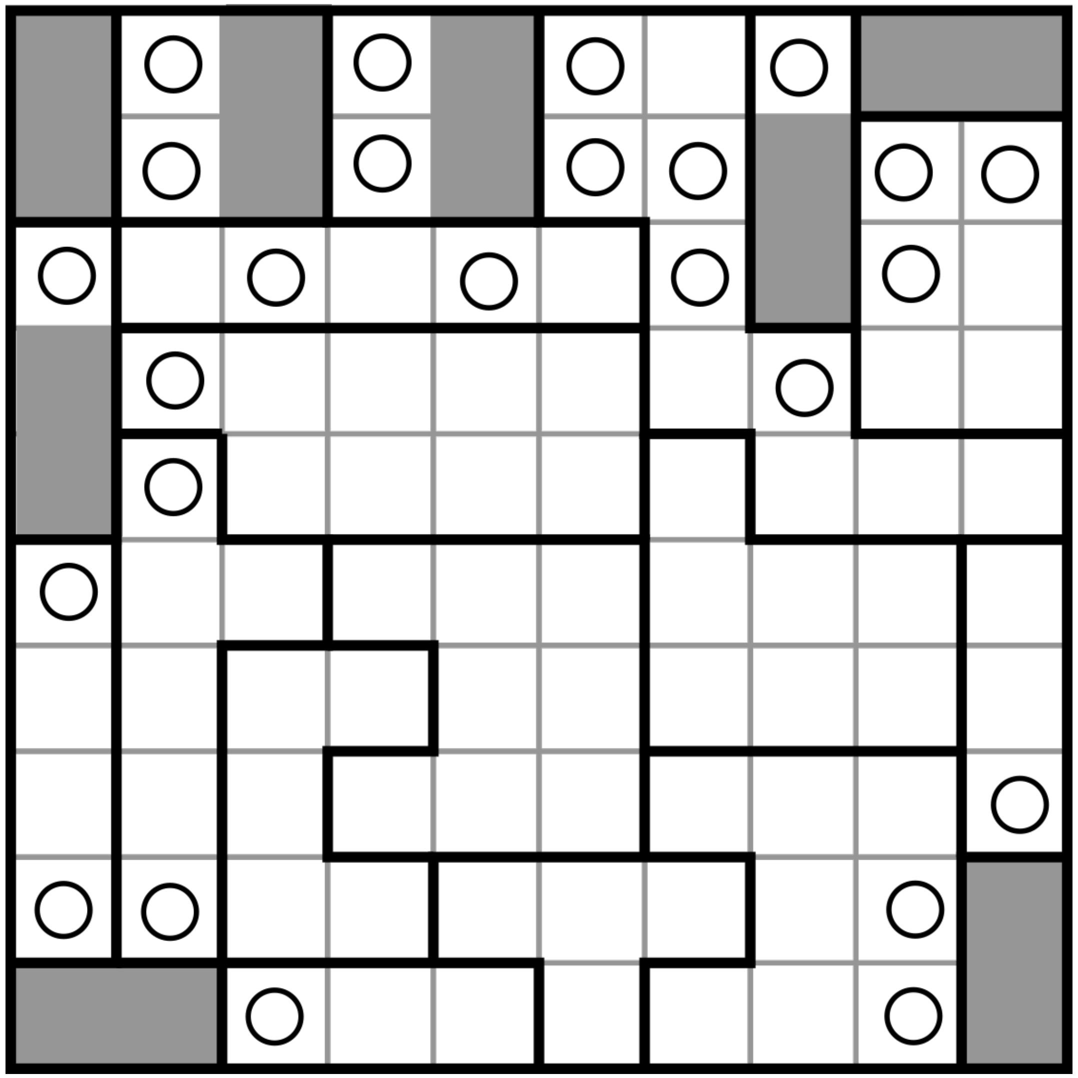

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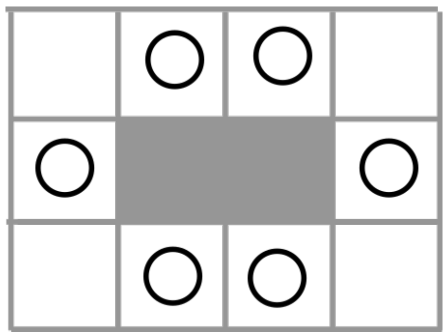

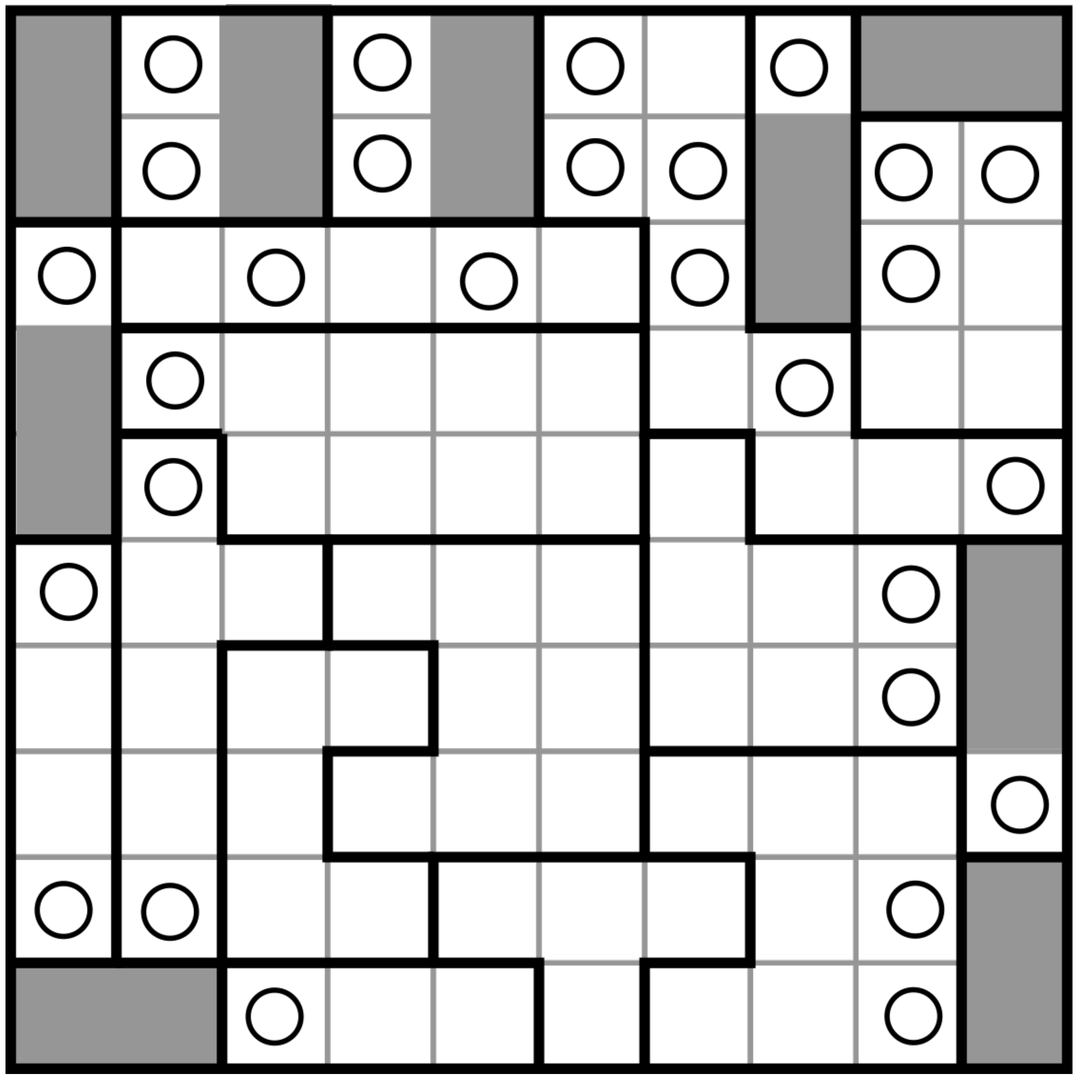

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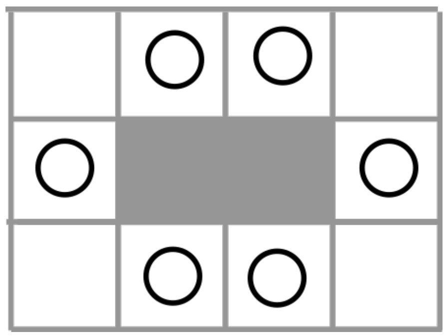

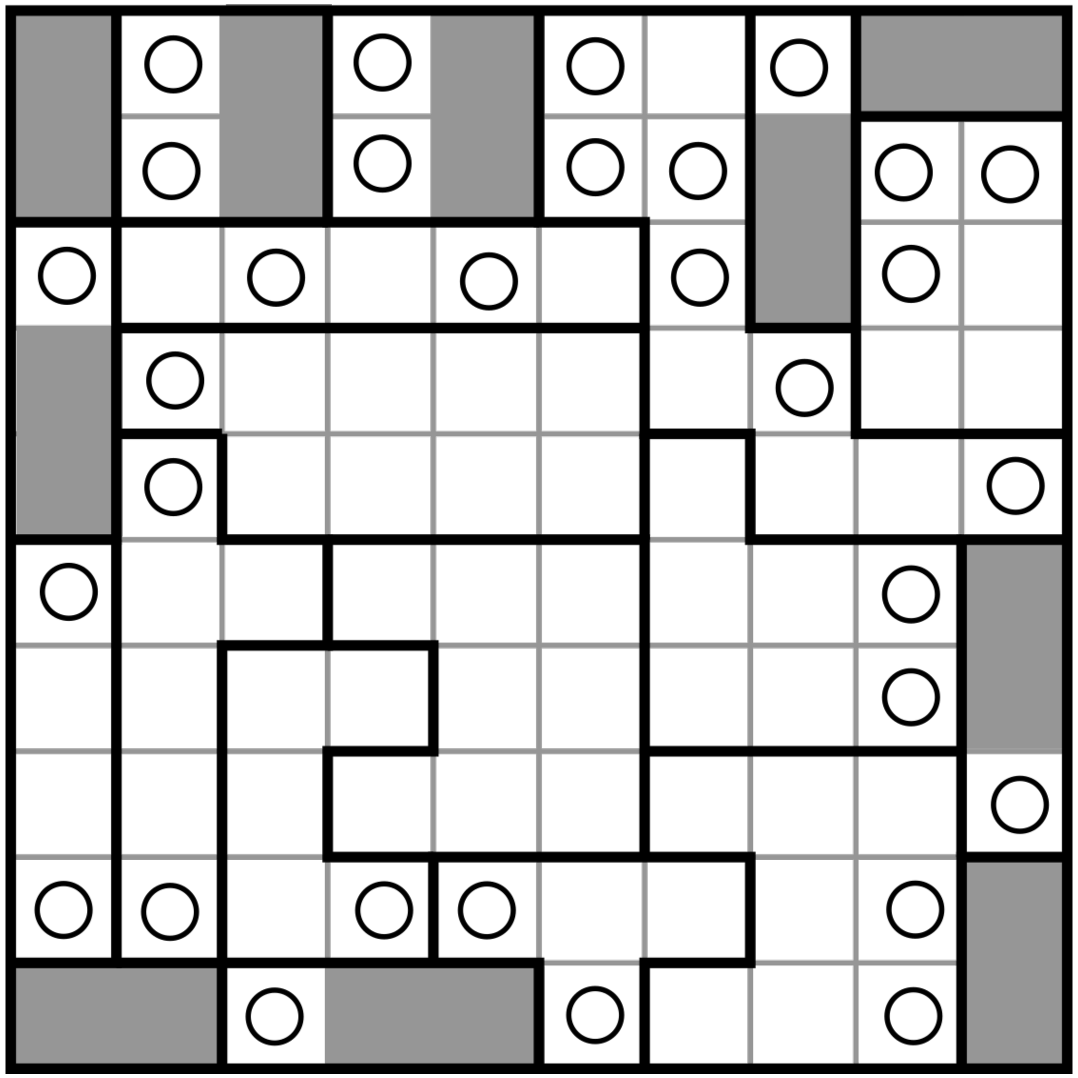

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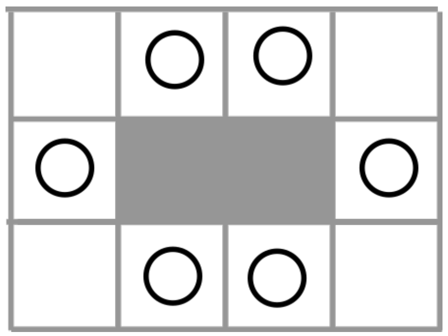

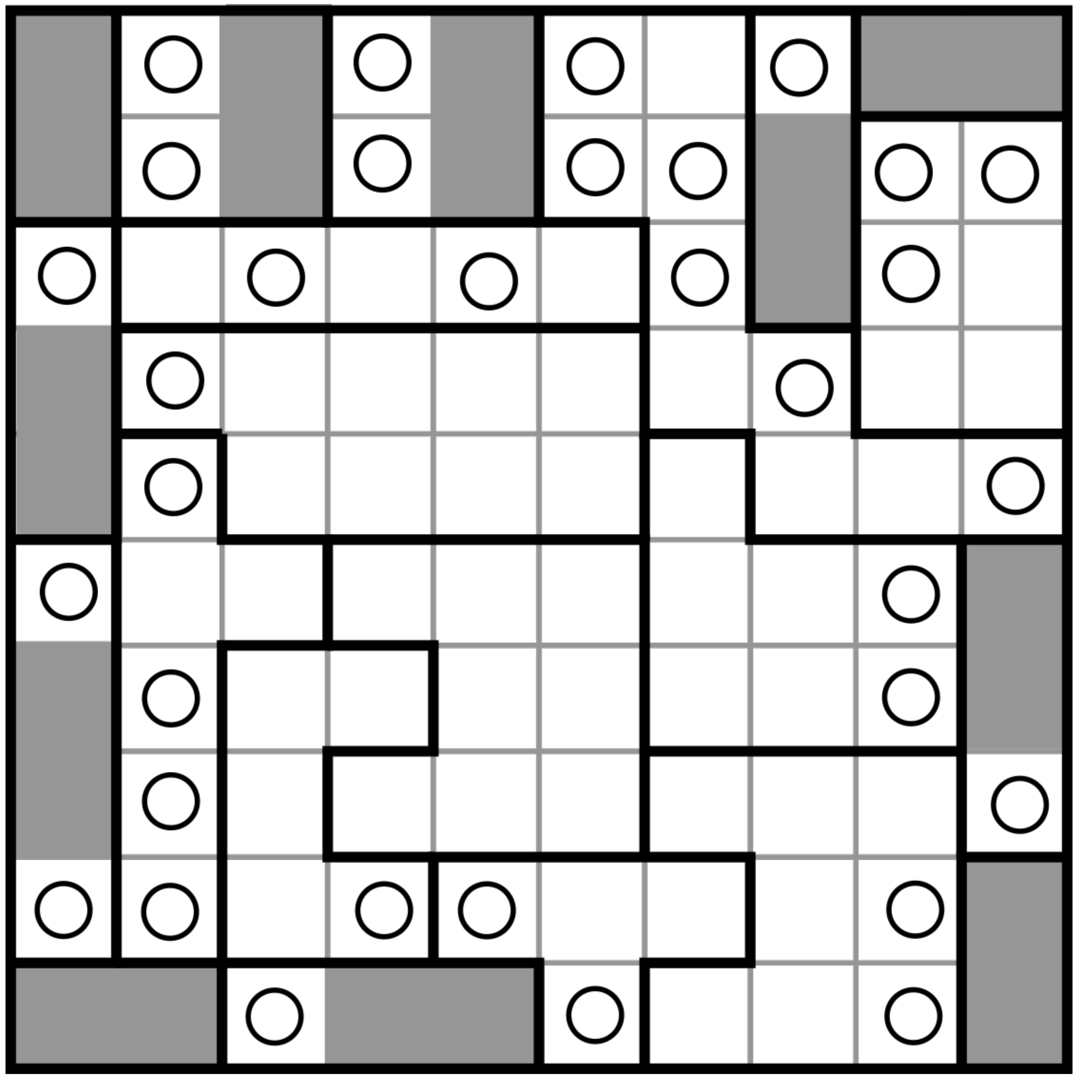

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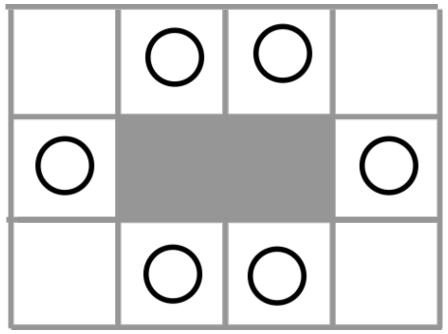

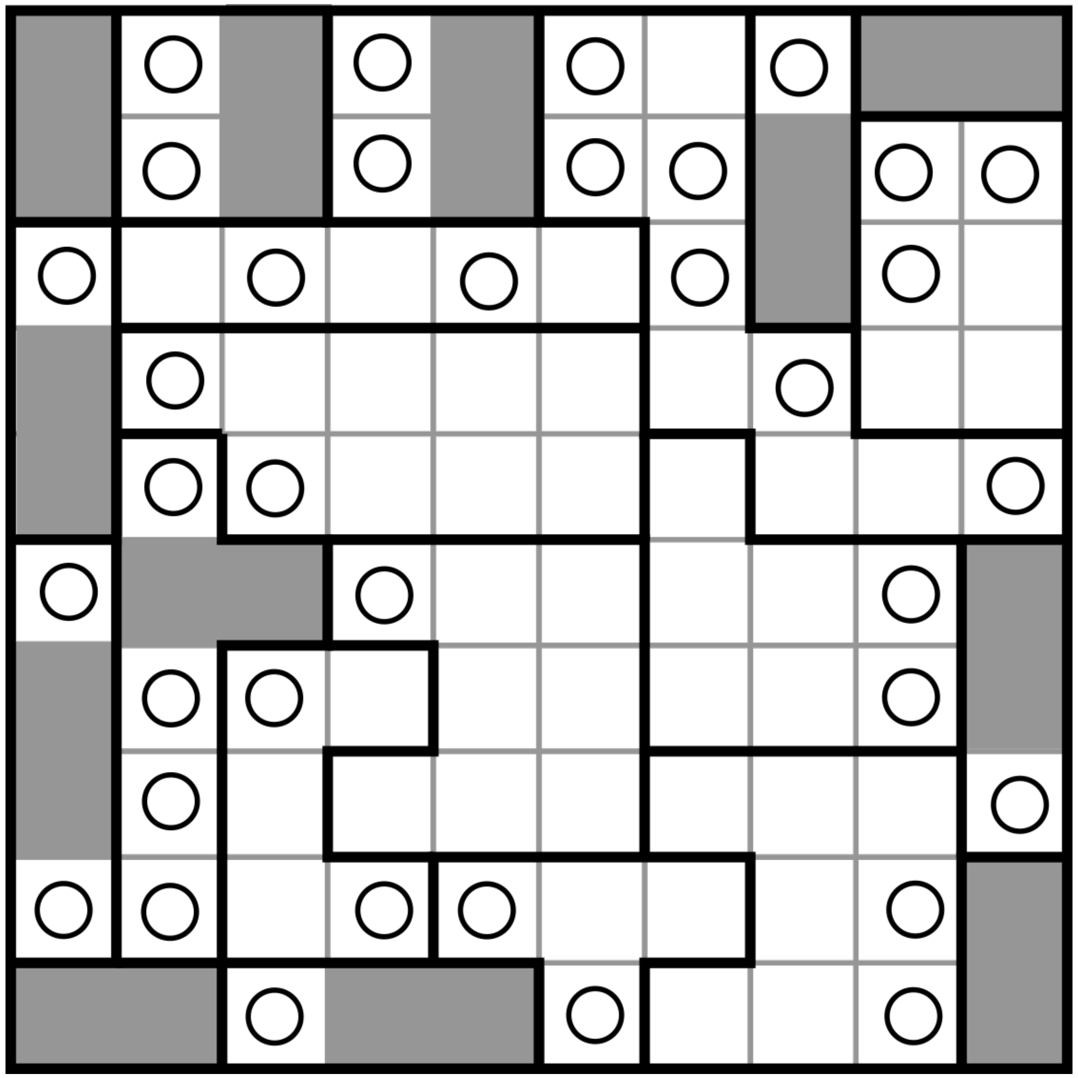

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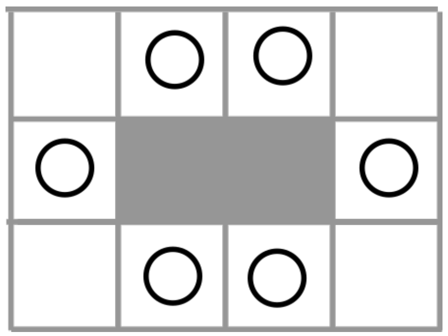

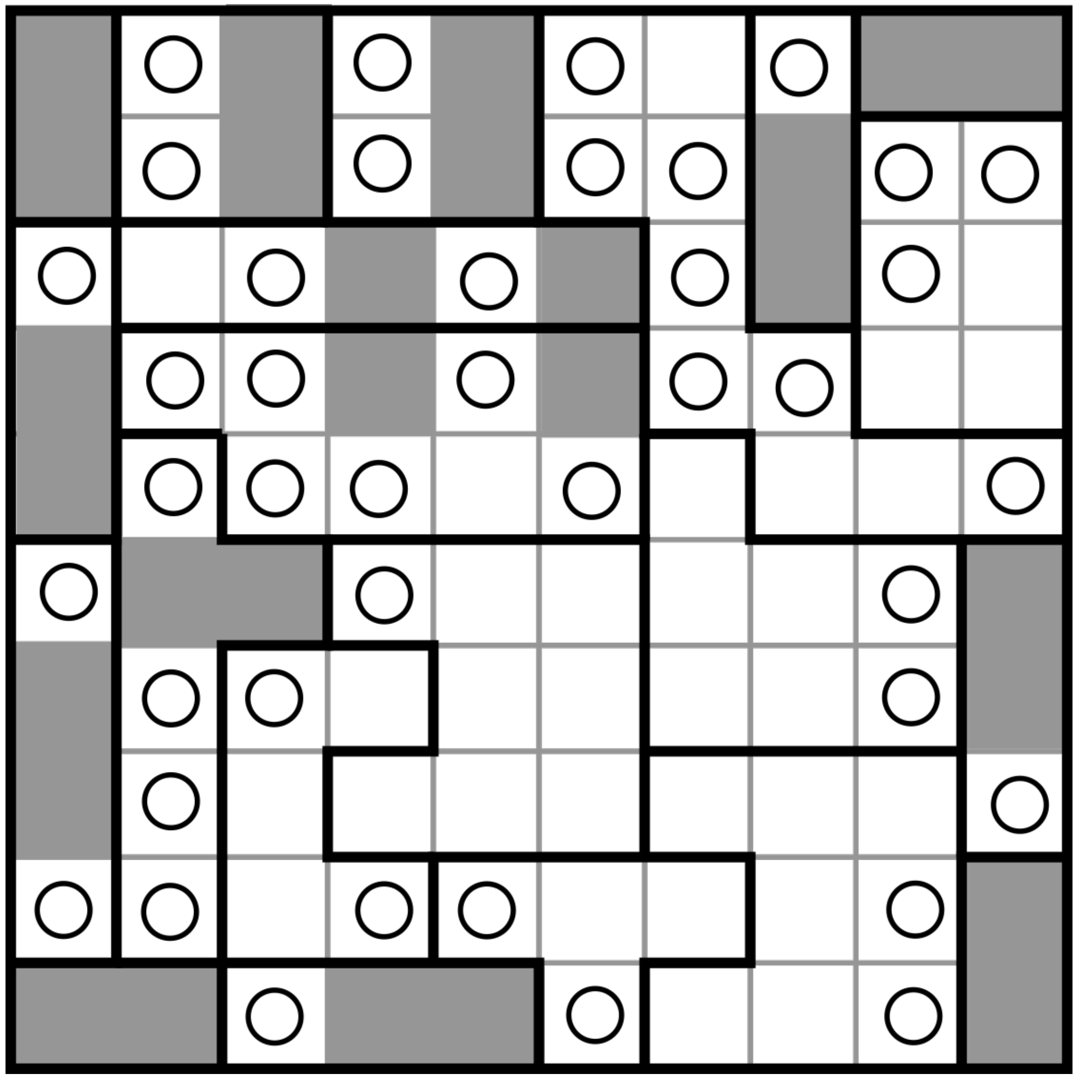

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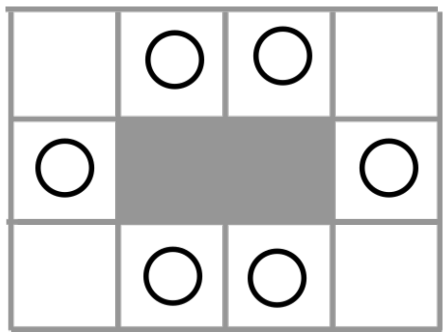

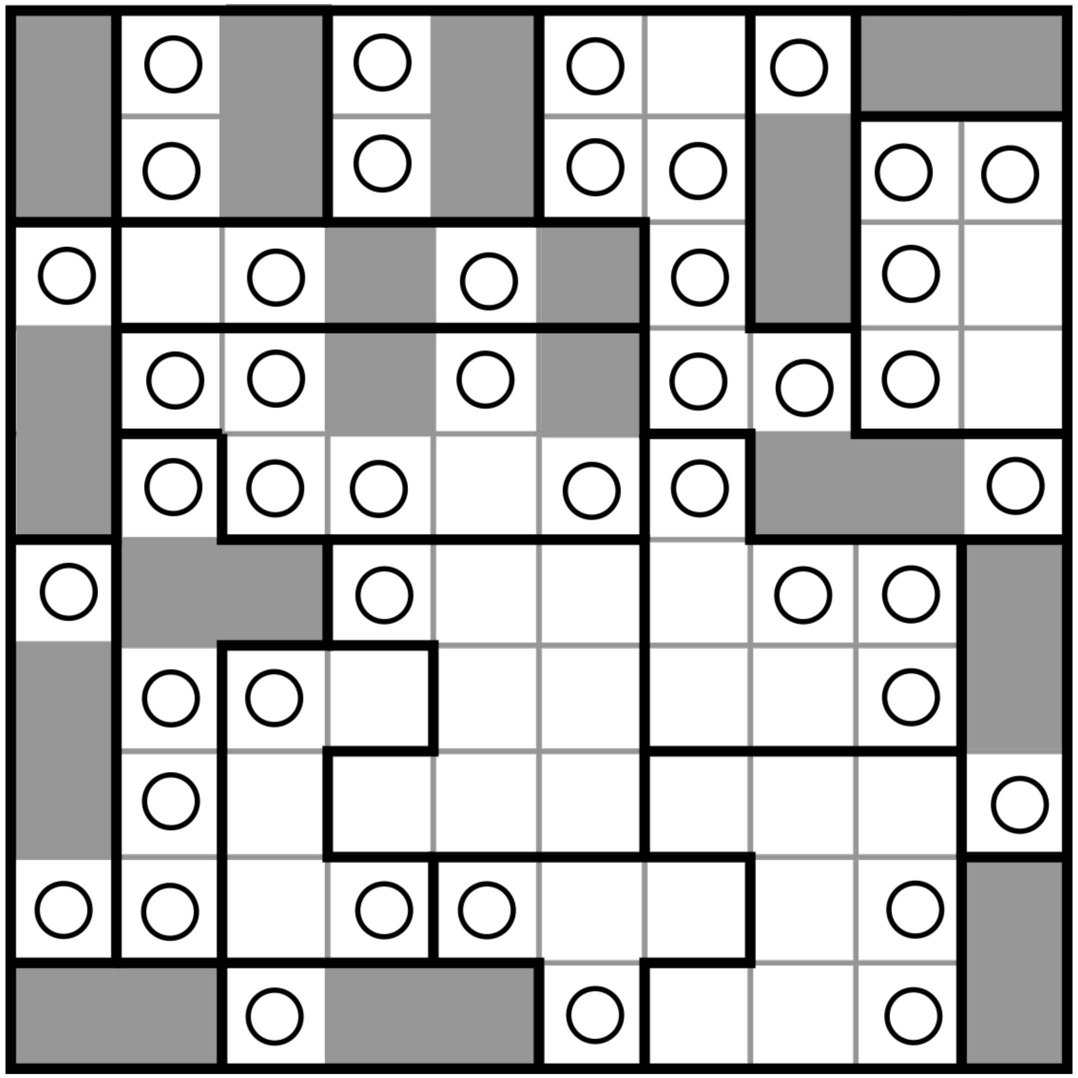

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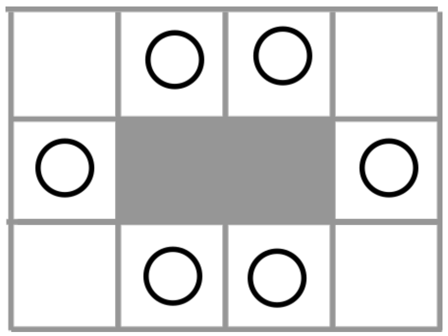

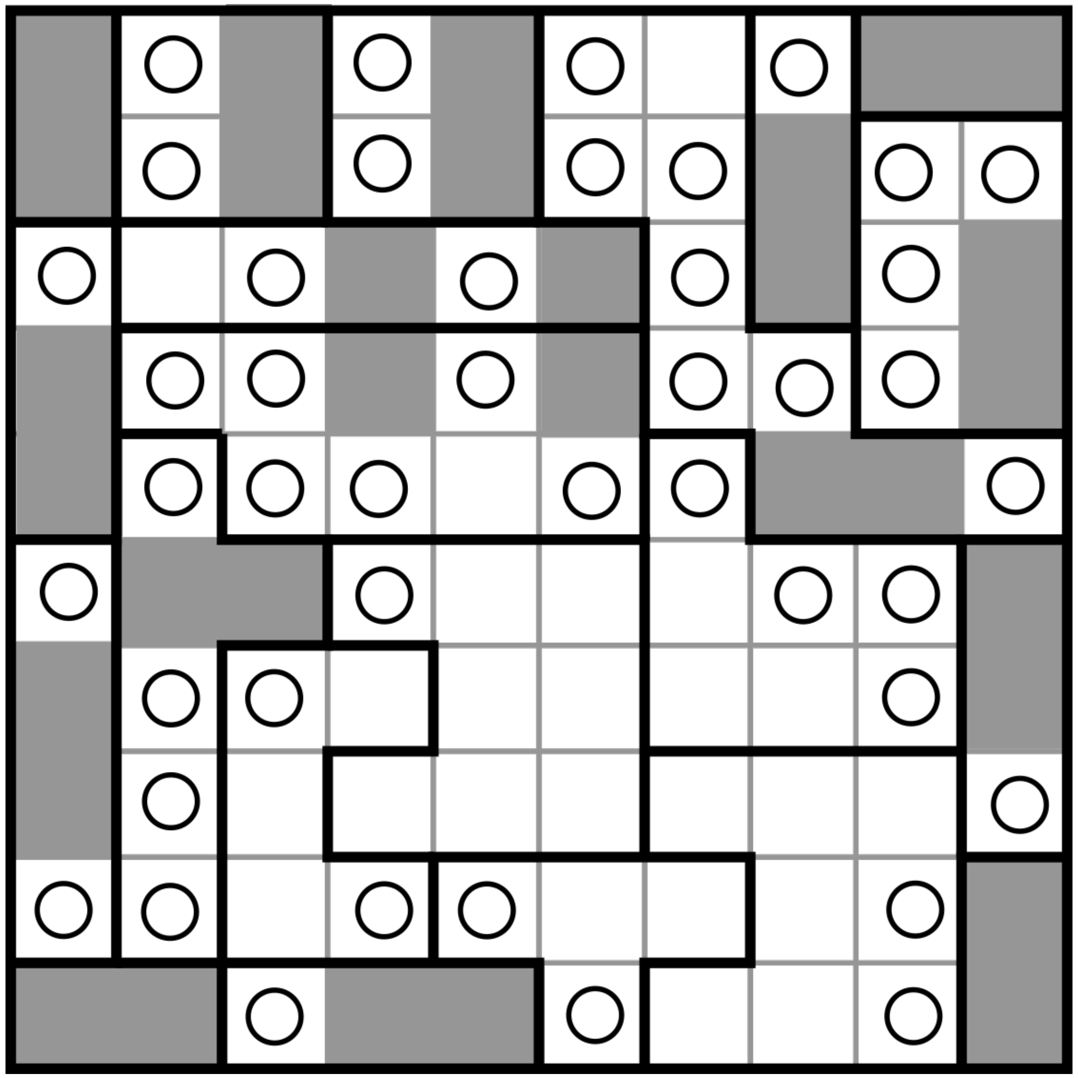

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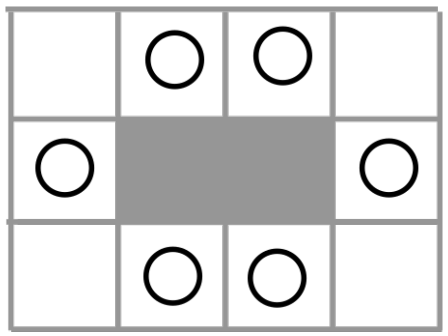

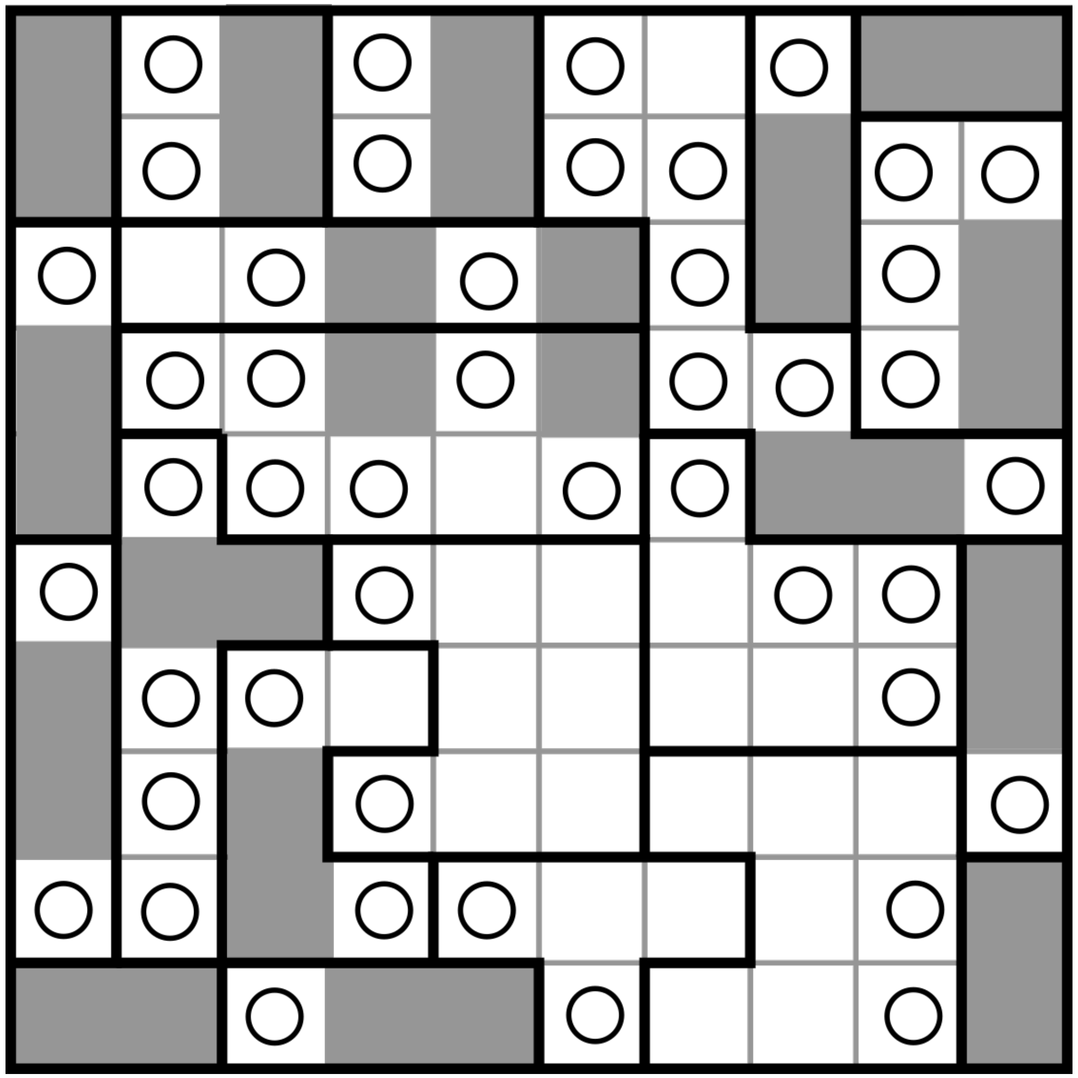

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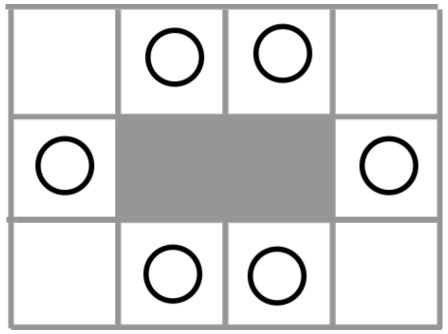

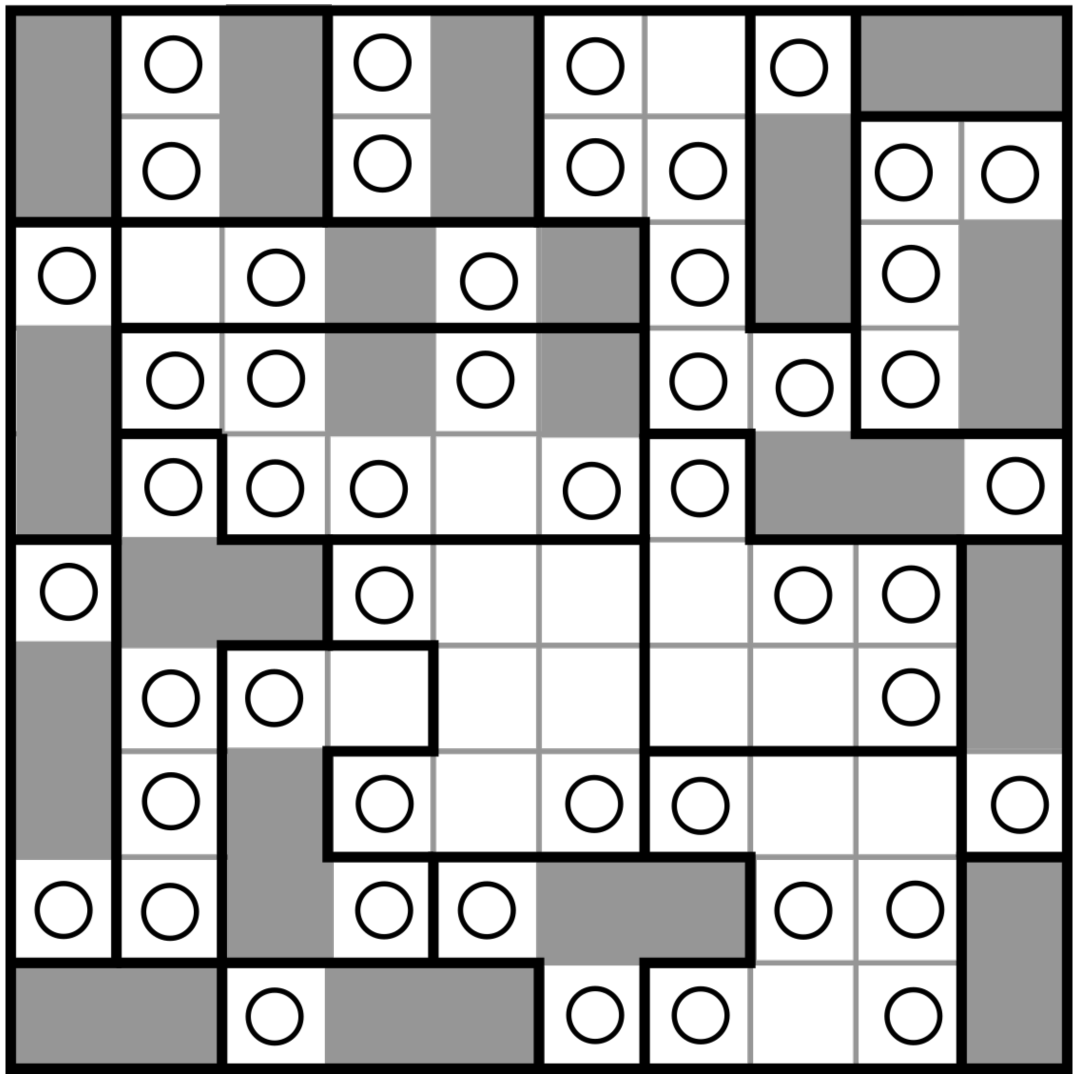

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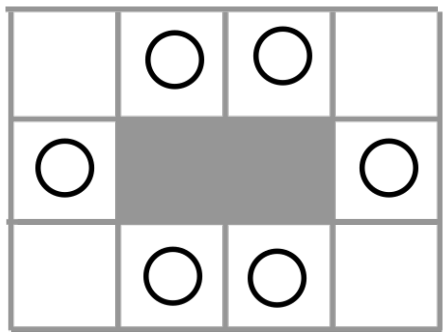

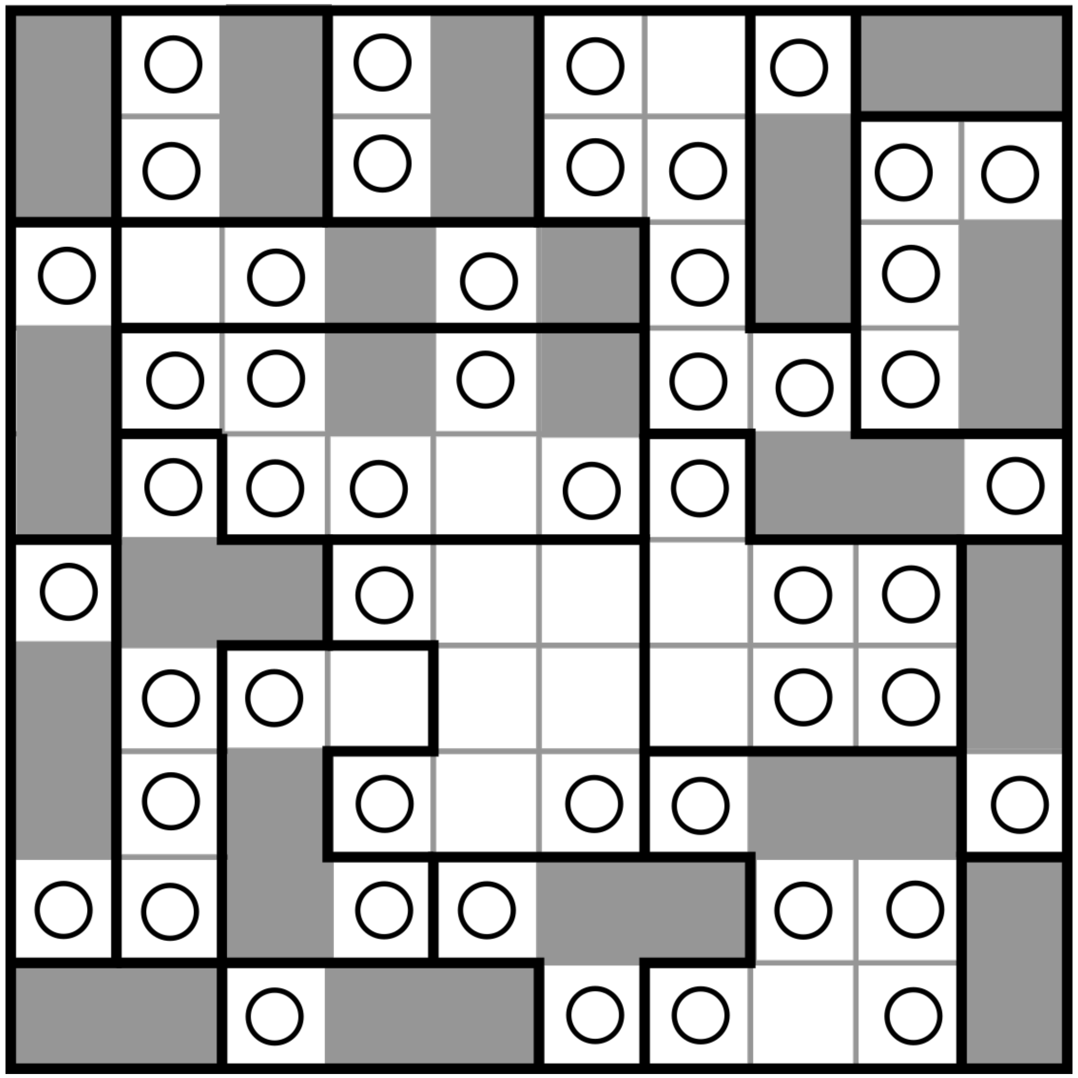

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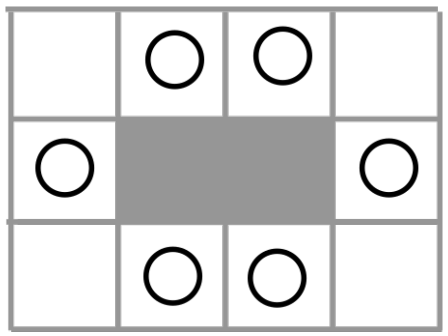

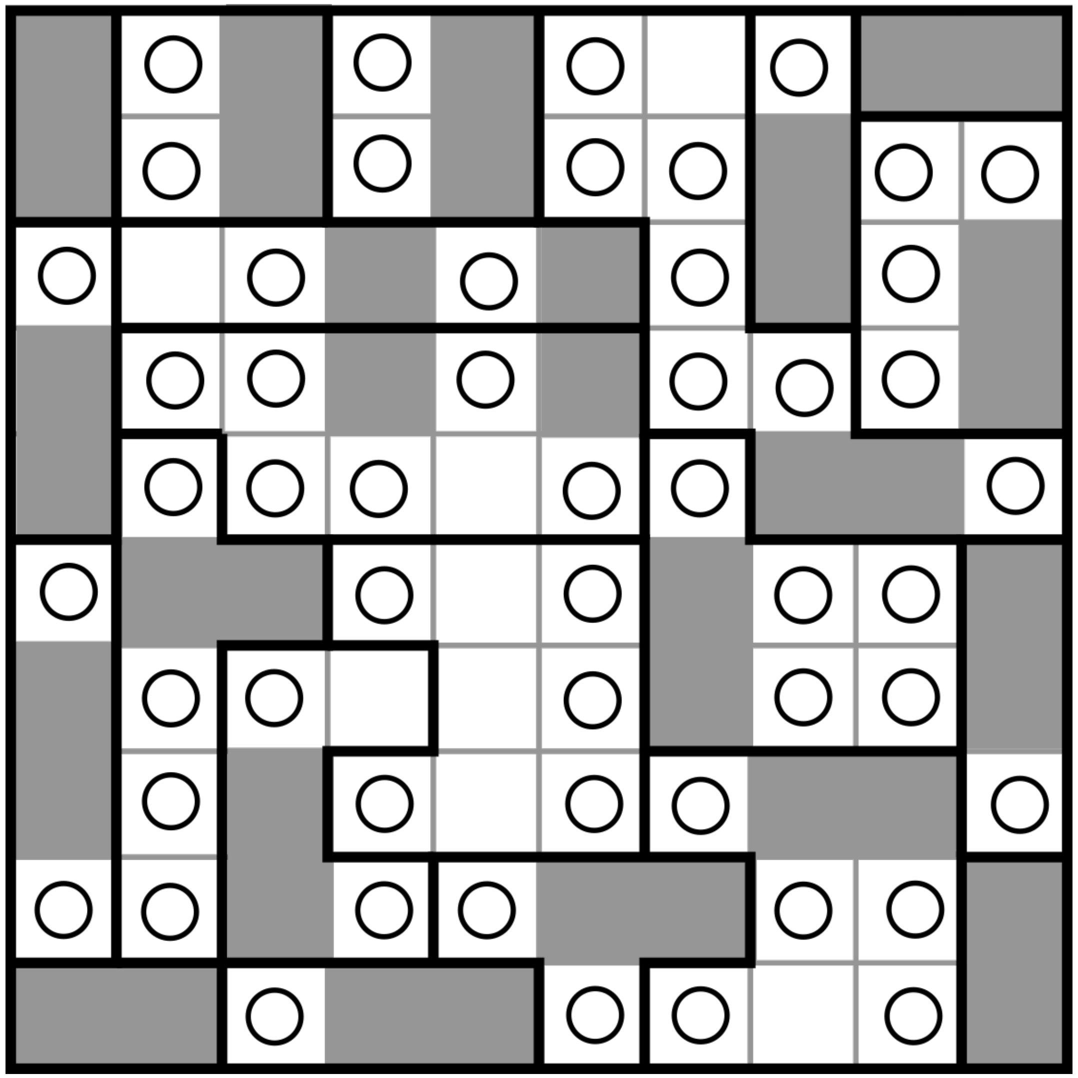

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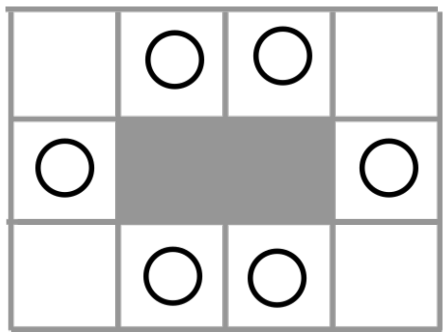

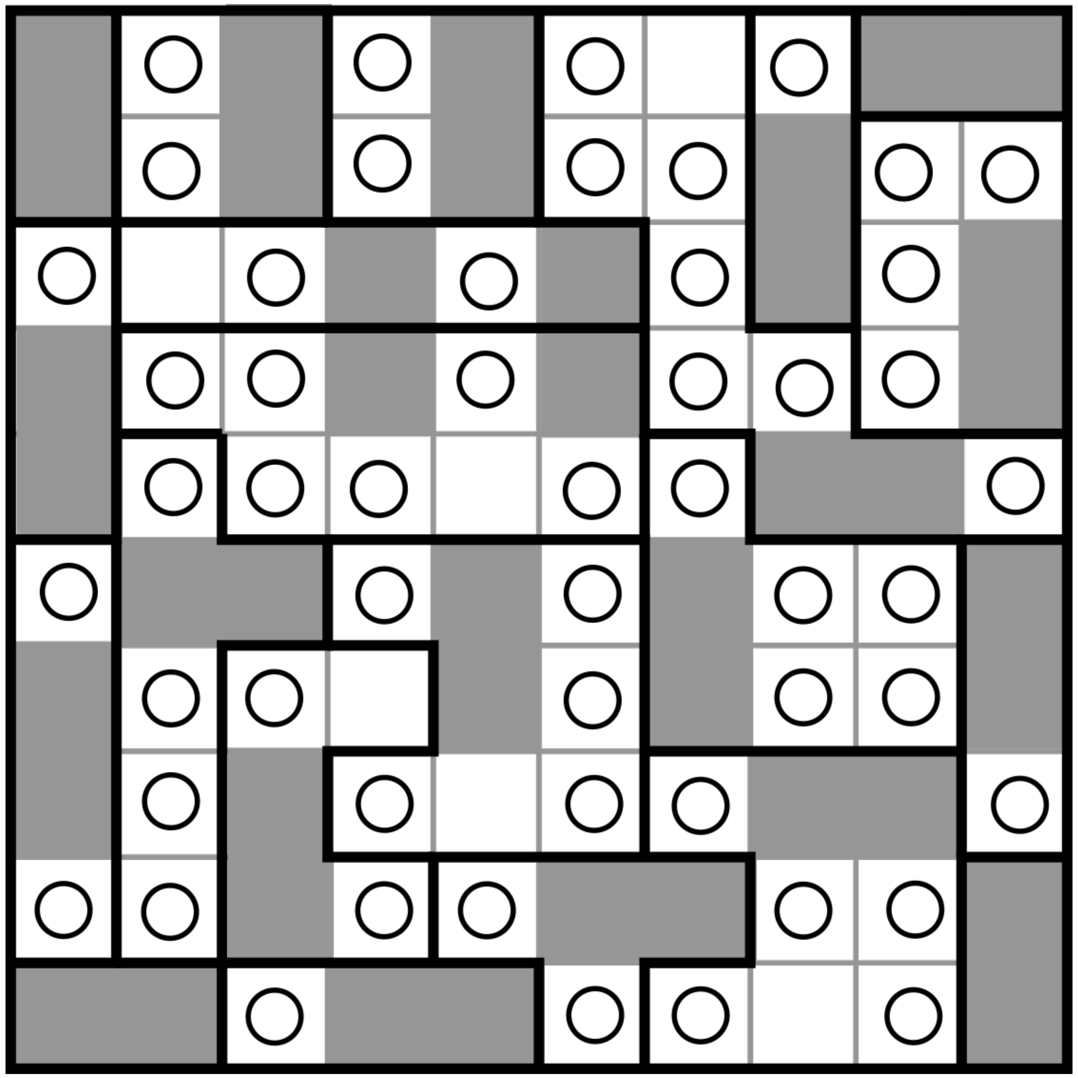

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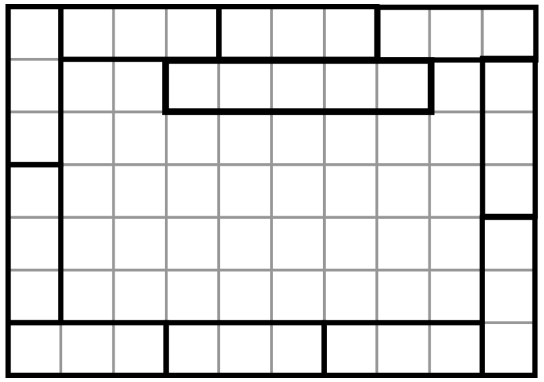

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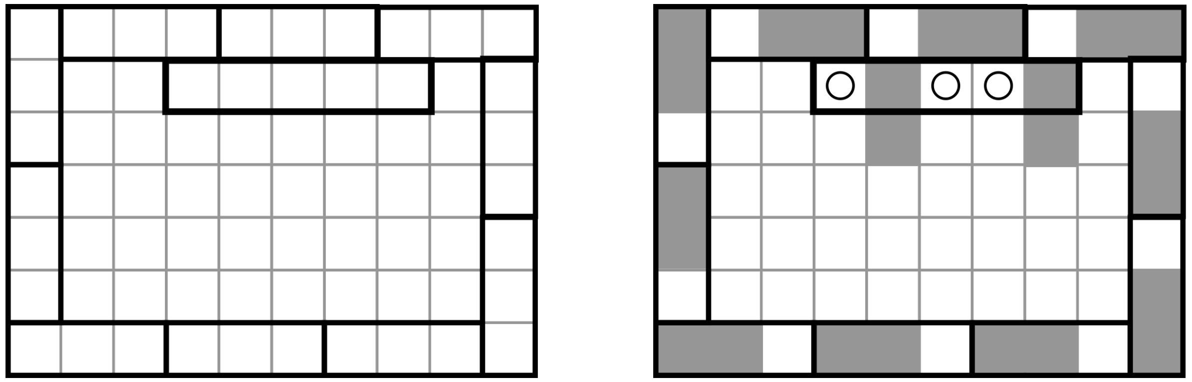

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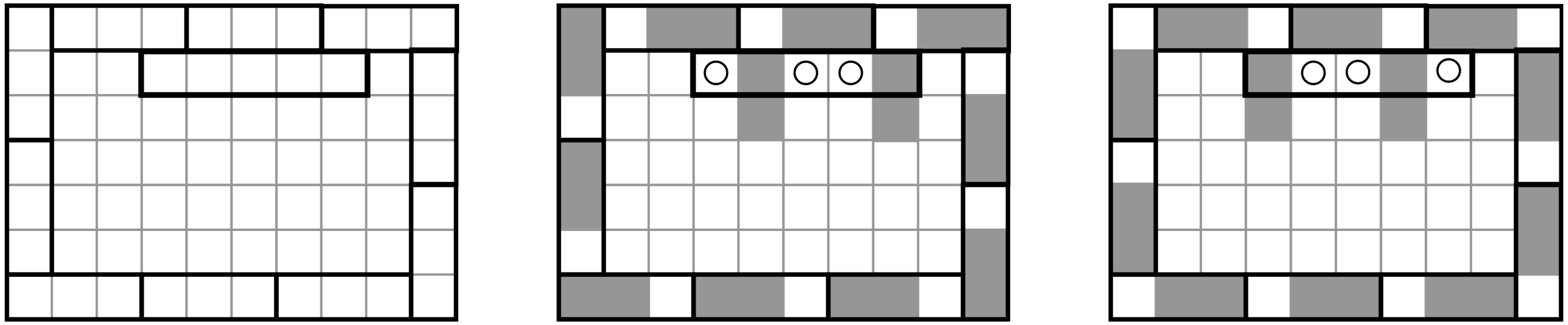

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Fixes squares in center face,
and makes third solution to the loop infeasible

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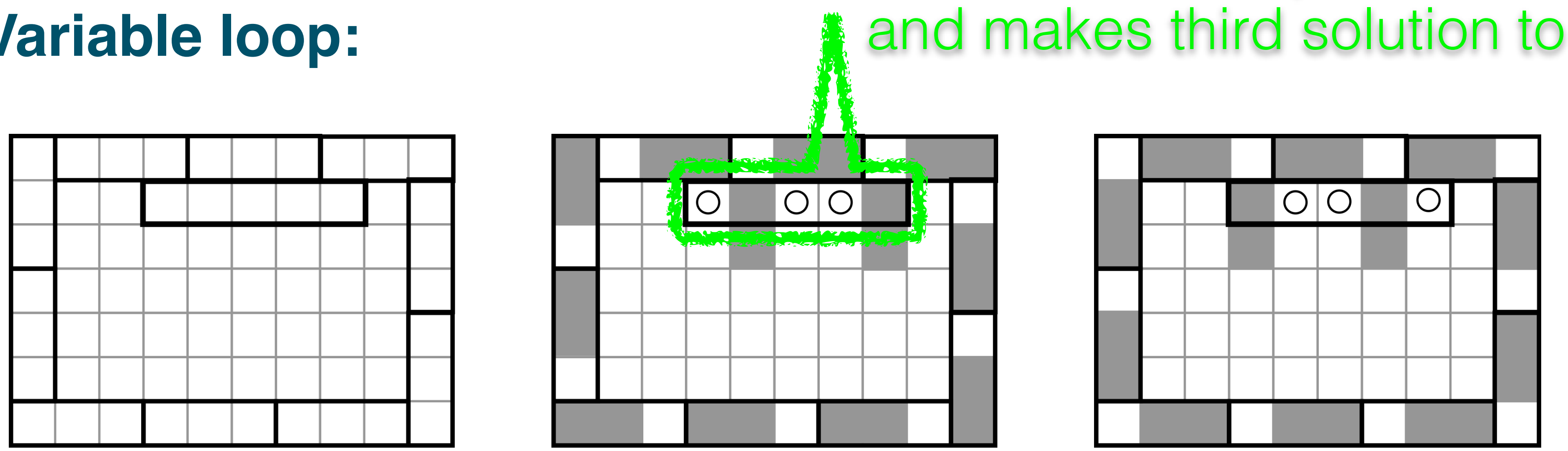

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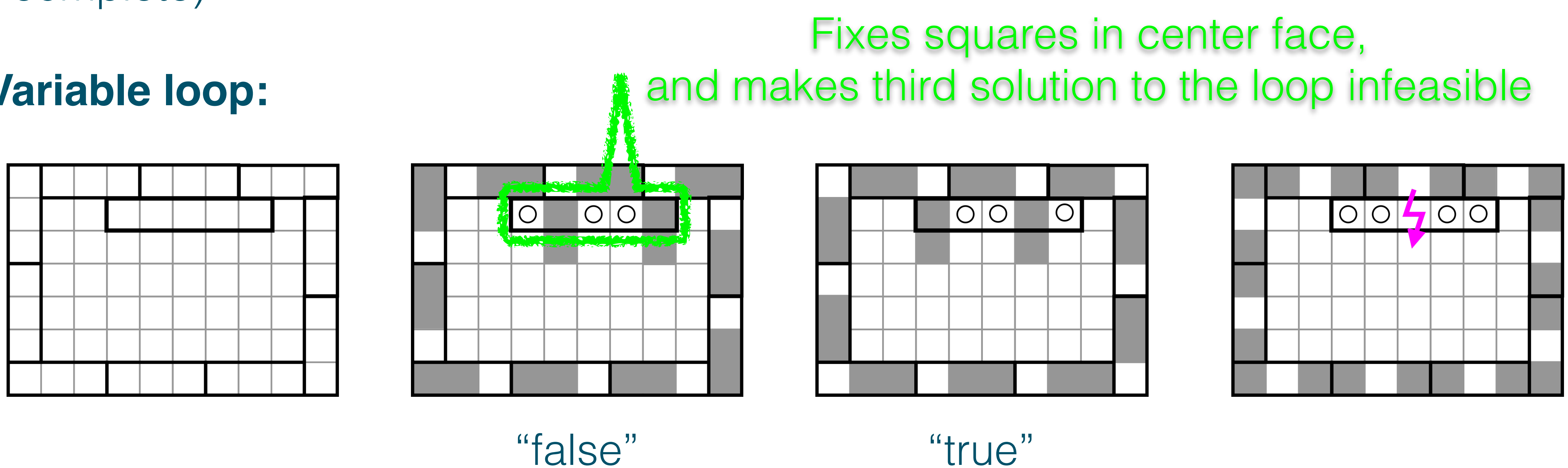

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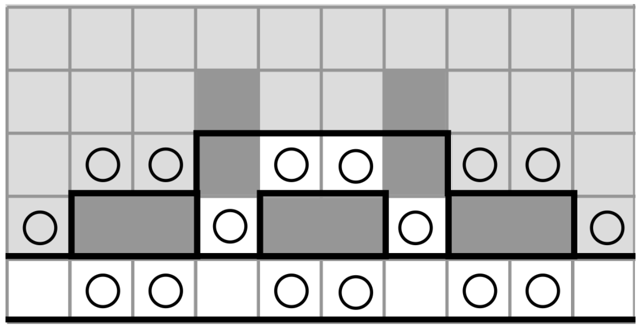

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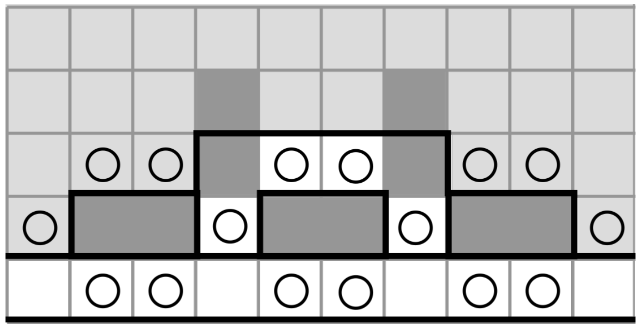

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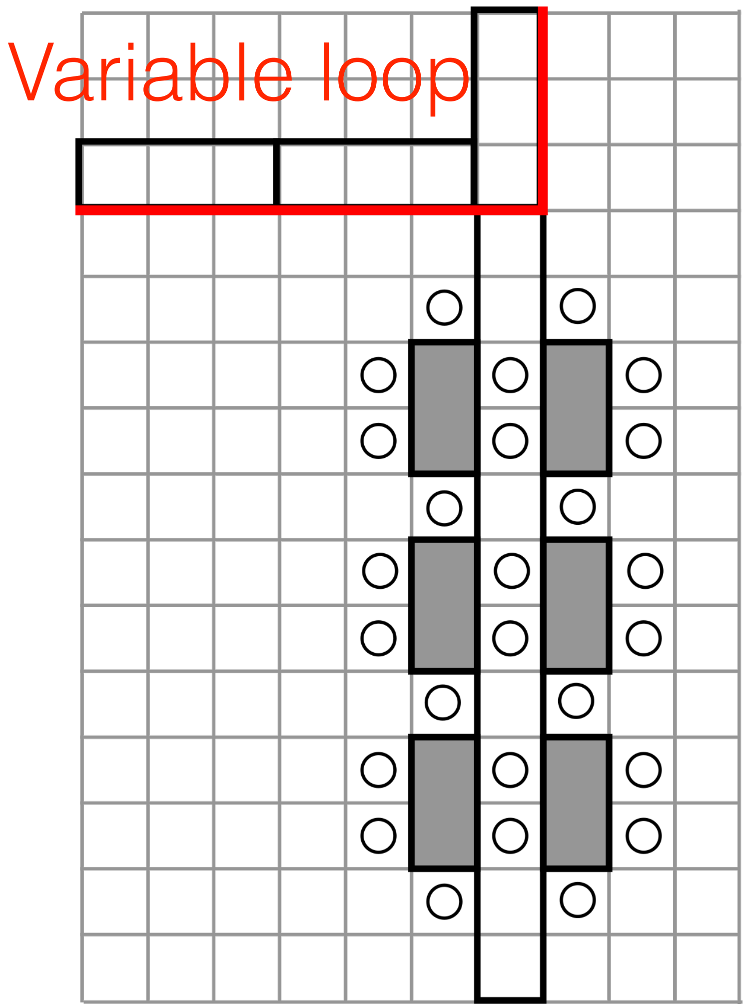

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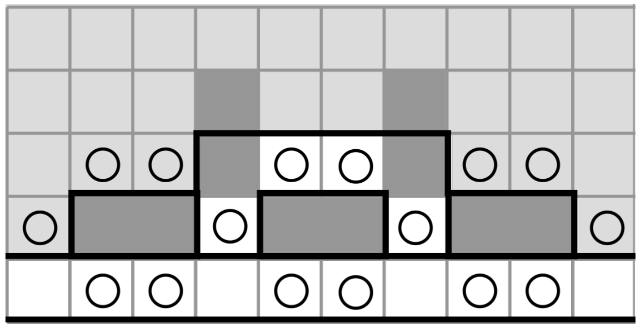

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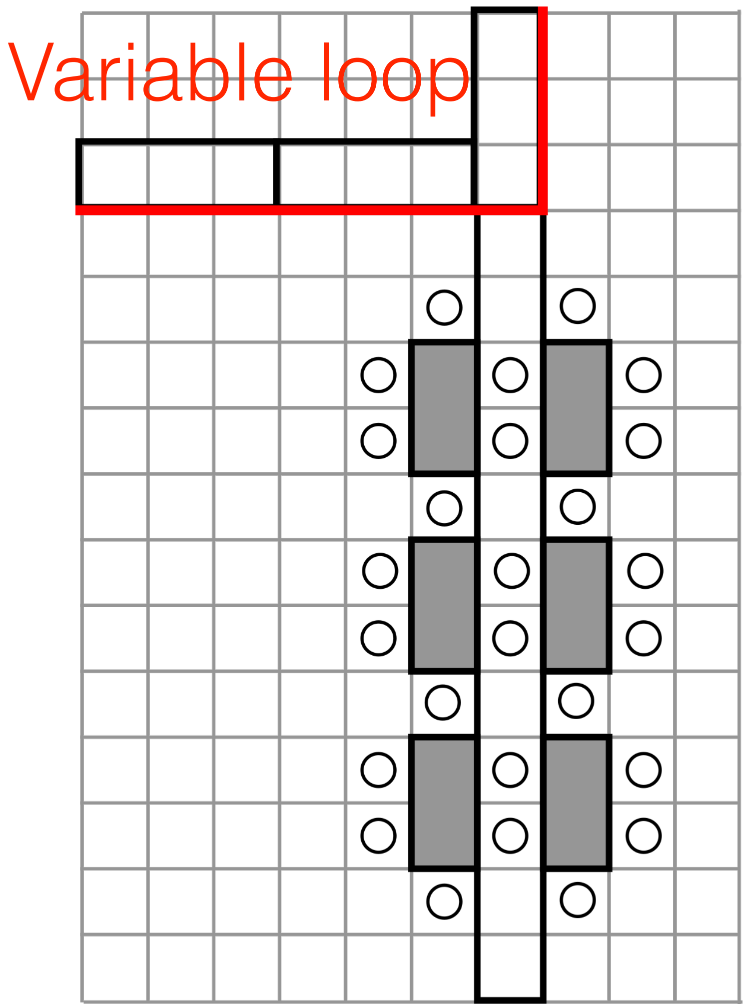


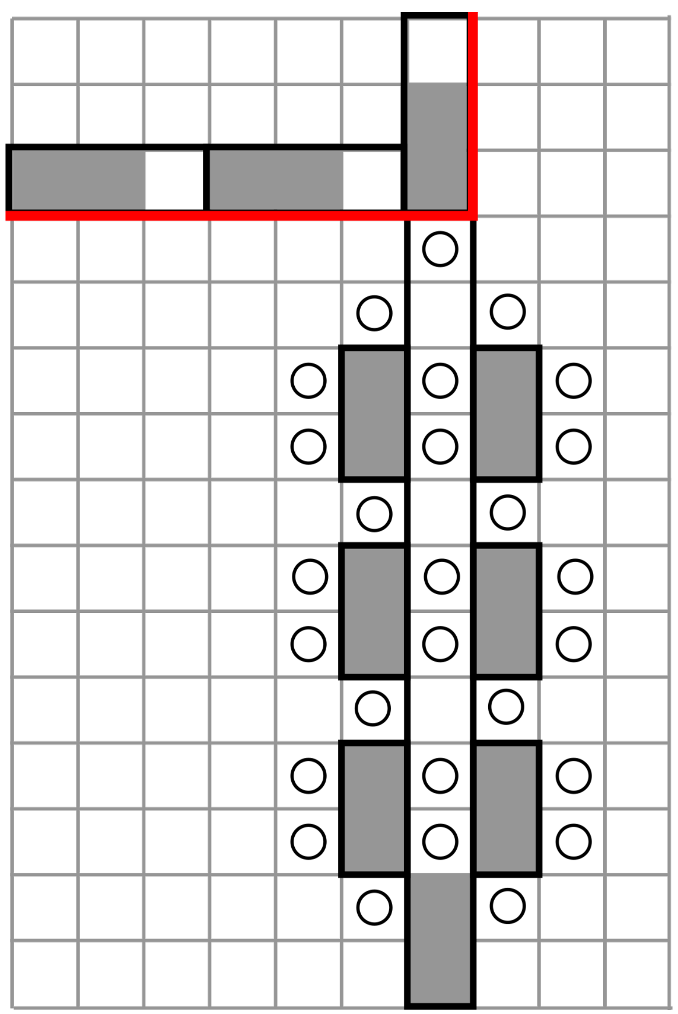

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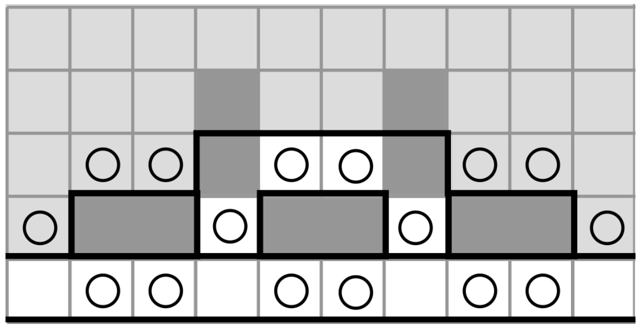
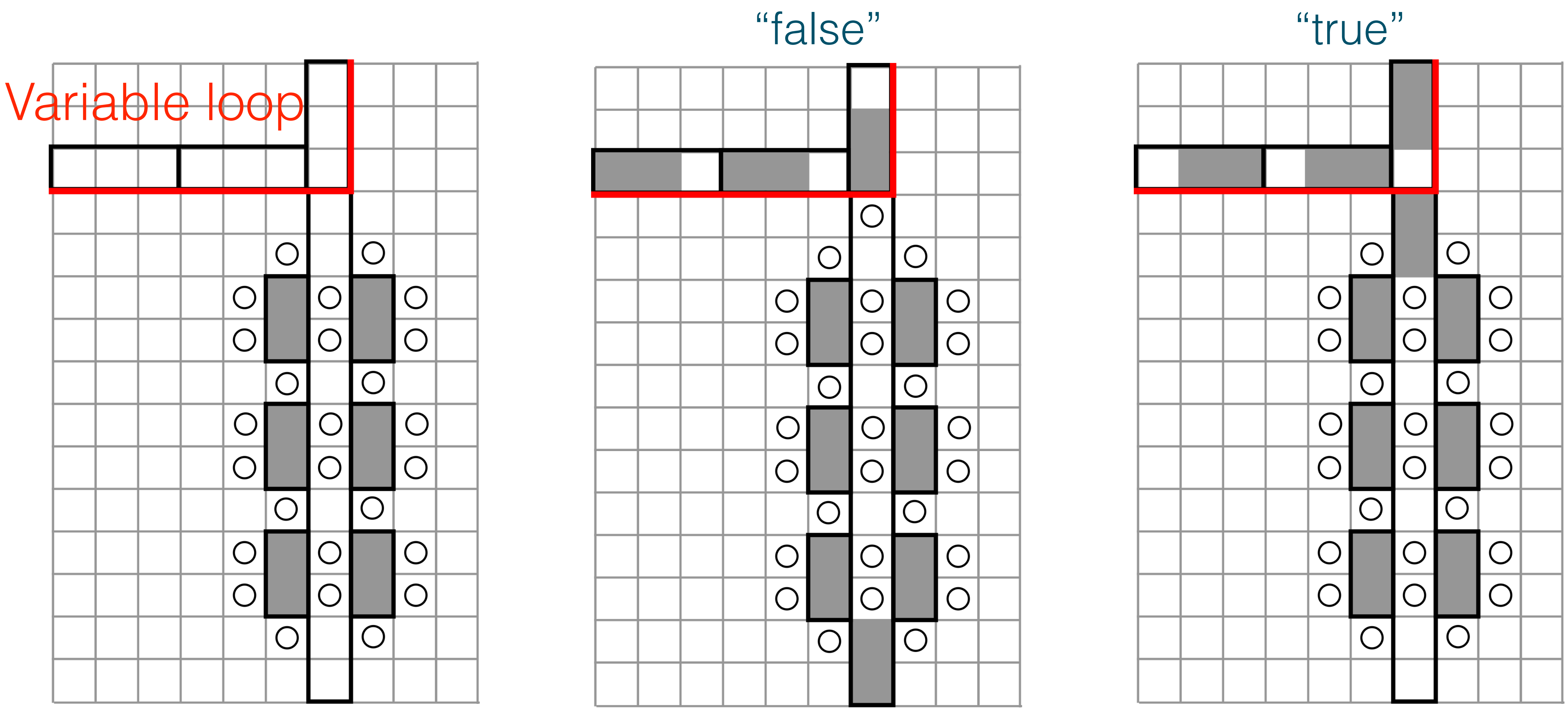
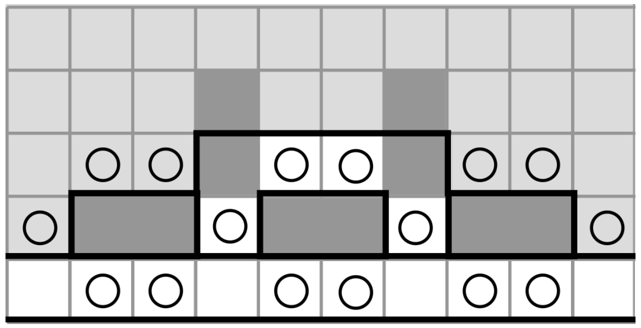


**Corridor gadget**, propagates variable value:



**Face gadget**, for any open region:



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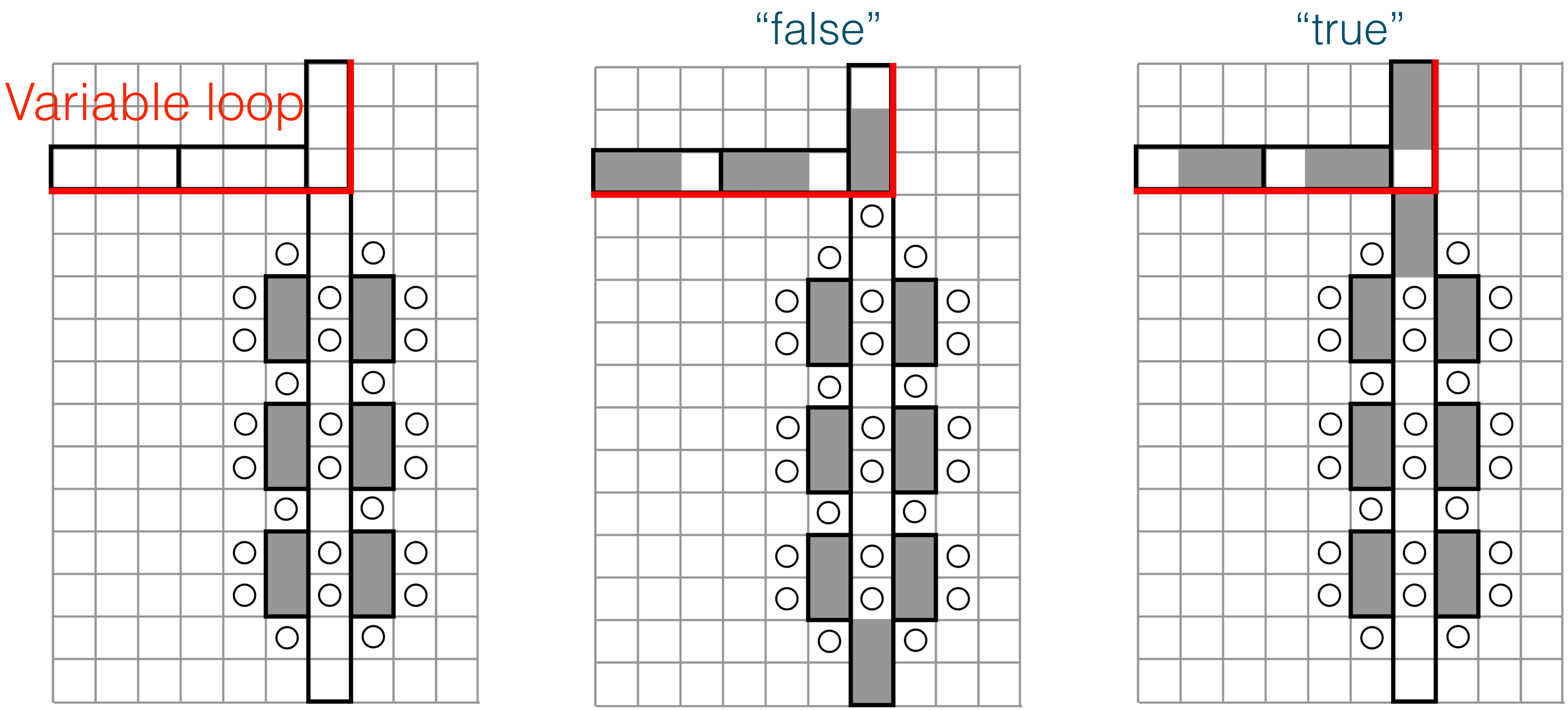
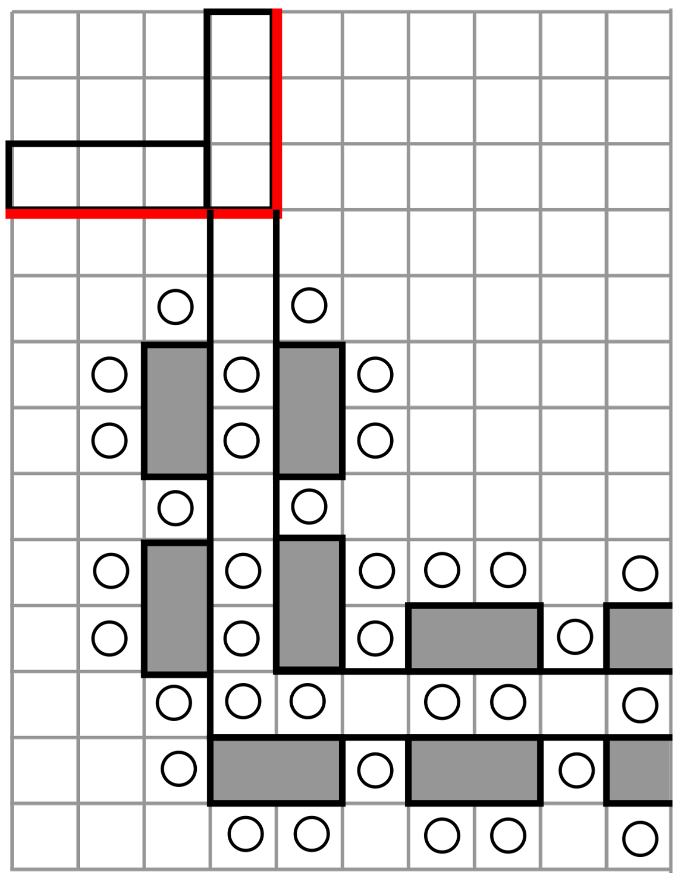
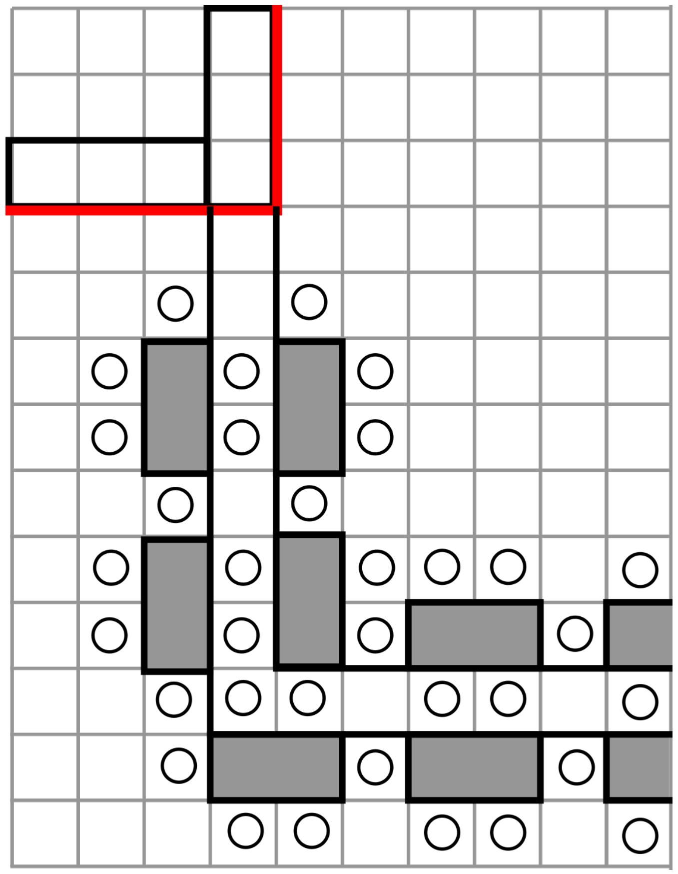



Wires for both variable and its negation: connect to appropriate place of variable loop.

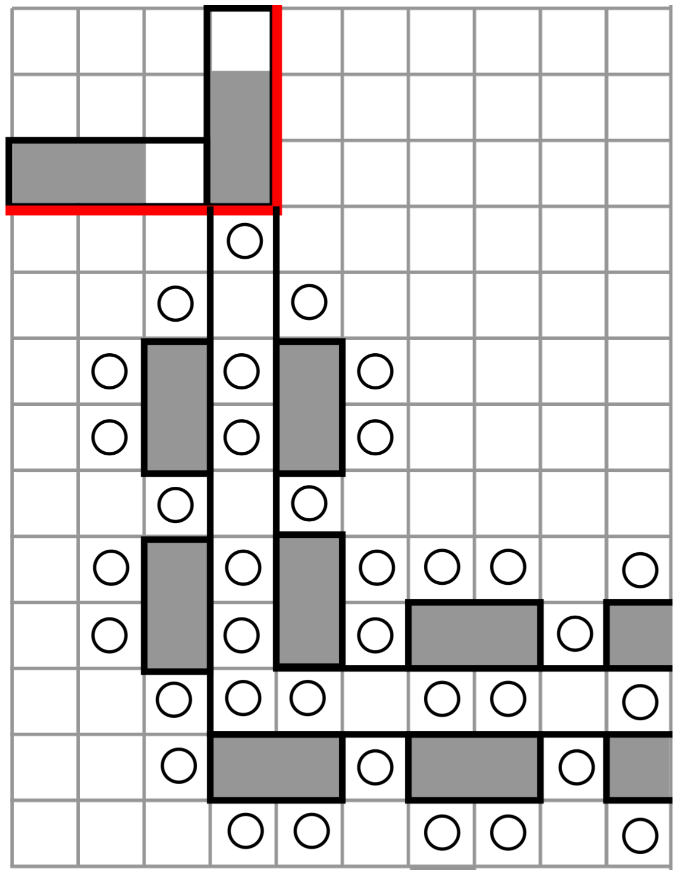
**Bend gadget:**



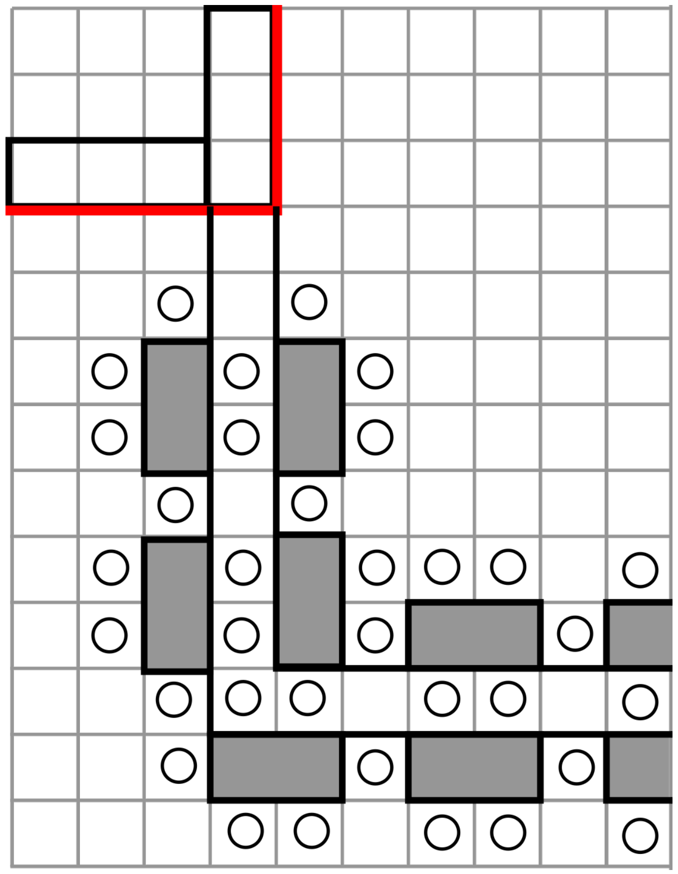
## Bend gadget:

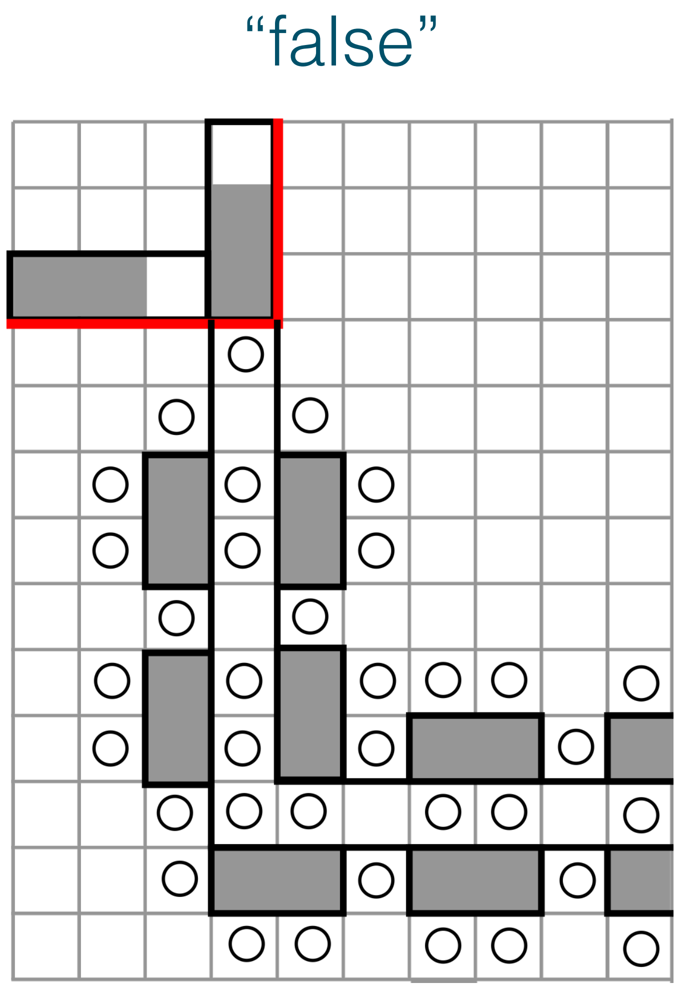


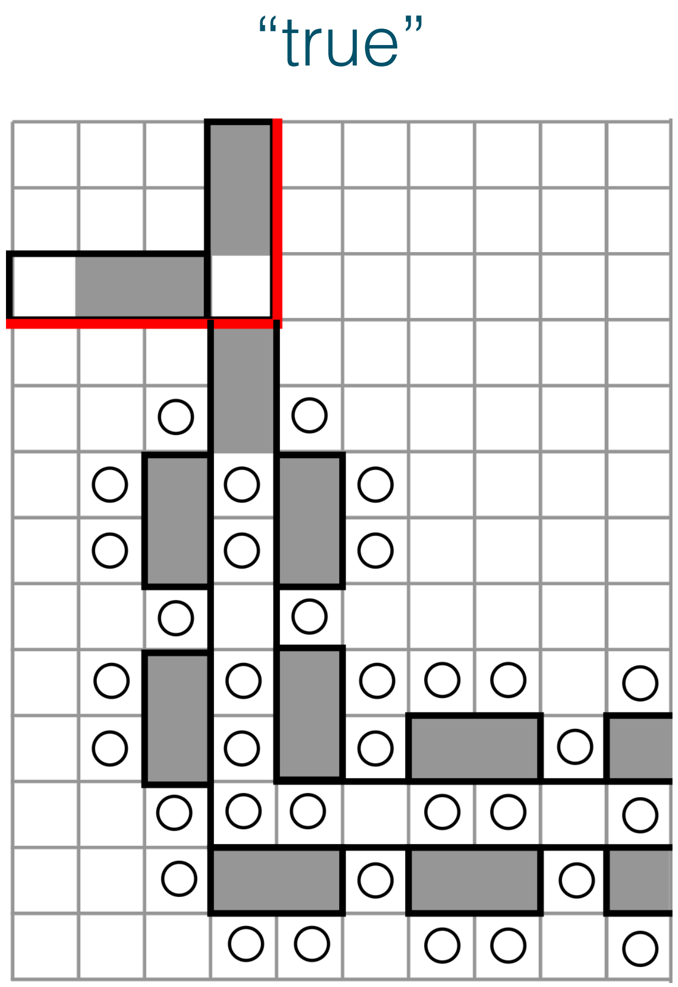
“false”



**Bend gadget:**

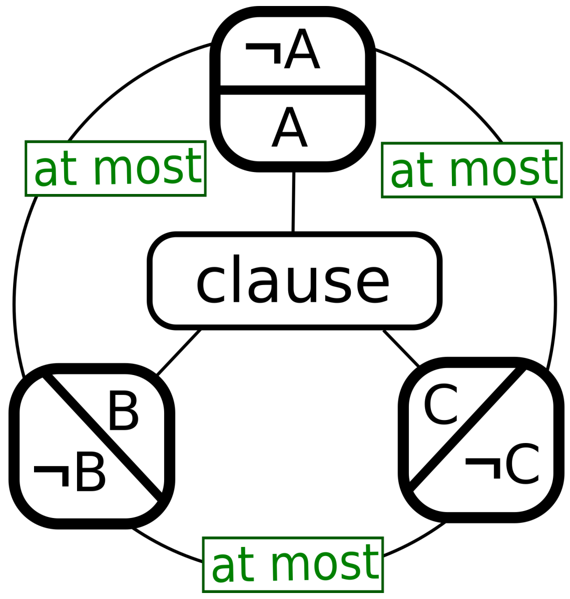




# Norinori

**1-in-3 gadget:**



# Norinori

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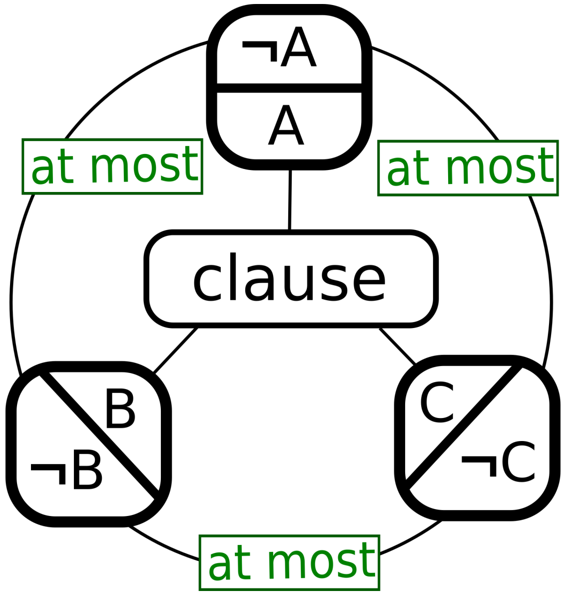
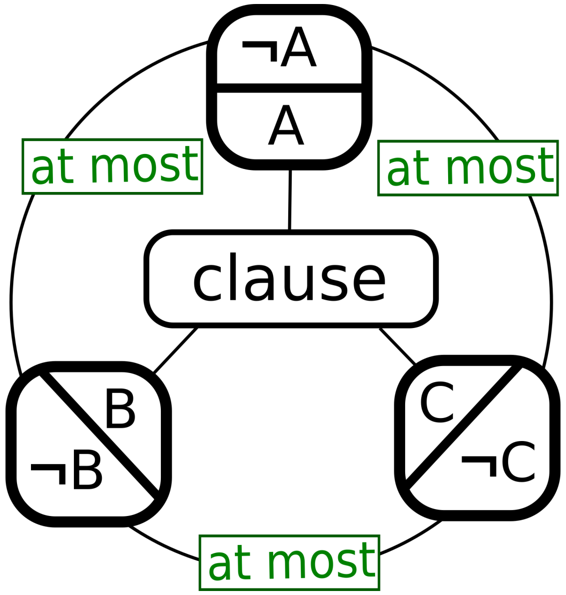



**At-most gadget** (connects corridors from two negated variables):

# Norinori

**1-in-3 gadget:**

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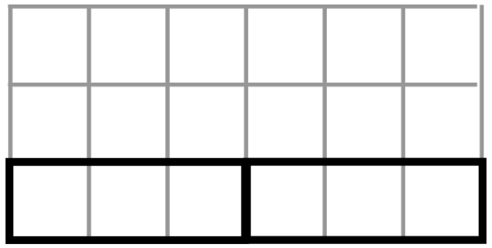

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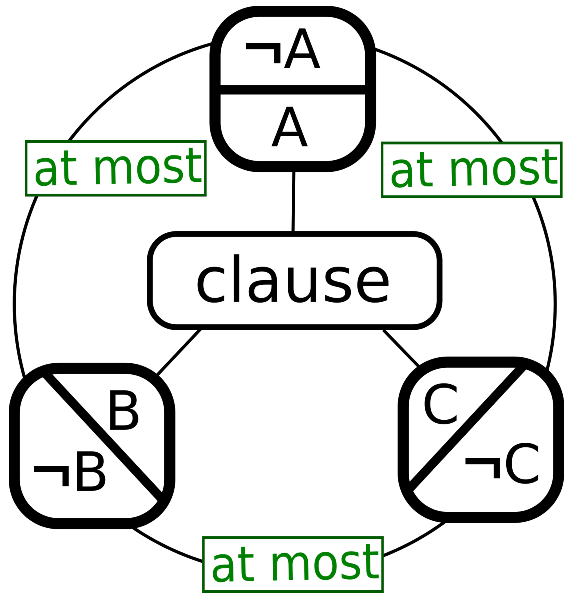


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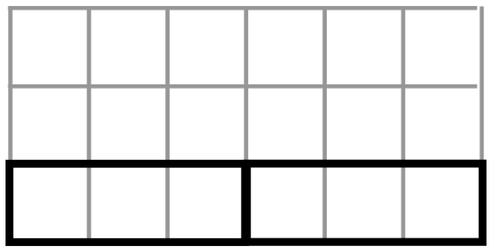

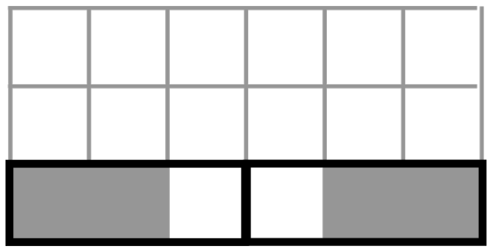

both "true"
(both variables "false")

# Norinori

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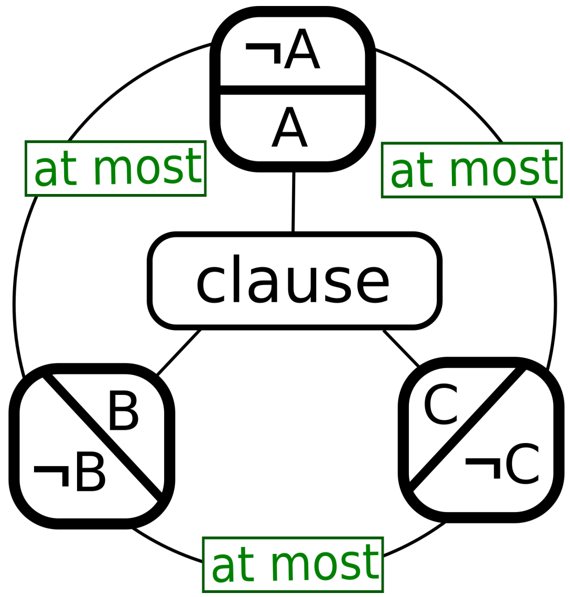


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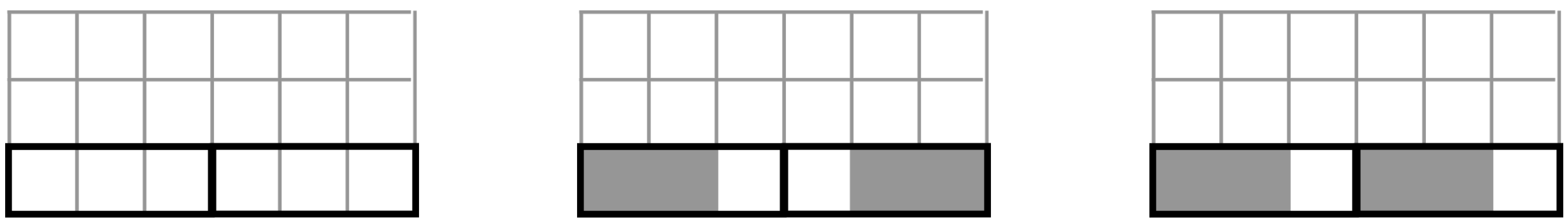
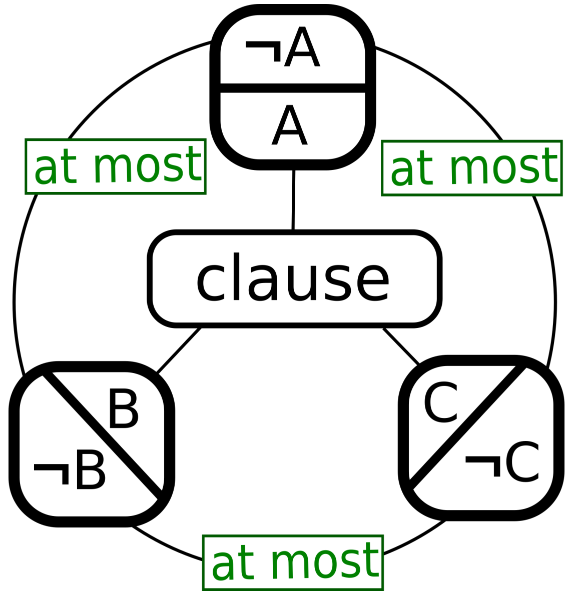


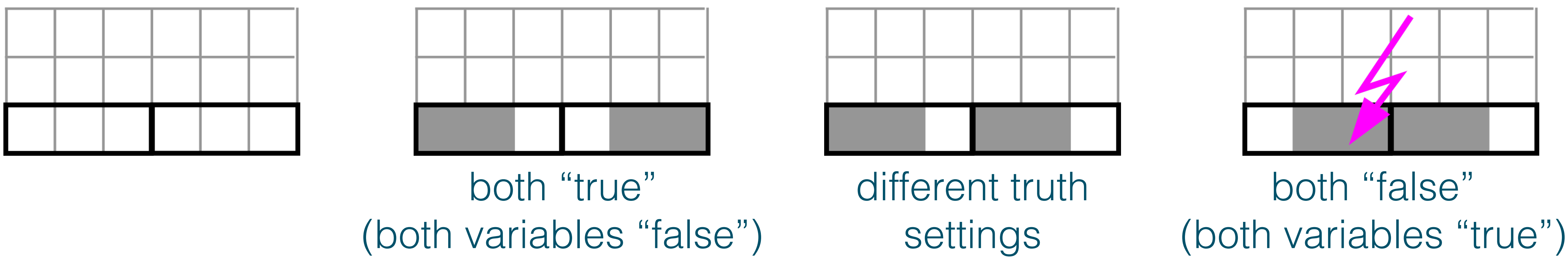
both “true”
(both variables “false”)

different truth
settings

**1-in-3 gadget:**




**At-most gadget** (connects corridors from two negated variables):



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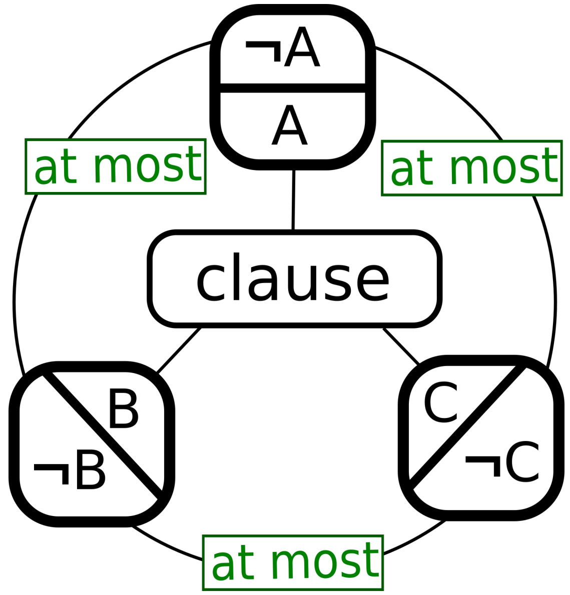


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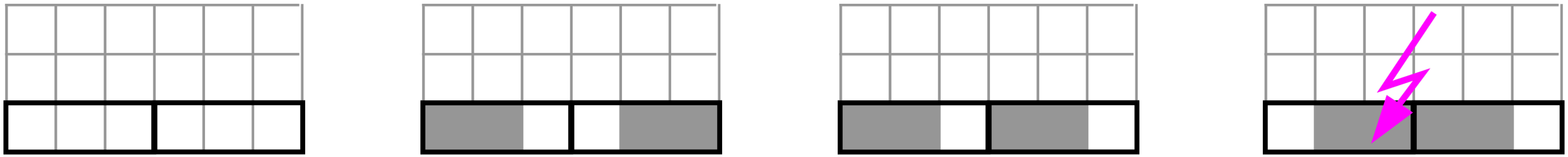
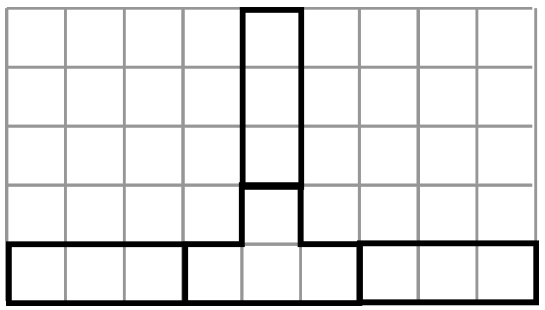


both "true"
(both variables "false")

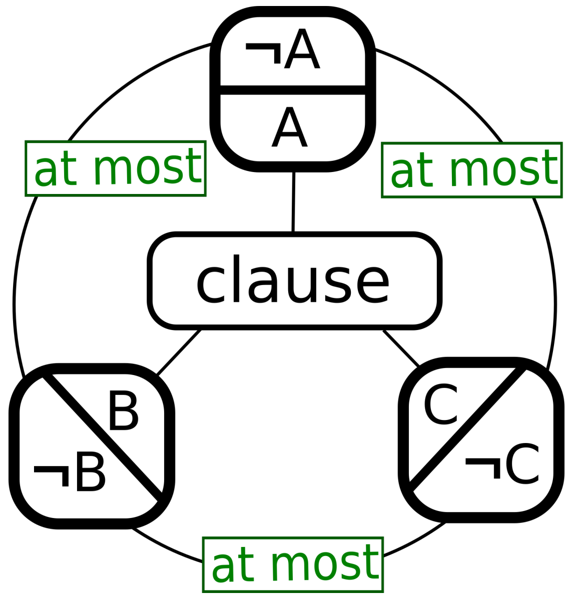
different truth
settings

both "false"
(both variables "true")

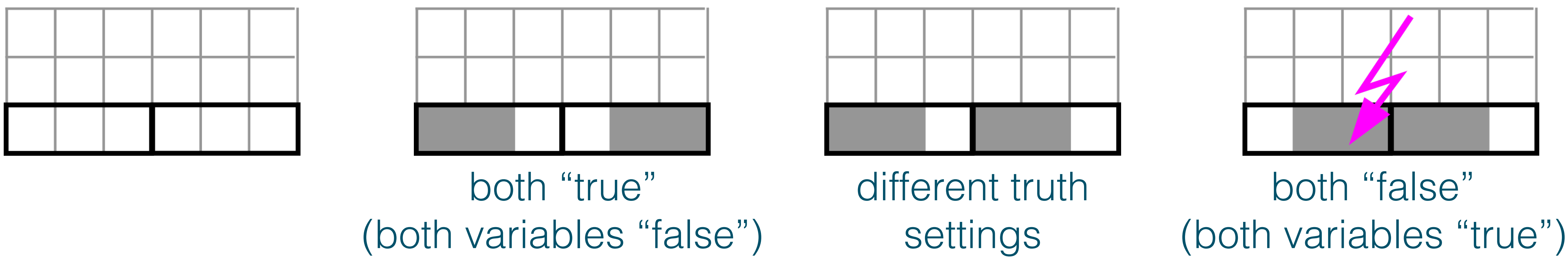
**Clause gadget:**



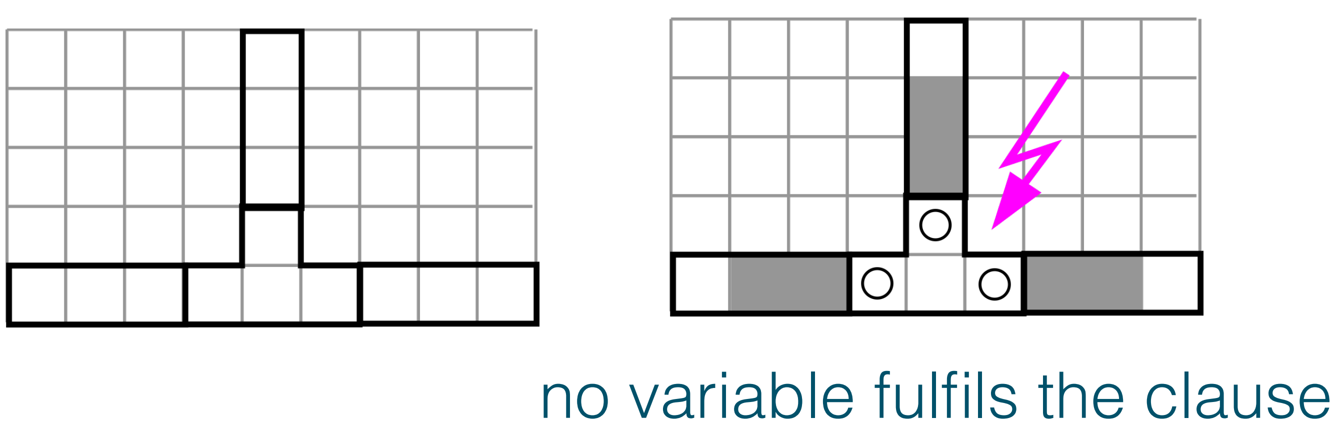
**1-in-3 gadget:**




**At-most gadget** (connects corridors from two negated variables):




**Clause gadget:**



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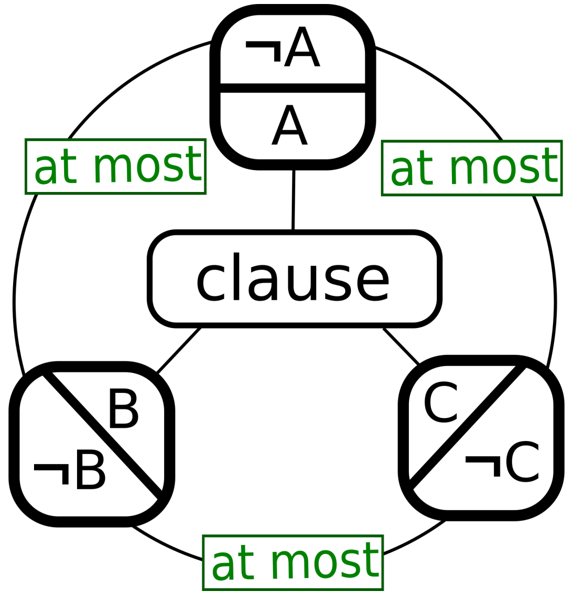


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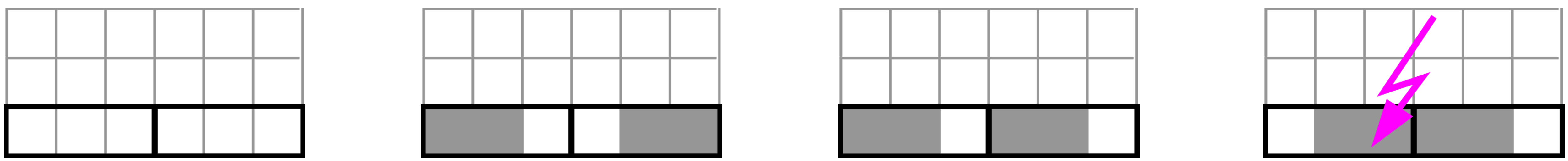
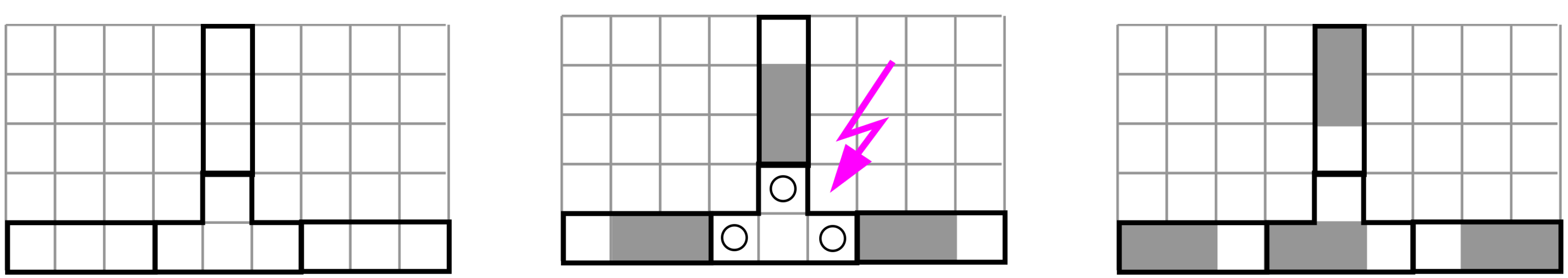


both “true”
(both variables “false”)

different truth
settings

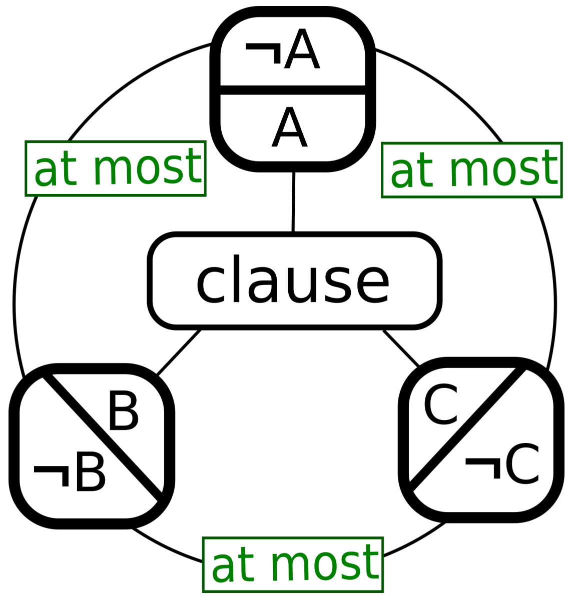
both “false”
(both variables “true”)

**Clause gadget:**



no variable fulfils the clause

**1-in-3 gadget:**

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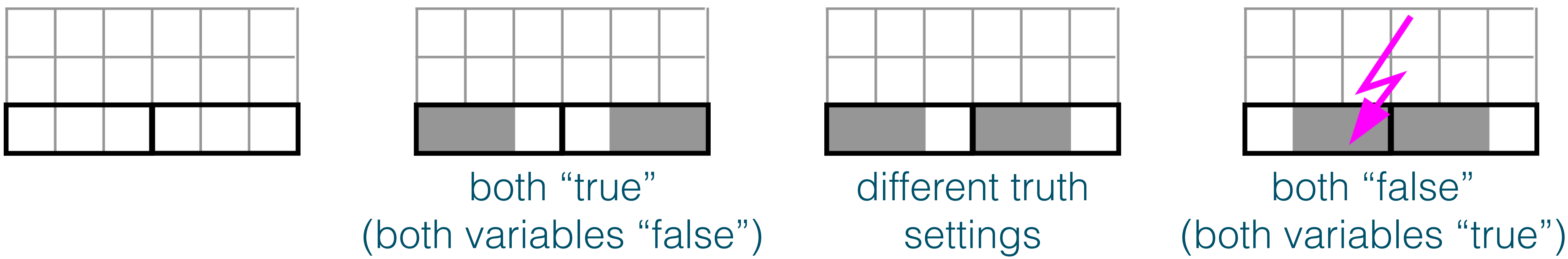


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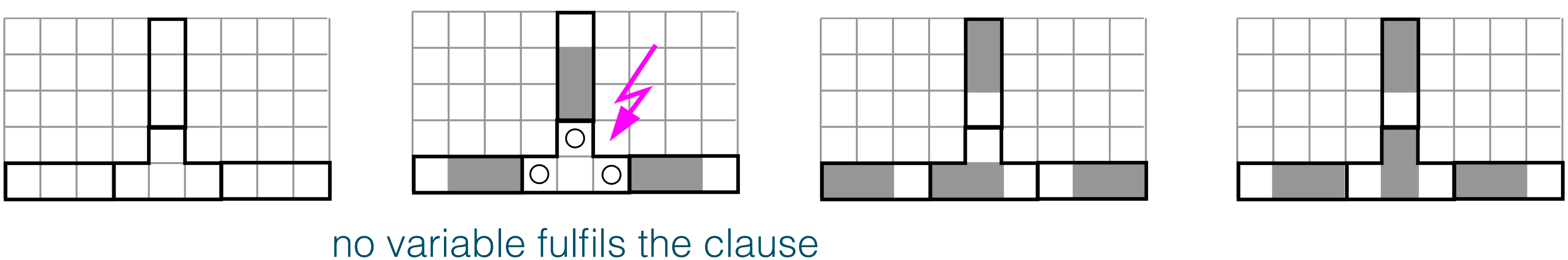
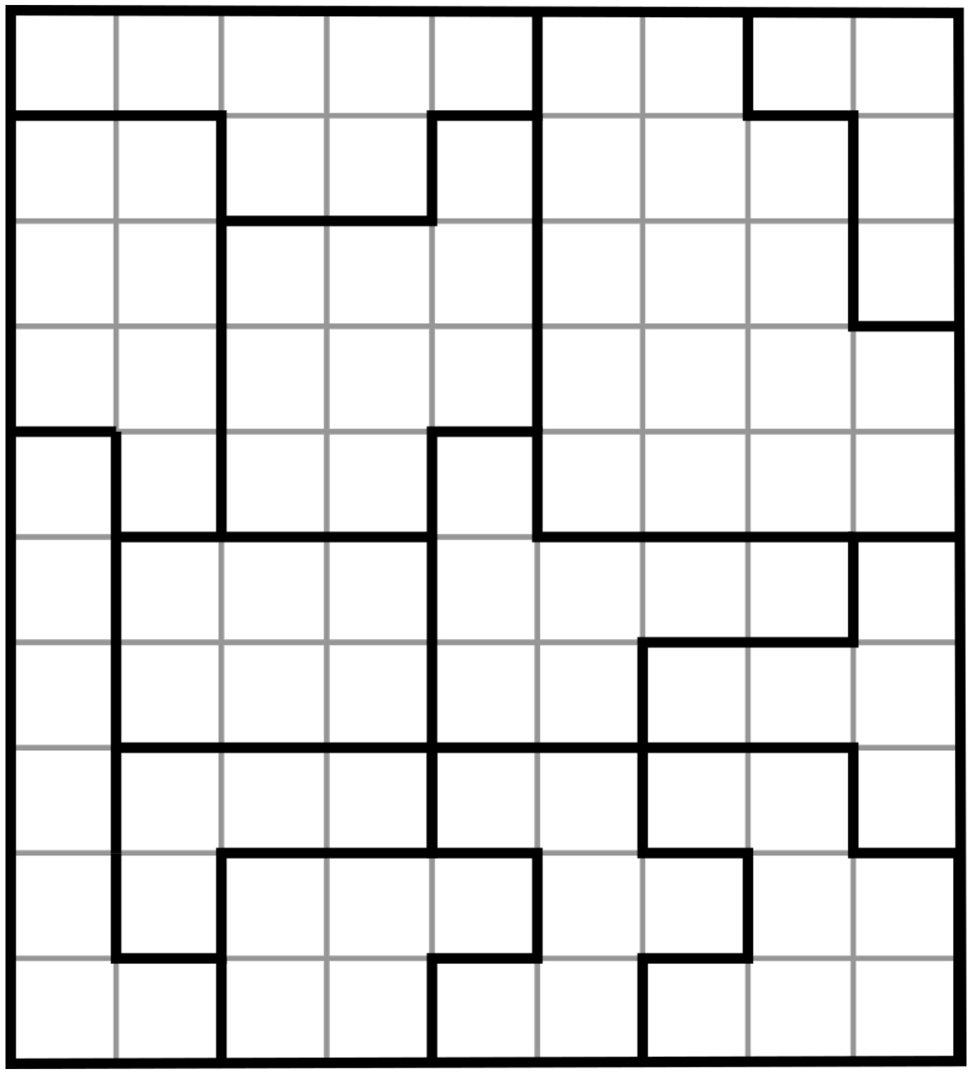


# LITS

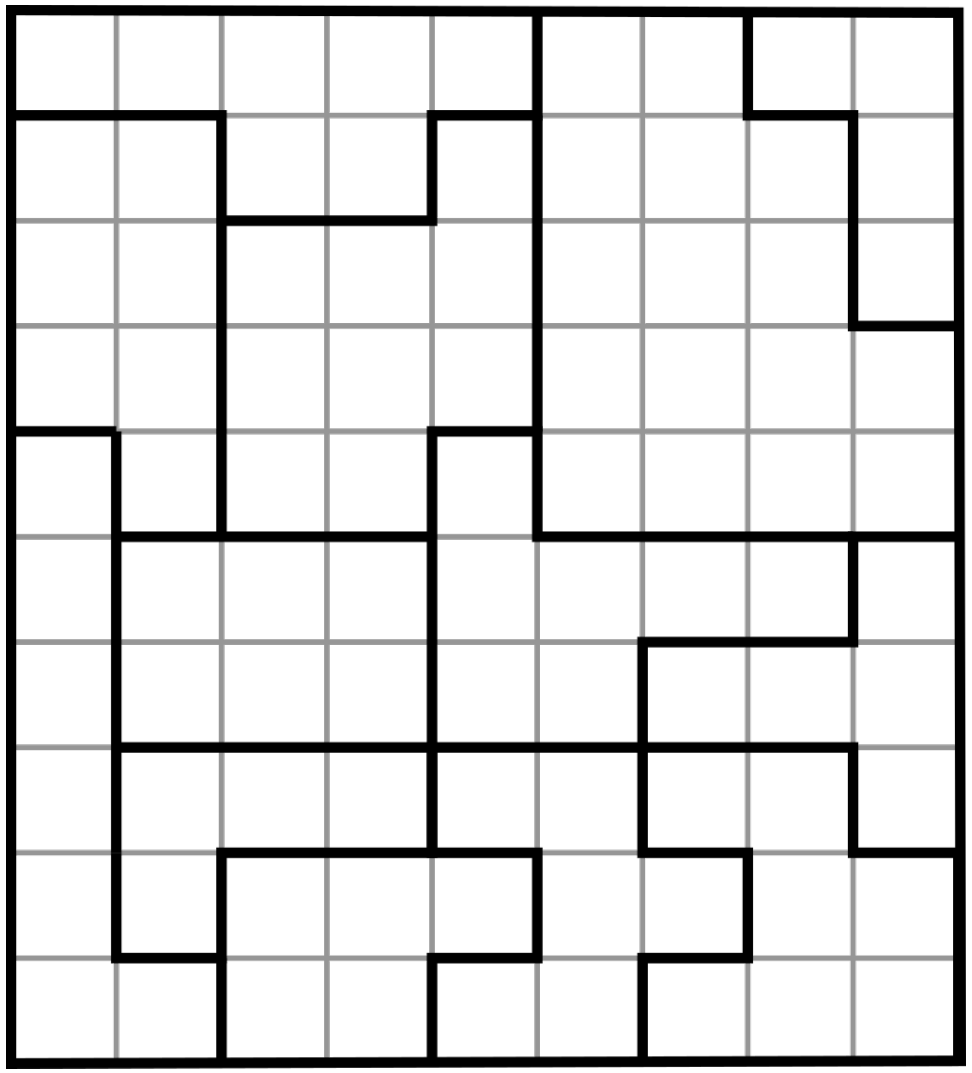
Place black squares in the polyominoes, such that the final board satisfies



# LITS

Place black squares in the polyominoes, such that the final board satisfies

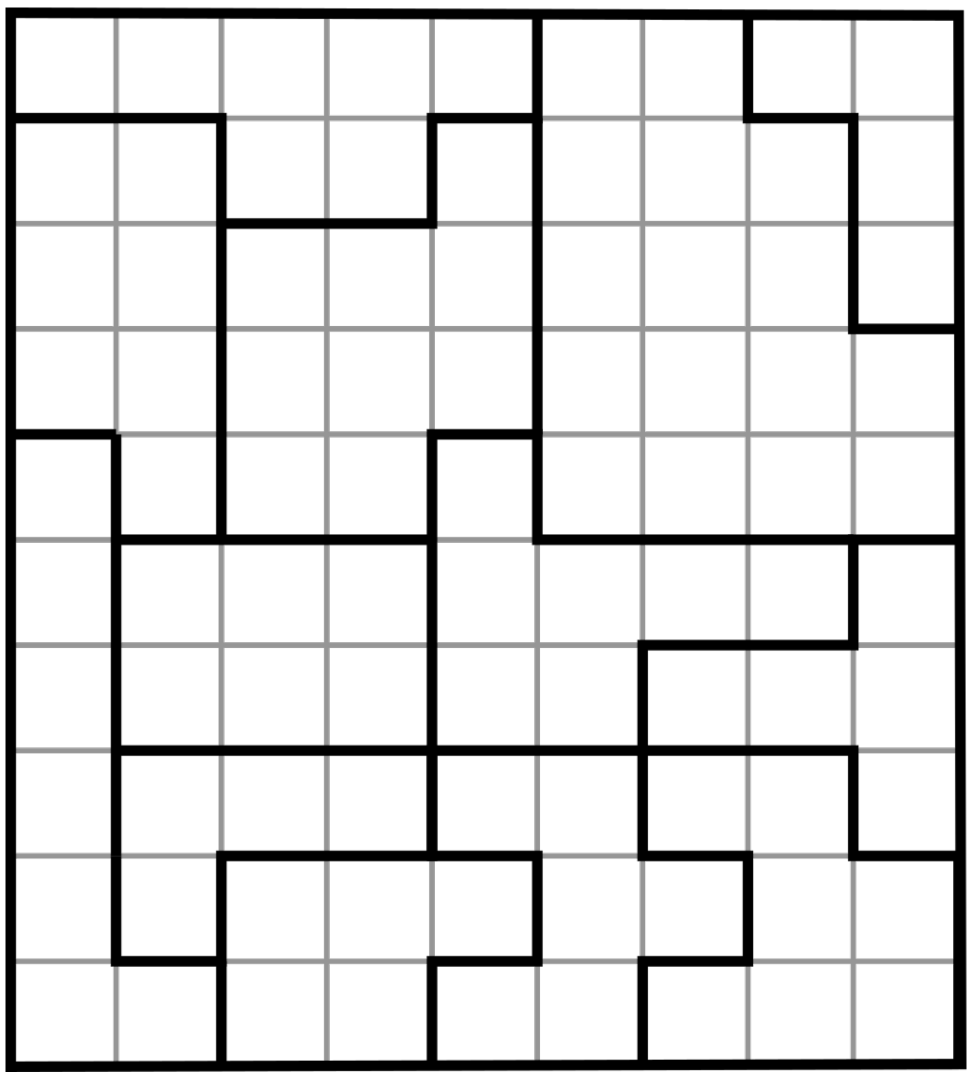
- The black squares form a connected polyomino.



# LITS

Place black squares in the polyominoes, such that the final board satisfies

- The black squares form a connected polyomino.
- Each polyomino region contains a connected black tetromino.



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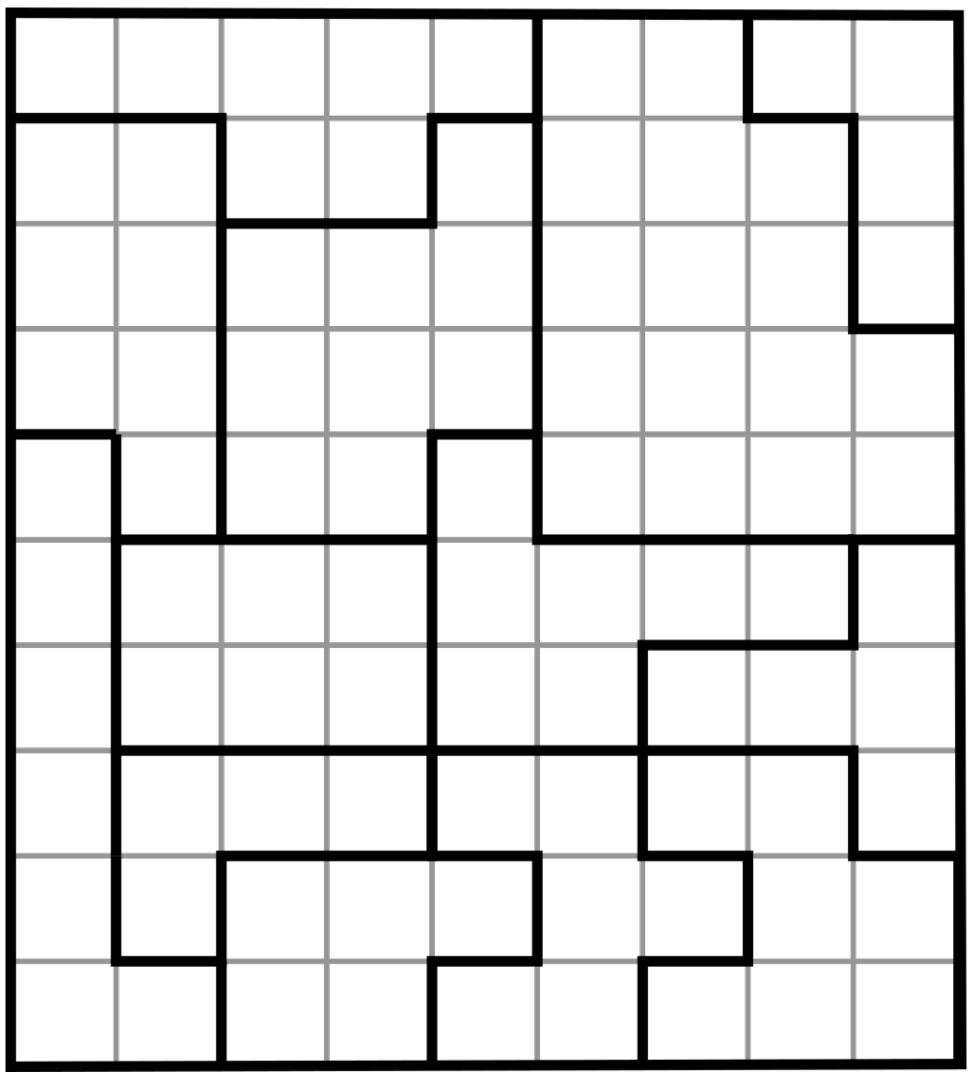
- No two congruent tetrominoes are adjacent.



# LITS

Place black squares in the polyominoes, such that the final board satisfies

- The black squares form a connected polyomino.
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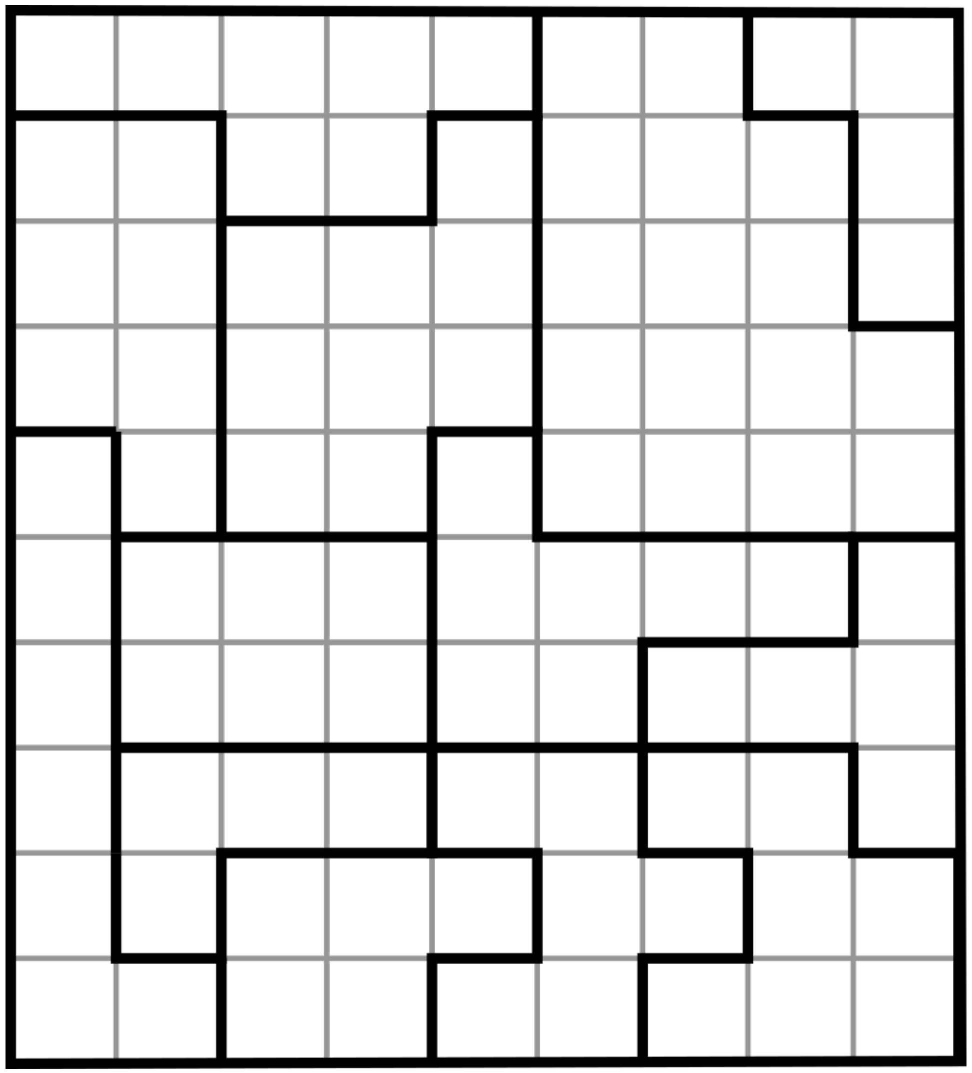
- Black squares may not build 2x2 squares.



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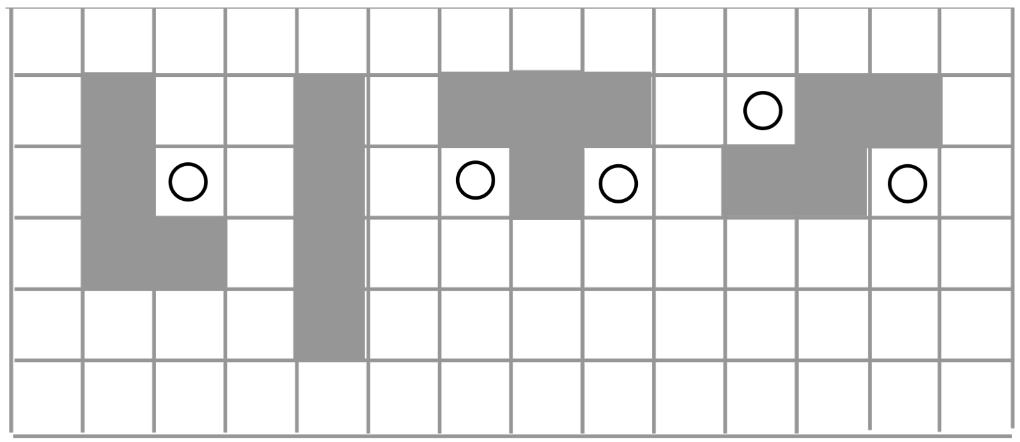

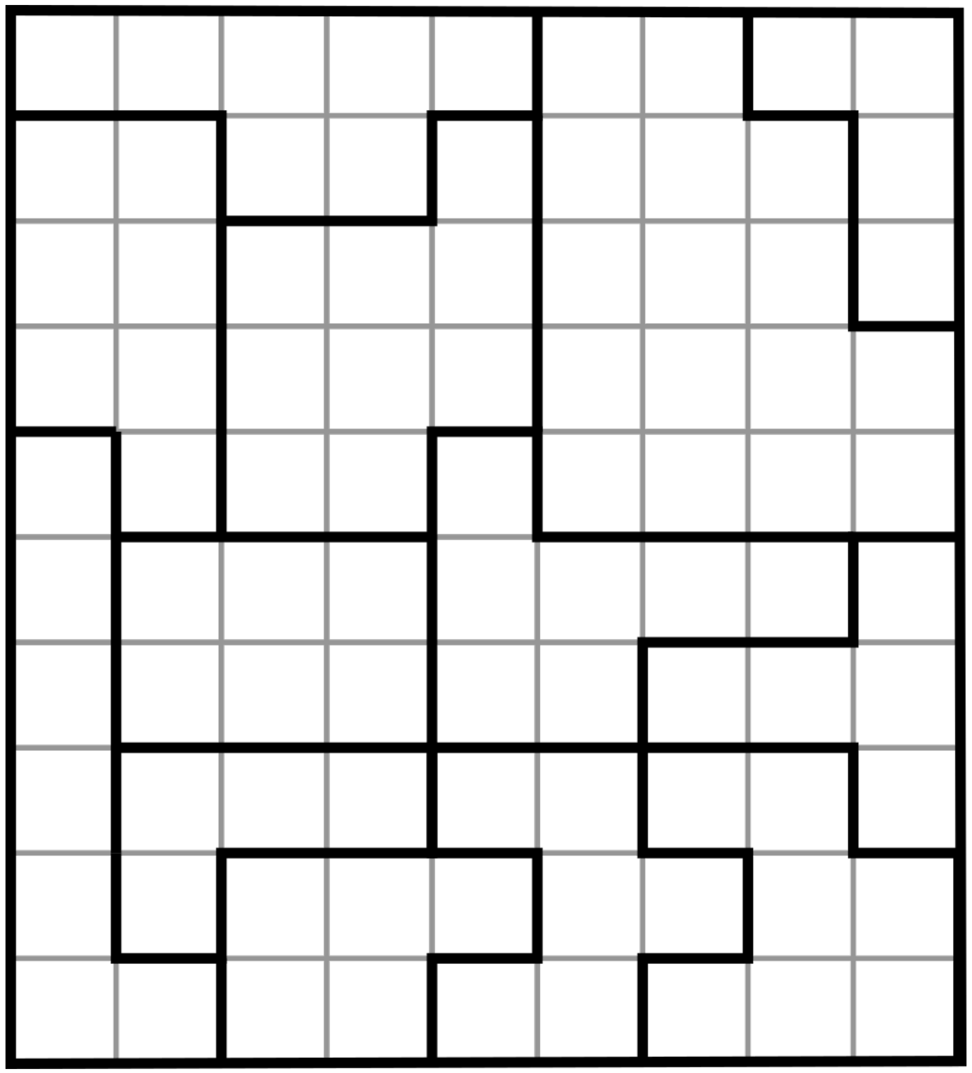

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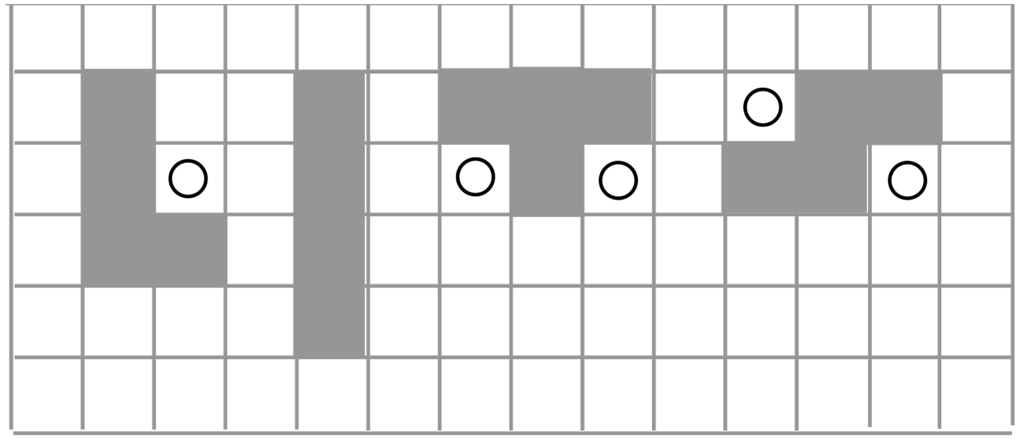

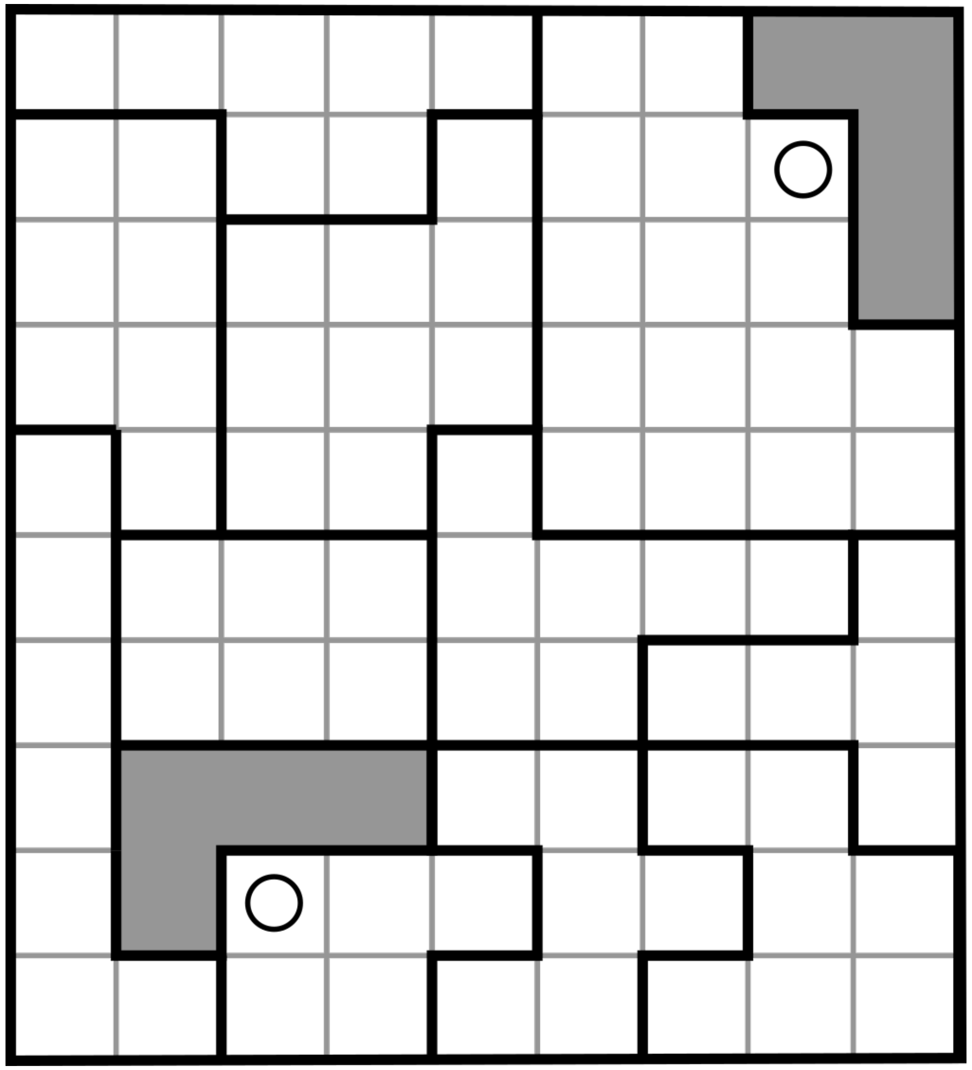

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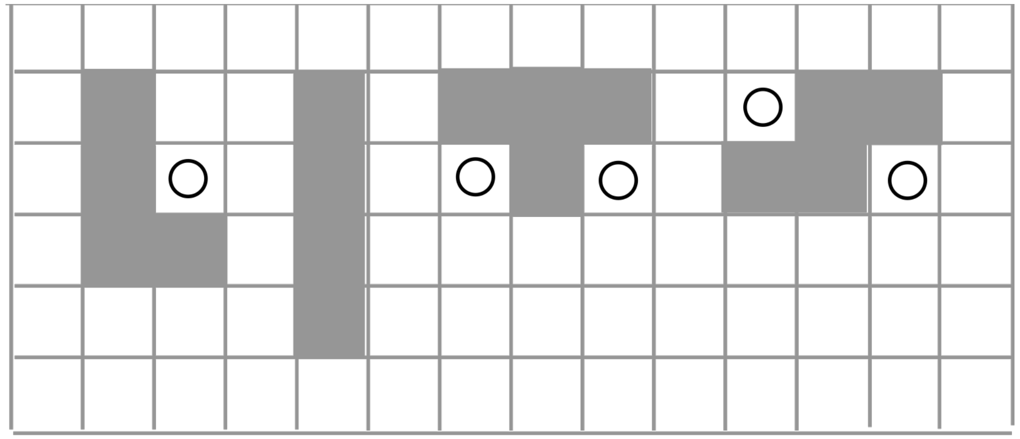

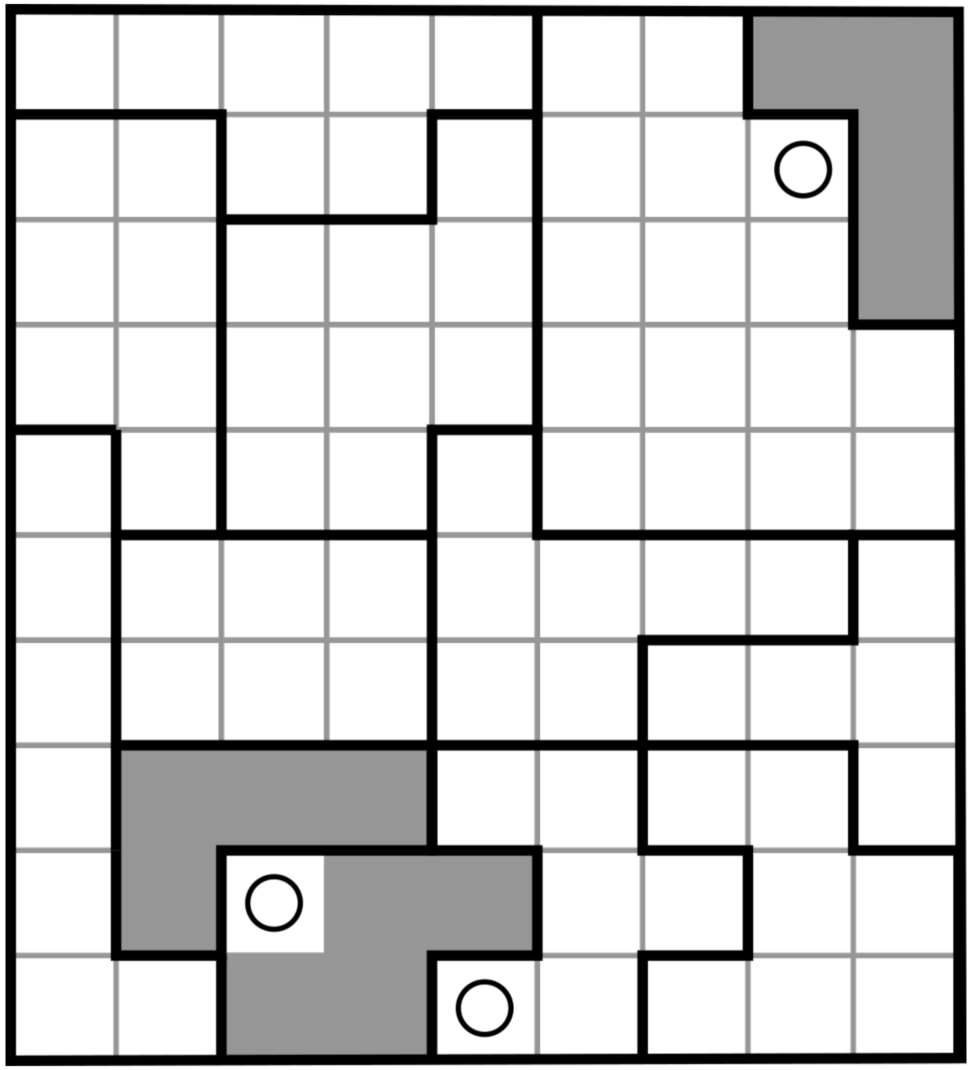

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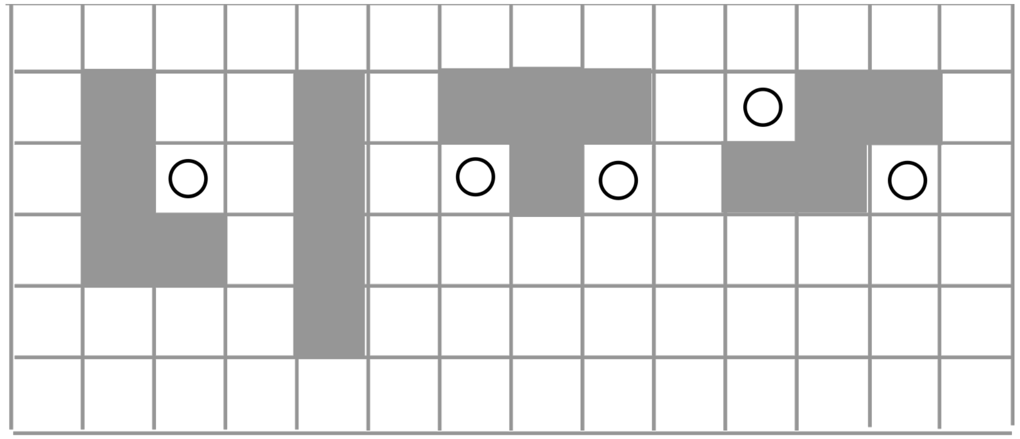

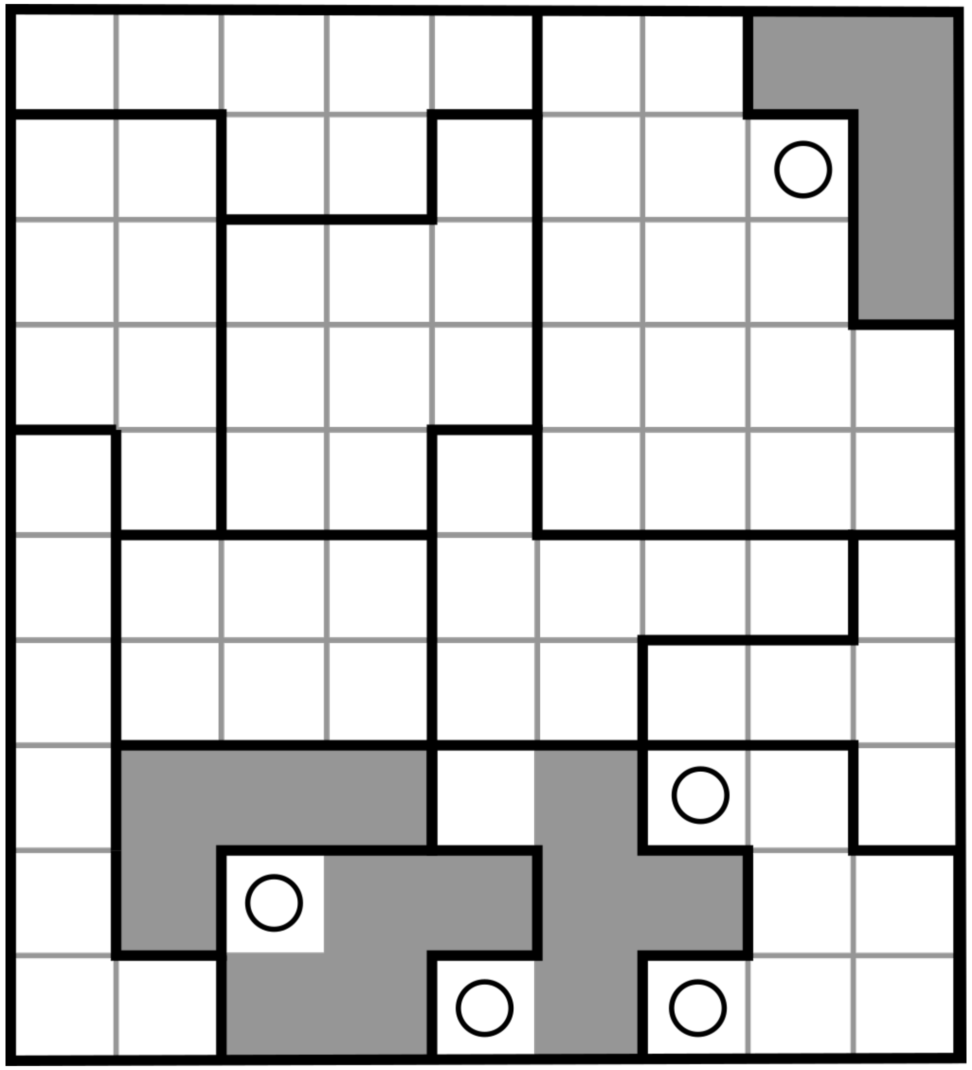

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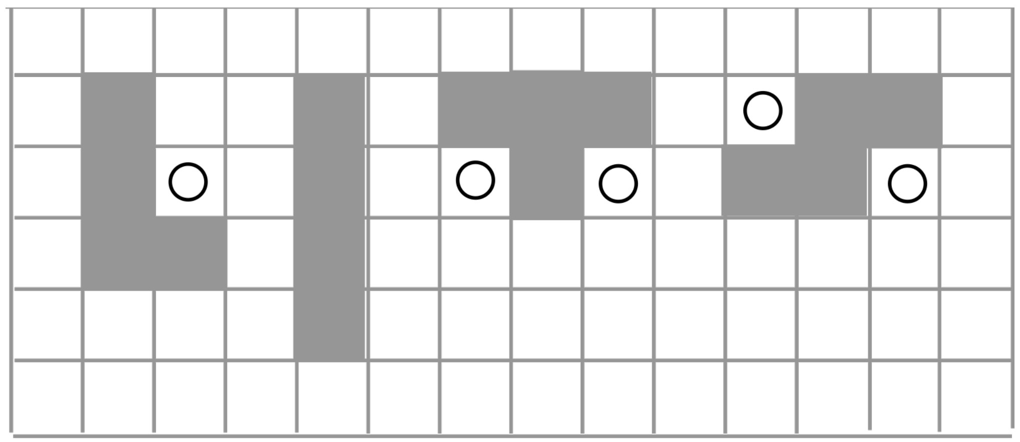

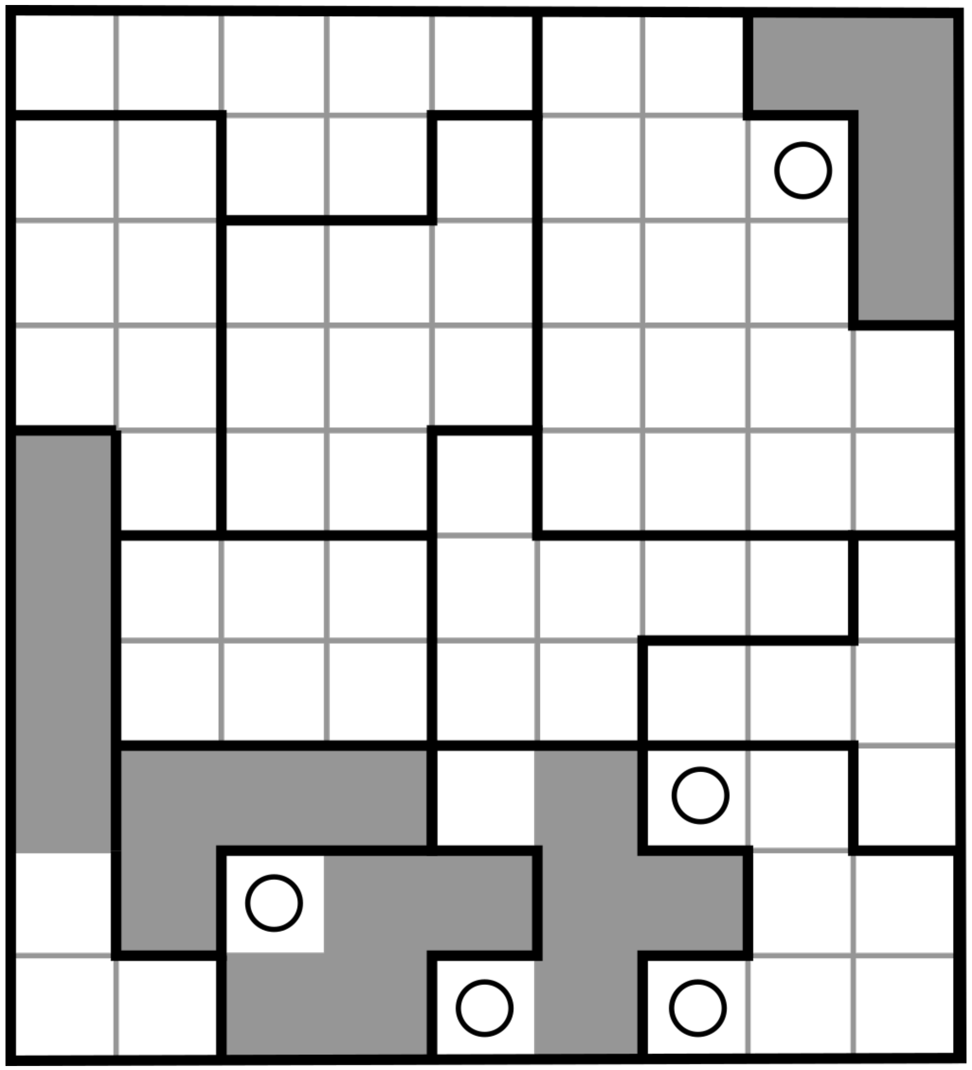

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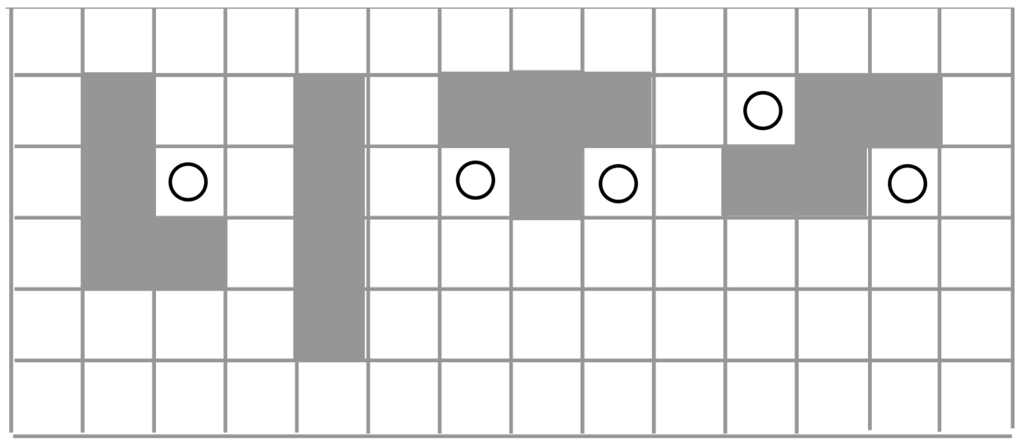

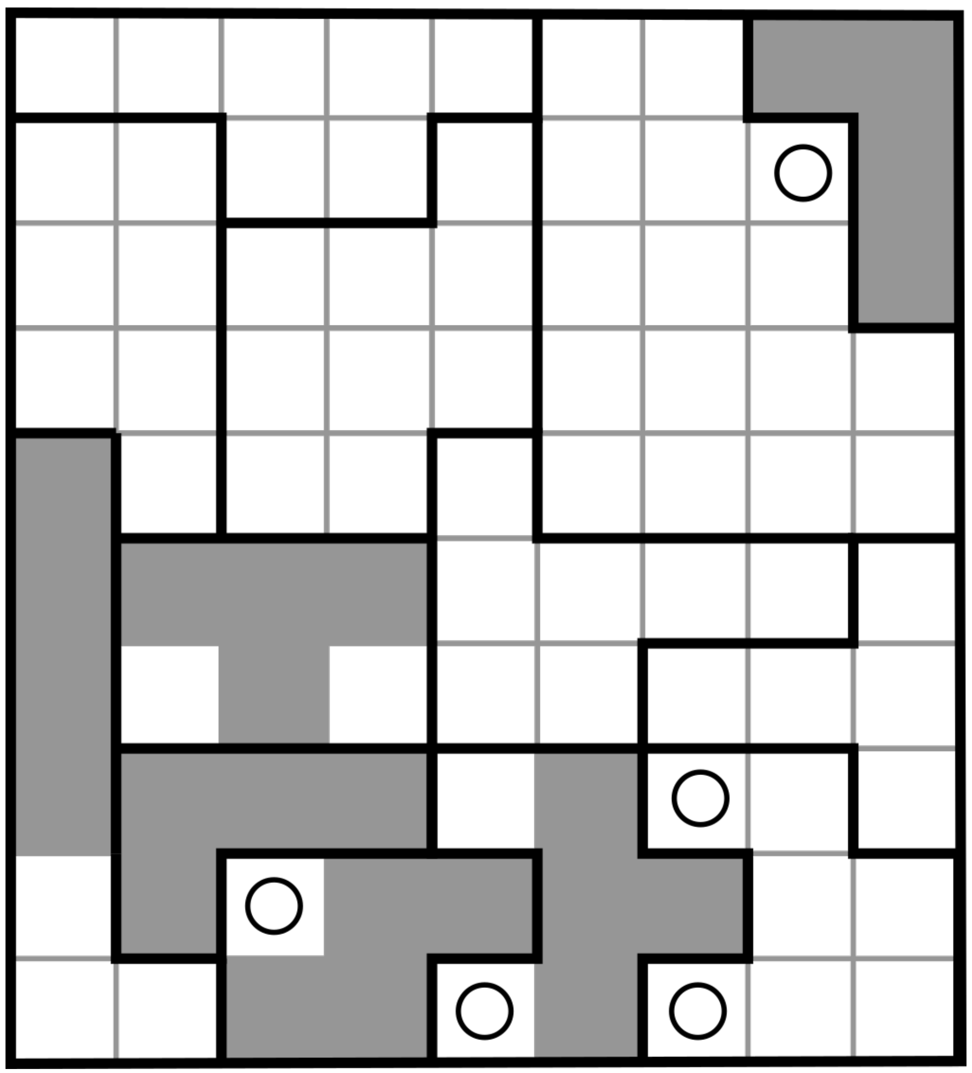

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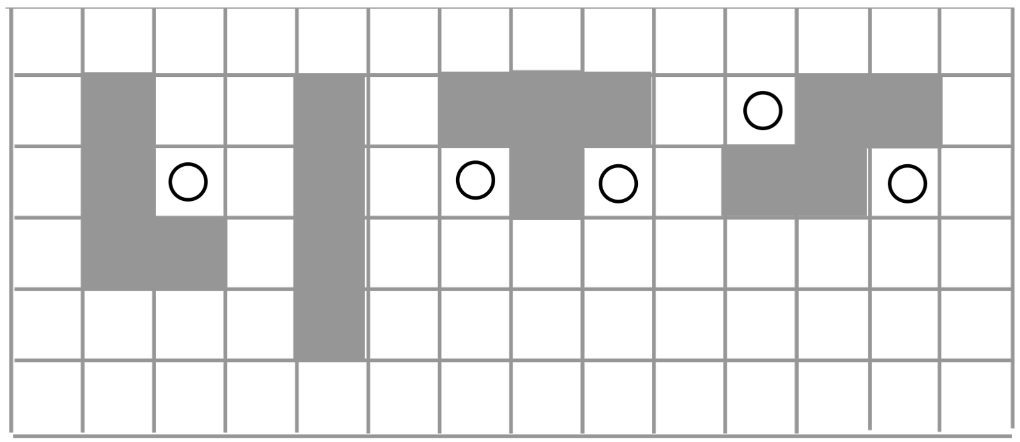

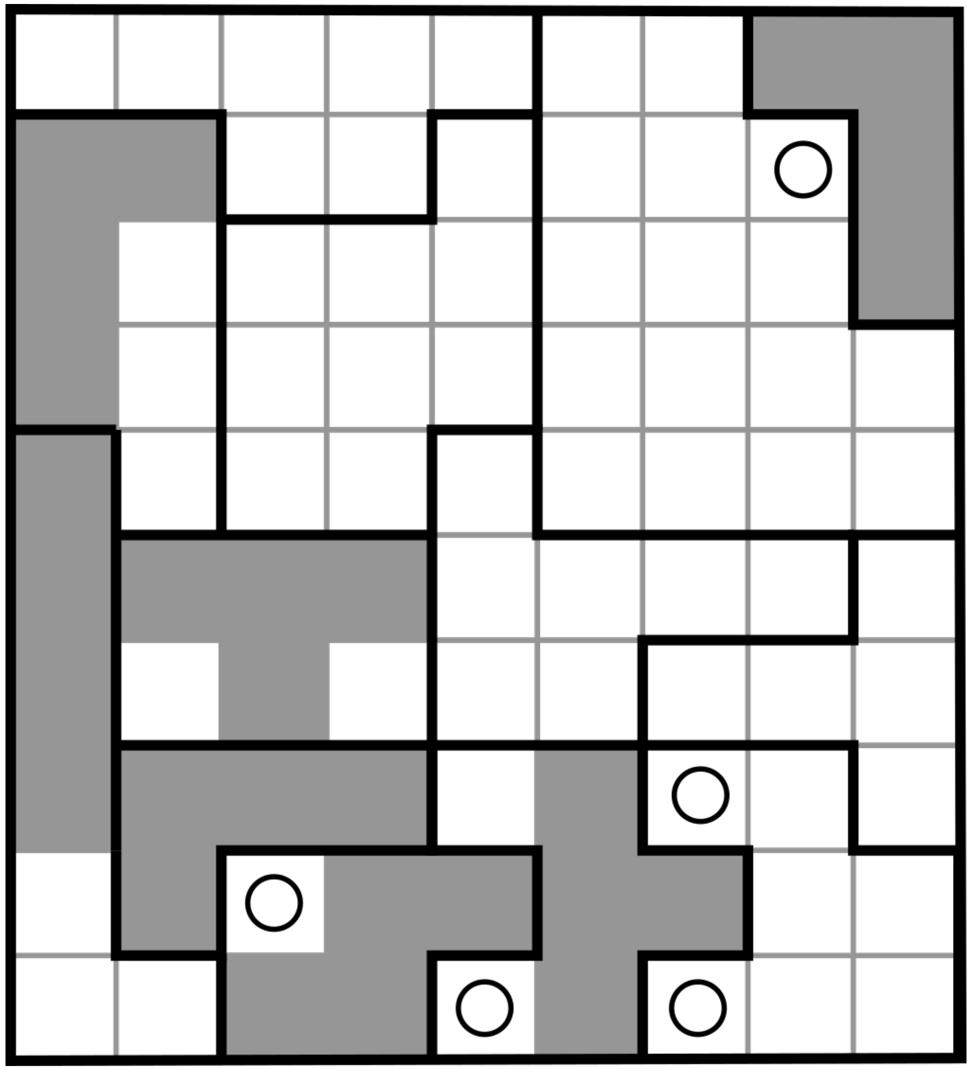

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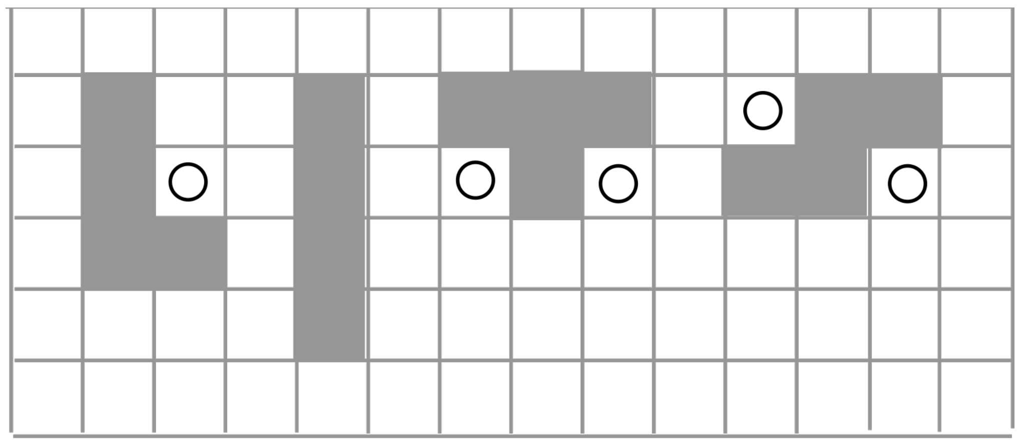

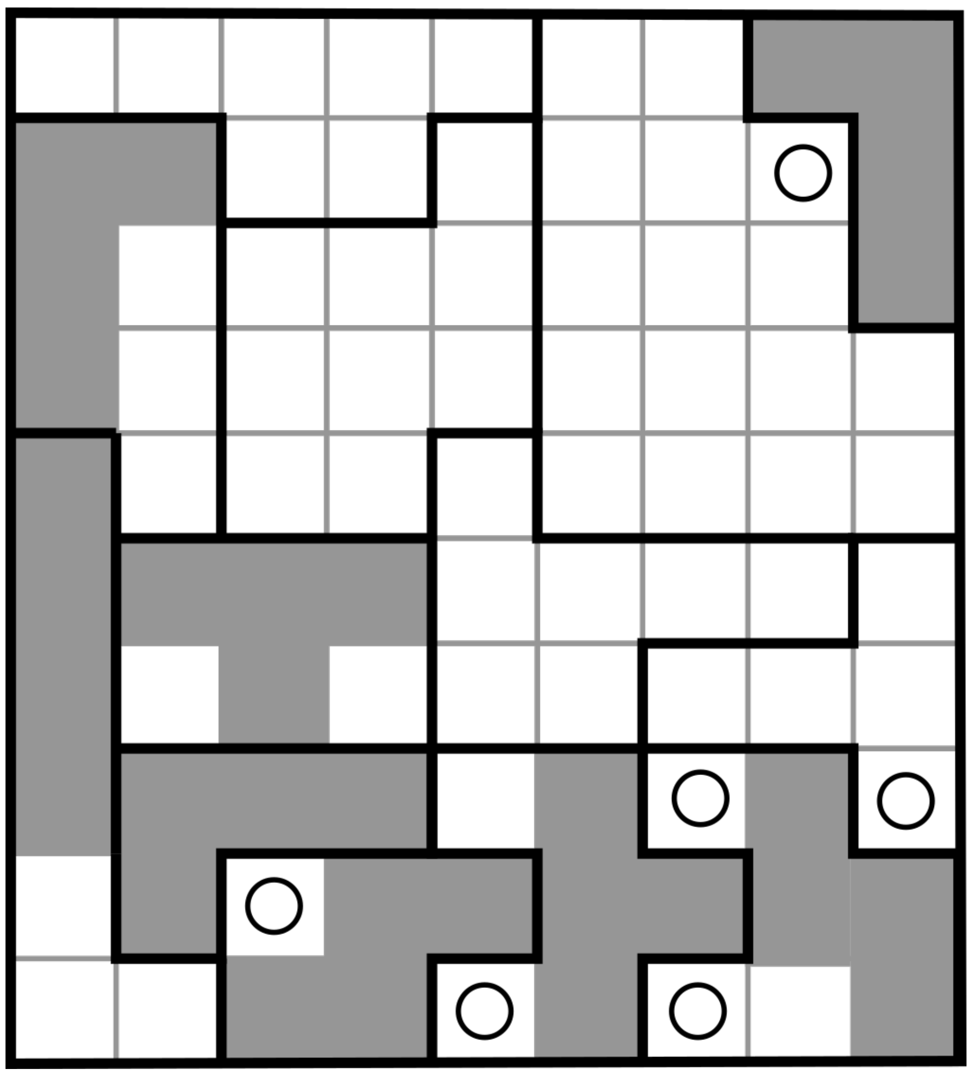

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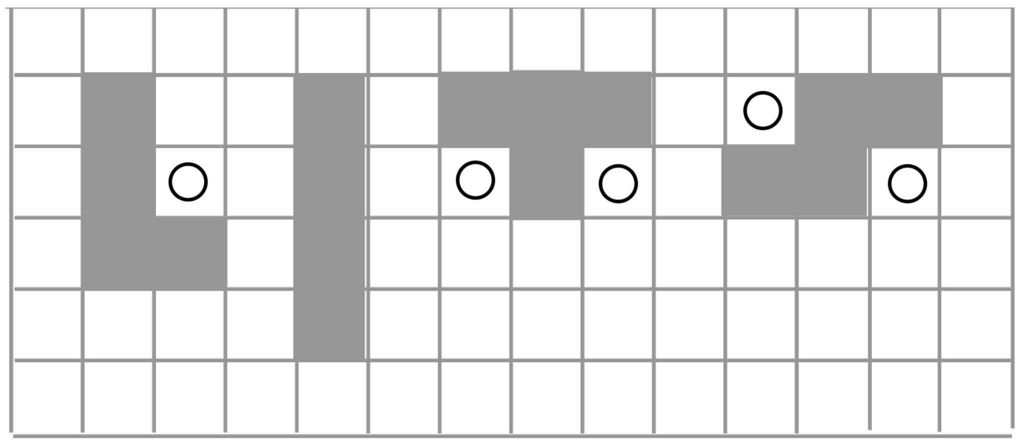

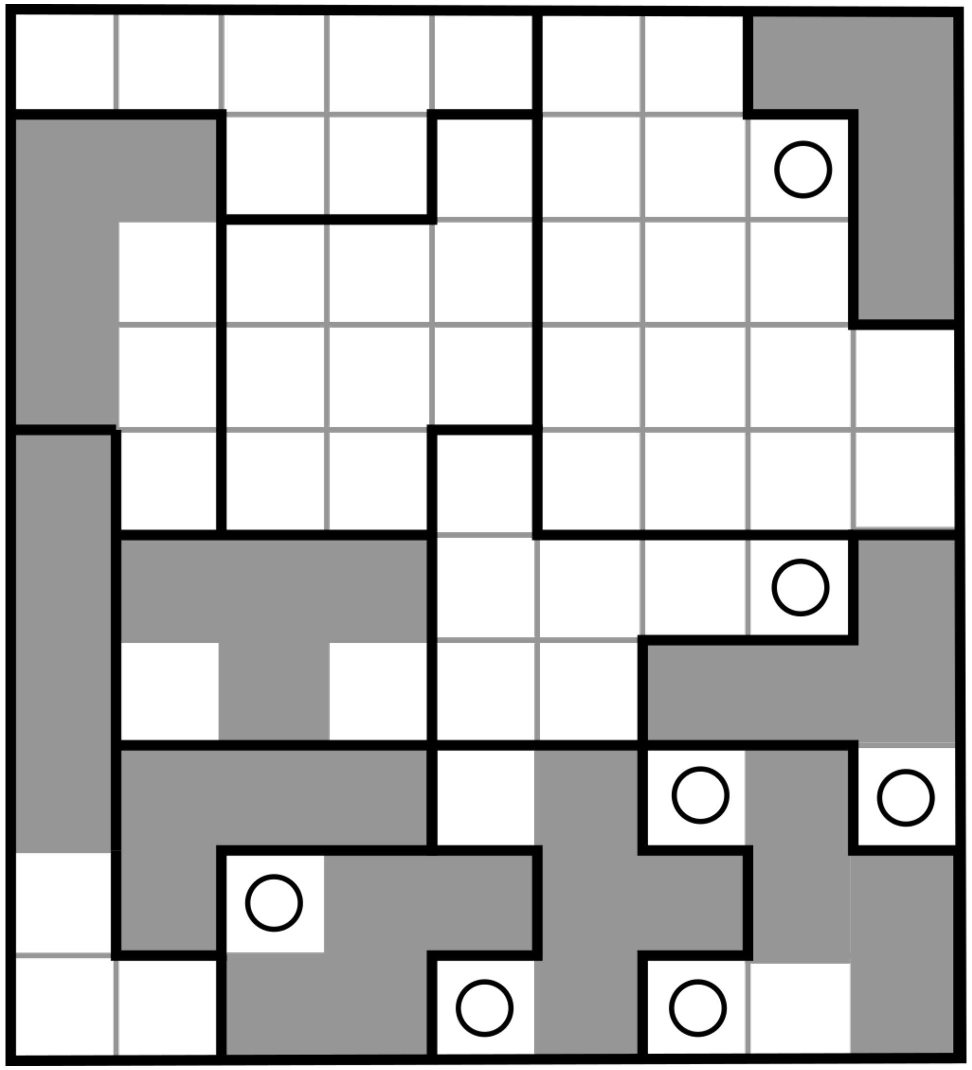

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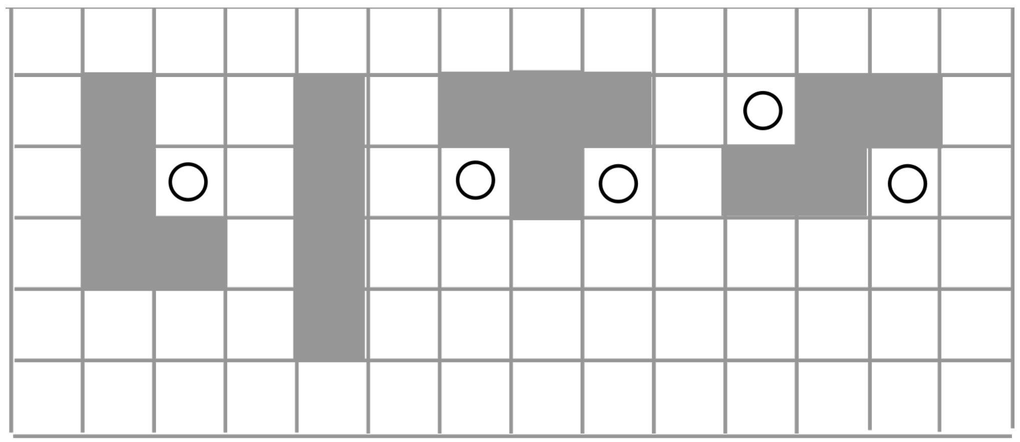

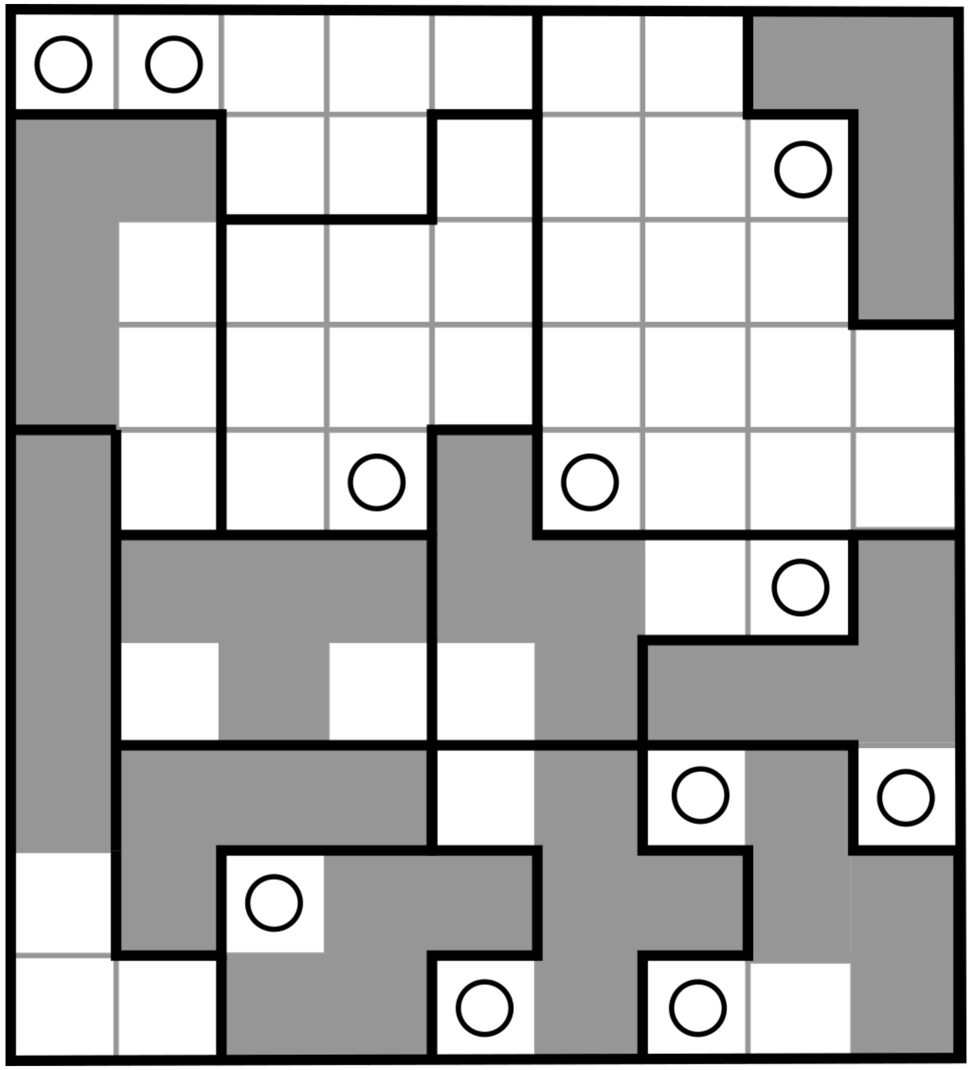

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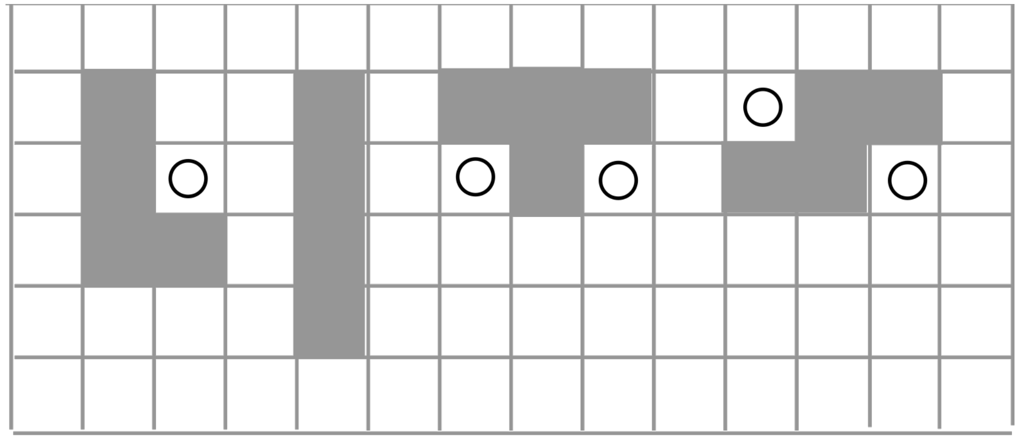

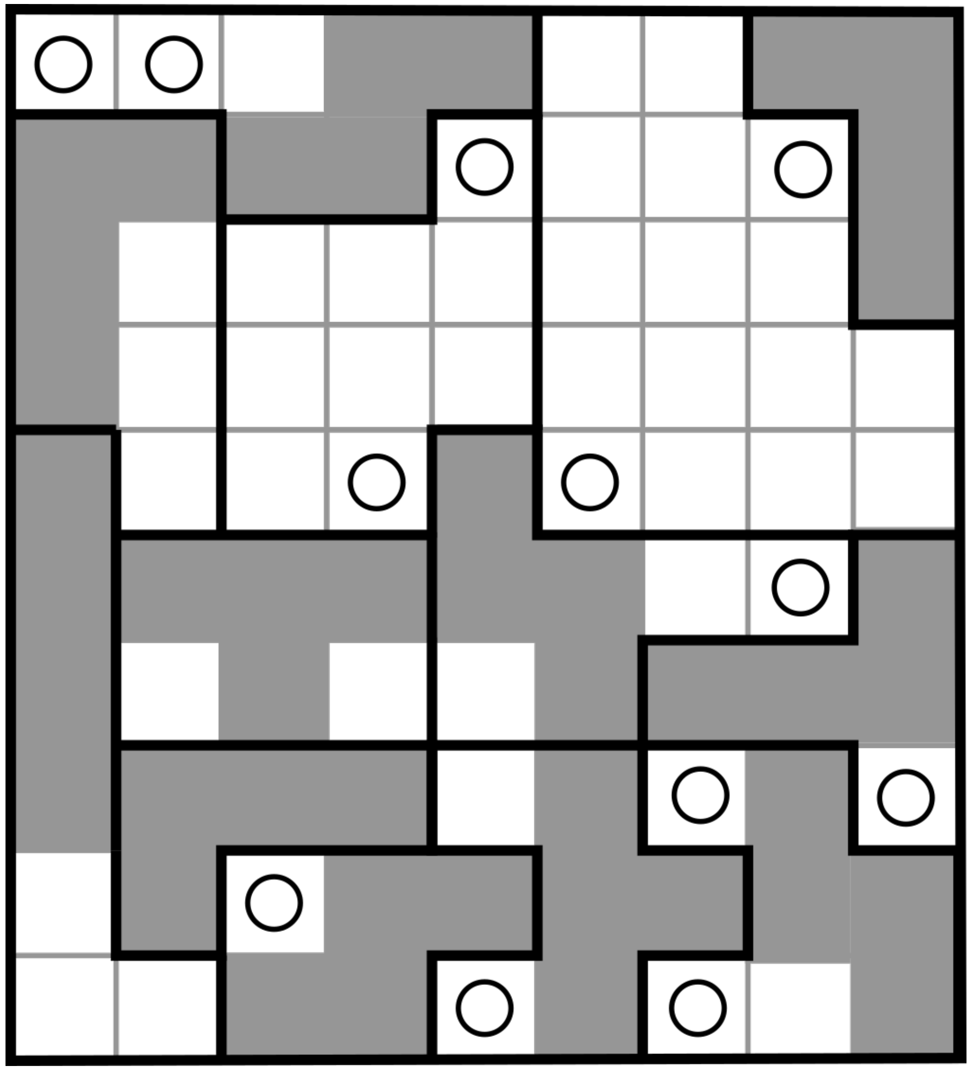

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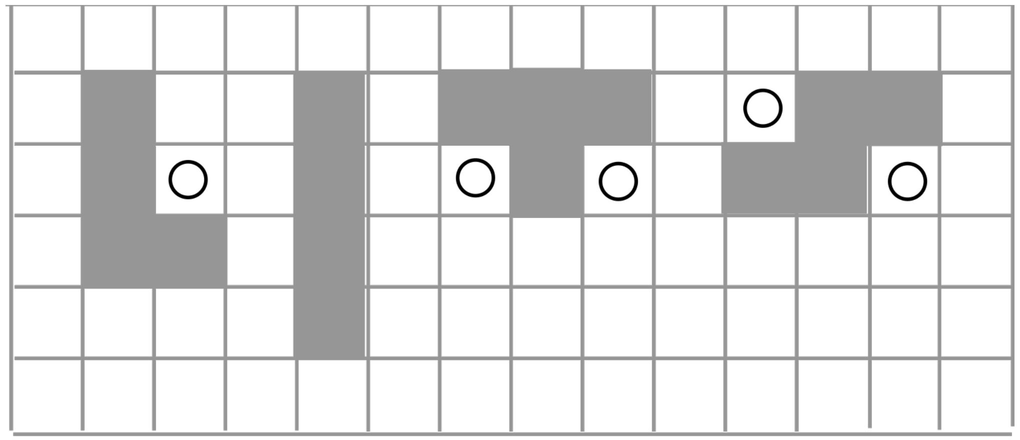
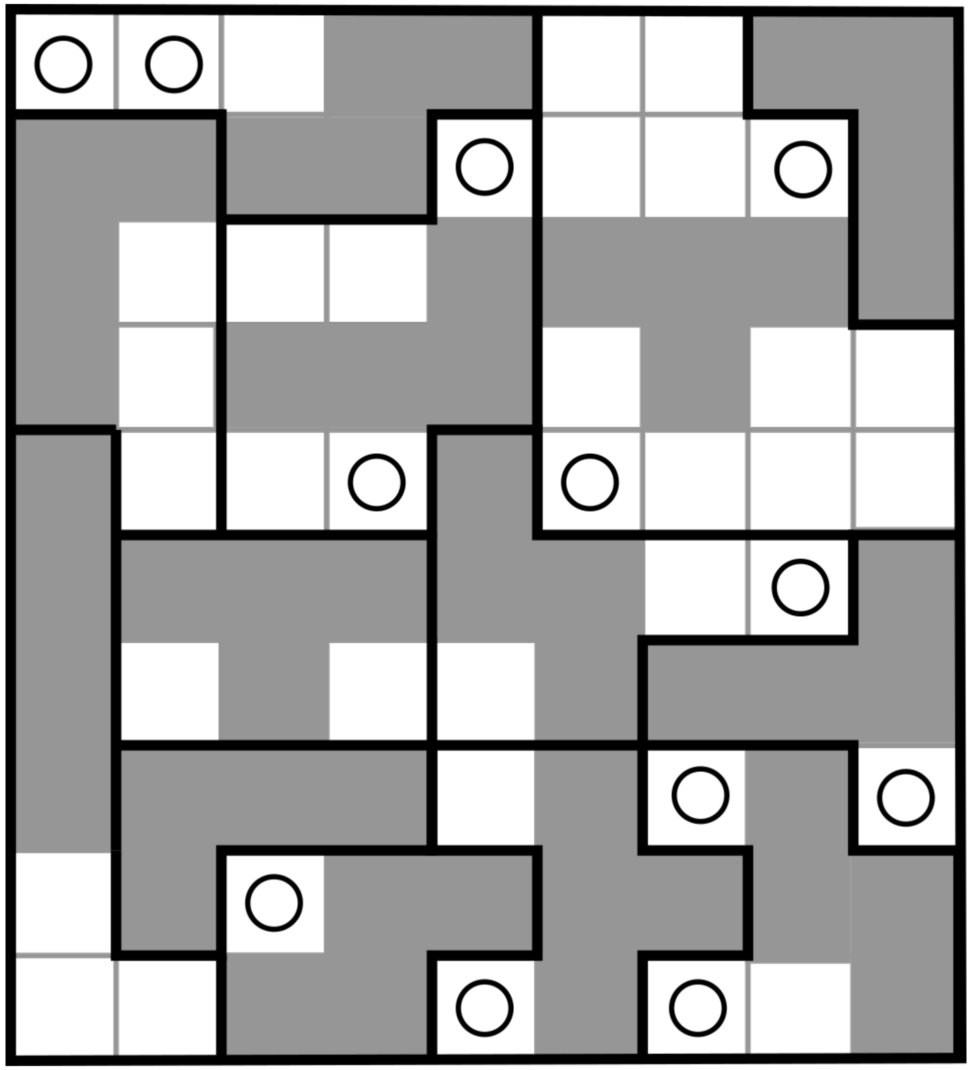
- No two congruent tetrominoes are adjacent.
- Black squares may not build 2x2 squares.





# LITS

**Theorem 2:**
Determining if a LITS board is solvable is NP-complete and counting the number of solutions is #P-complete.

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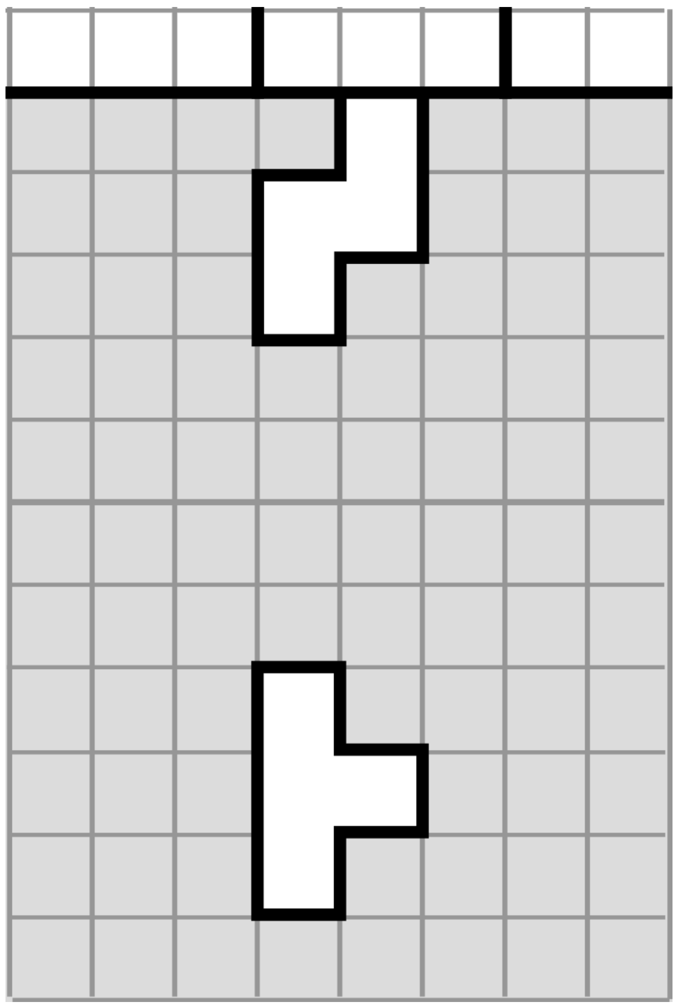
**Face gadget:**



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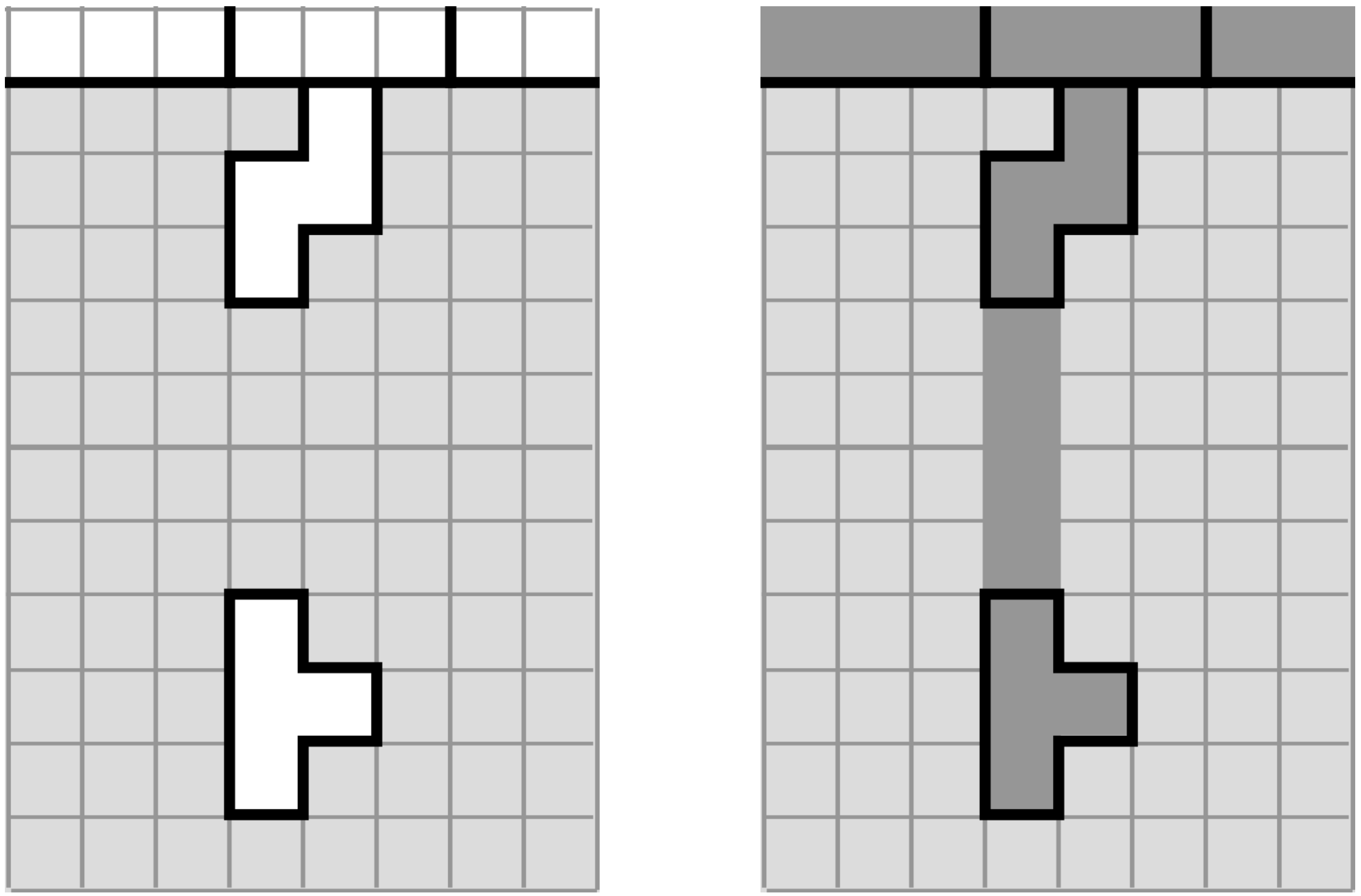
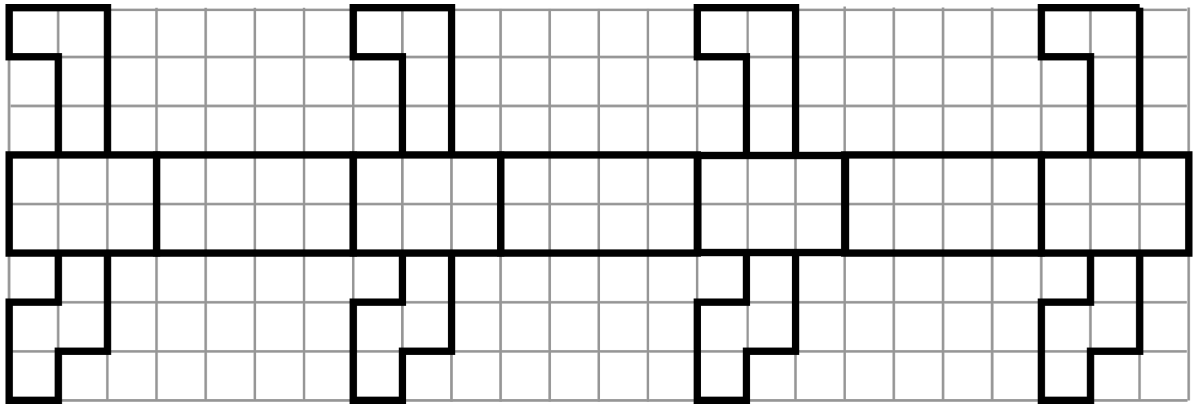
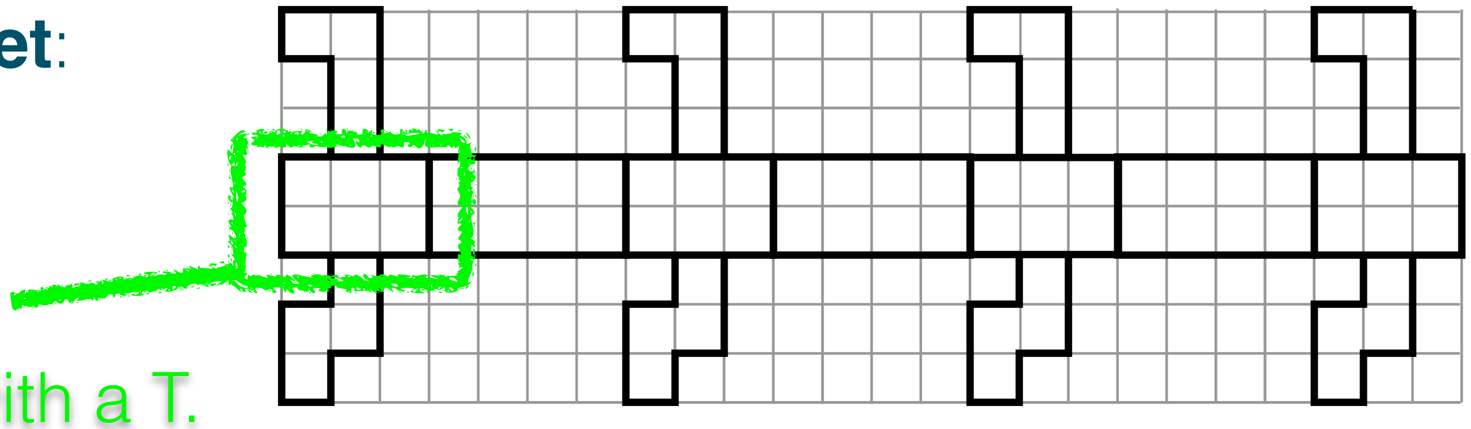


# LITS

**Variable gadget**:

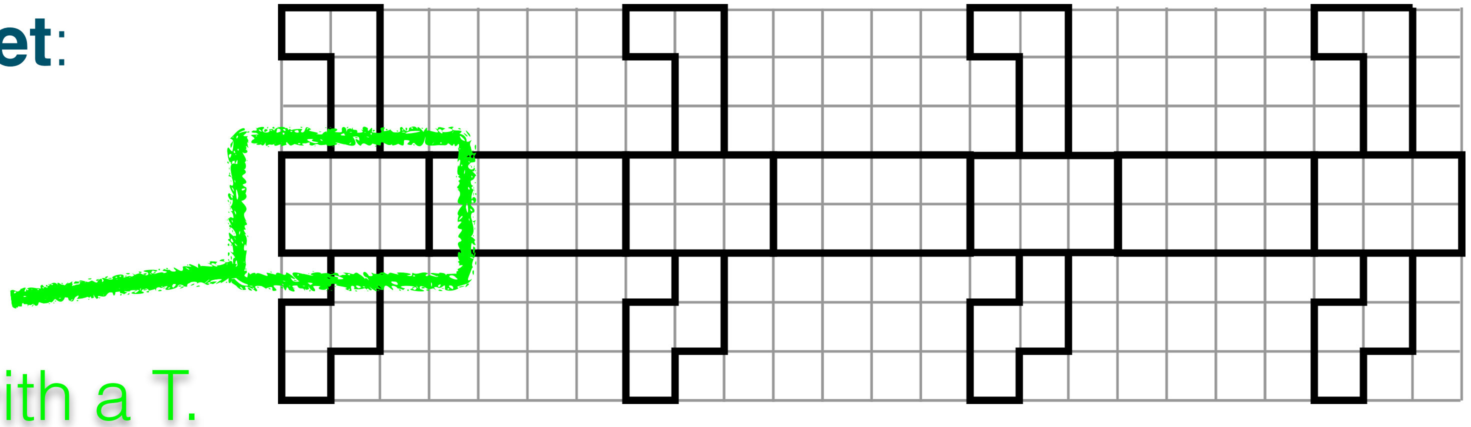


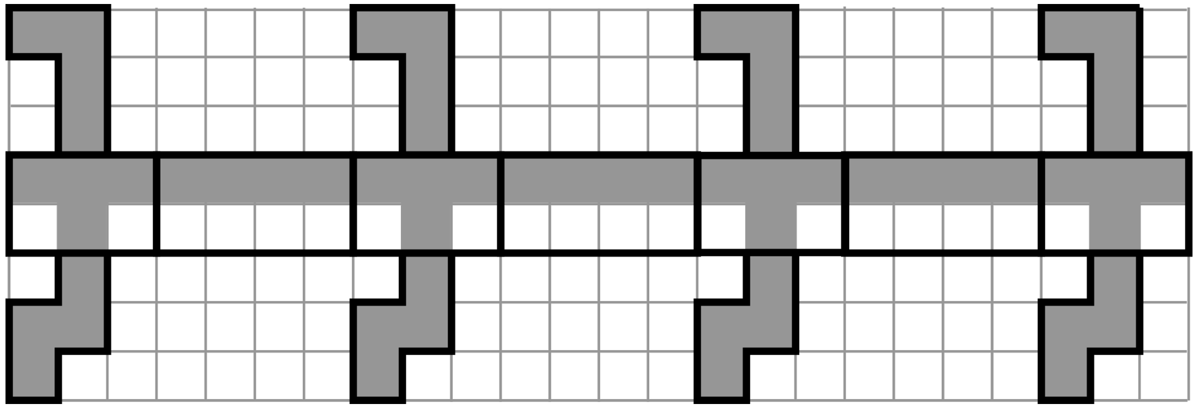
**Variable gadget**:

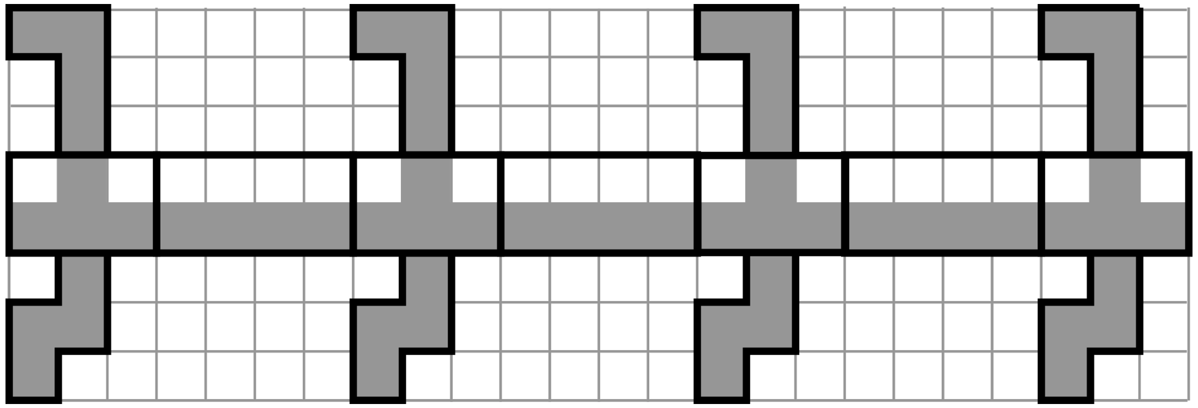


Must be filled with a T.

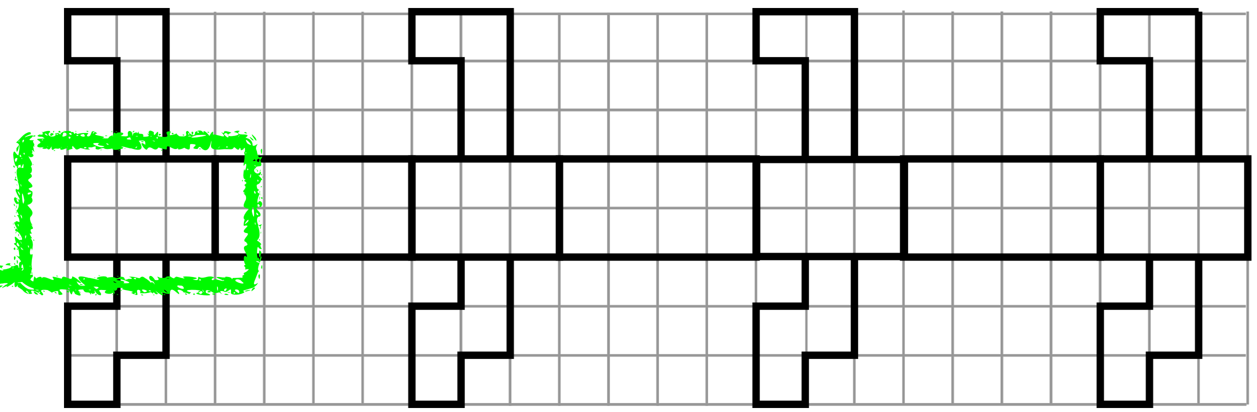
**Variable gadget**:



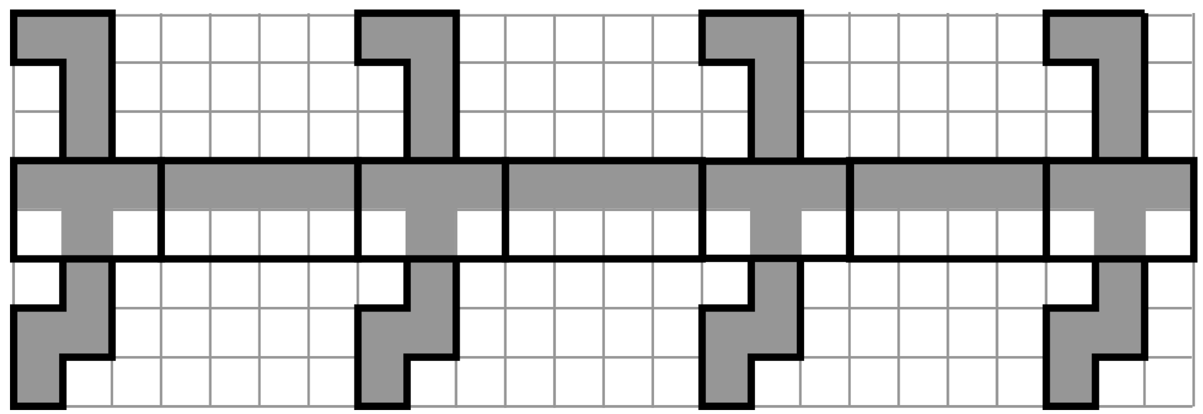



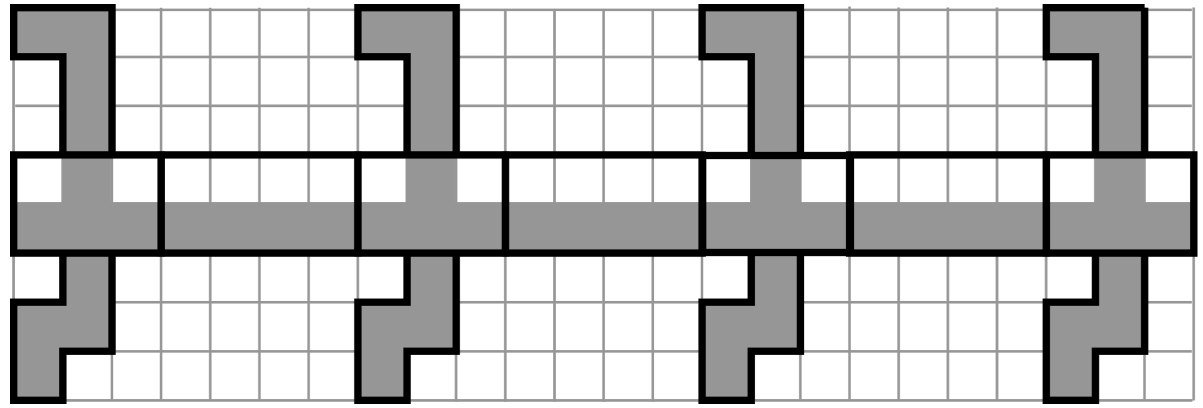
**Variable gadget**:



Must be filled with a T.



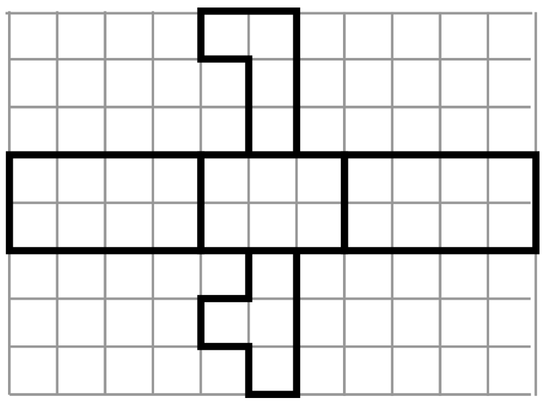
"true"



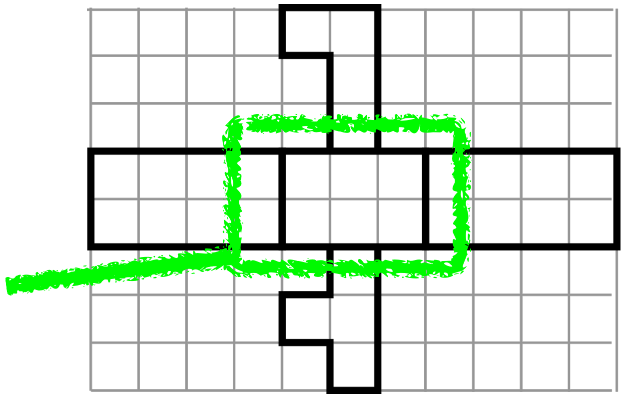
"false"

**Corridor gadget:** linearly repeat this pattern.

**NOT gadget**:



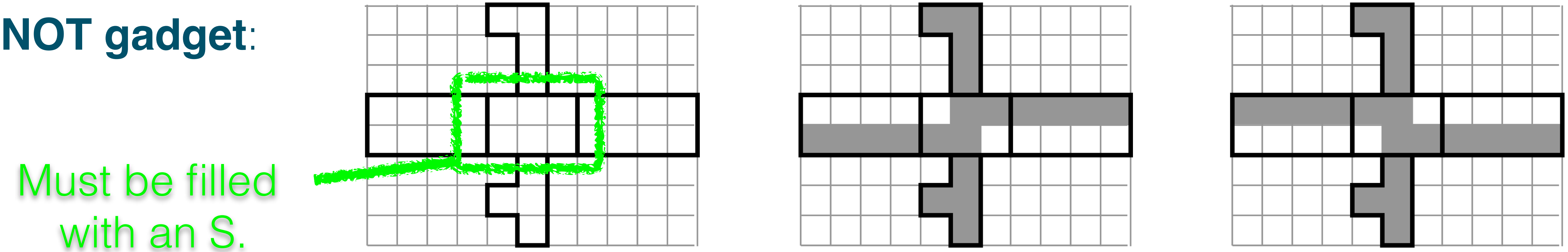
**NOT gadget**:



Must be filled with an S.

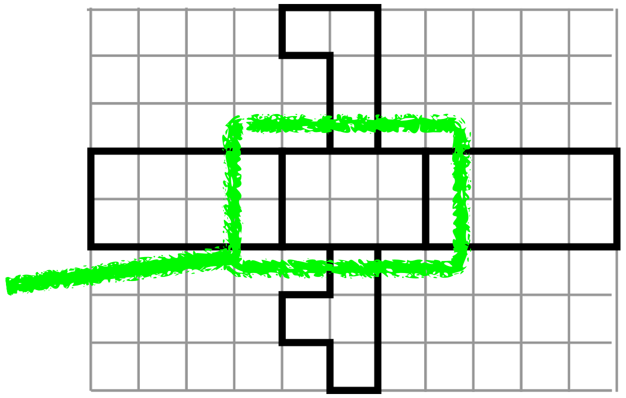
# LITS

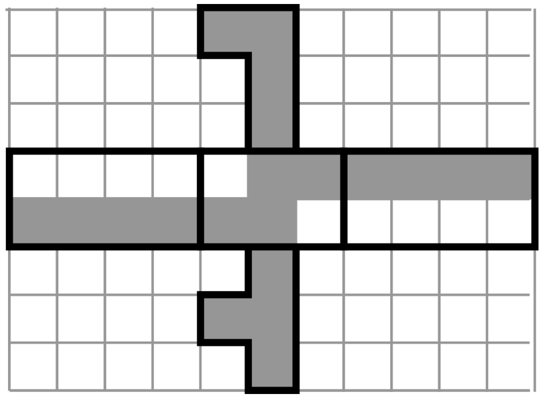
**NOT gadget**:

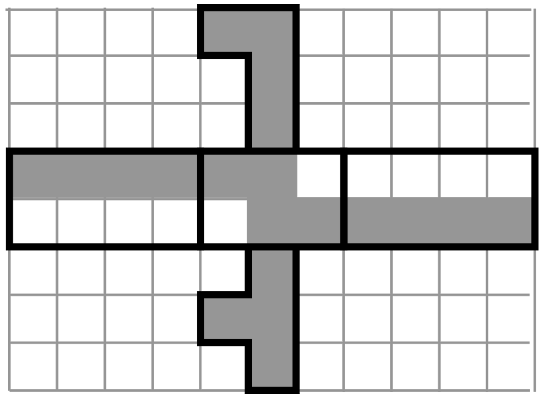


**NOT gadget**:

Must be filled with an S.

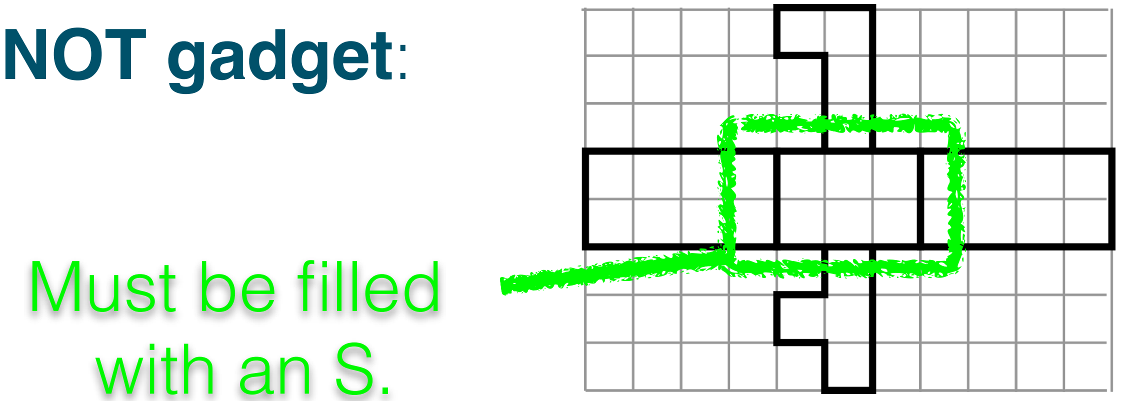


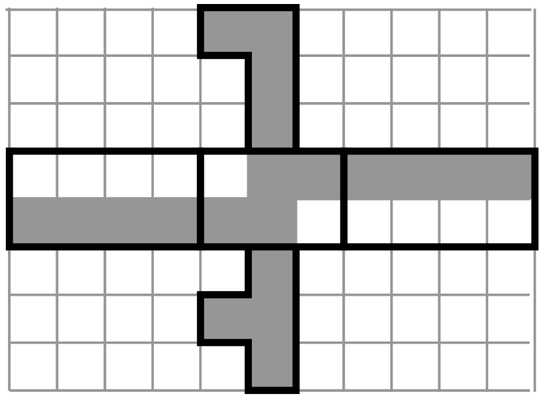


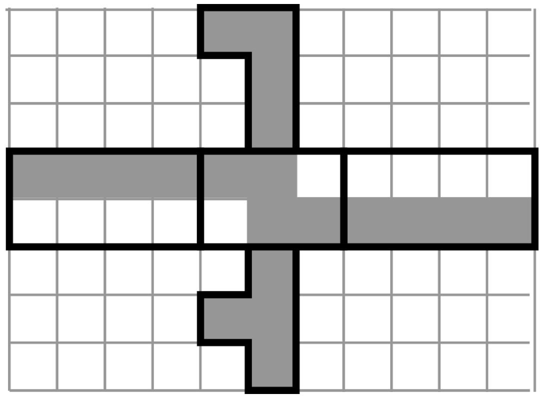


The wires connected by the gadget always satisfy opposite truth assignments.

**NOT gadget**:





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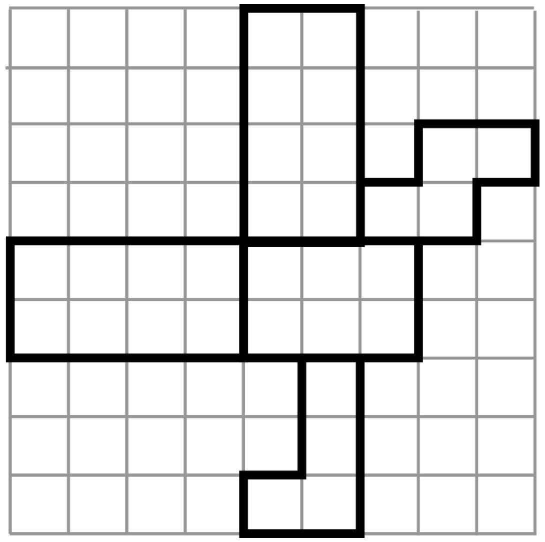
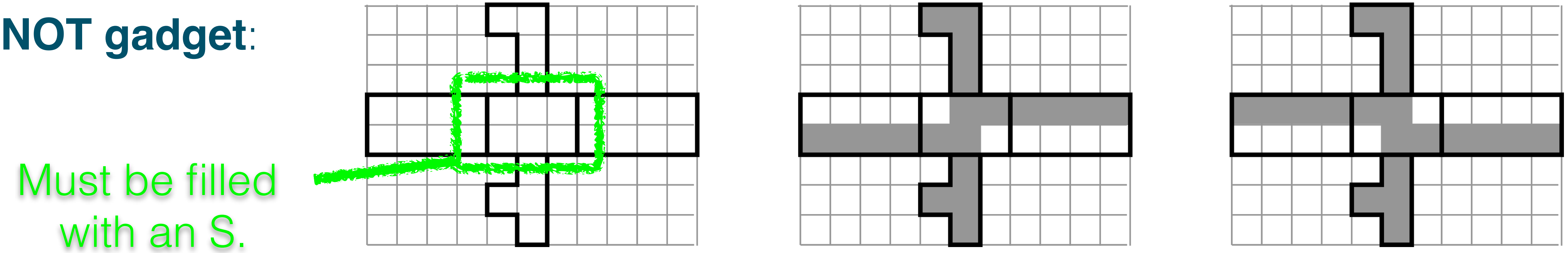


**NOT gadget**:

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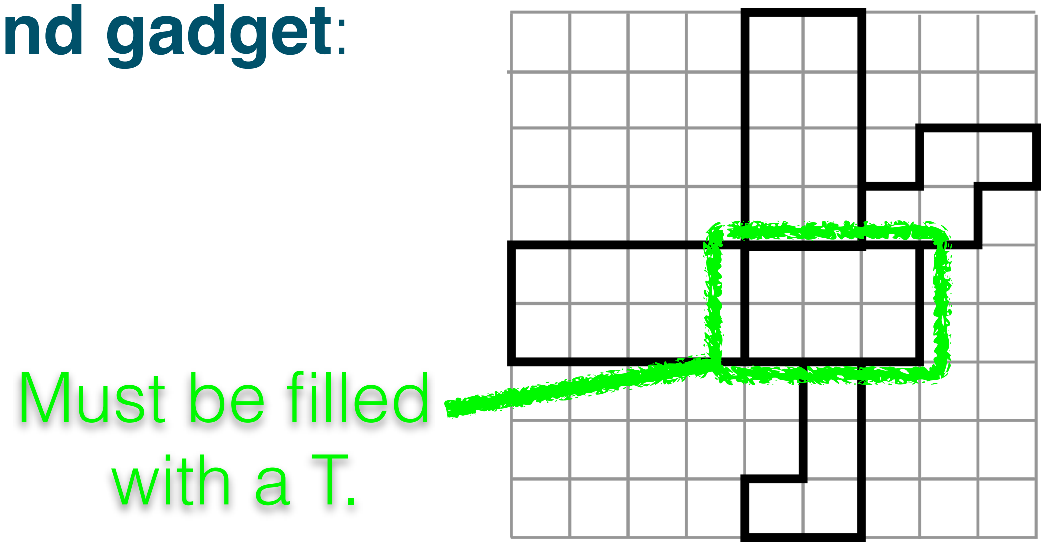
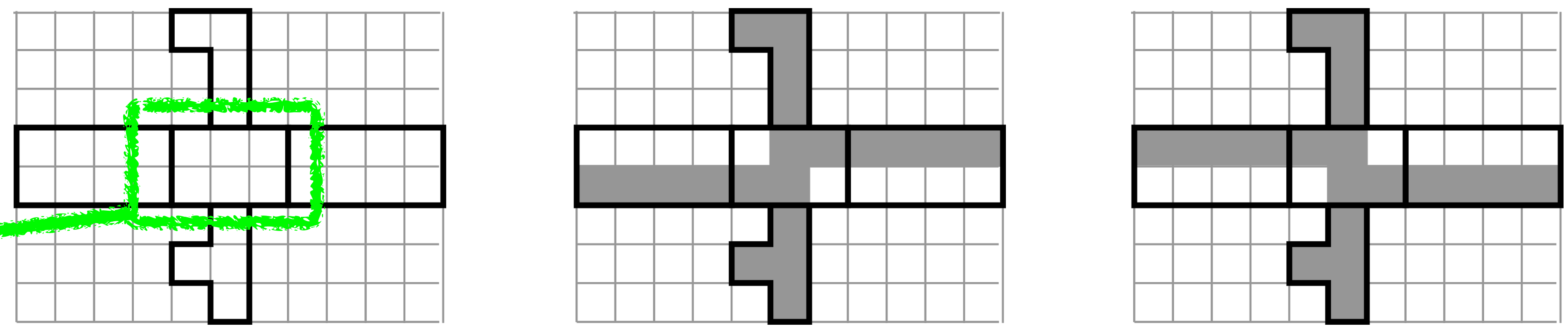


# LITS

**NOT gadget**:

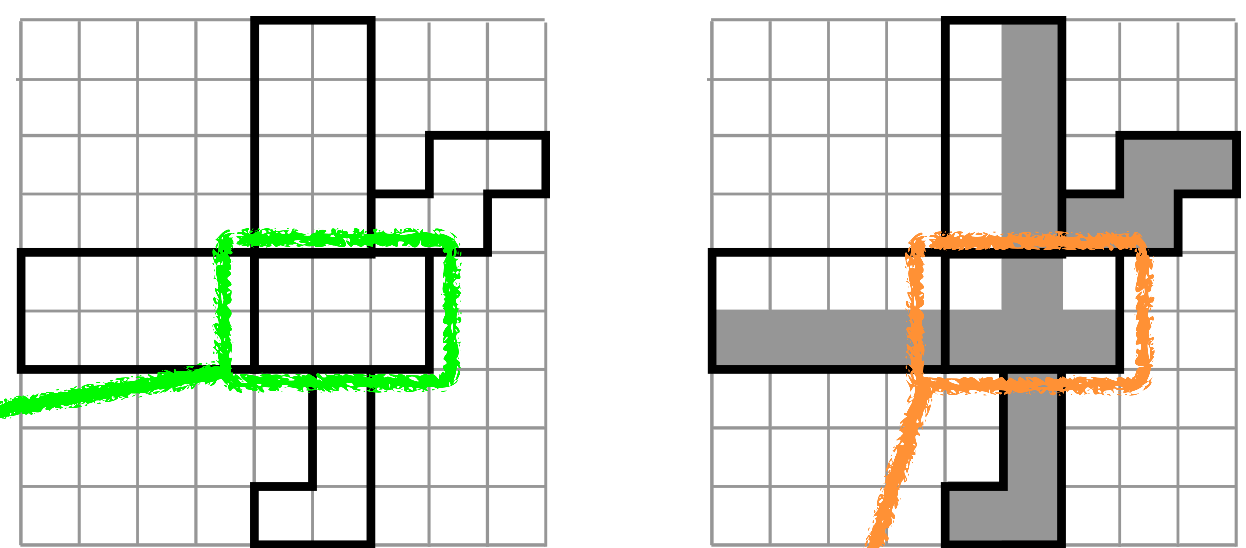
Must be filled with an S.



The wires connected by the gadget always satisfy opposite truth assignments.

**Bend gadget**:

Must be filled with a T.

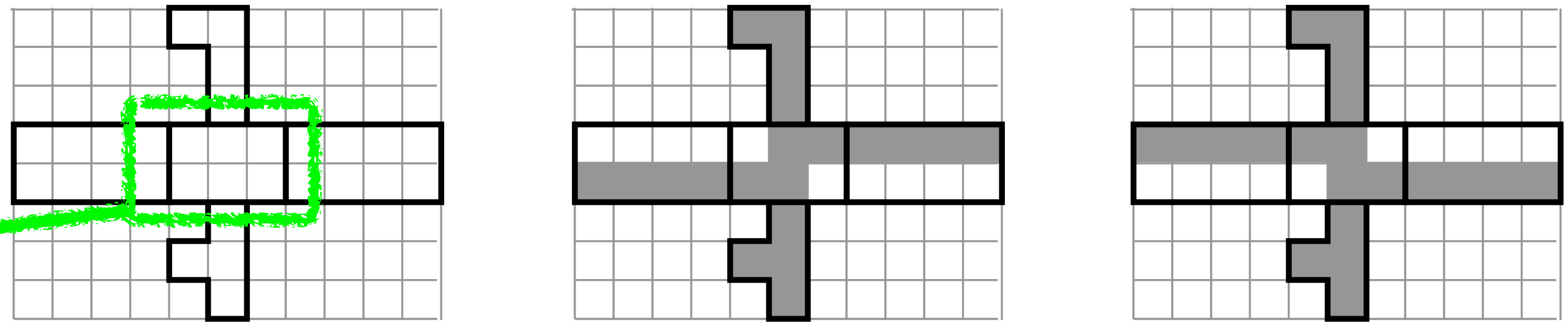


“false”

The other T wouldn’t connect to the incoming I.

# LITS

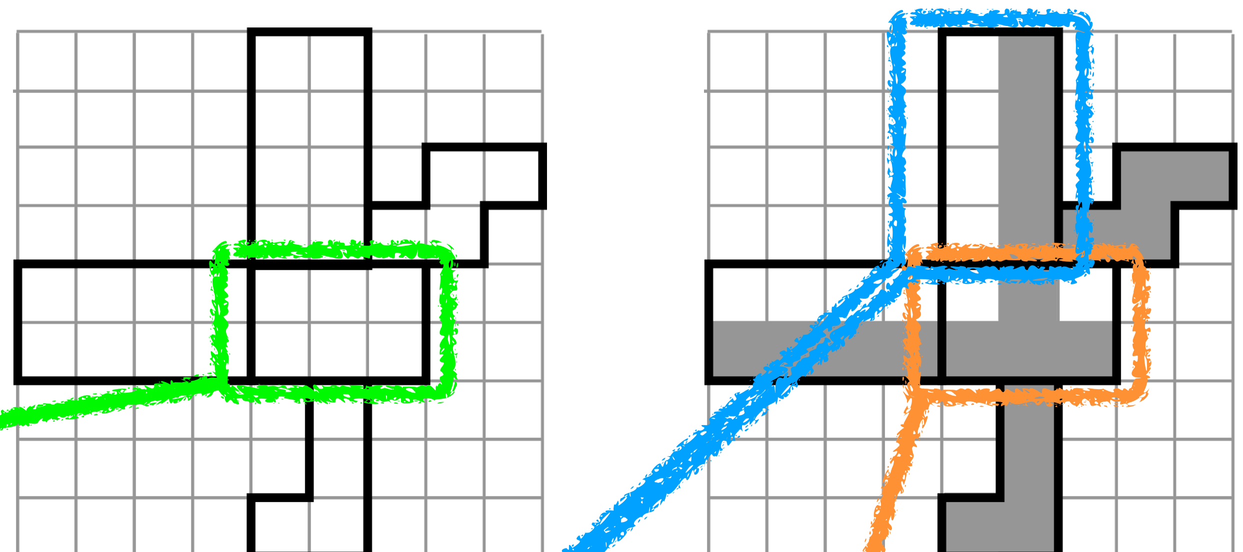
**NOT gadget**:



Must be filled with an S.

The wires connected by the gadget always satisfy opposite truth assignments.

**Bend gadget**:



Must be filled with a T.

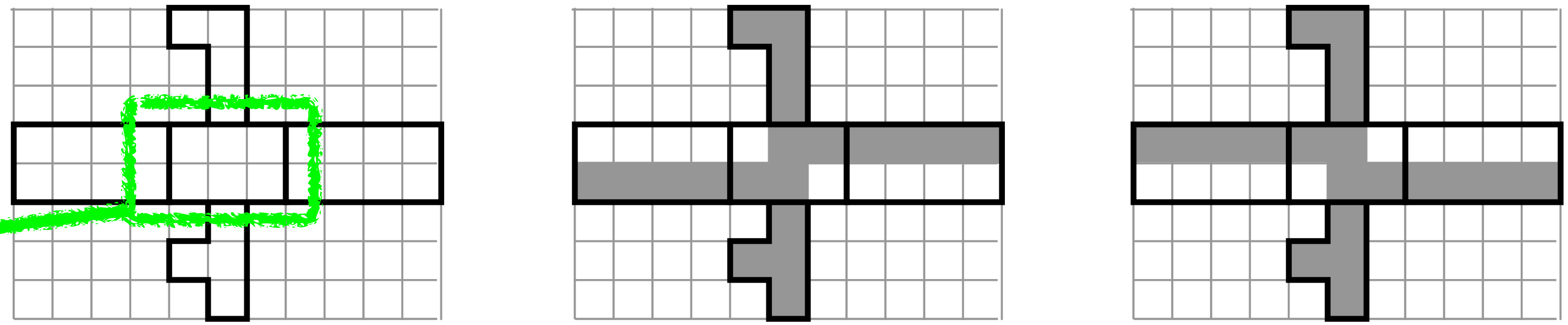
“false”

The other T wouldn’t connect to the incoming I.

Other I would leave S disconnected.

# LITS

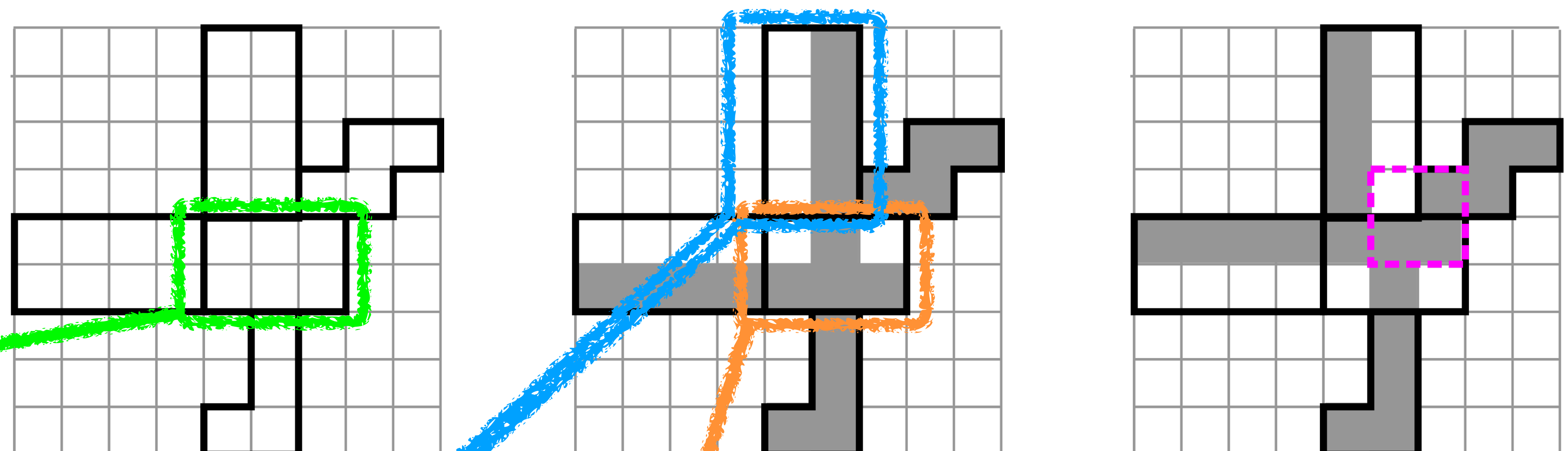
**NOT gadget**:



Must be filled with an S.

The wires connected by the gadget always satisfy opposite truth assignments.

**Bend gadget**:



Must be filled with a T.

"false"

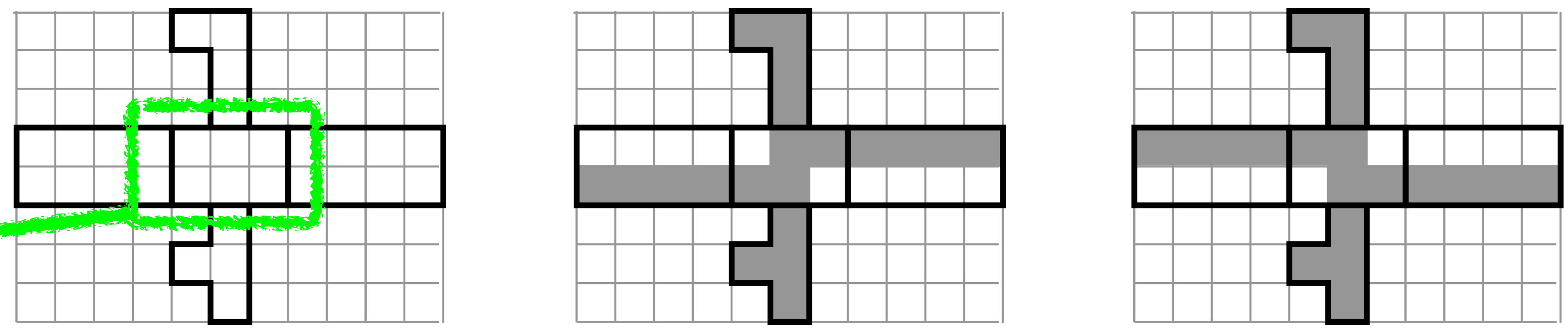
"true"

The other T wouldn't connect to the incoming I.

Other I would leave S disconnected.

# LITS

**NOT gadget**:



Must be filled with an S.

The wires connected by the gadget always satisfy opposite truth assignments.

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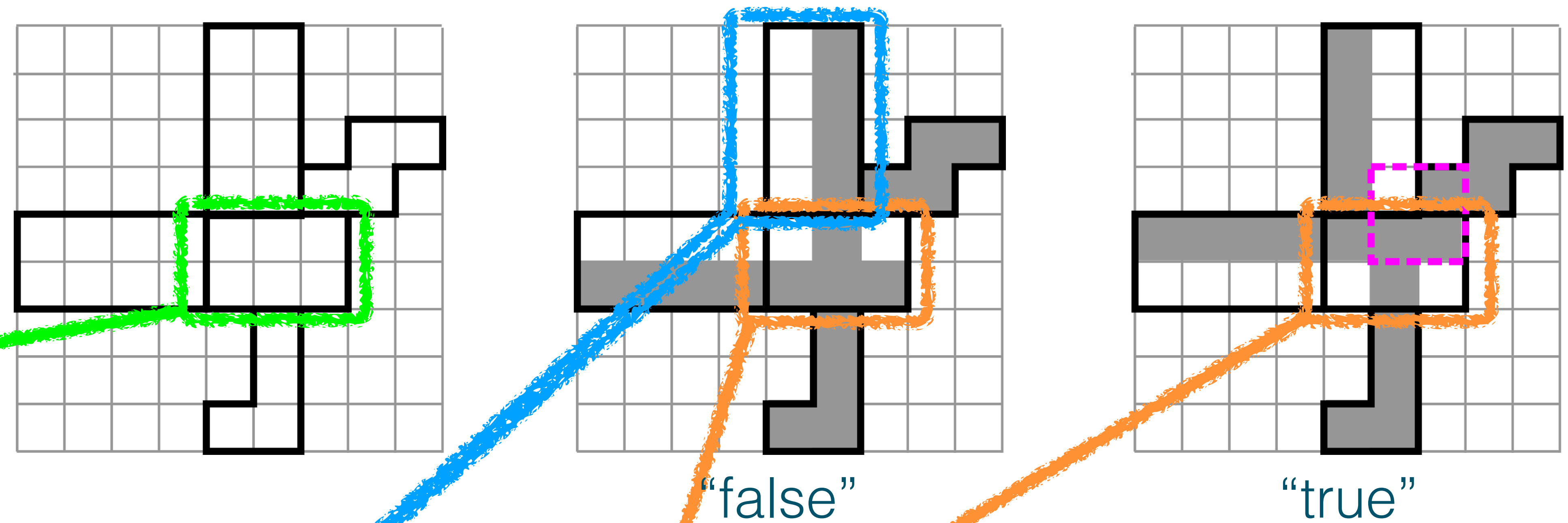
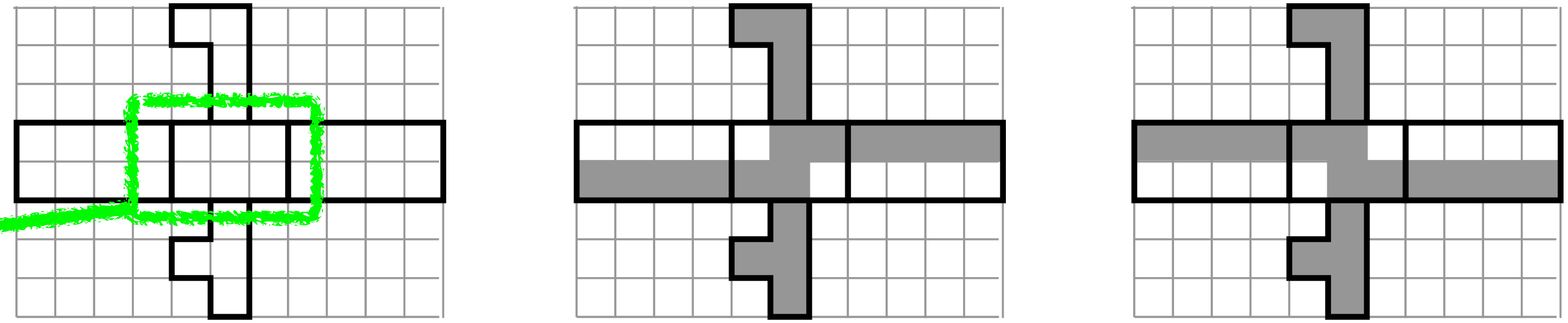



Must be filled with a T.

The other T wouldn't connect to the incoming I.

Other I would leave S disconnected.

**NOT gadget**:



Must be filled with an S.

The wires connected by the gadget always satisfy opposite truth assignments.

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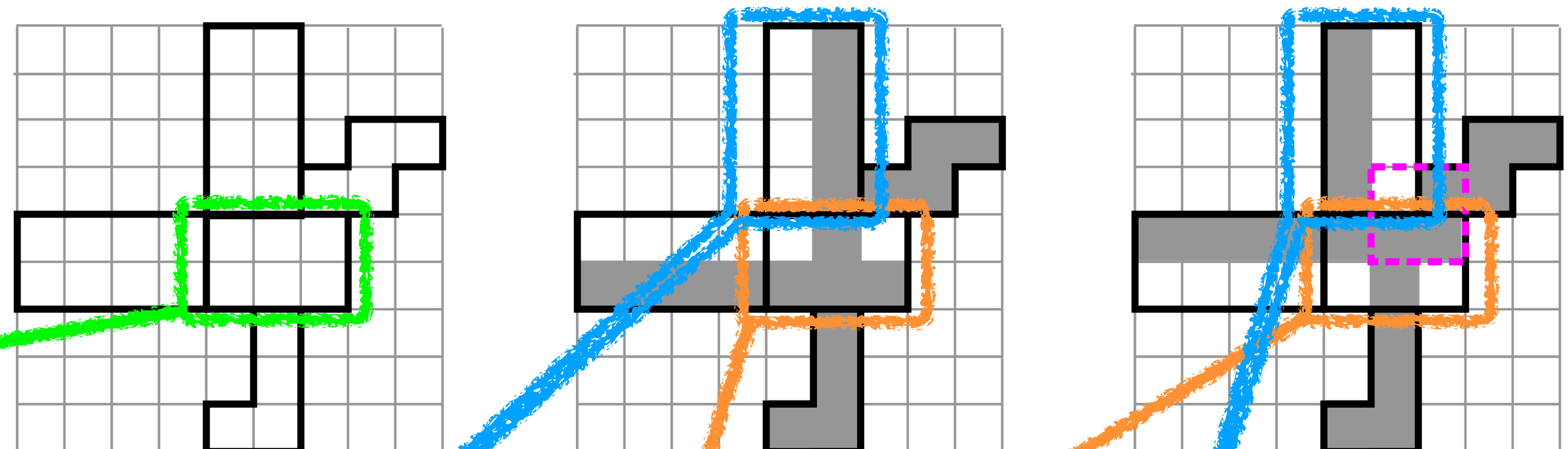


Must be filled with a T.

“false”

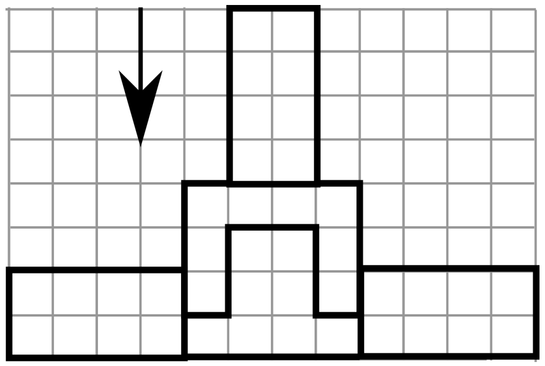
“true”

The other T wouldn’t connect to the incoming I.

Other I would leave S disconnected.

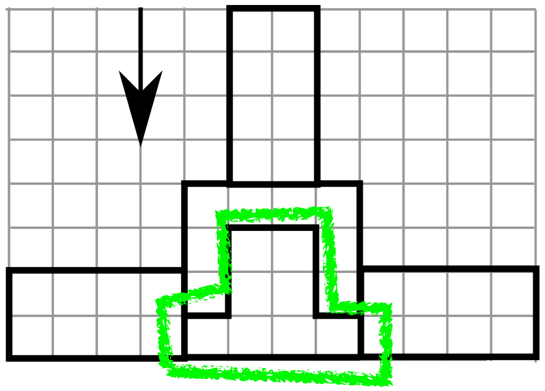
Other I would result in **2x2 block.**

**Split gadget**:



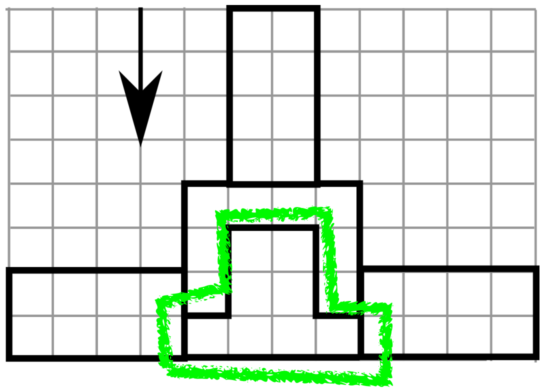
# LITS

**Split gadget**:



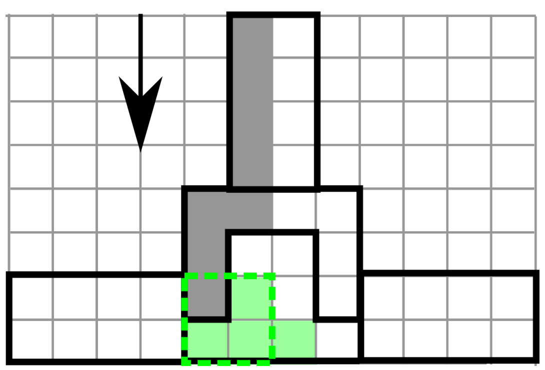
Must be filled with an S or a T.

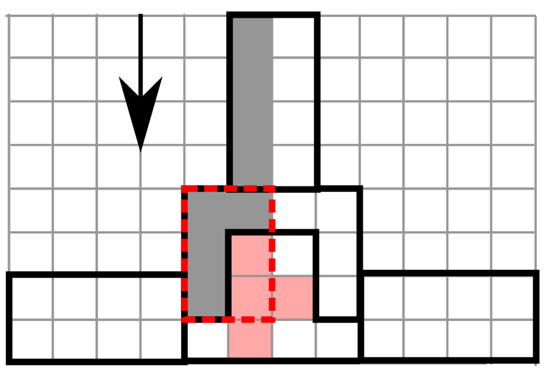
**Split gadget**:

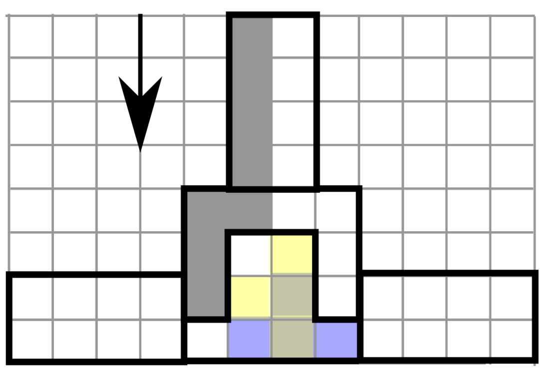


Must be filled with an S or a T.

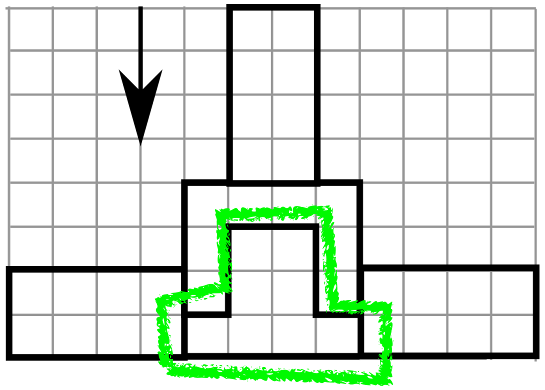
No position of T possible:

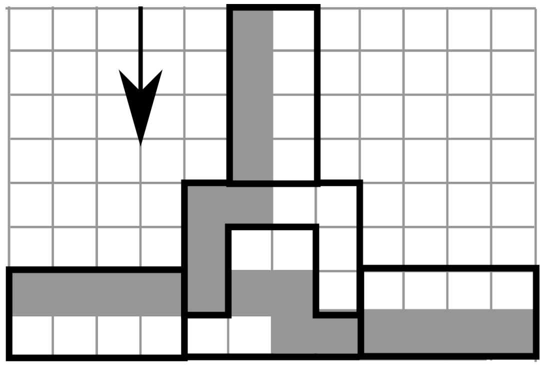






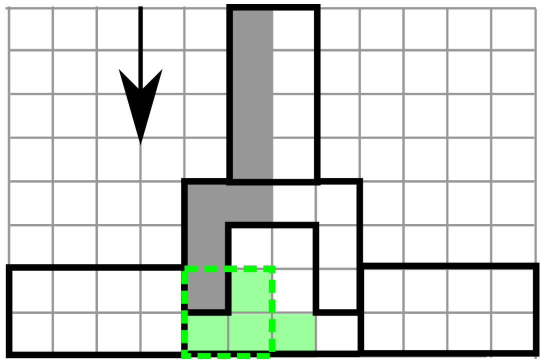
**Split gadget**:

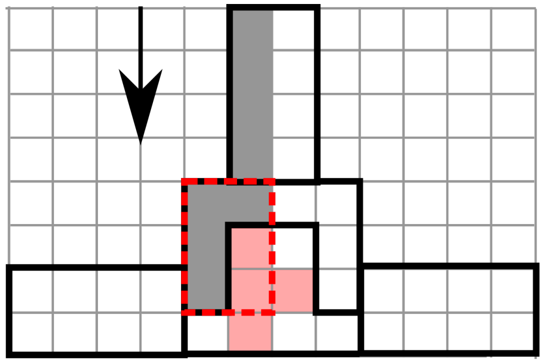


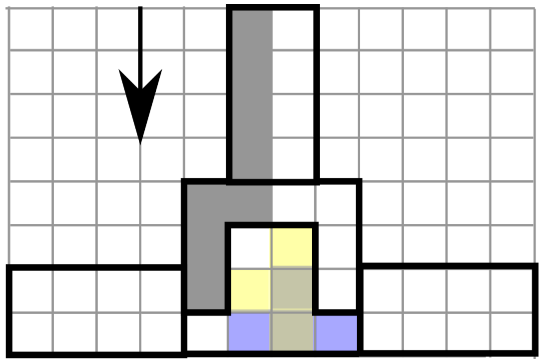


Must be filled with an S or a T.

No position of T possible:







# LITS

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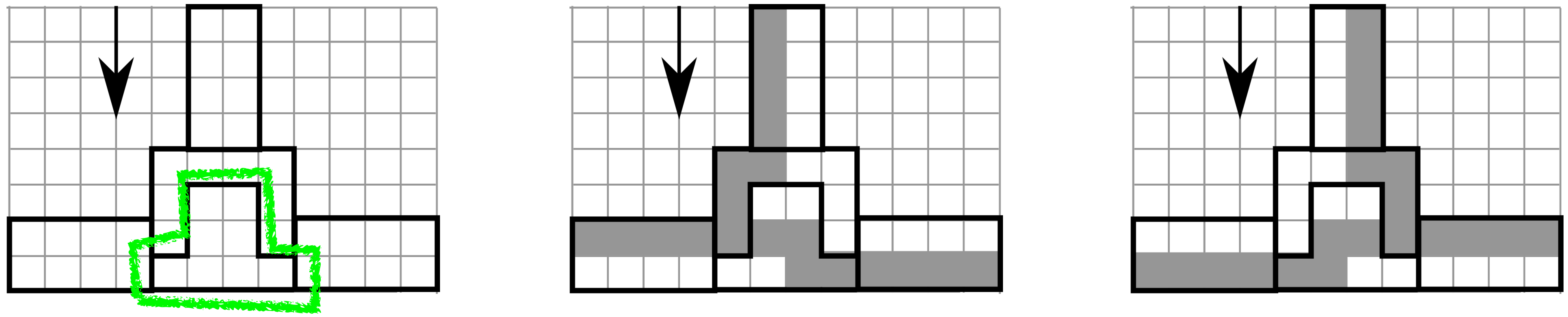
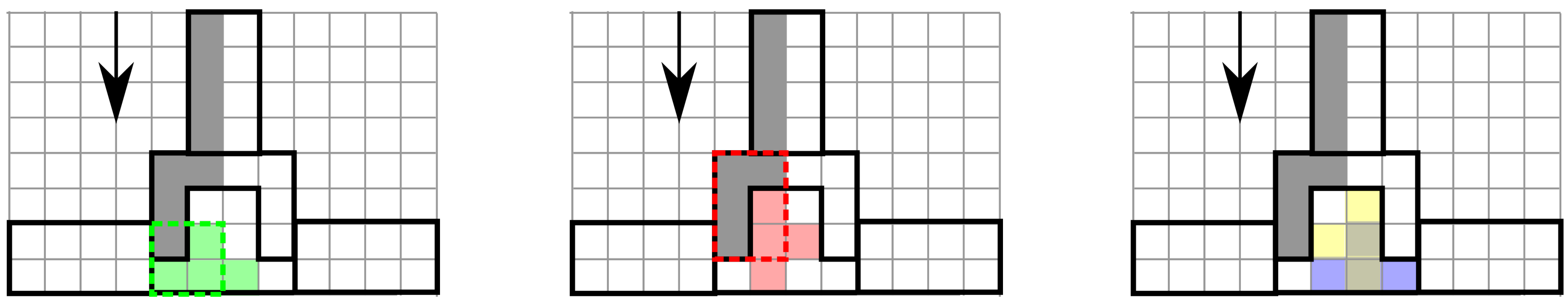
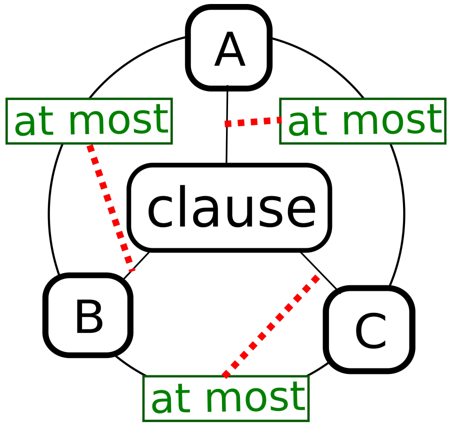


Must be filled with an S or a T.

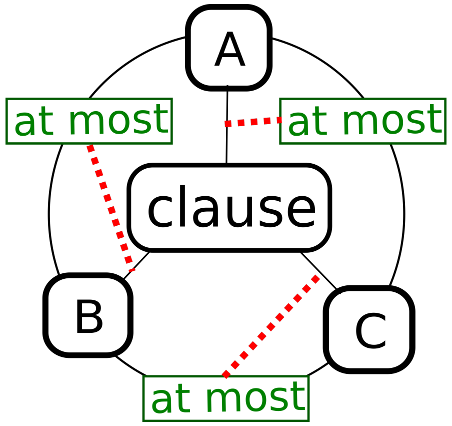
No position of T possible:



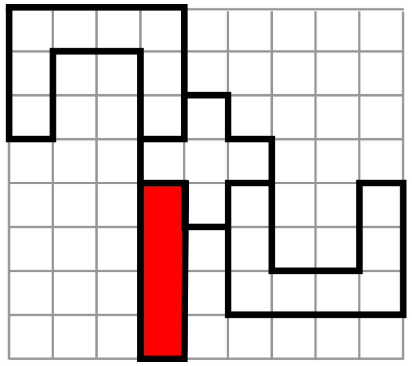
**1-in-3 gadget:**



**1-in-3 gadget:**

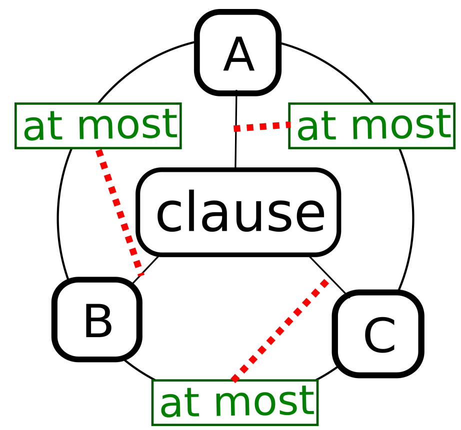



**At-most gadget** (Two C-shaped regions connect to variable corridors):

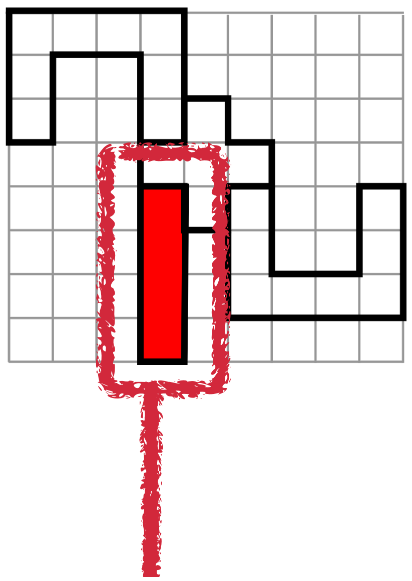


# LITS

**1-in-3 gadget:**

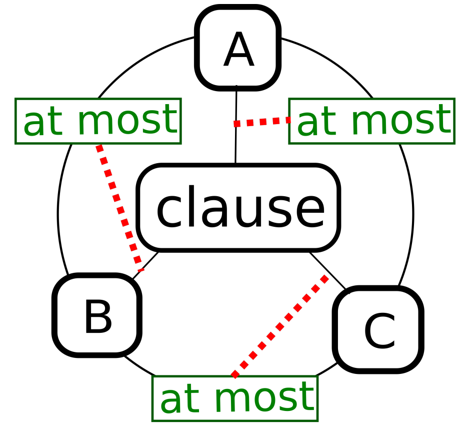



**At-most gadget** (Two C-shaped regions connect to variable corridors):

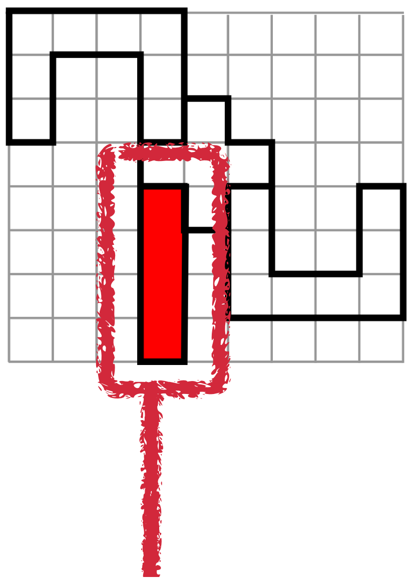


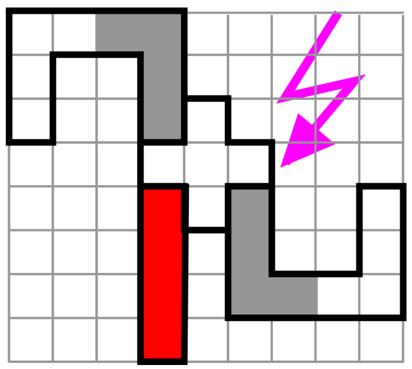
Corridor of enforced I and T shapes that connect to an S or L of a corridor on that face.

**1-in-3 gadget:**




**At-most gadget** (Two C-shaped regions connect to variable corridors):

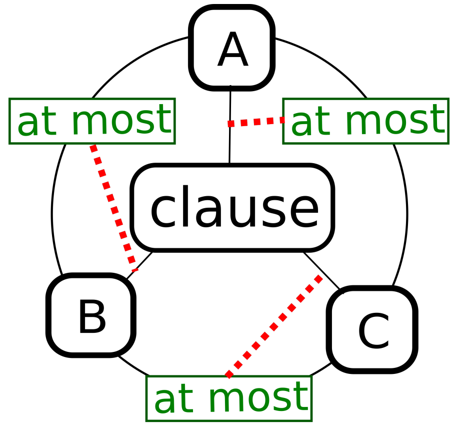




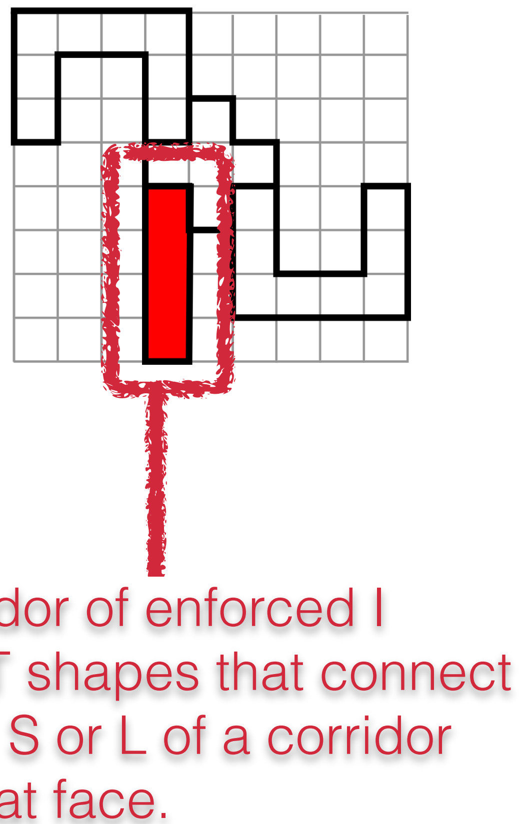
Both variables truth setting that fulfils the clause.

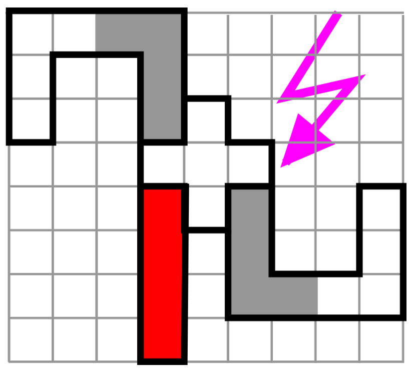
Corridor of enforced I and T shapes that connect to an S or L of a corridor on that face.

**1-in-3 gadget:**

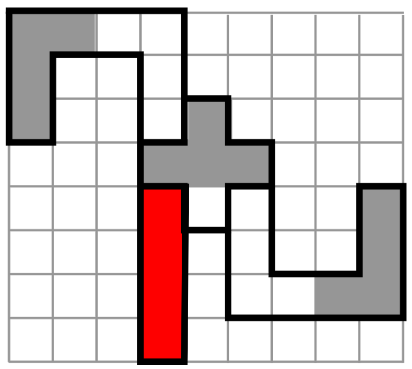



**At-most gadget** (Two C-shaped regions connect to variable corridors):



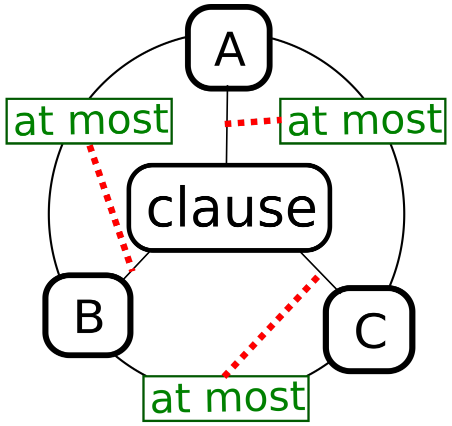



Both variables truth setting that fulfils the clause.

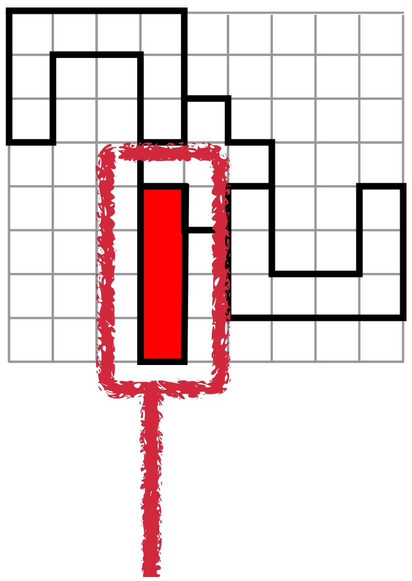


Both variables truth setting that does not fulfil the clause.

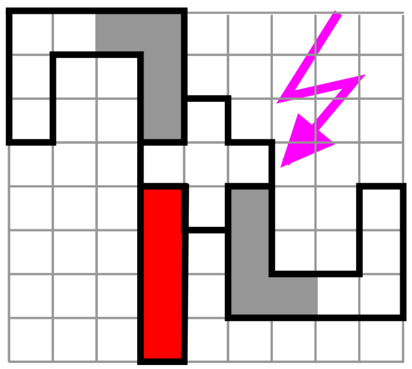
**1-in-3 gadget:**



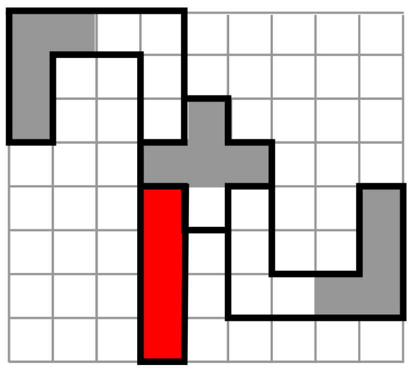

**At-most gadget** (Two C-shaped regions connect to variable corridors):



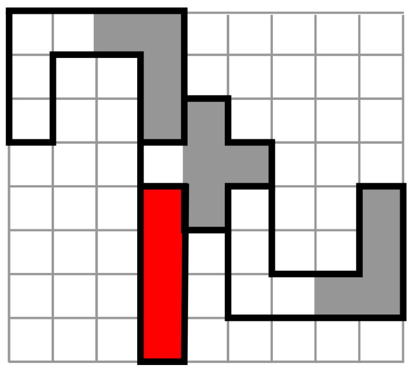
Corridor of enforced I and T shapes that connect to an S or L of a corridor on that face.



Both variables truth setting that fulfils the clause.

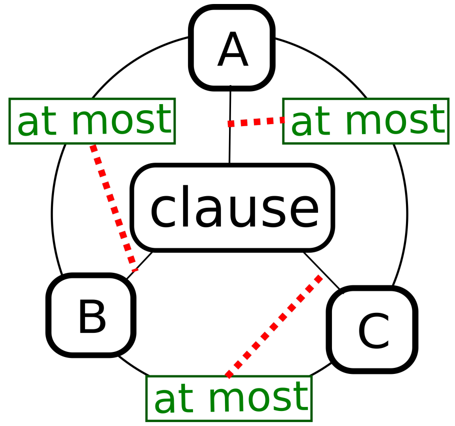


Both variables truth setting that does not fulfil the clause.

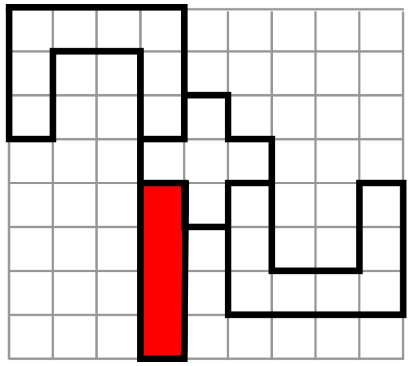


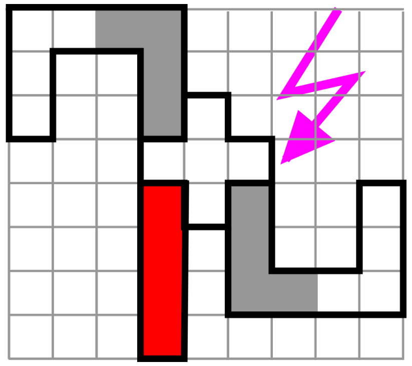
Only one variable fulfils the clause.

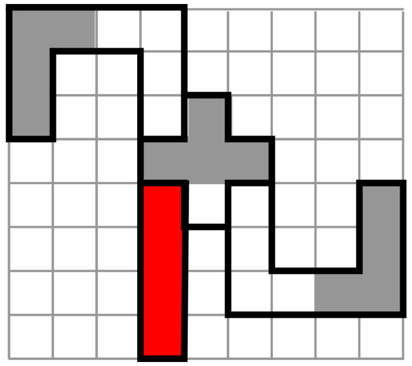
**1-in-3 gadget:**

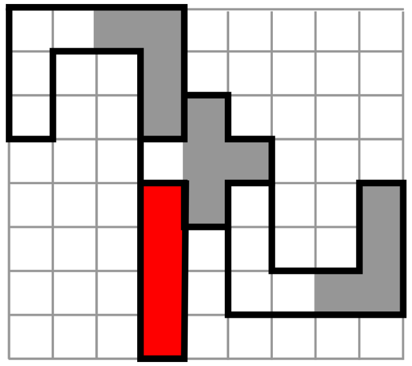



**At-most gadget** (Two C-shaped regions connect to variable corridors):




Both variables truth setting that fulfils the clause.


Both variables truth setting that does not fulfil the clause.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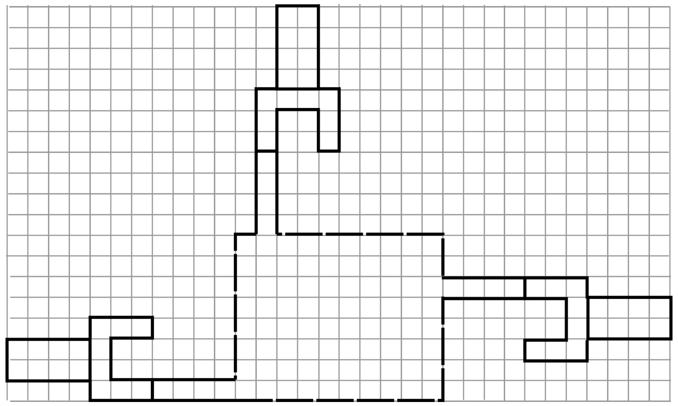
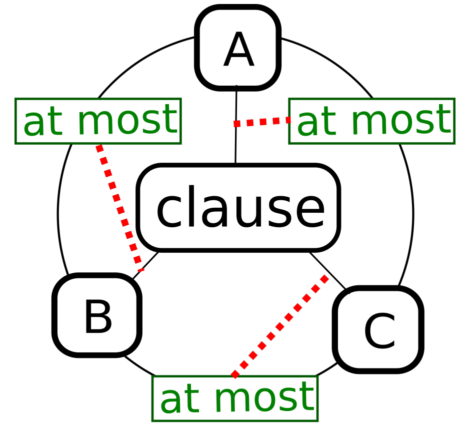
**Clause gadget:**



# LITS

**1-in-3 gadget:**

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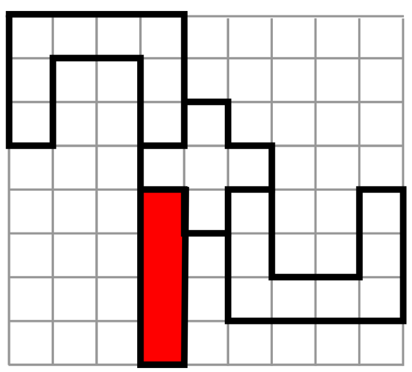

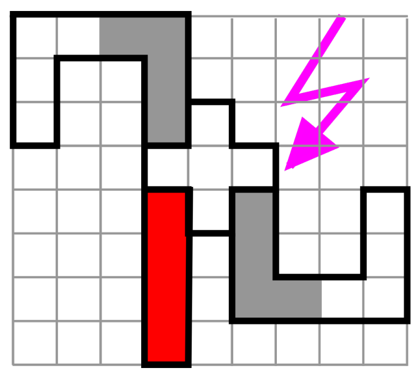

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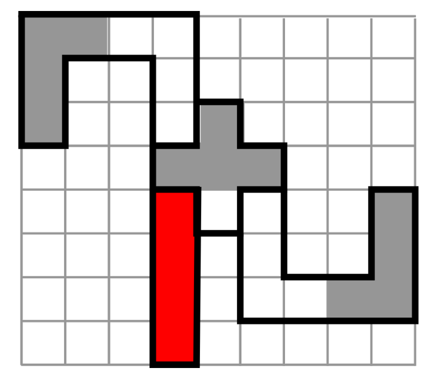
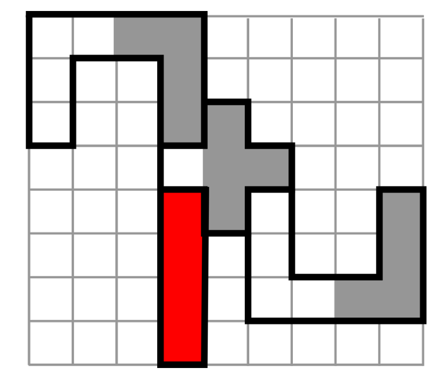


Both variables truth setting that does not fulfil the clause.



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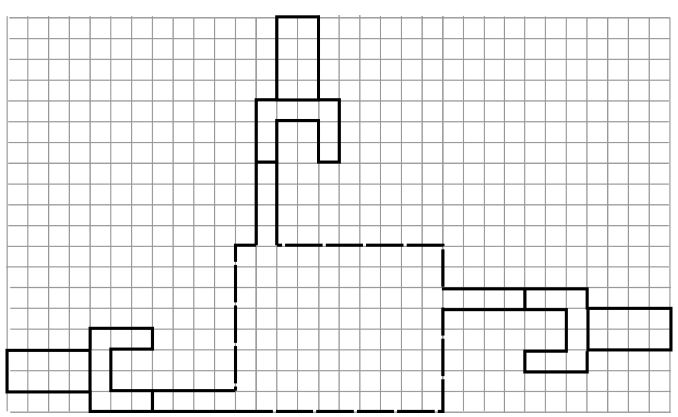
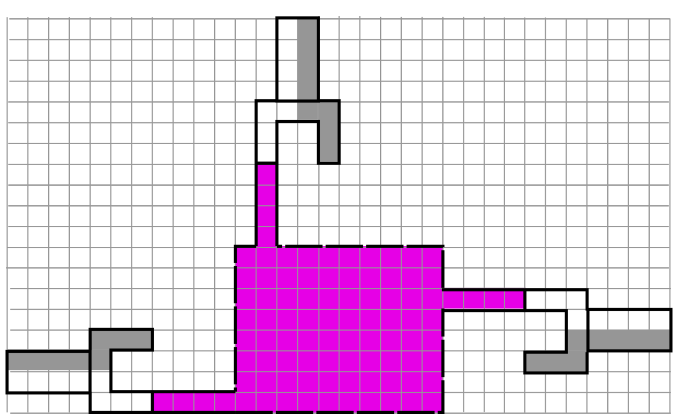
**Clause gadget:**





All variables do not fulfil the clause ➜ no tetromino in the **pink region** can be connected.

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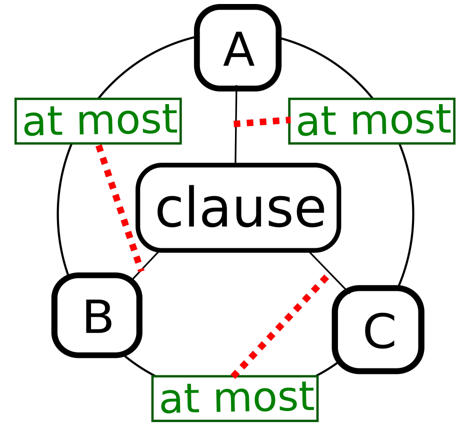
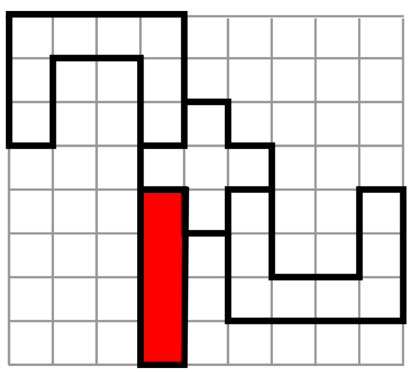
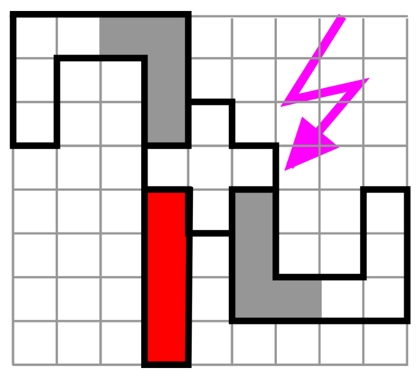



## At-most gadget (Two C-shaped regions connect to variable corridors):





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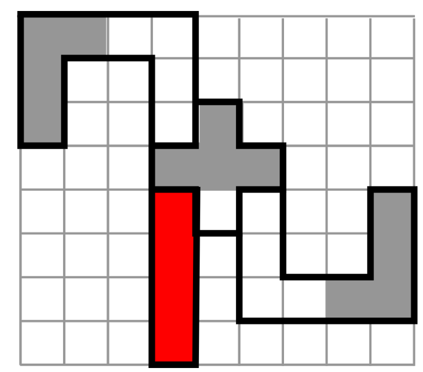

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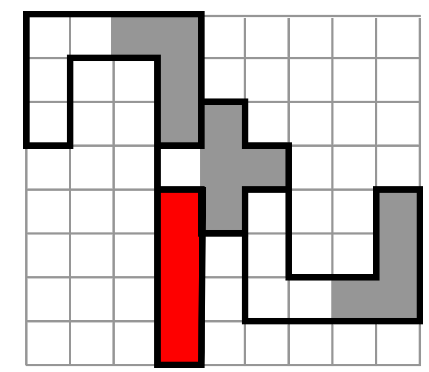

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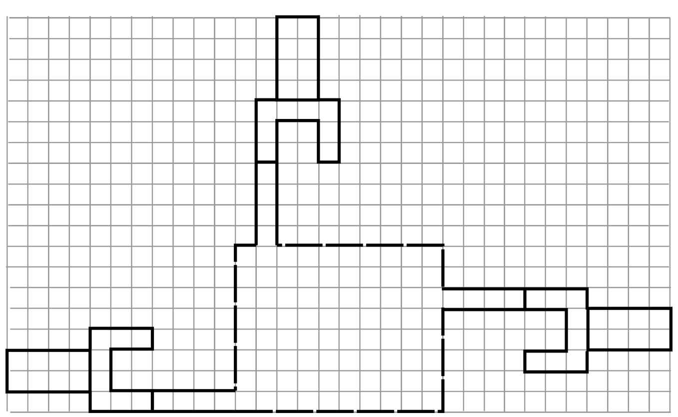
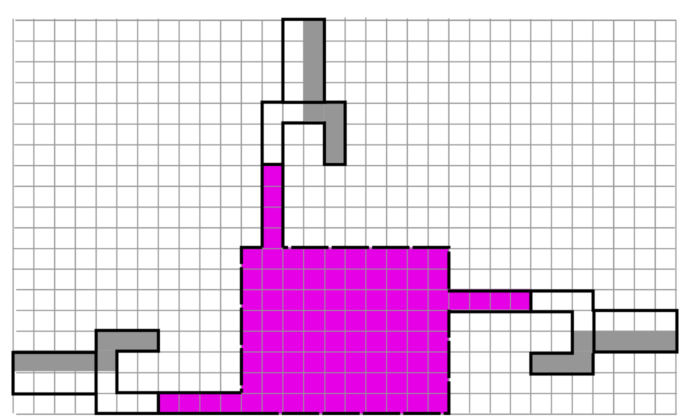
## Clause gadget:





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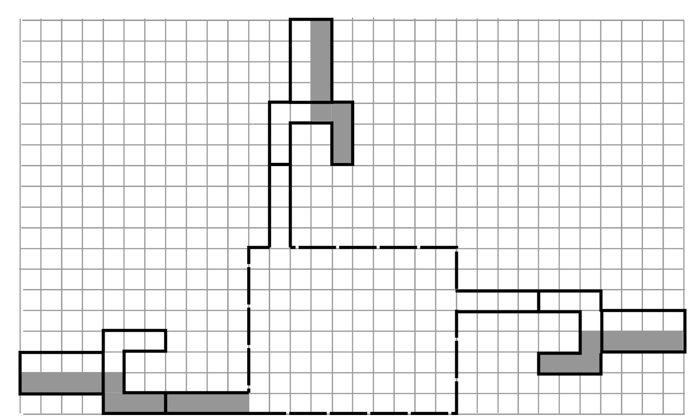

At least one fulfils the clause ➜ An I can connect to other tetrominoes.

# Boards with Unique Solutions

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In other words, 3 regions suffice to completely determine an nxm LITS board, as long as $n \geq 10$ and $m \geq 2$.

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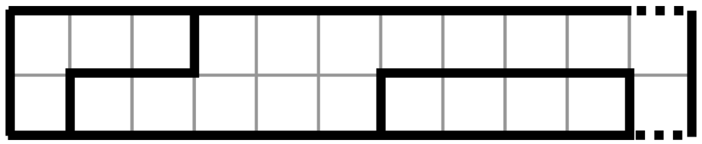

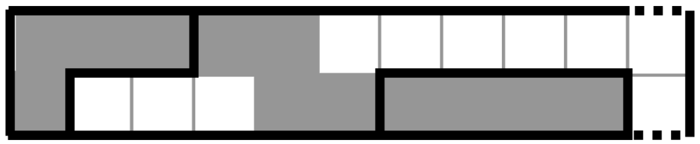

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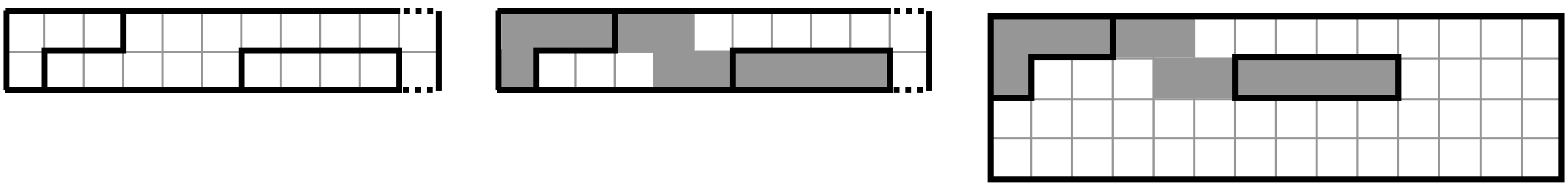

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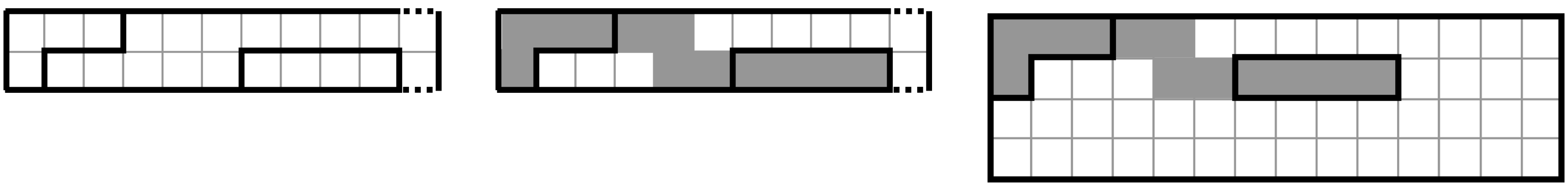

**Theorem 4:**

1. $U_N(n,1) = 0$ for $n \not\equiv 2 \mod 3$
2. $U_N(n,1) = \frac{n+1}{3}$ for $n \equiv 2 \mod 3$
3. $U_N(n,2) \leq \lceil \frac{n}{4} \rceil$ for $n \geq 3$
4. $U_N(n,m) = 3$ for all $n \geq 5, m \geq 3$.

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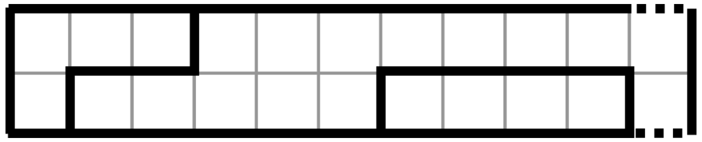
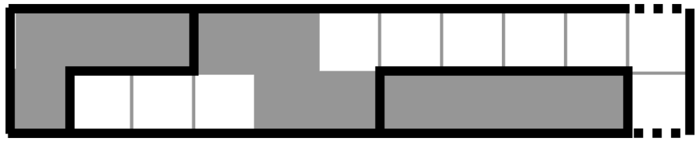
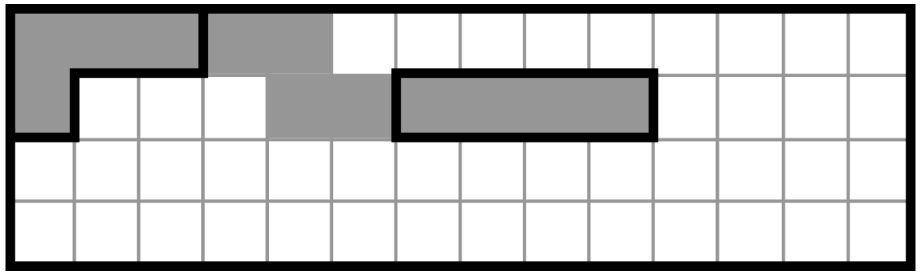

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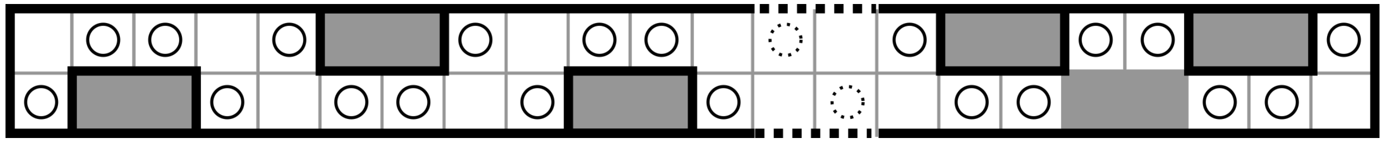

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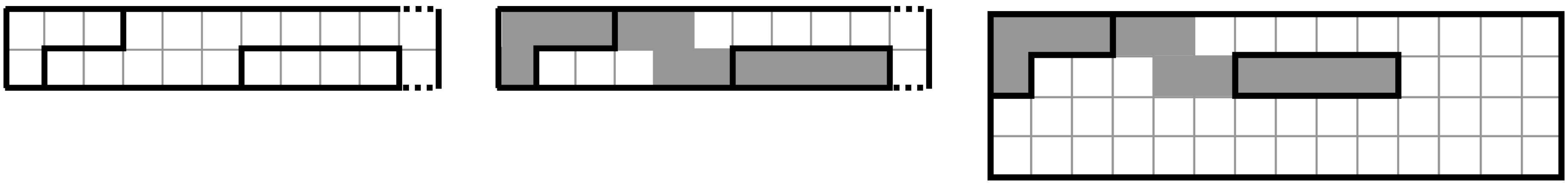

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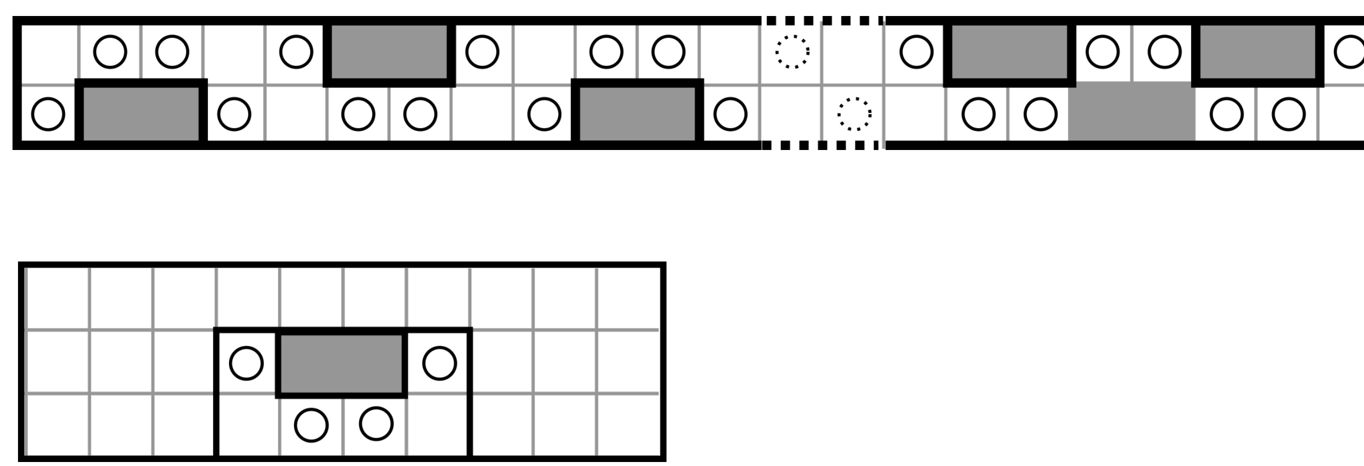

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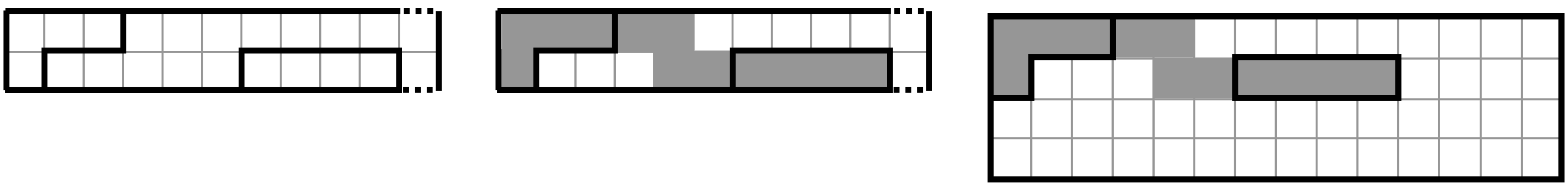

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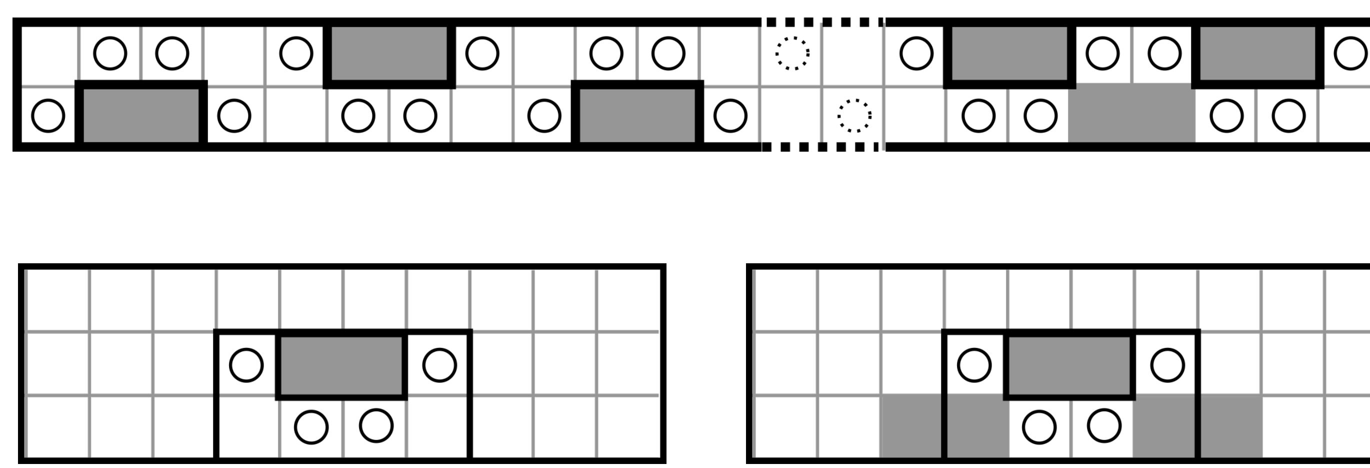
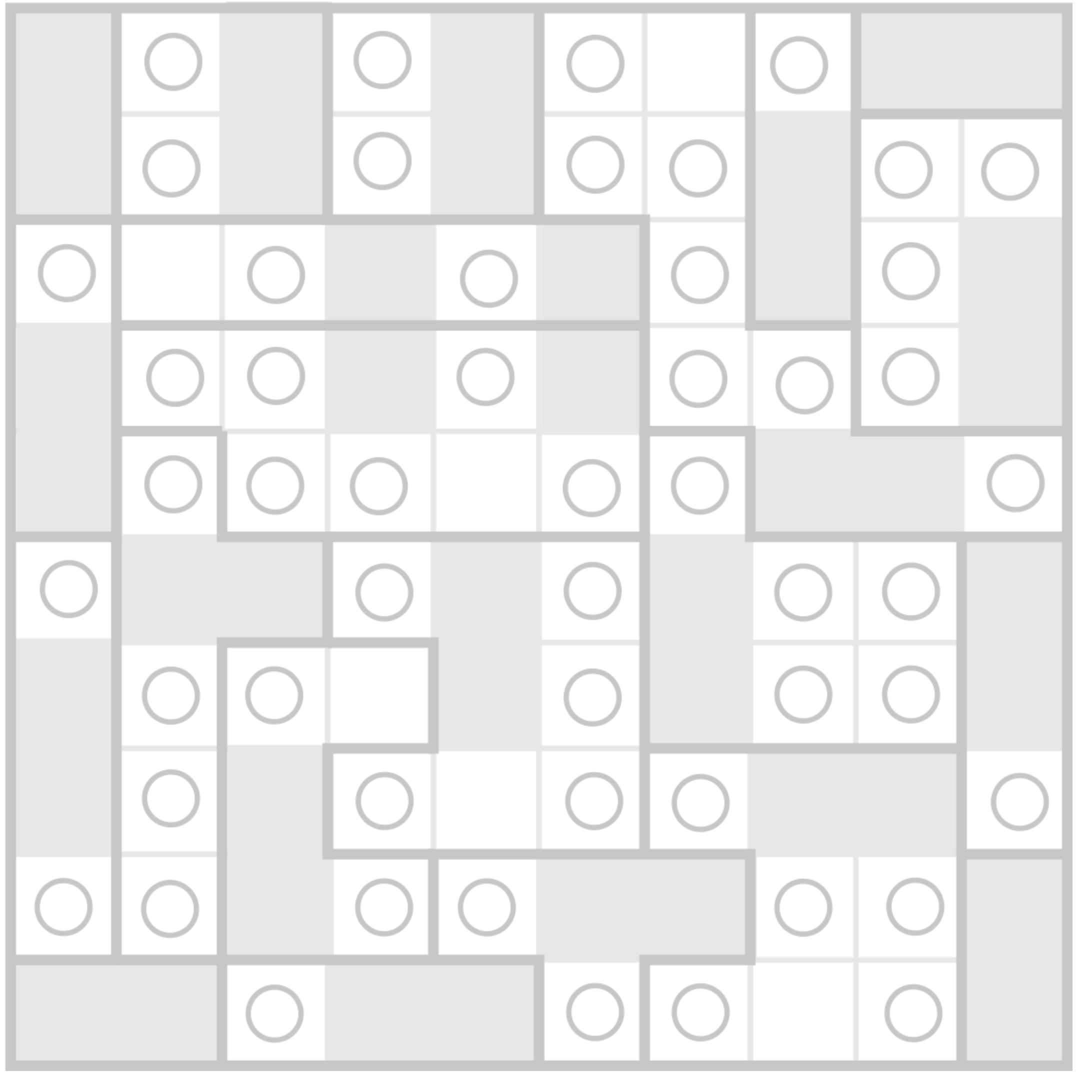
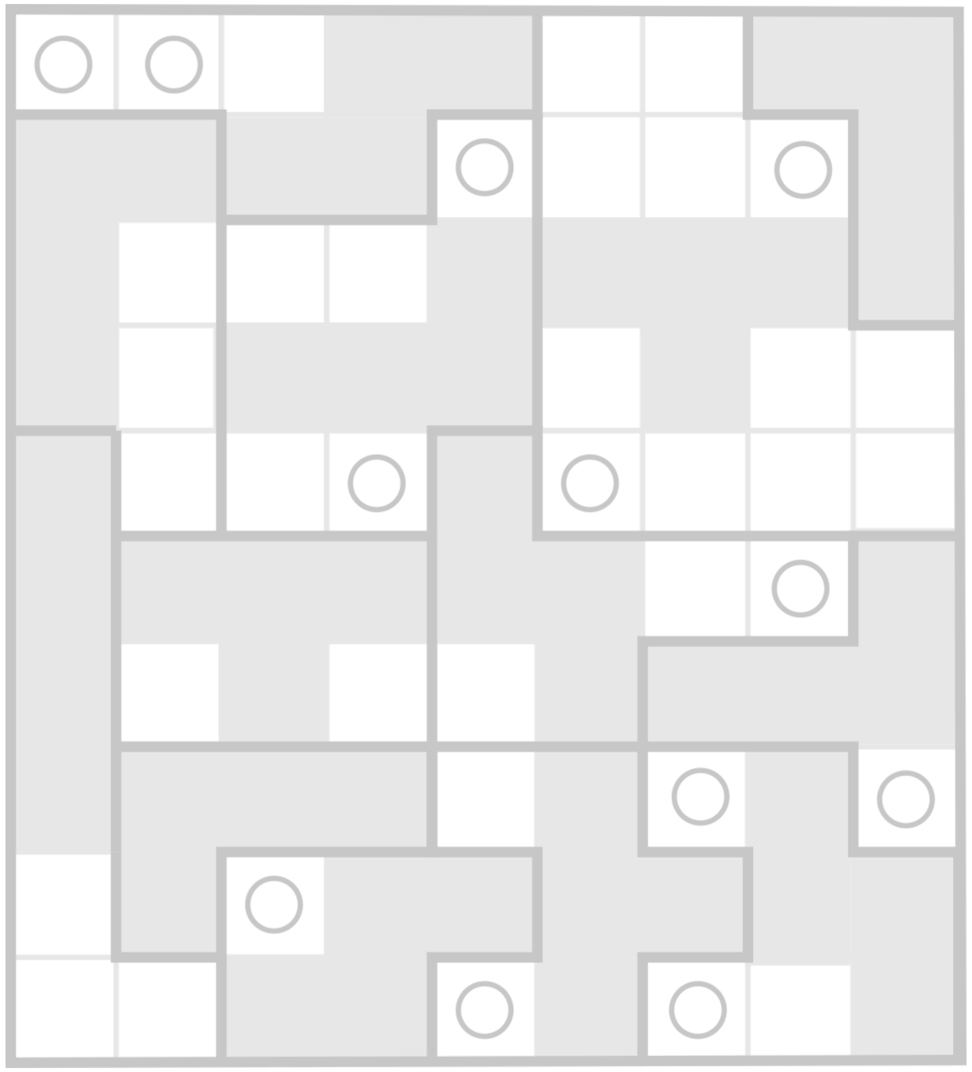


# THANK YOU.





- ✳Determining if a Norinori board is solvable is NP-complete and counting the number of solutions is #P-complete.
- ✳Determining if a LITS board is solvable is NP-complete and counting the number of solutions is #P-complete.
- ✳Bounds on the minimum number of regions among all nxm Norinori/LITS boards with unique solutions.