Altitude Terrain Guarding and Guarding Uni-Monotone Polygons

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Christiane Schmidt
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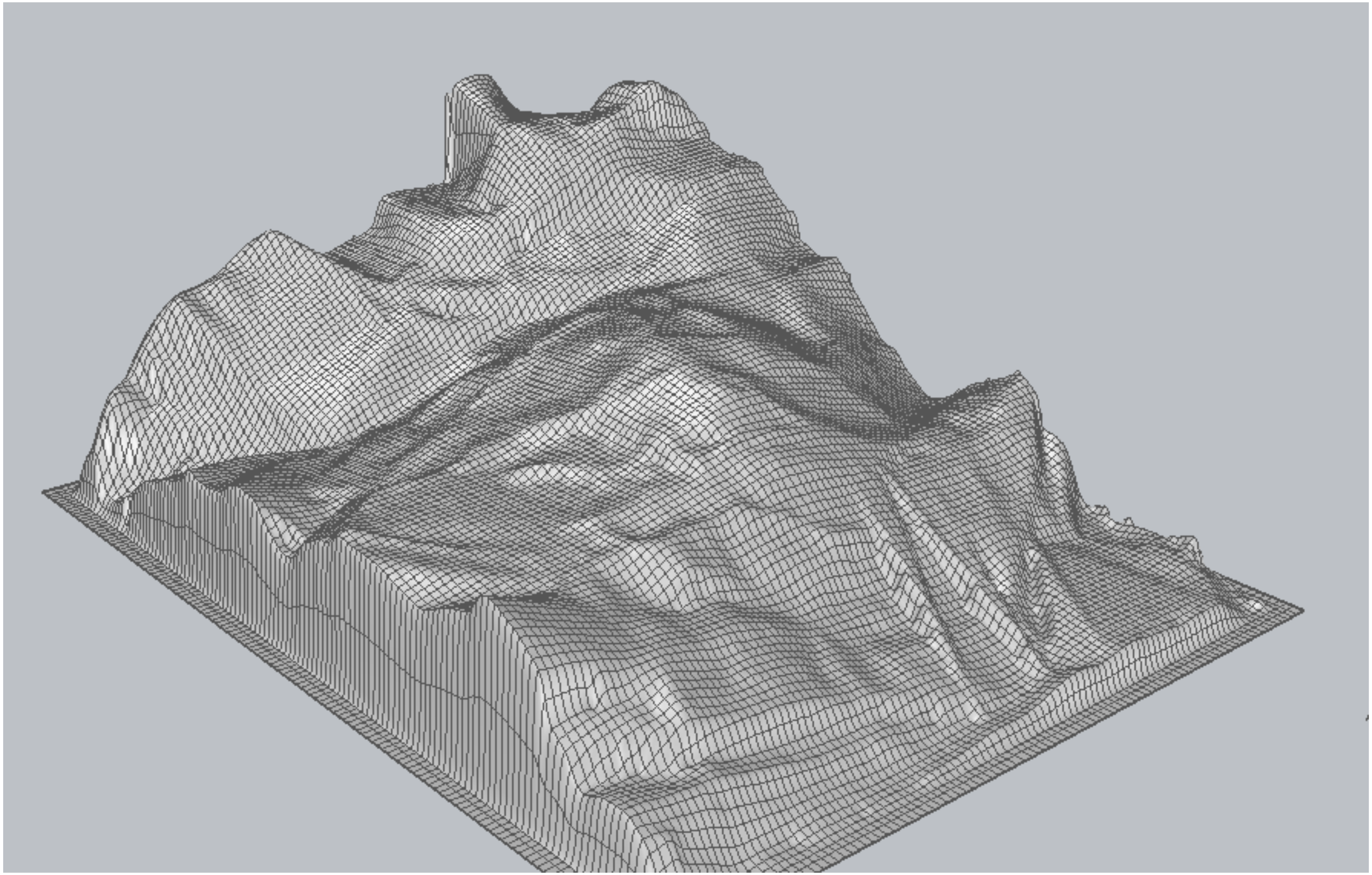

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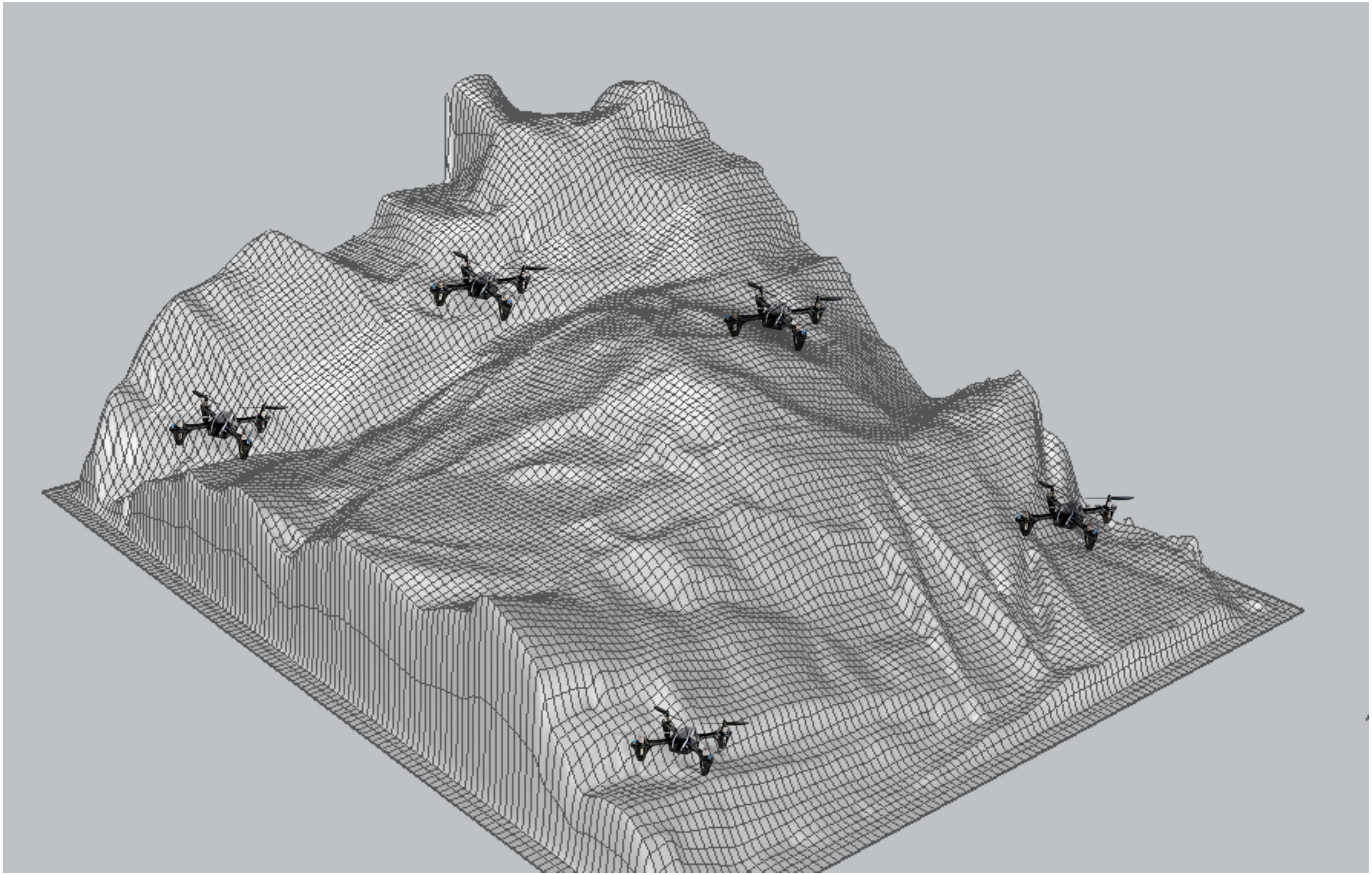

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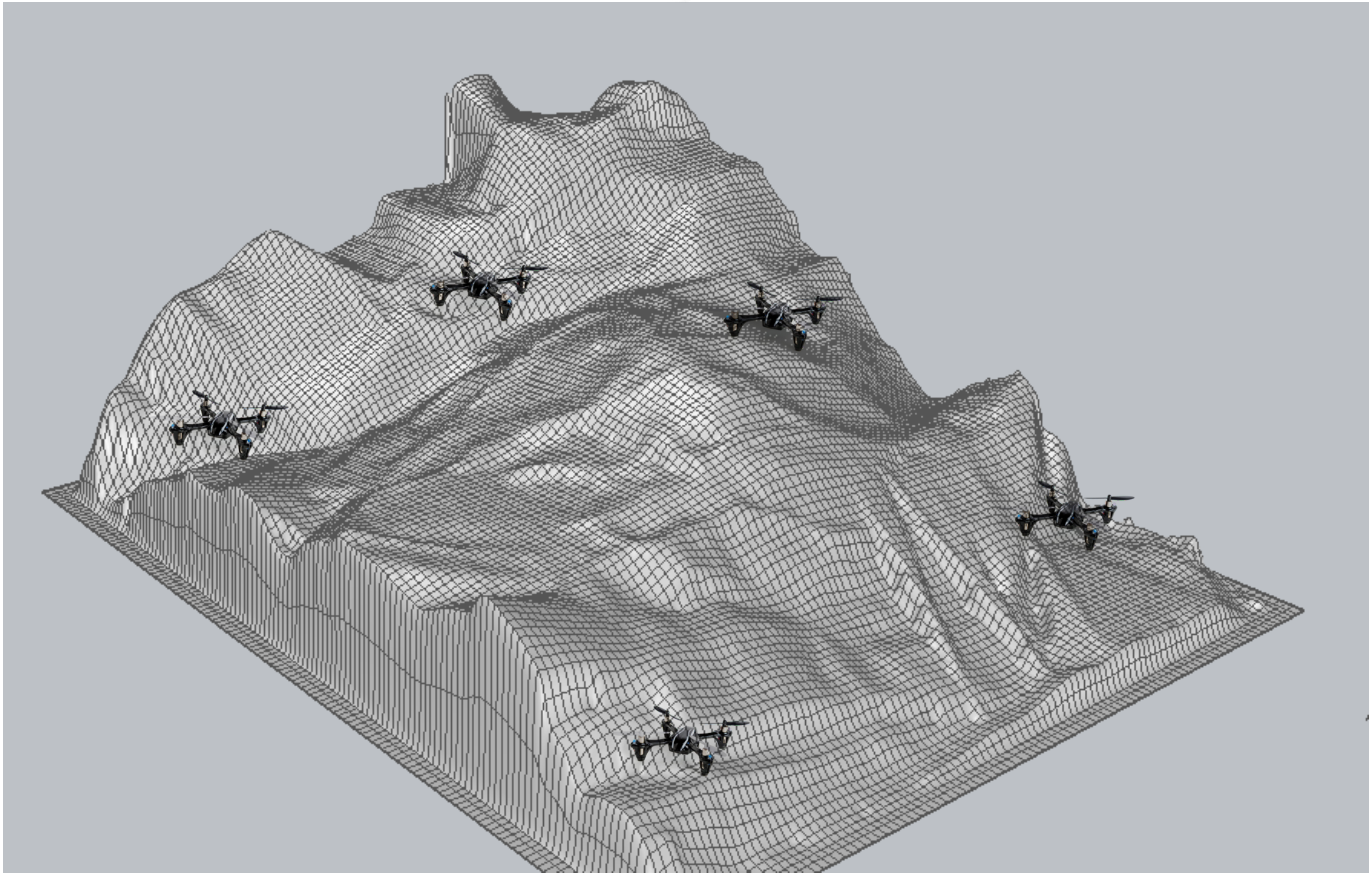

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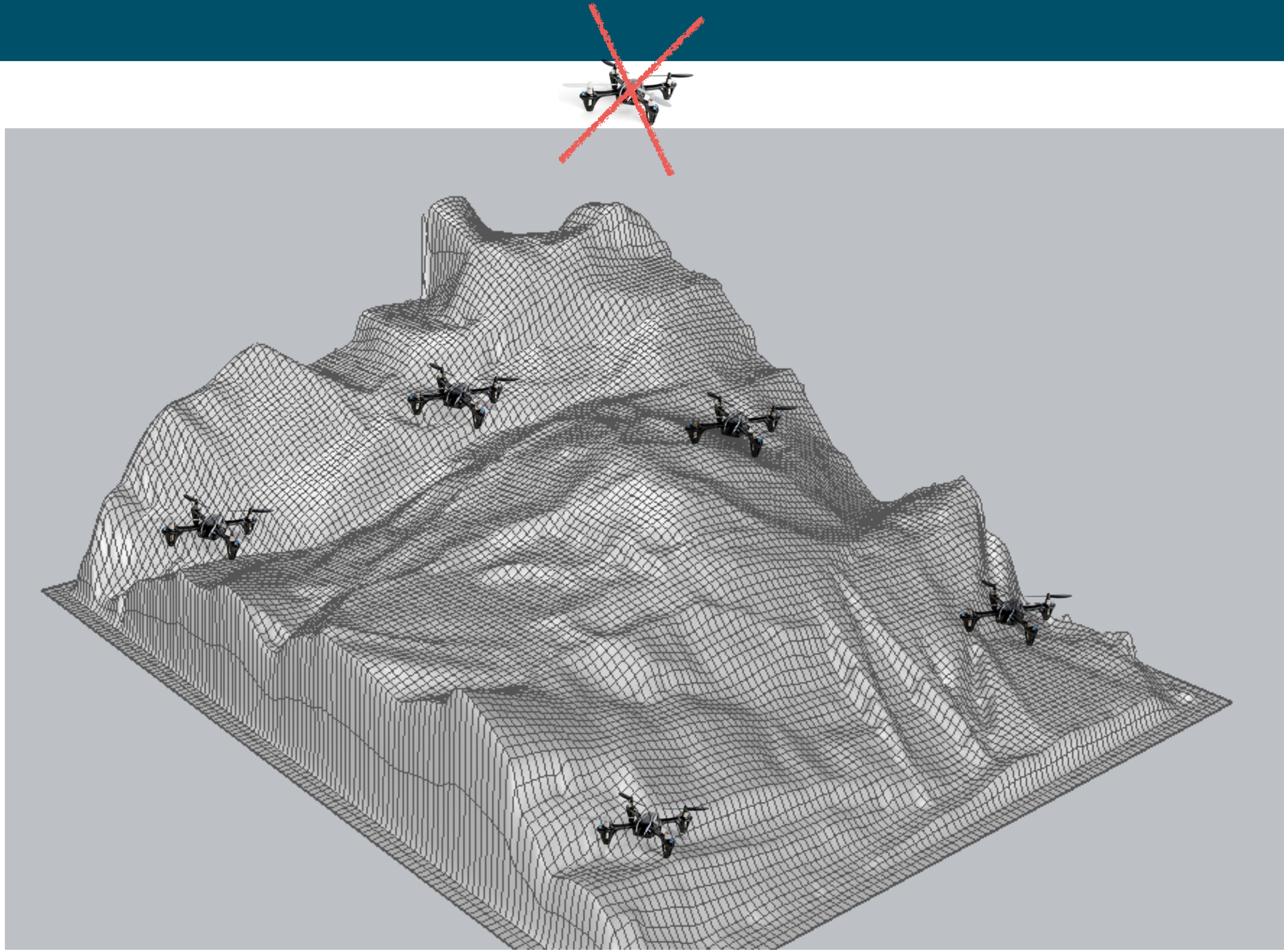

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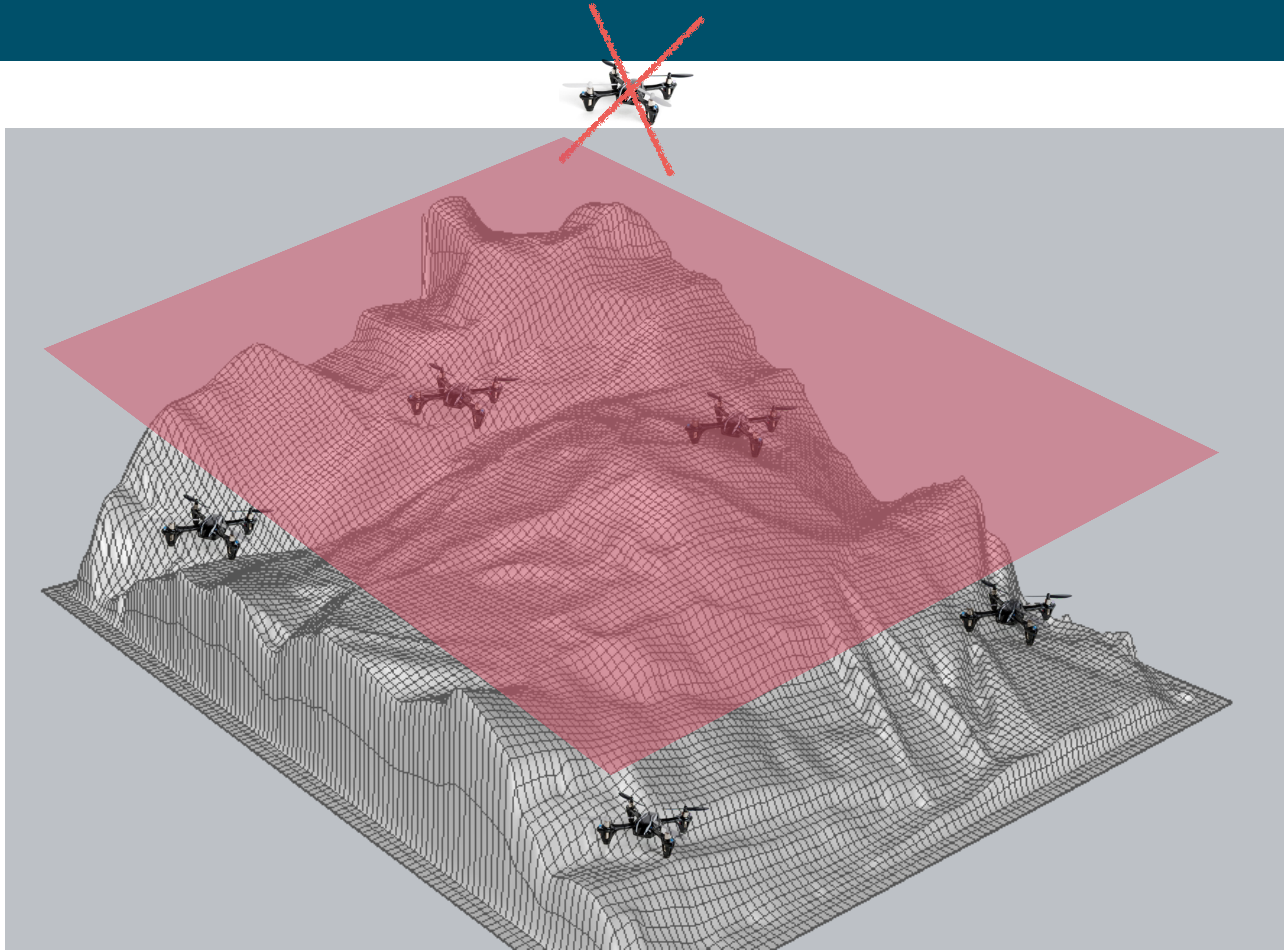

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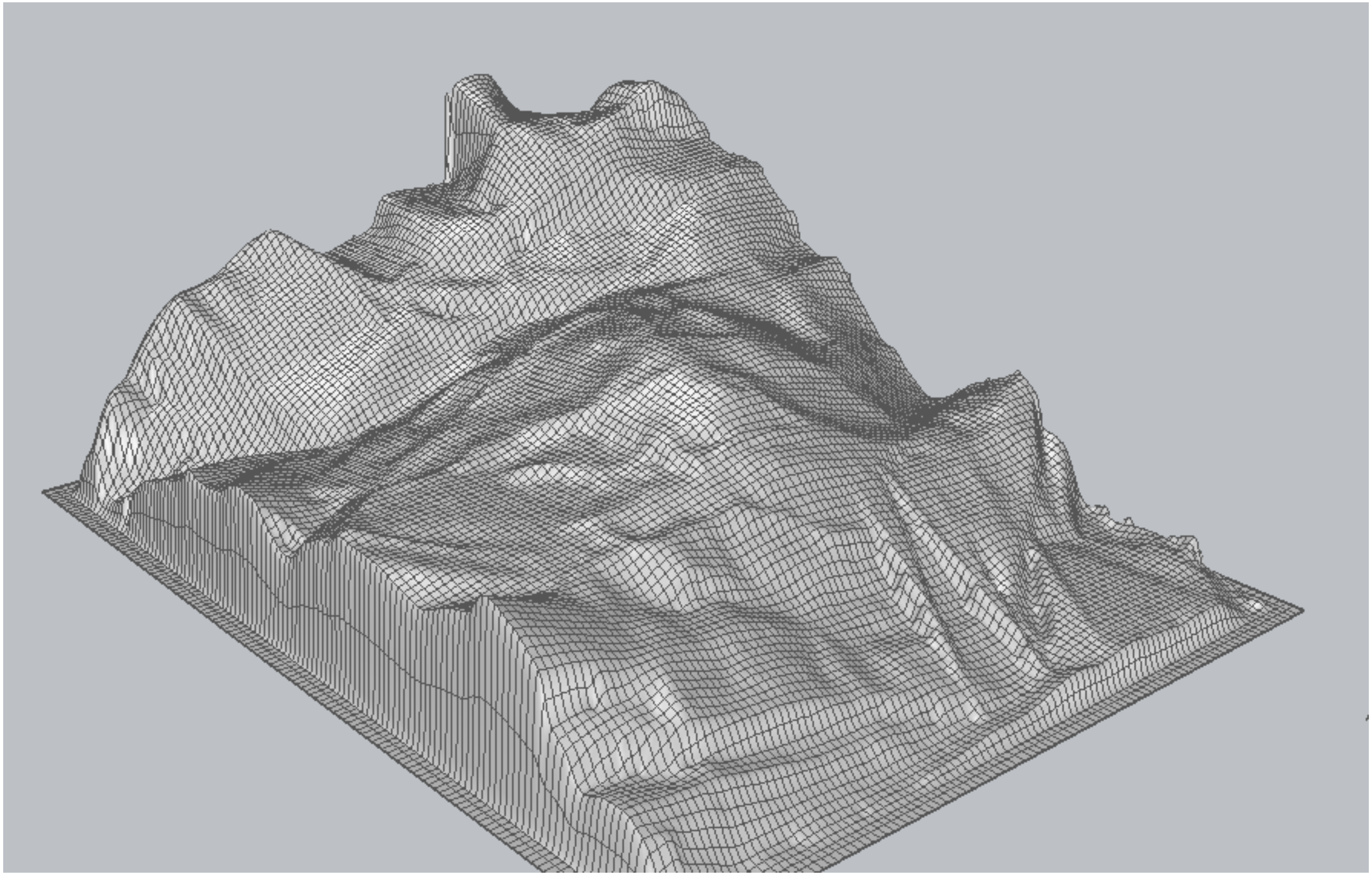

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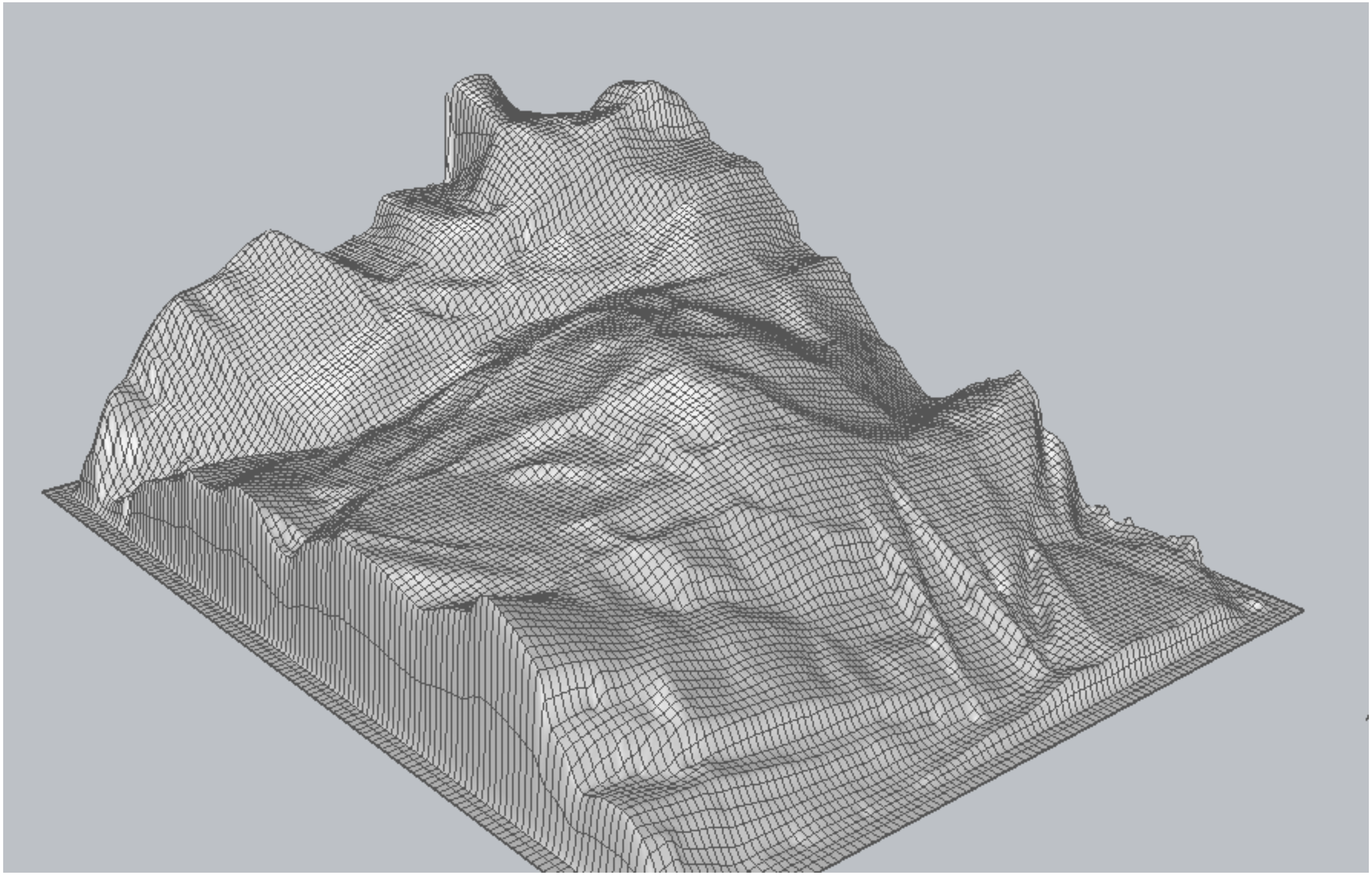
2.5D



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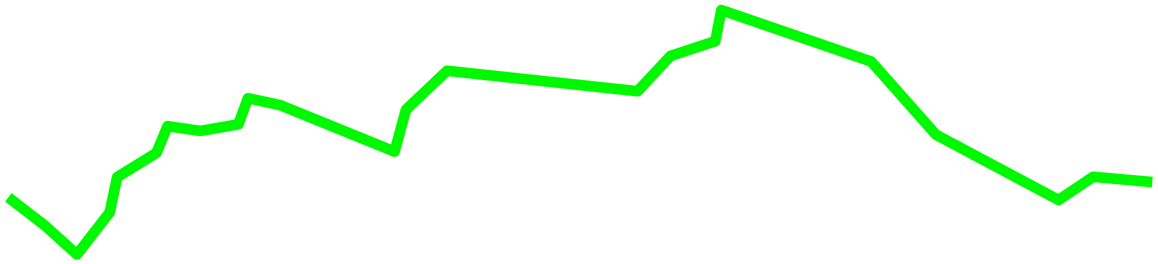
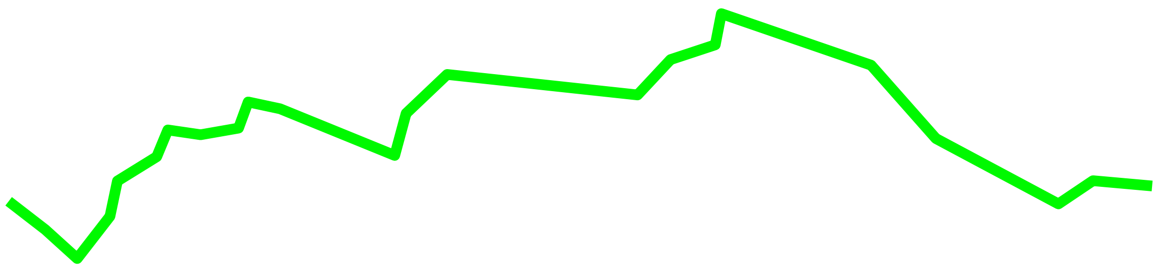


1.5D

Altitude Terrain Guarding Problem



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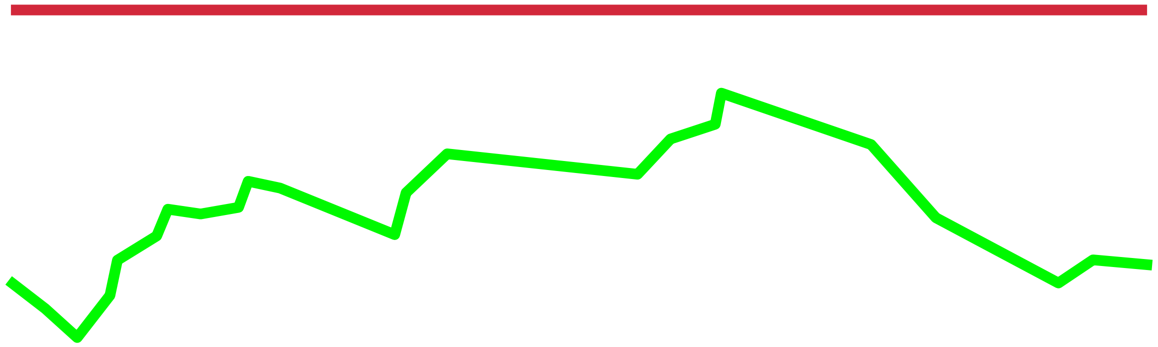

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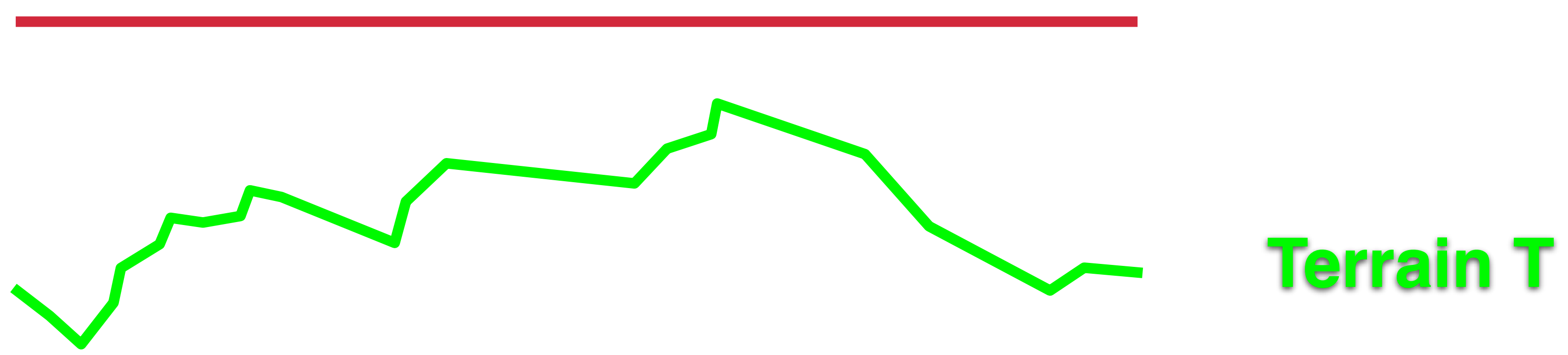

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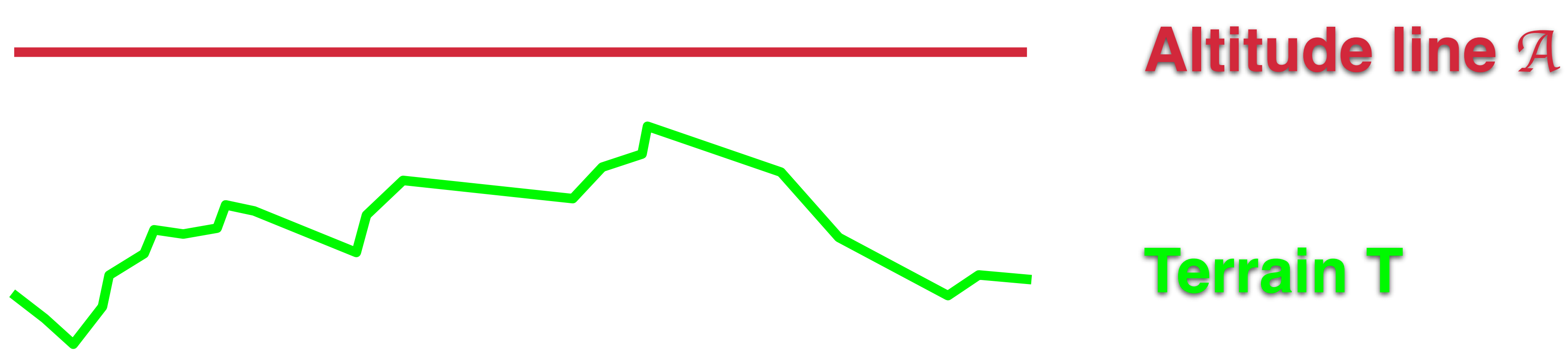

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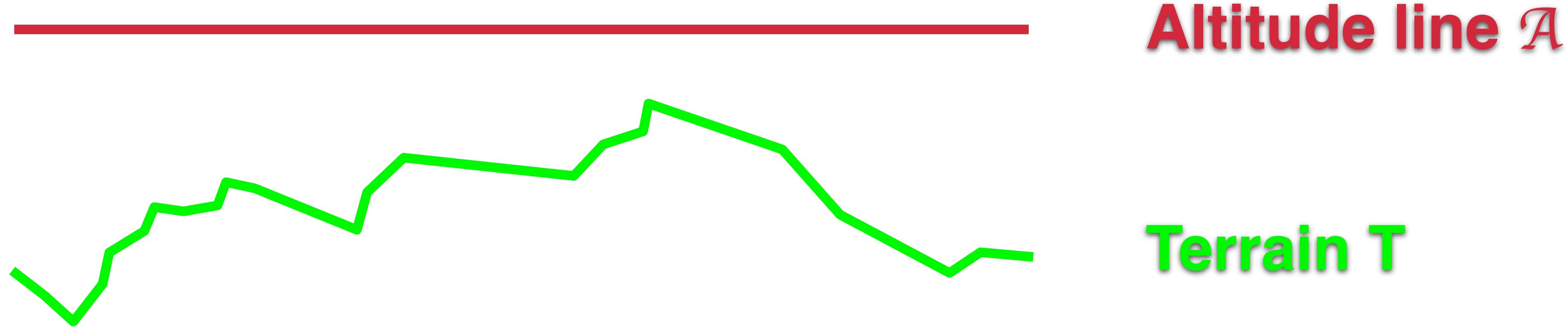

x-monotone chain of line segments in R^2
defined by its vertices $V(T) = \{ v_1 , \ldots , v_n\}$

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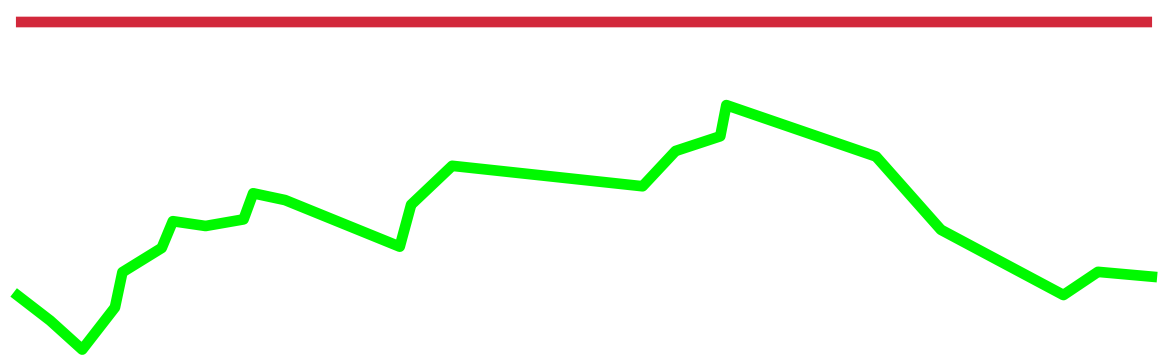

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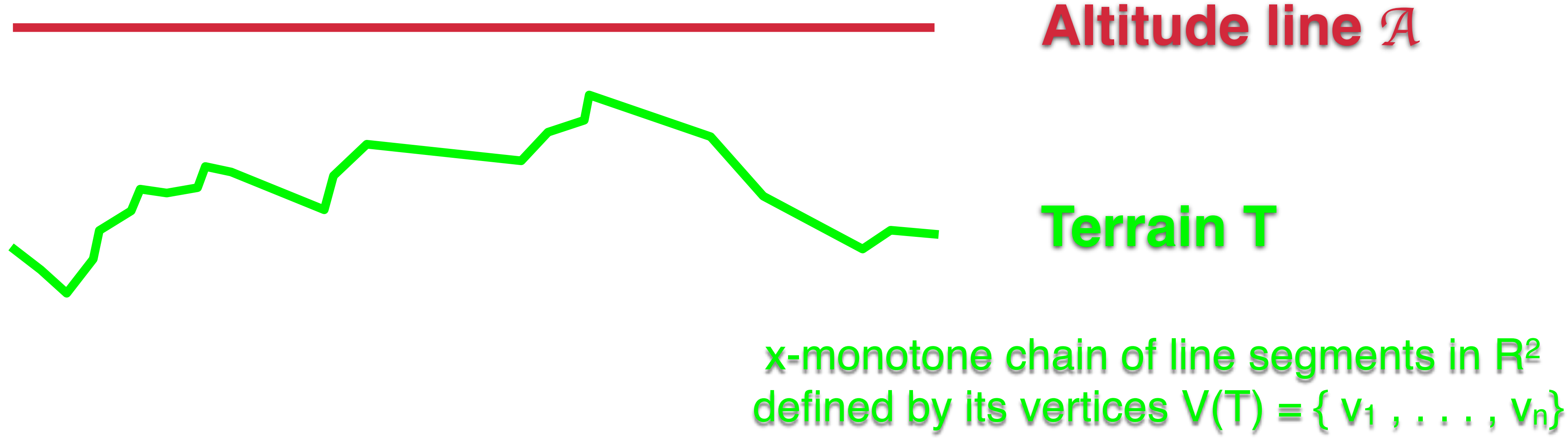
Monotonicity

Altitude Terrain Guarding Problem

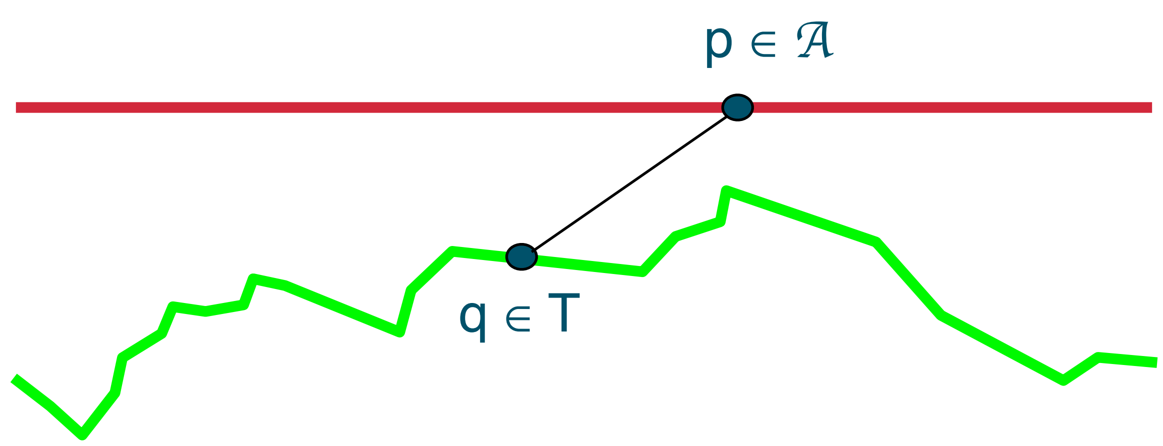


Monotonicity

➡ Points on T are totally ordered wrt to x-coordinate: $p<q$.

Altitude Terrain Guarding Problem

A point p ∈ $\mathcal{A}$ sees or covers q ∈ T if and only if pq is nowhere below T (i.e. pq lies on or above T).



Altitude line $\mathcal{A}$

Terrain T

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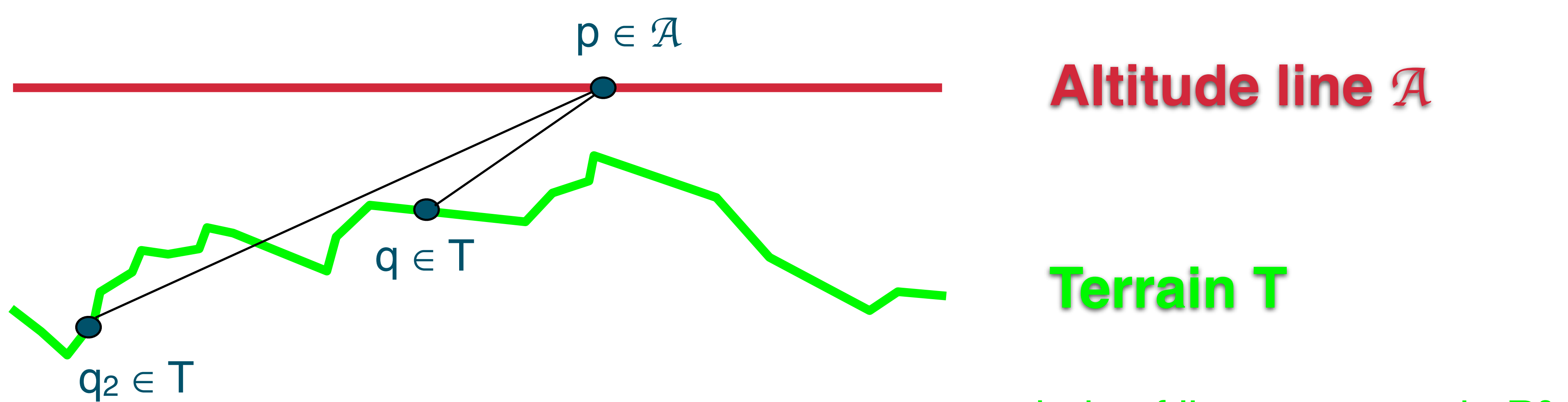

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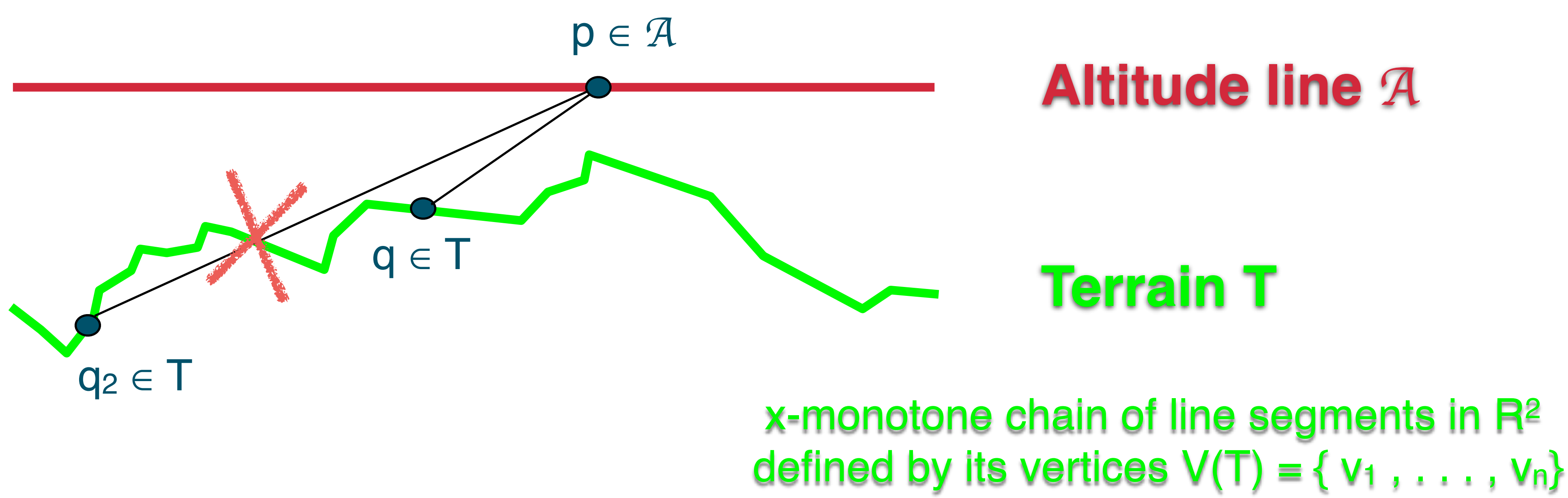

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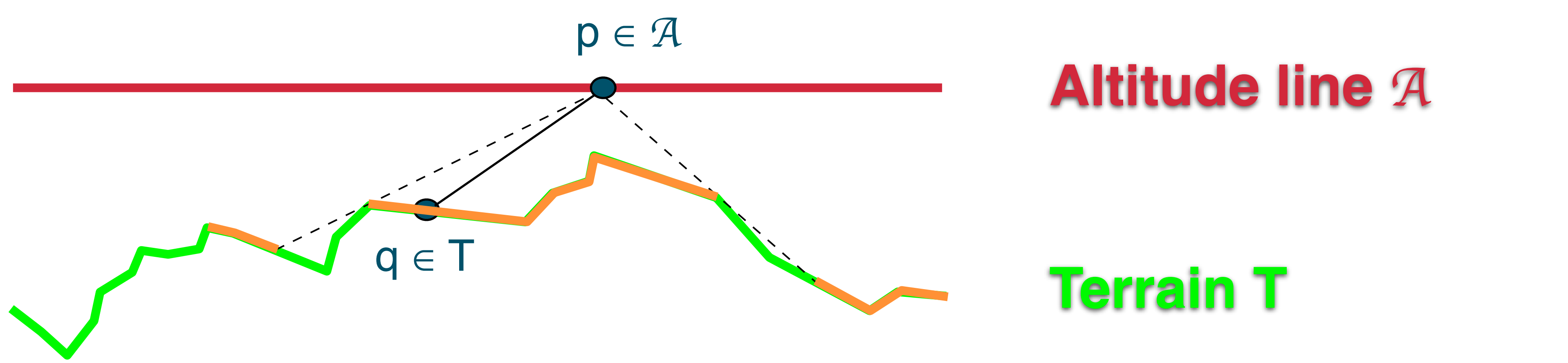
A point $p \in \mathcal{A}$ sees or covers $q \in T$ if and only if pq is nowhere below T (i.e. pq lies on or above T).

$V_T(p)$ is the visibility region of p with $V_T(p) := \{q \in T \mid p \text{ sees } q\}$.



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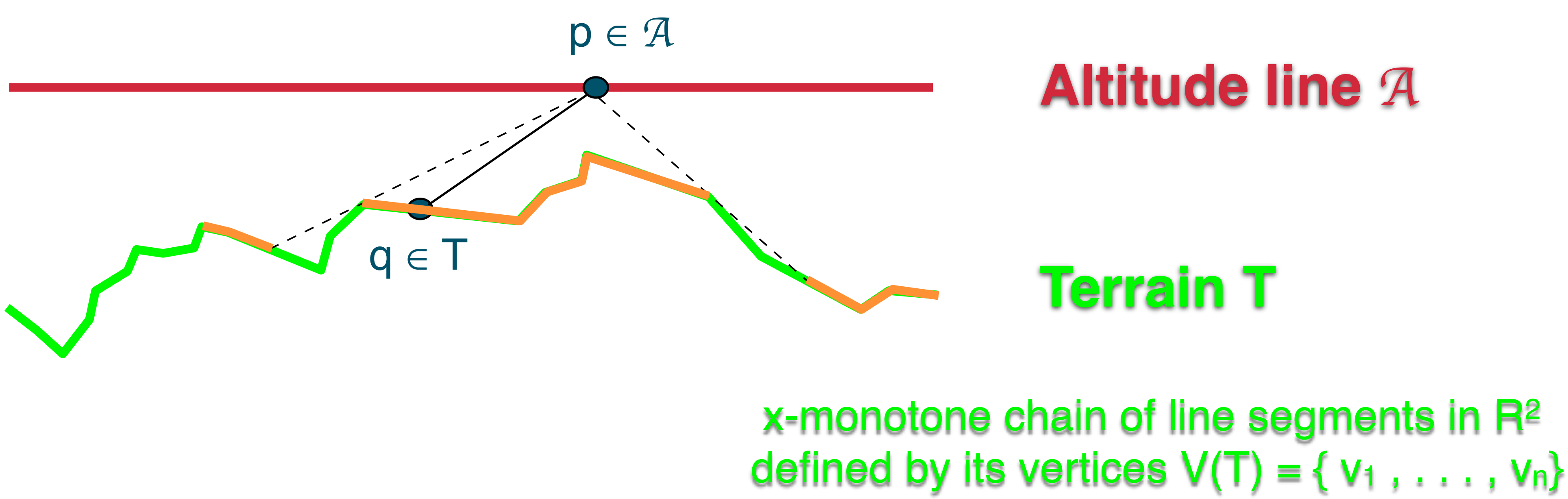

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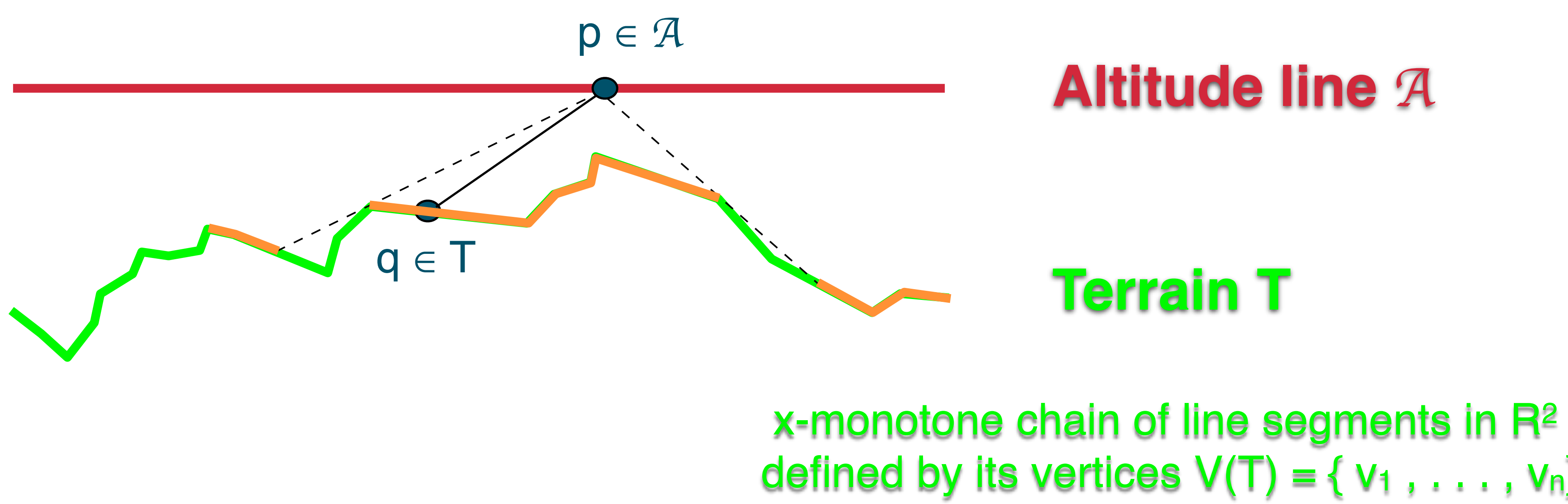

Altitude Terrain Guarding Problem (ATGP) ATGP(T,$\mathcal{A}$)

Given: a terrain T and an altitude line $\mathcal{A}$.

A minimum set of guards that see all of T.

(Formally: A guard set $G \subset \mathcal{A}$ is optimal w.r.t. ATGP(T,$\mathcal{A}$) if G is feasible, that is, $T \subseteq V_T(G)$, and

$|G| = OPT(T,\mathcal{A}) := \min\{|C| \mid C \subset \mathcal{A}$ is feasible w.r.t. ATGP(T,$\mathcal{A}$)$\}$.)

AGP in uni-monotone polygons

uni-monotone polygon?

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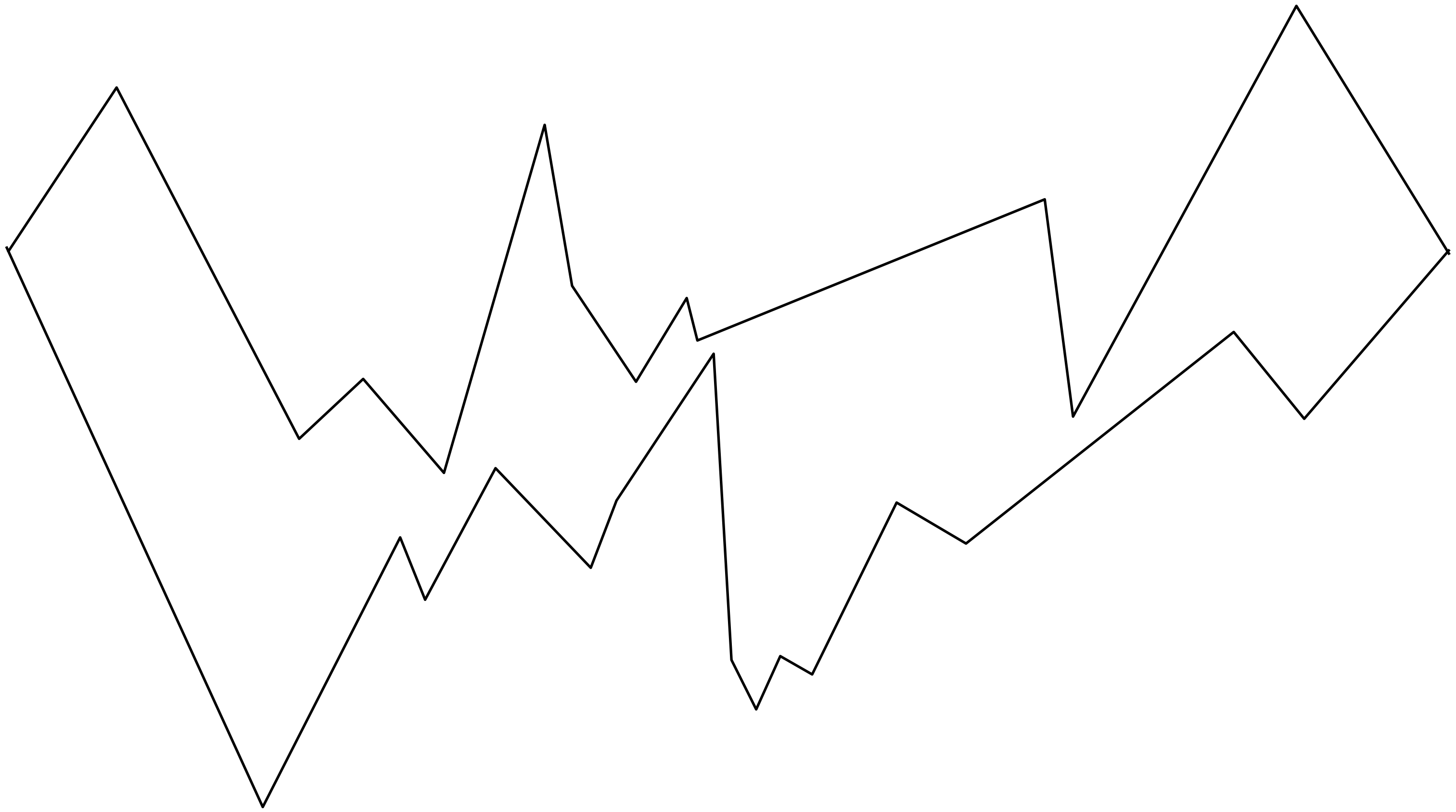
uni-monotone polygon?

monotone polygon



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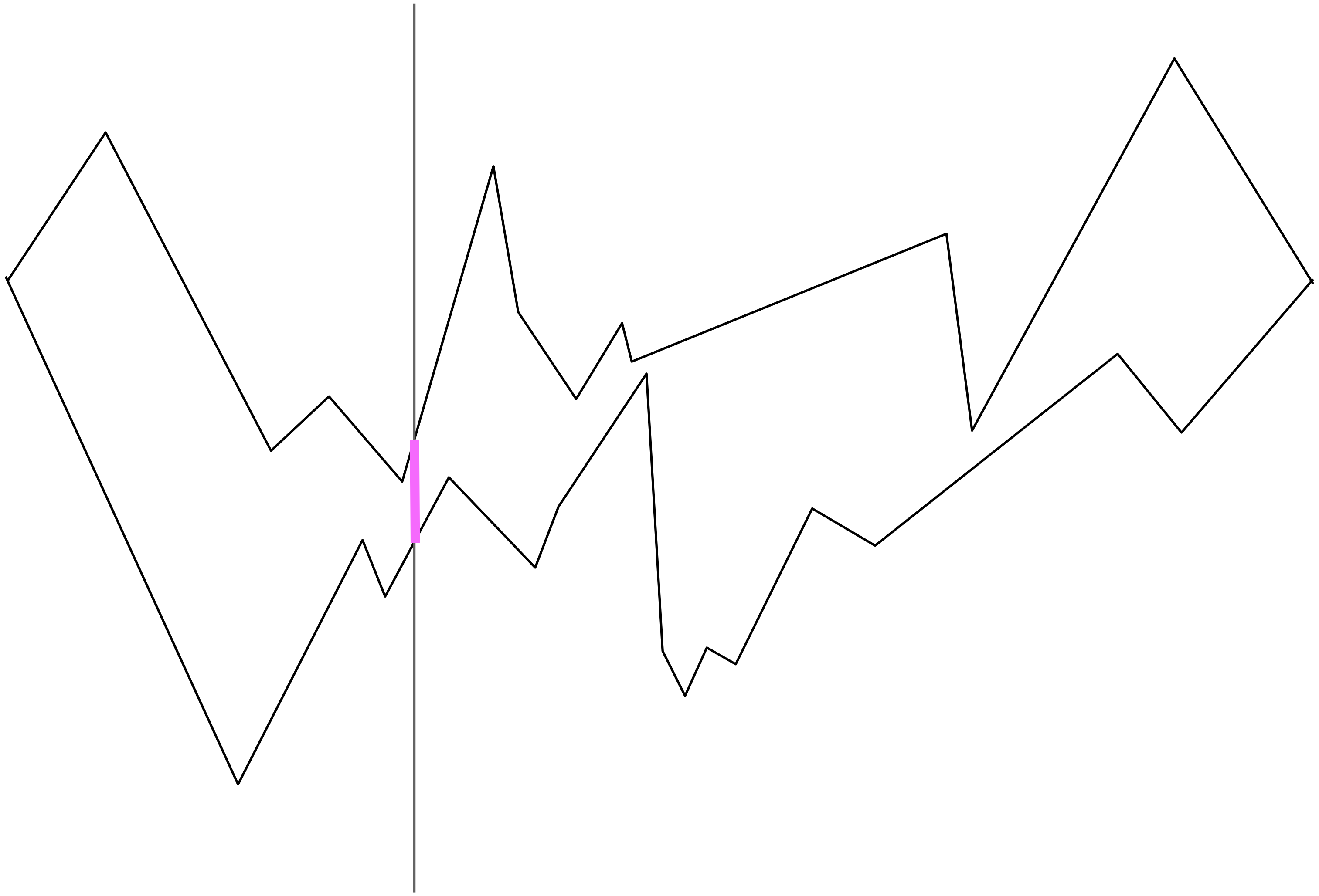

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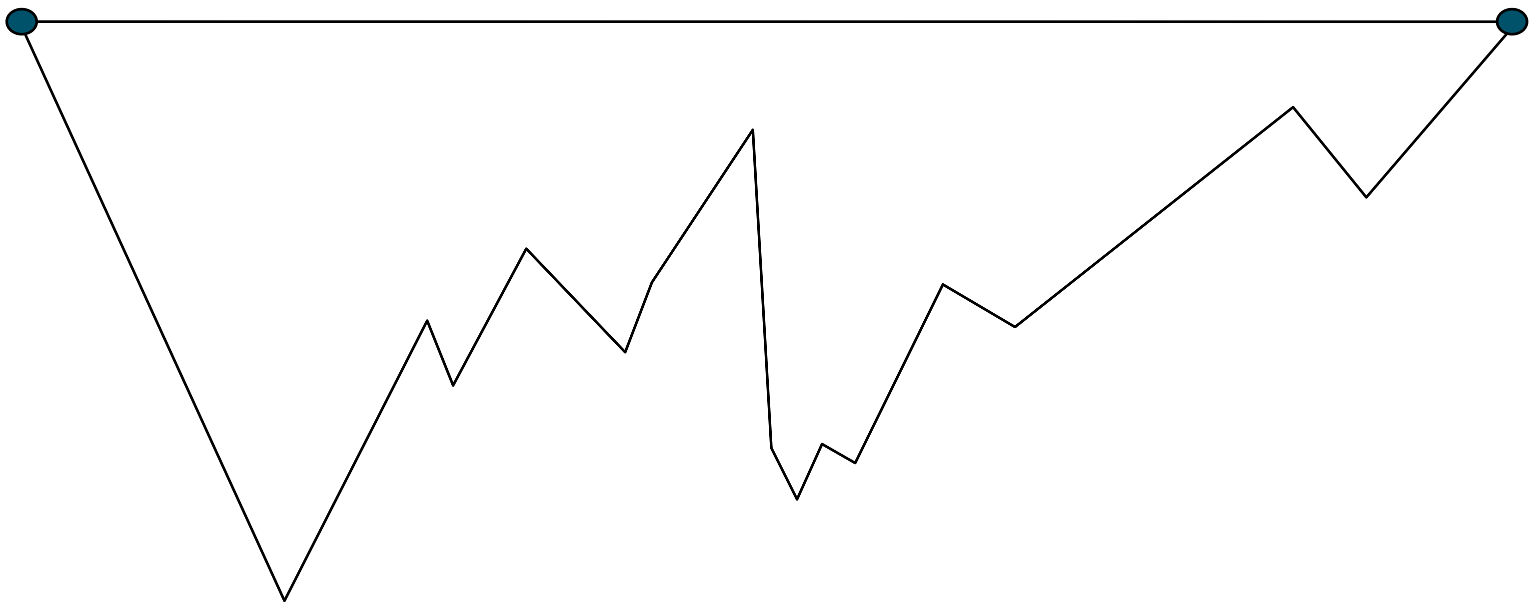

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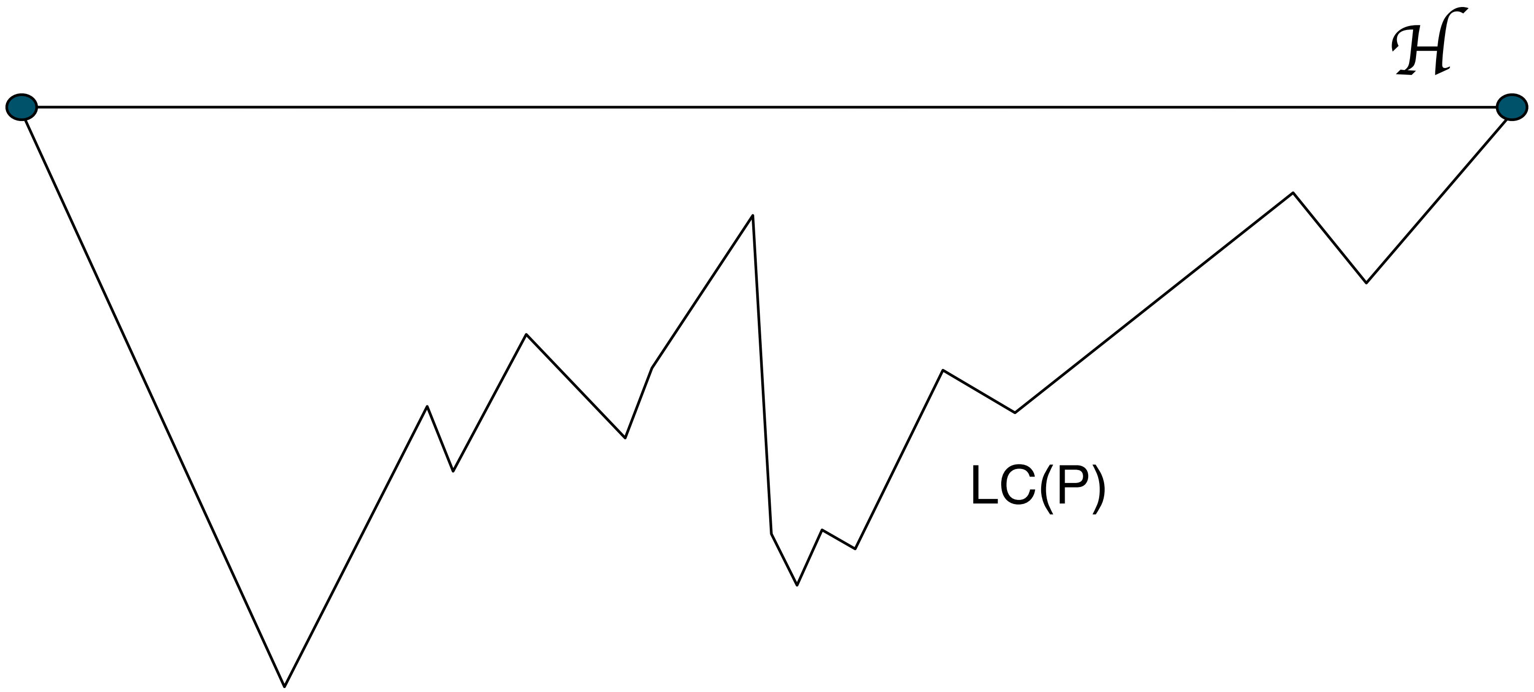

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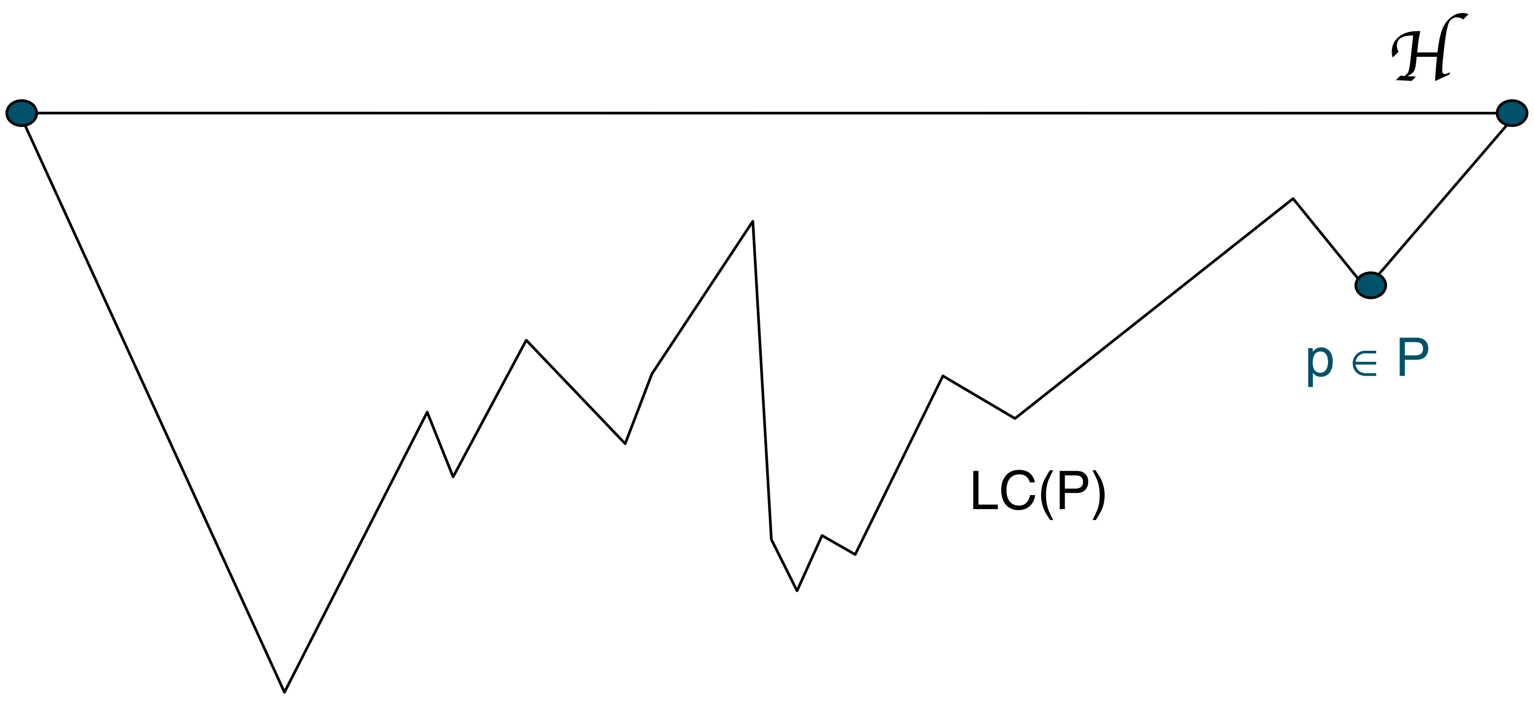

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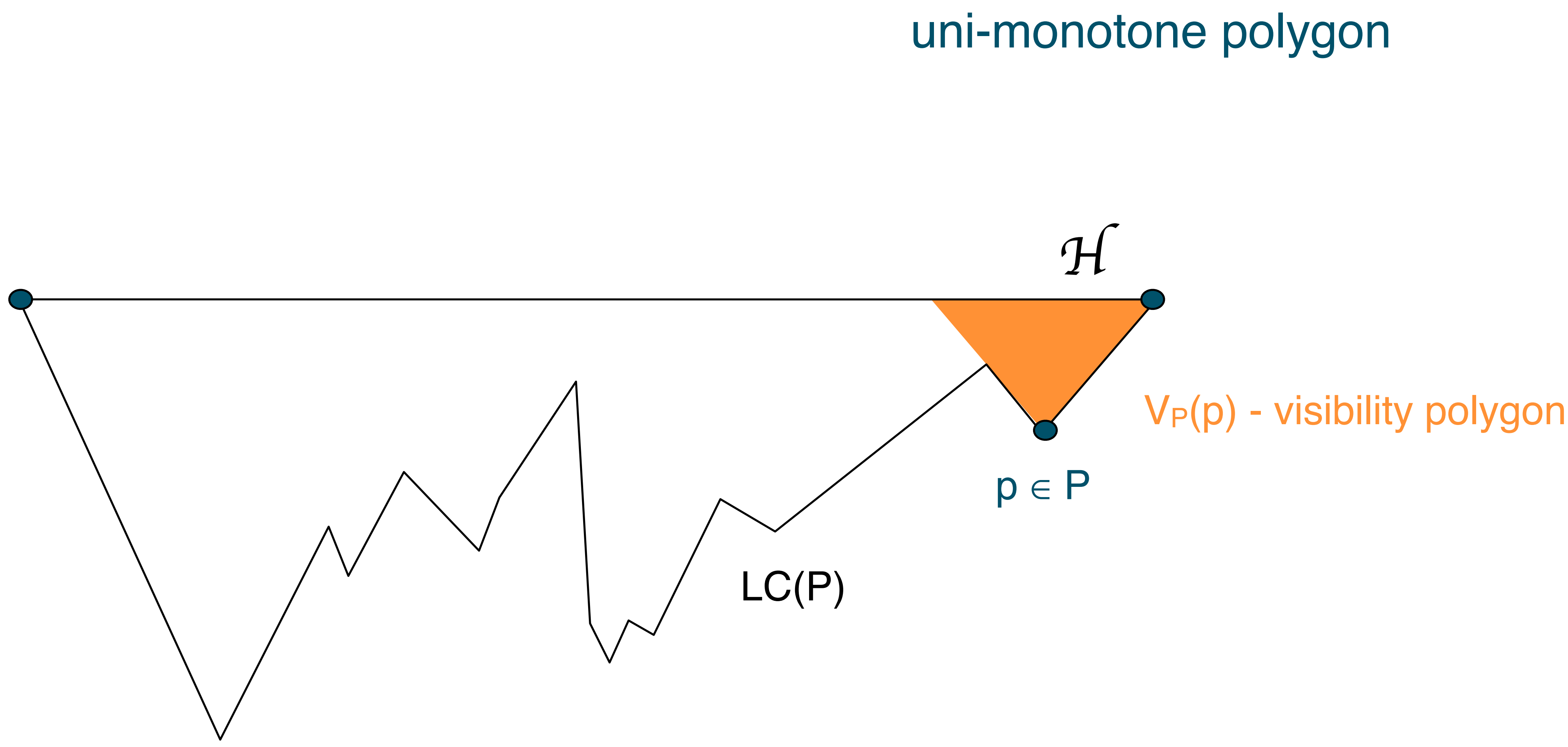

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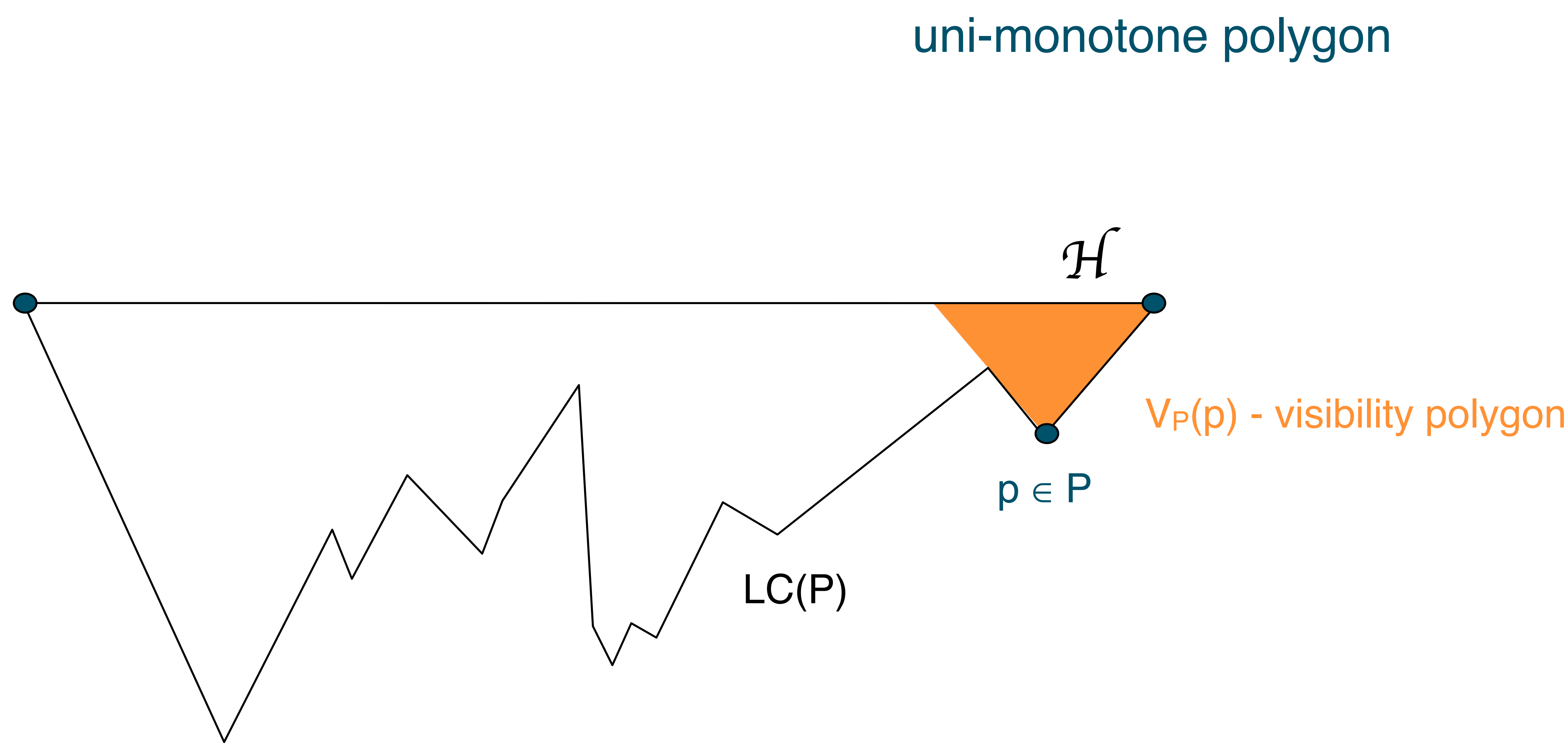

Formally:

Art Gallery Problem (AGP) AGP(G,W)

Given: a polygon P and sets of guard candidates and points to cover $G, W \subseteq P$.

A minimum guard set $C \subseteq G$ that covers W (that is, $W \subseteq V_P(C)$).

We want to solve AGP(P,P).

AGP in uni-monotone polygons

If we want to solve the AGP for a uni-monotone polygon, w.l.o.g. we can restrict our guards to be located on $\mathcal{H}$.

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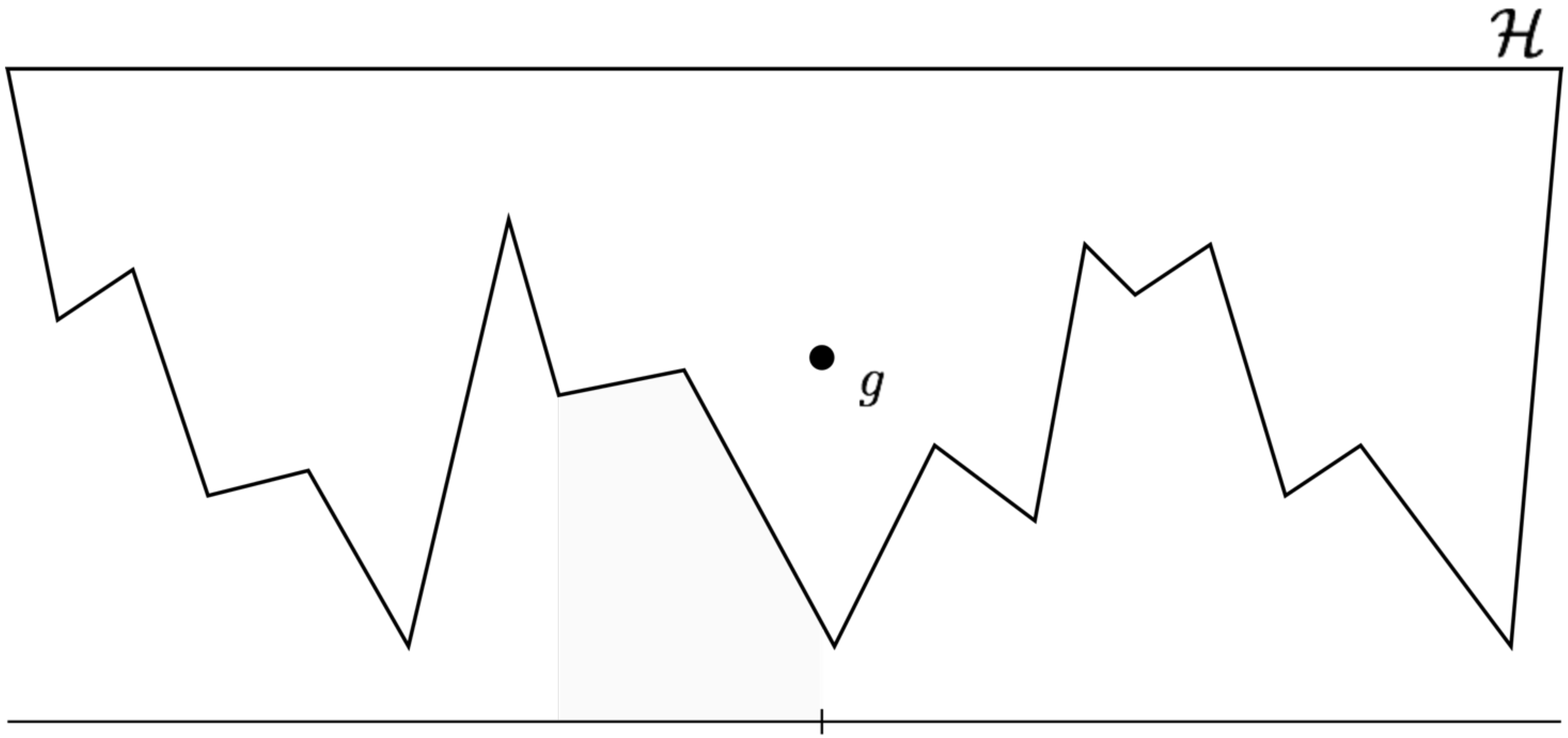
If we want to solve the AGP for a uni-monotone polygon, w.l.o.g. we can restrict our guards to be located on $\mathcal{H}$.

Proof: Consider any optimal guard set G, let g∈G be a guard not located on $\mathcal{H}$



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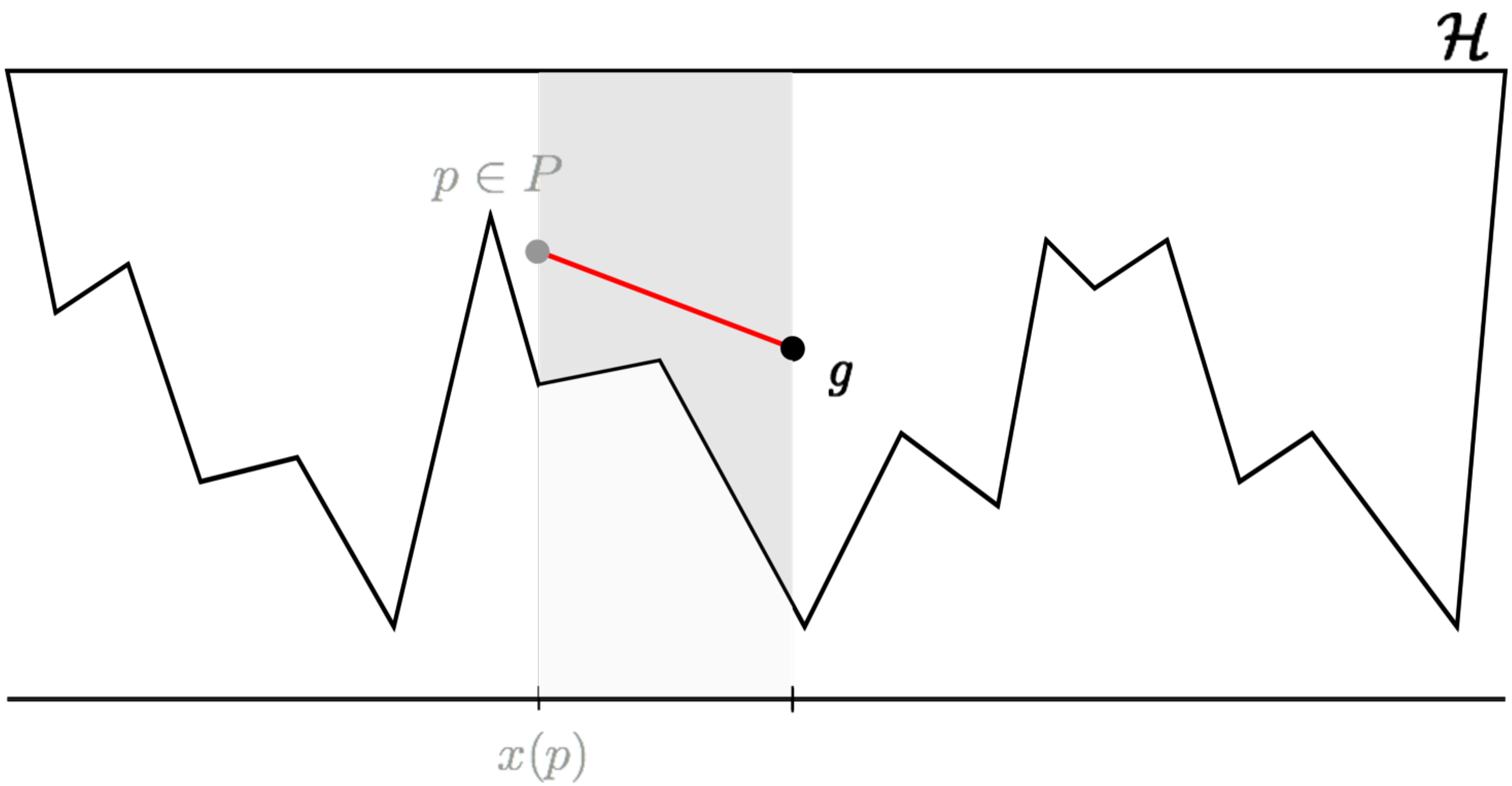

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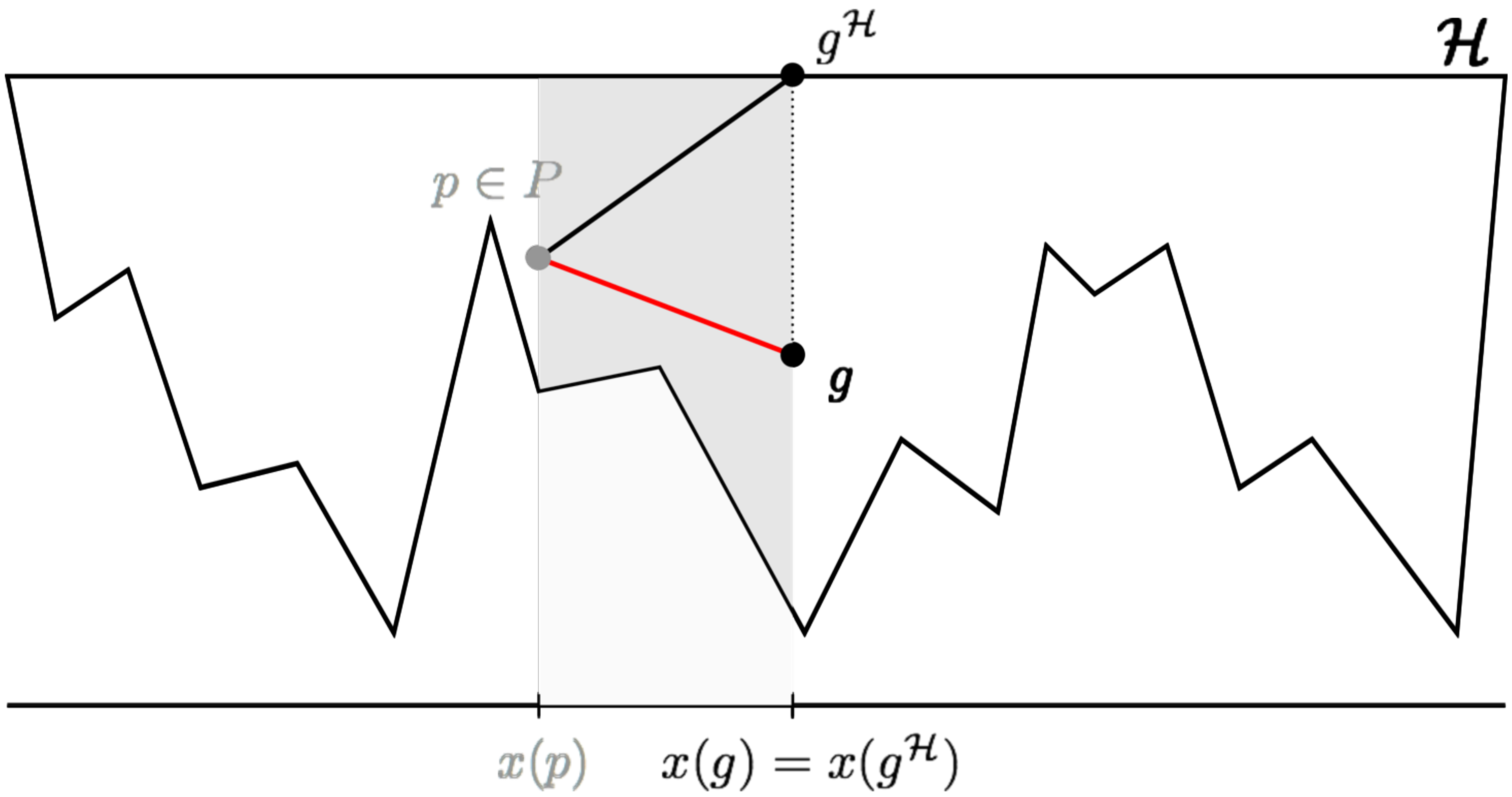

AGP in uni-monotone polygons

Let P be a uni-monotone polygon, let G be a guard set with $g \in \mathcal{H}$ $\forall g \in G$ that covers $LC(P)$, that is, $LC(P) \subset V_P(G)$. Then G covers all of P, that is, $P \subseteq V_P(G)$.

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Proof: Assume $p \in P$, $p \notin LC(P)$, $p \notin V_P(G)$

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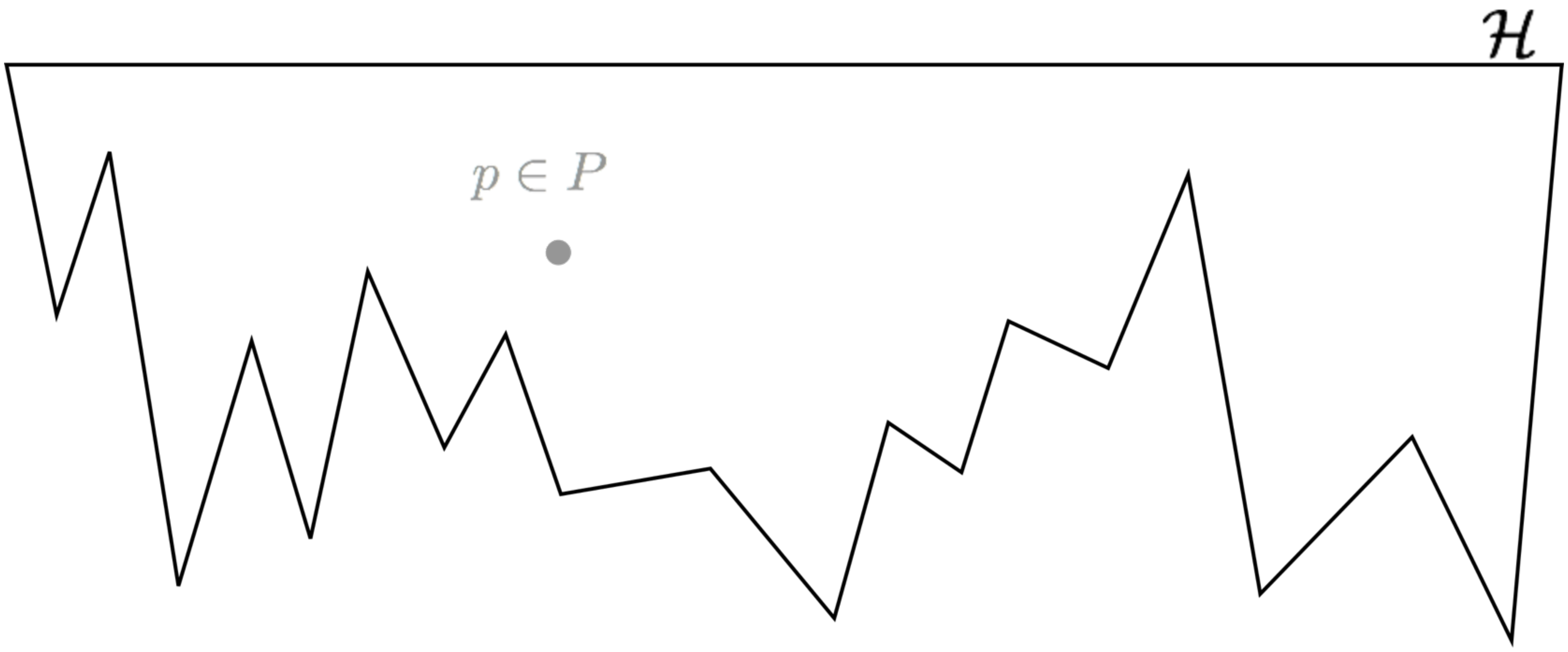

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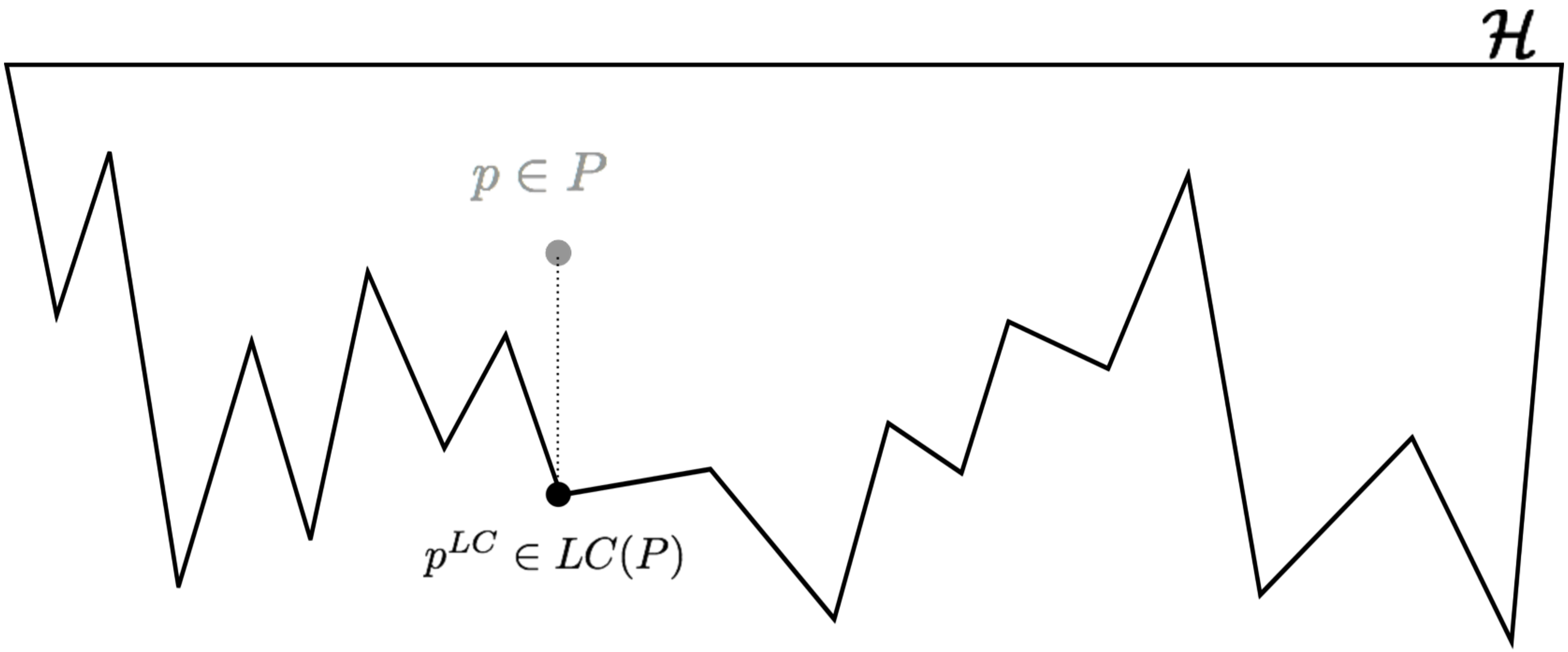

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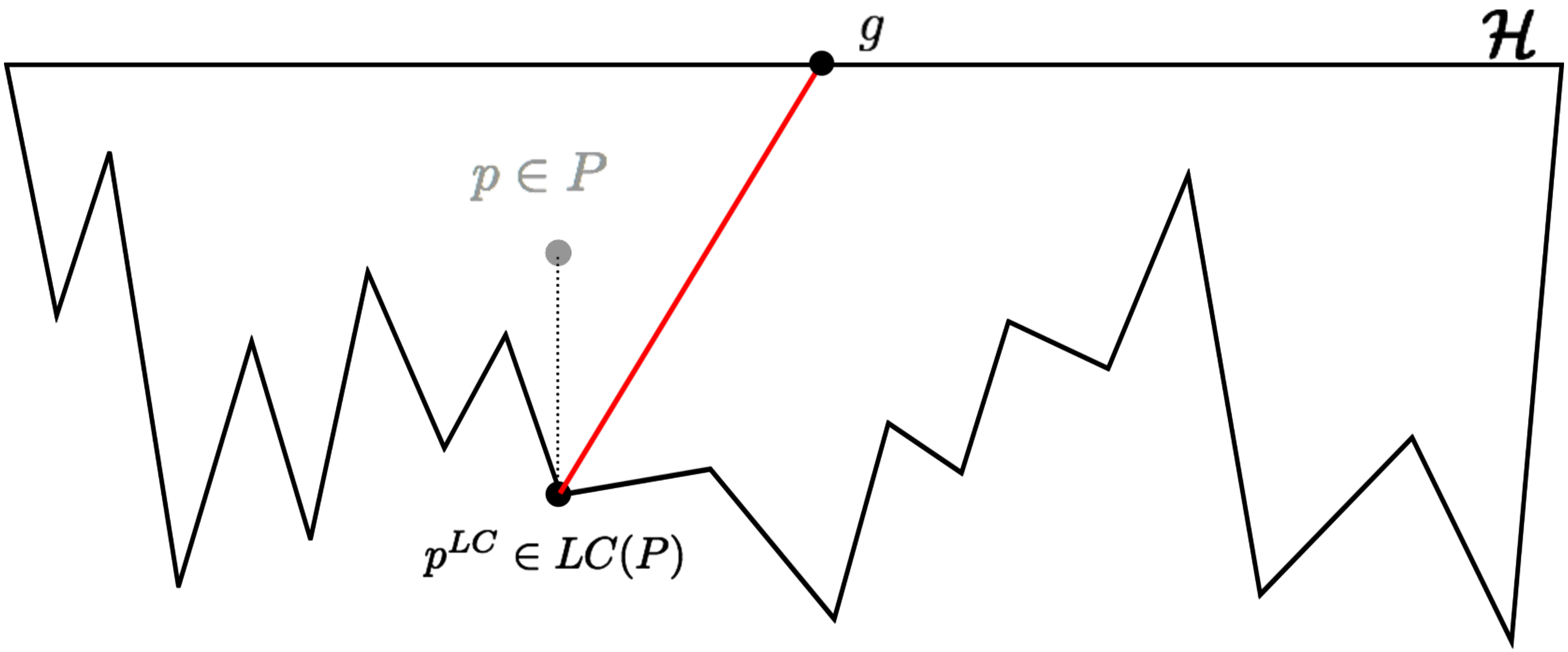

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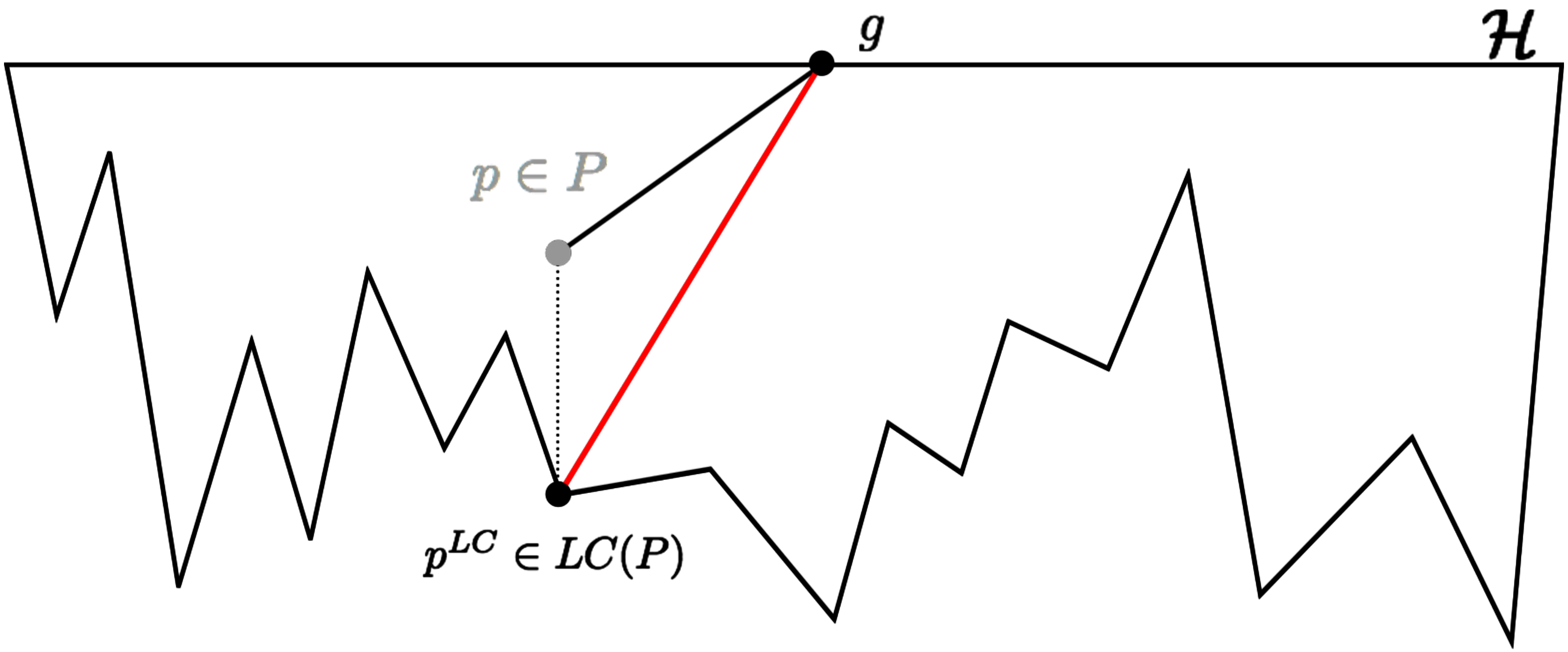

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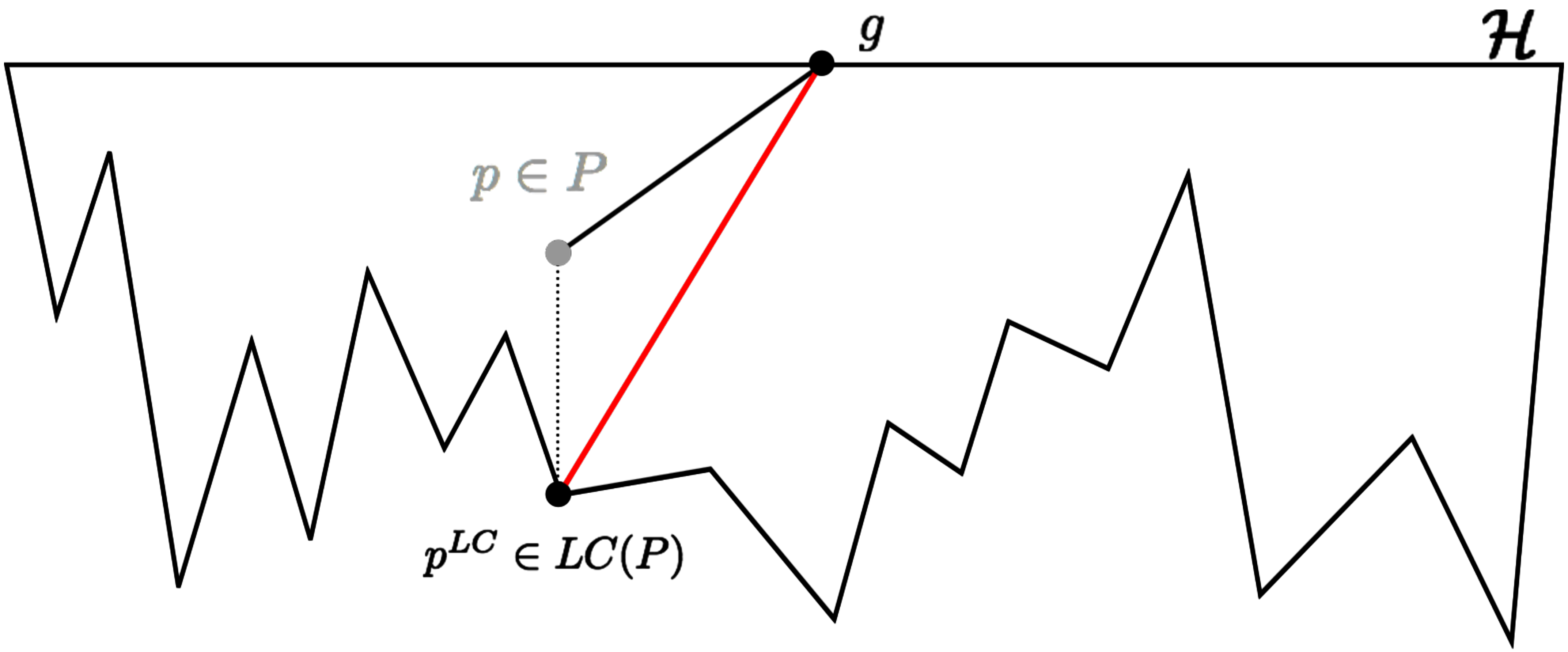
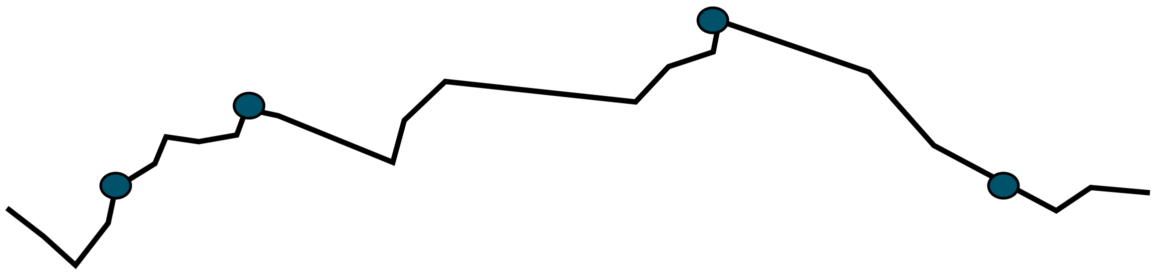


➜ATGP and AGP for uni-monotone polygons equivalent

AGP-TGP-ATGP

Art Gallery Problem (AGP)
Given: a polygon P.
A minimum guard set that covers P.



Terrain Guarding Problem (TGP)
Given: a terrain T.
Find: minimum number of guards located **on** T.

AGP-TGP-ATGP

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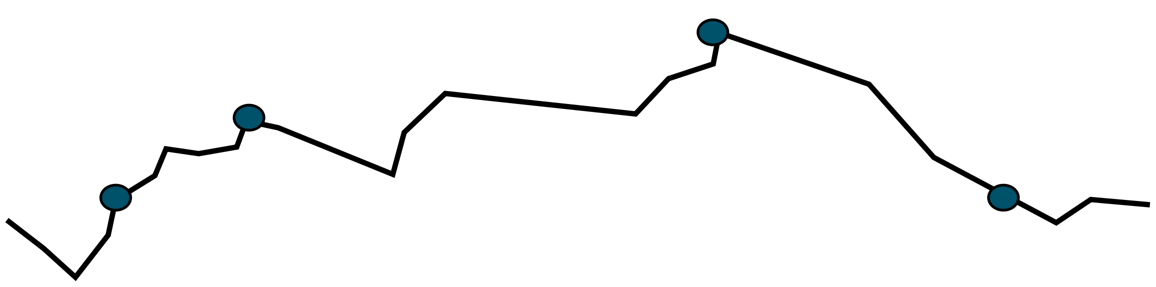

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$\boxed{\text{NP-hard}}$

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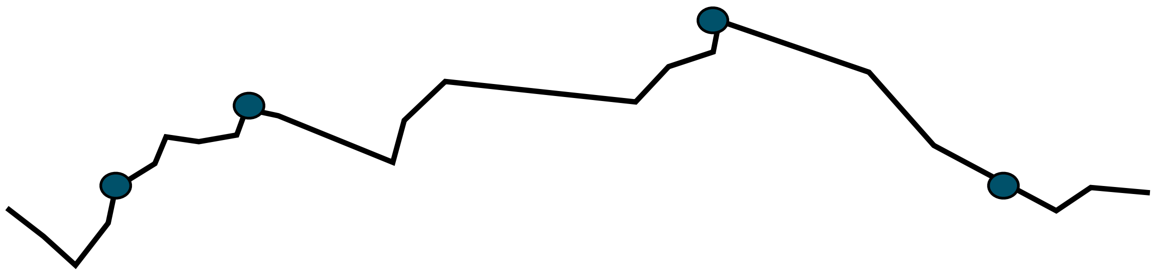
Given: a polygon P.
A minimum guard set that covers P.

NP-hard, even in monotone polygons



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NP-hard

Altitude Terrain Guarding Problem (ATGP)
Given: a terrain T and an altitude line $\mathcal{A}$.
A minimum set of guards that see all of T.

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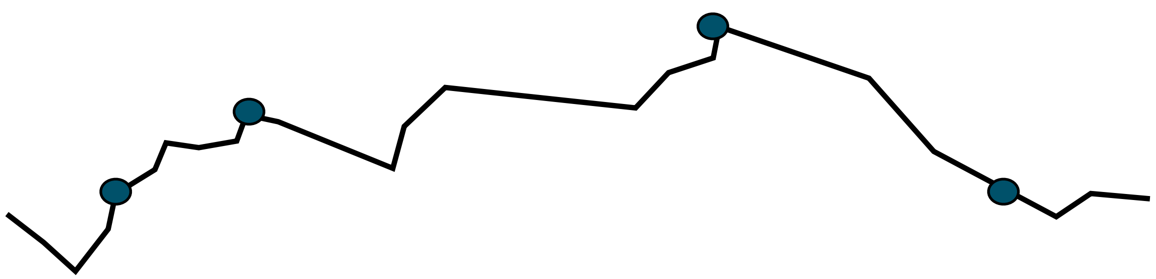

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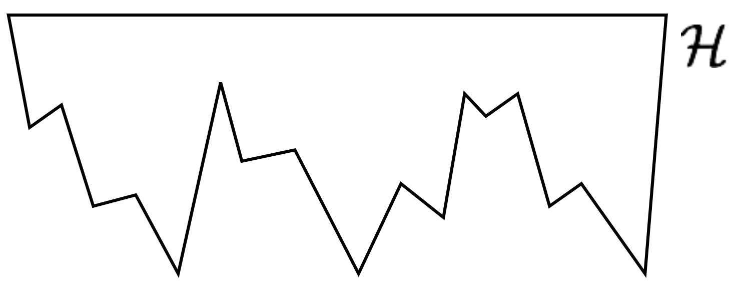

Art Gallery Problem (AGP) in Uni-Monotone Polygons
Given: a uni-monotone polygon P.
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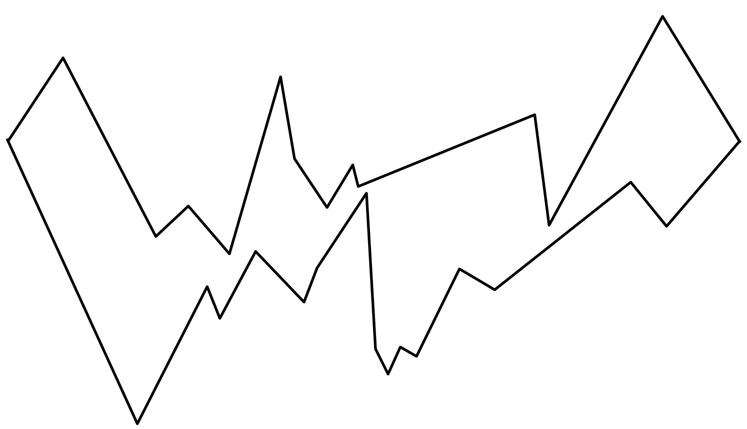

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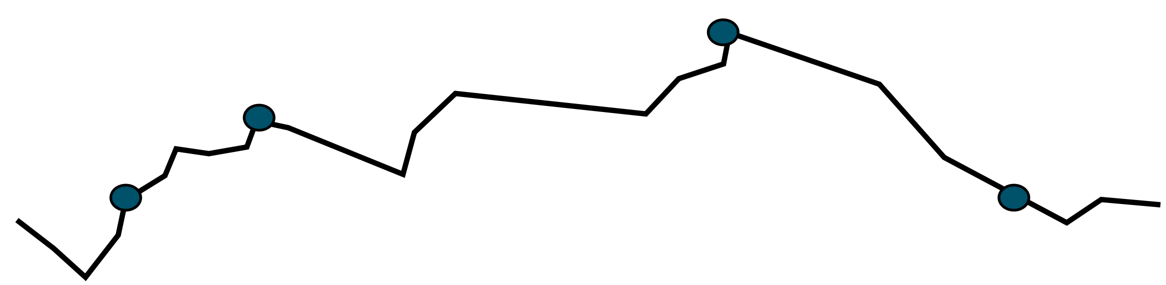

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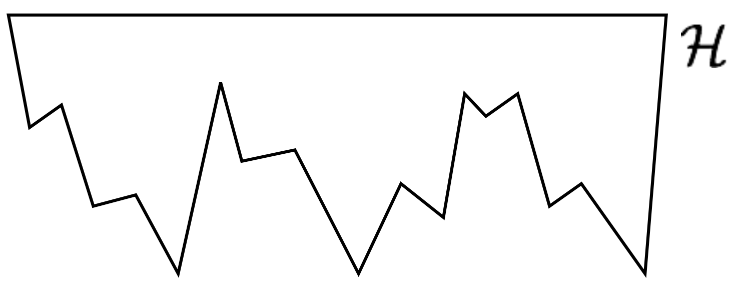

Art Gallery Problem (AGP) in Uni-Monotone Polygons
Given: a uni-monotone polygon P.
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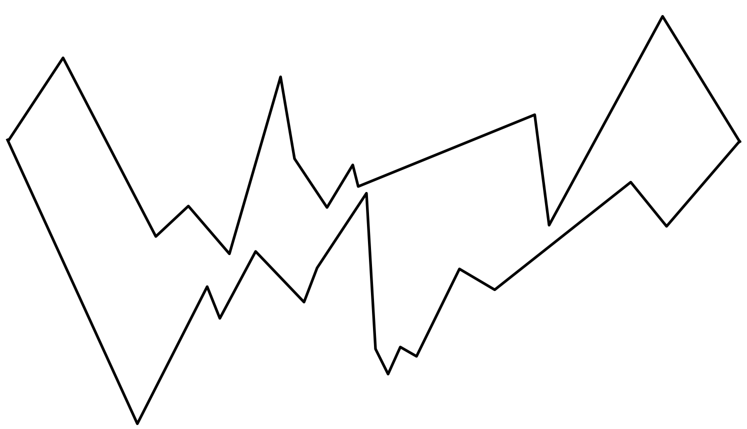
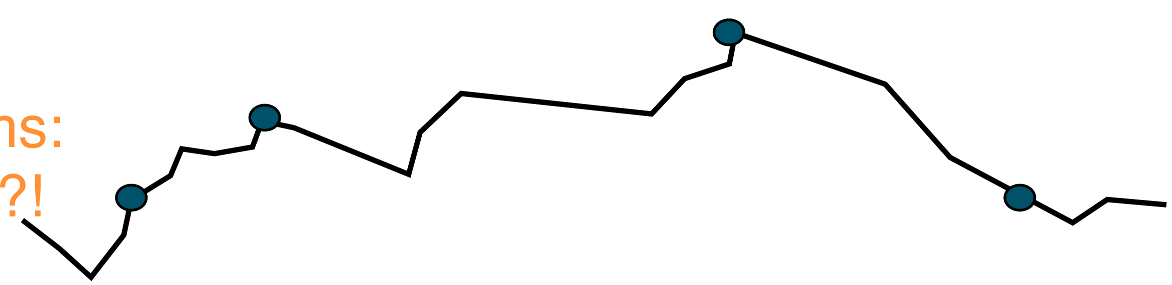


Our Problems:
“Inbetween”?!



Art Gallery Problem (AGP)
Given: a polygon P.
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NP-hard, even in monotone polygons

Terrain Guarding Problem (TGP)
Given: a terrain T.
Find: minimum number of guards located **on** T.

NP-hard

AGP-TGP-ATGP

Both polytime

Altitude Terrain Guarding Problem (ATGP)
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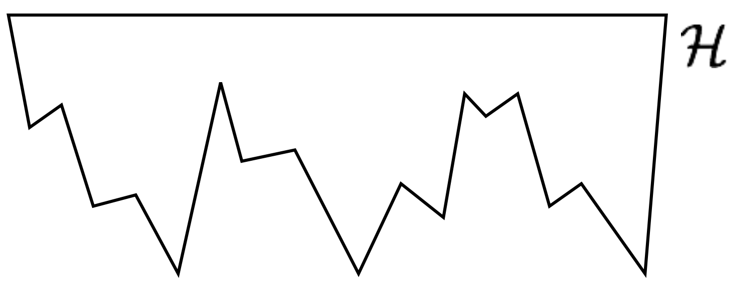

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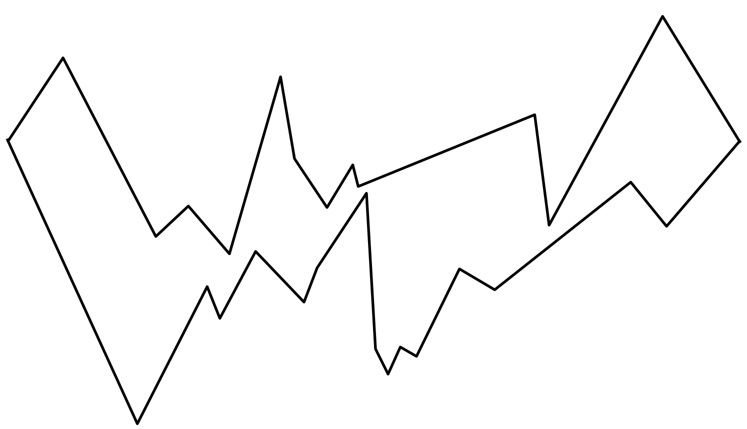

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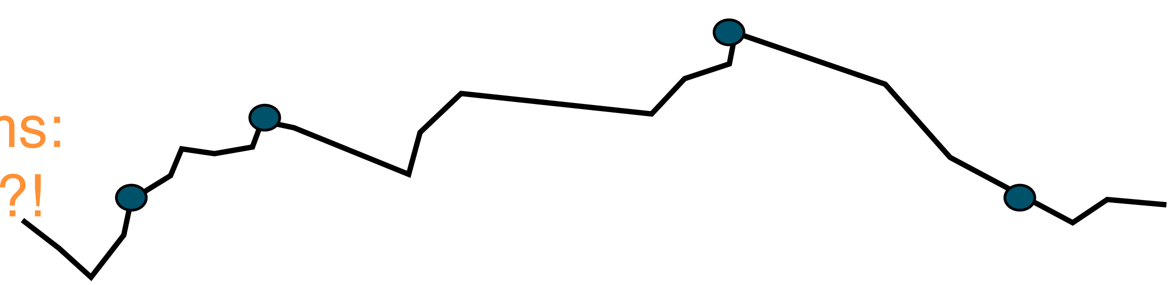

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NP-hard

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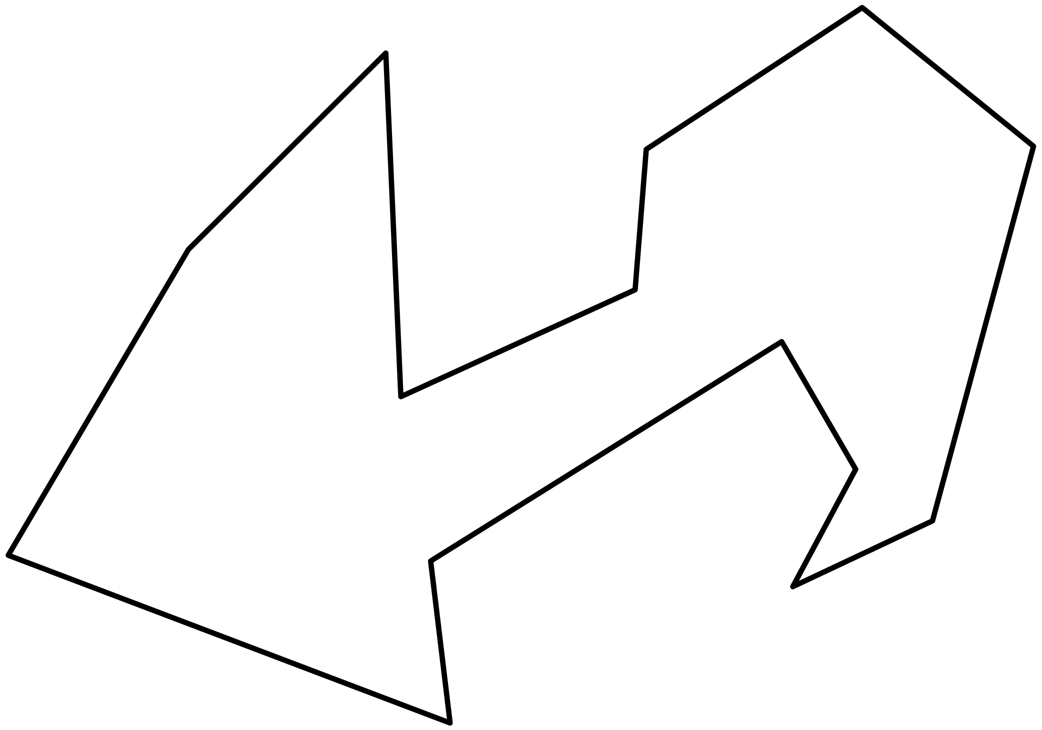

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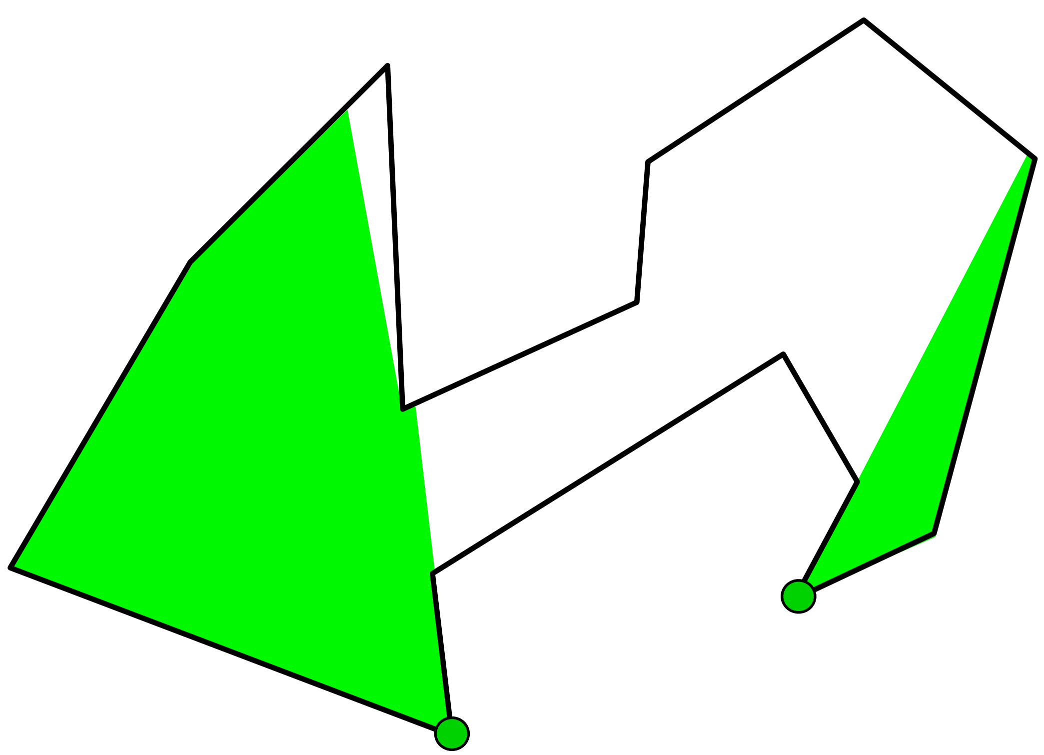

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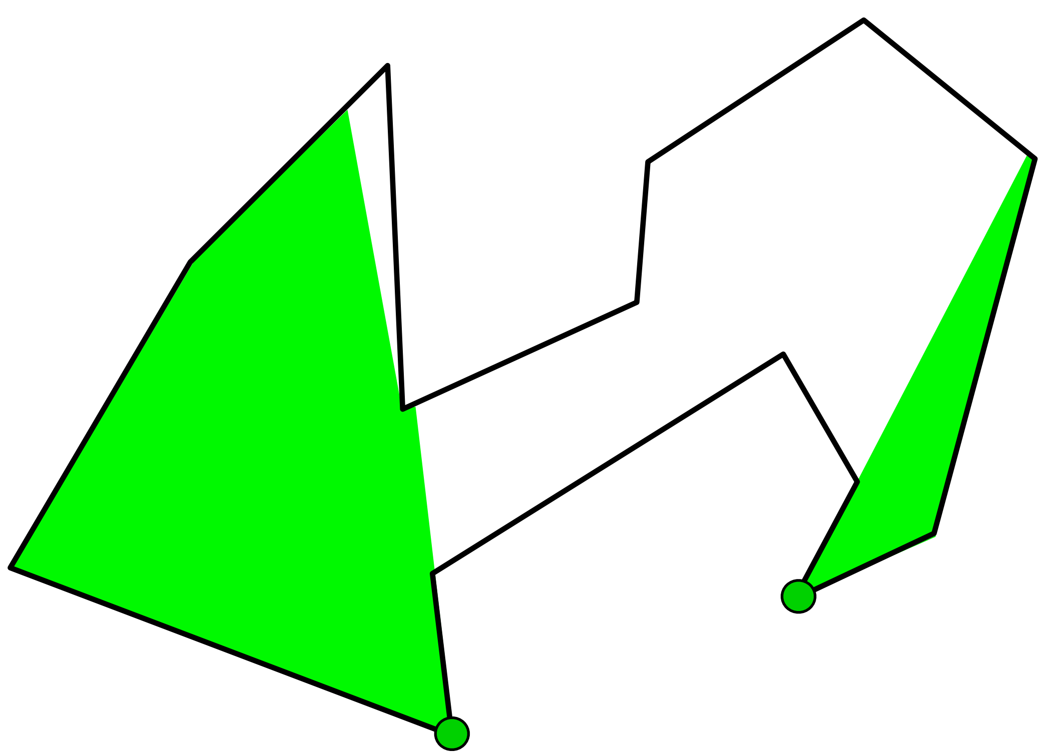

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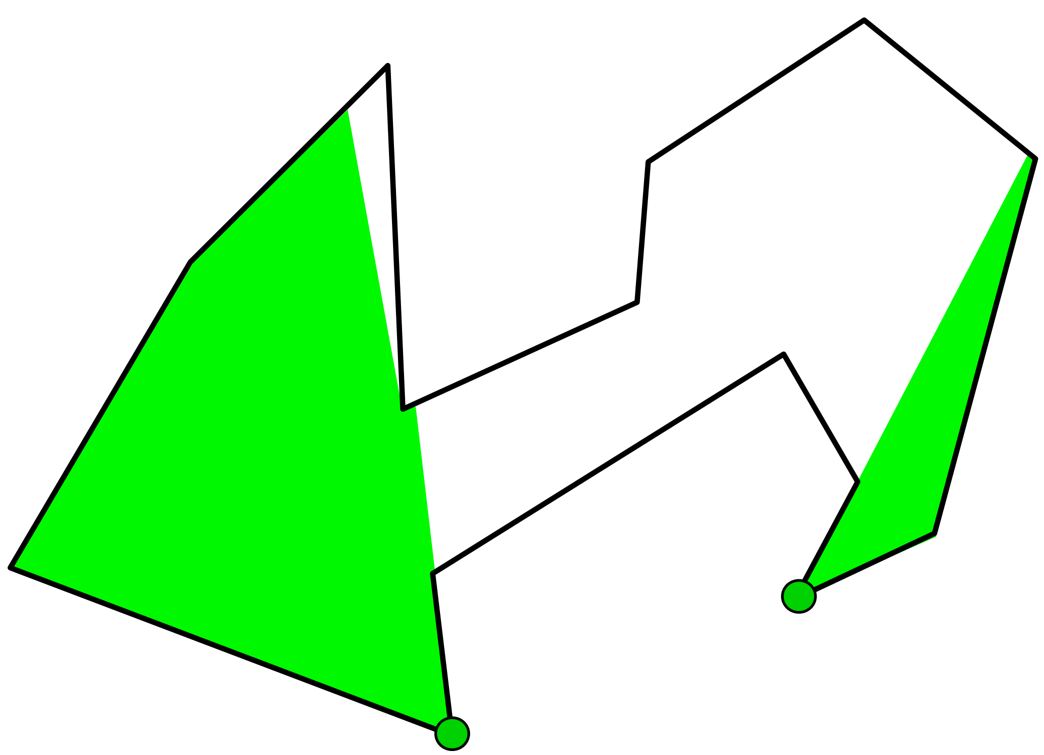

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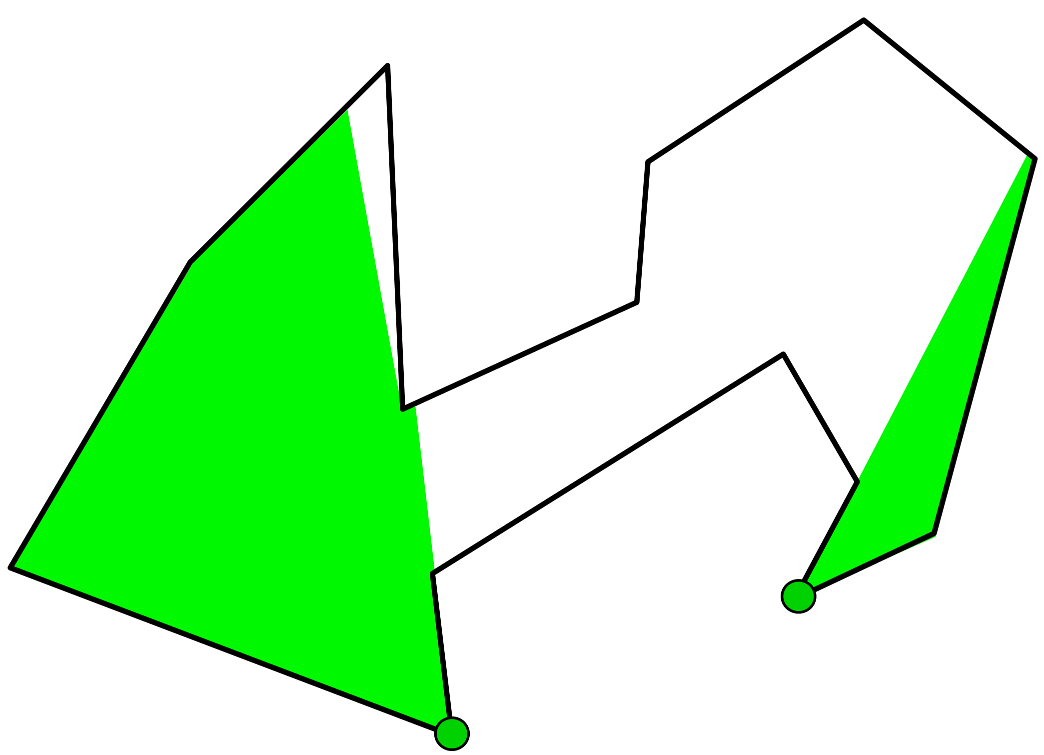
cardinality of an optimum guard set = cardinality of a maximum witness set $\forall P \in \mathcal{P}$



Altitude Terrain Guarding Problem

Guards to the left “don’t help”:

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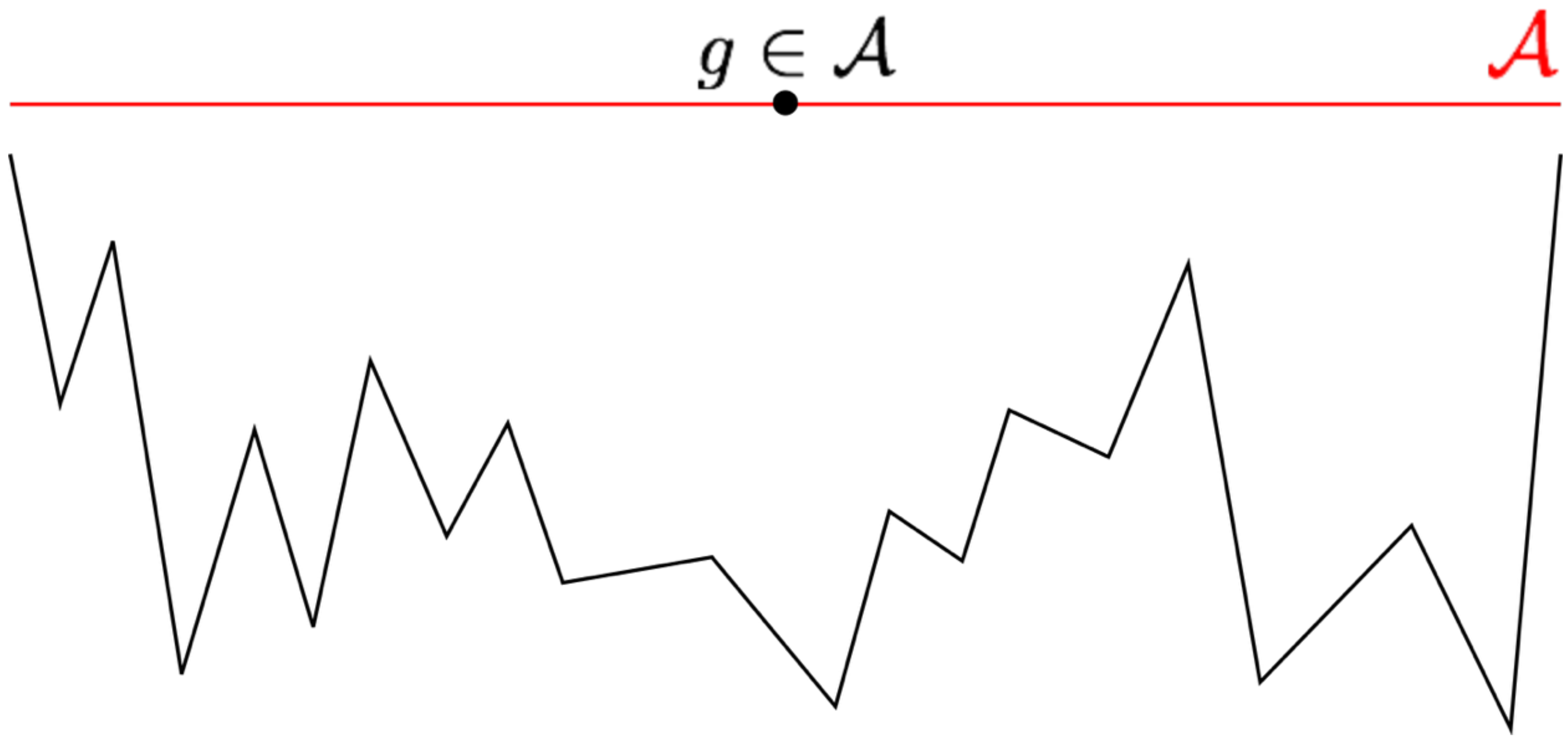

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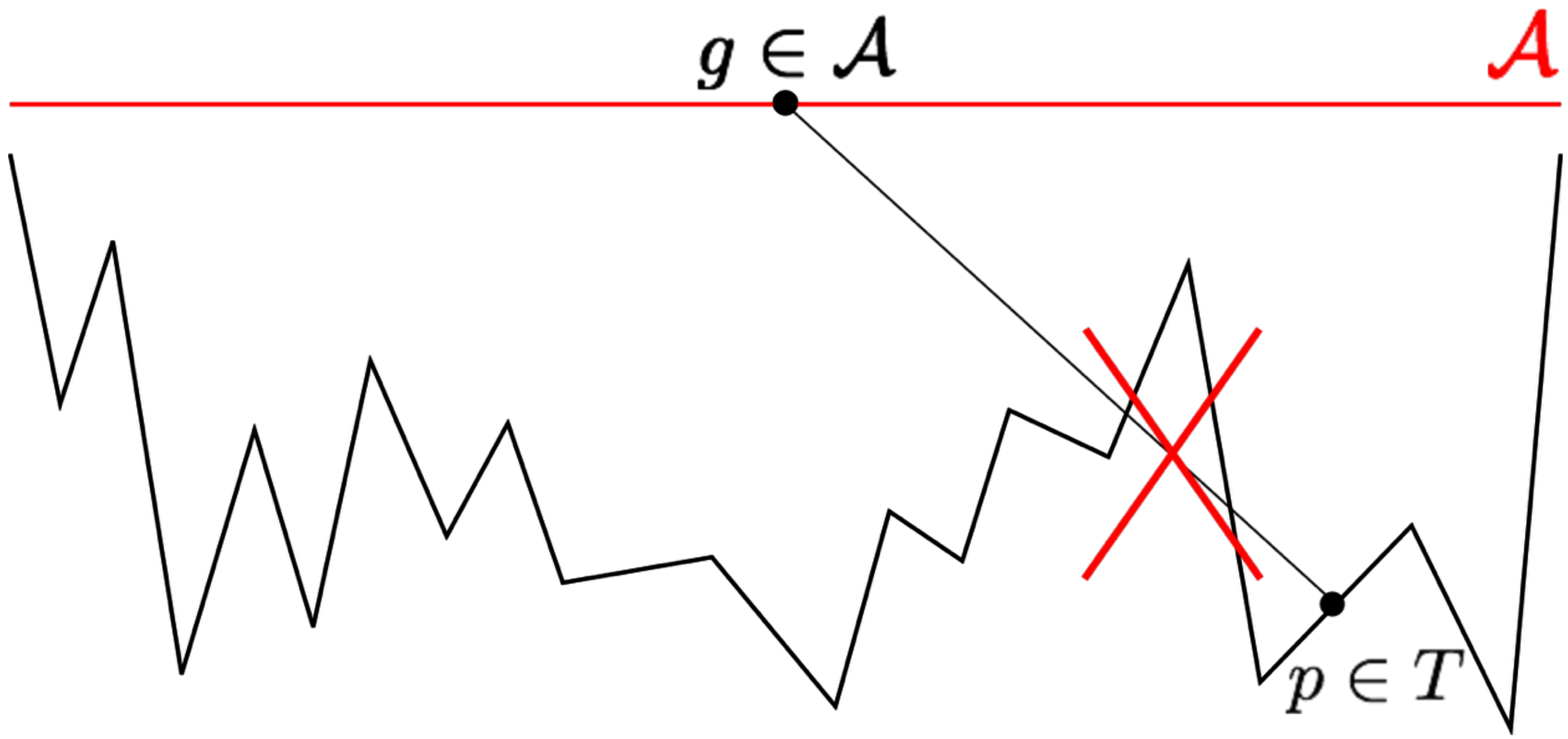

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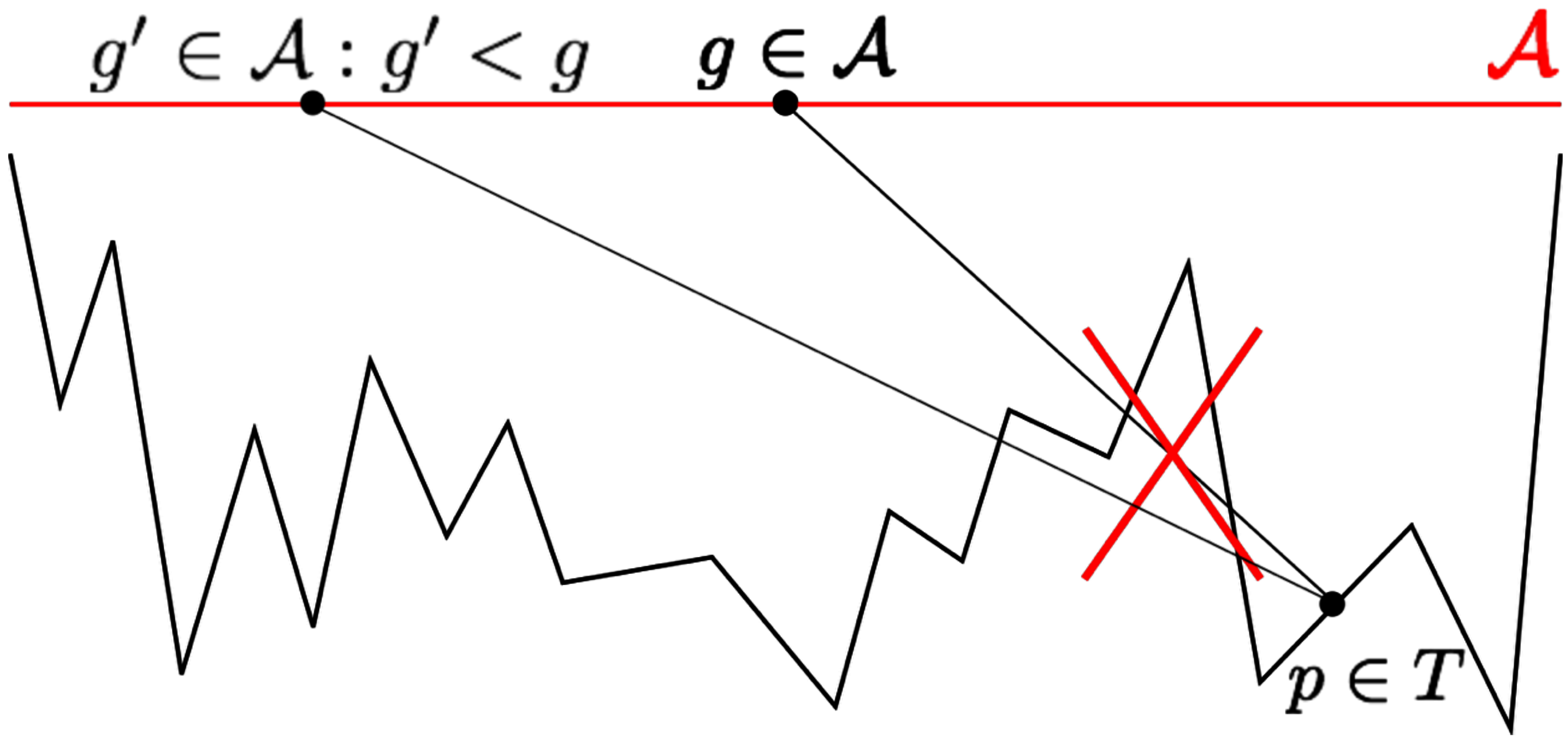

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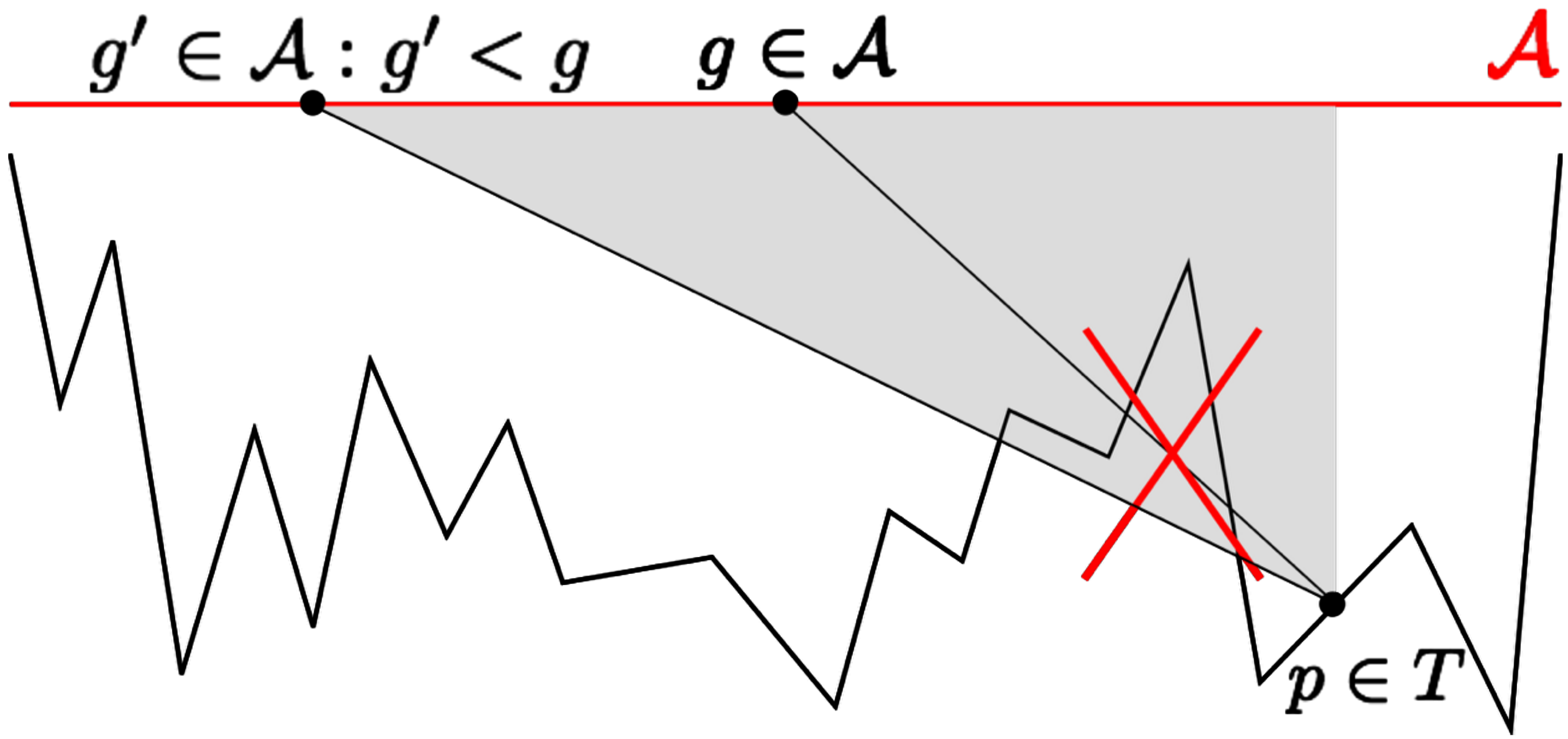

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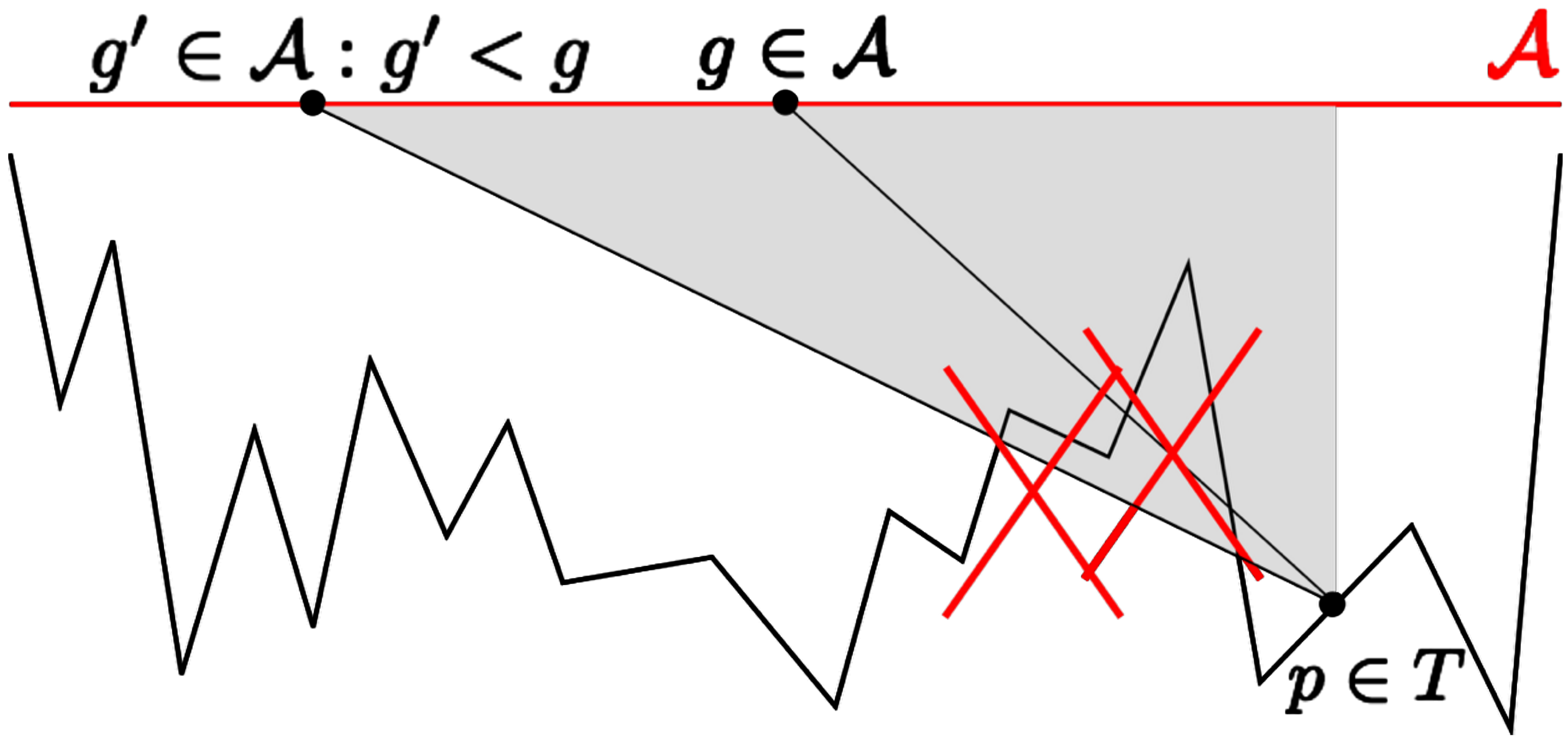

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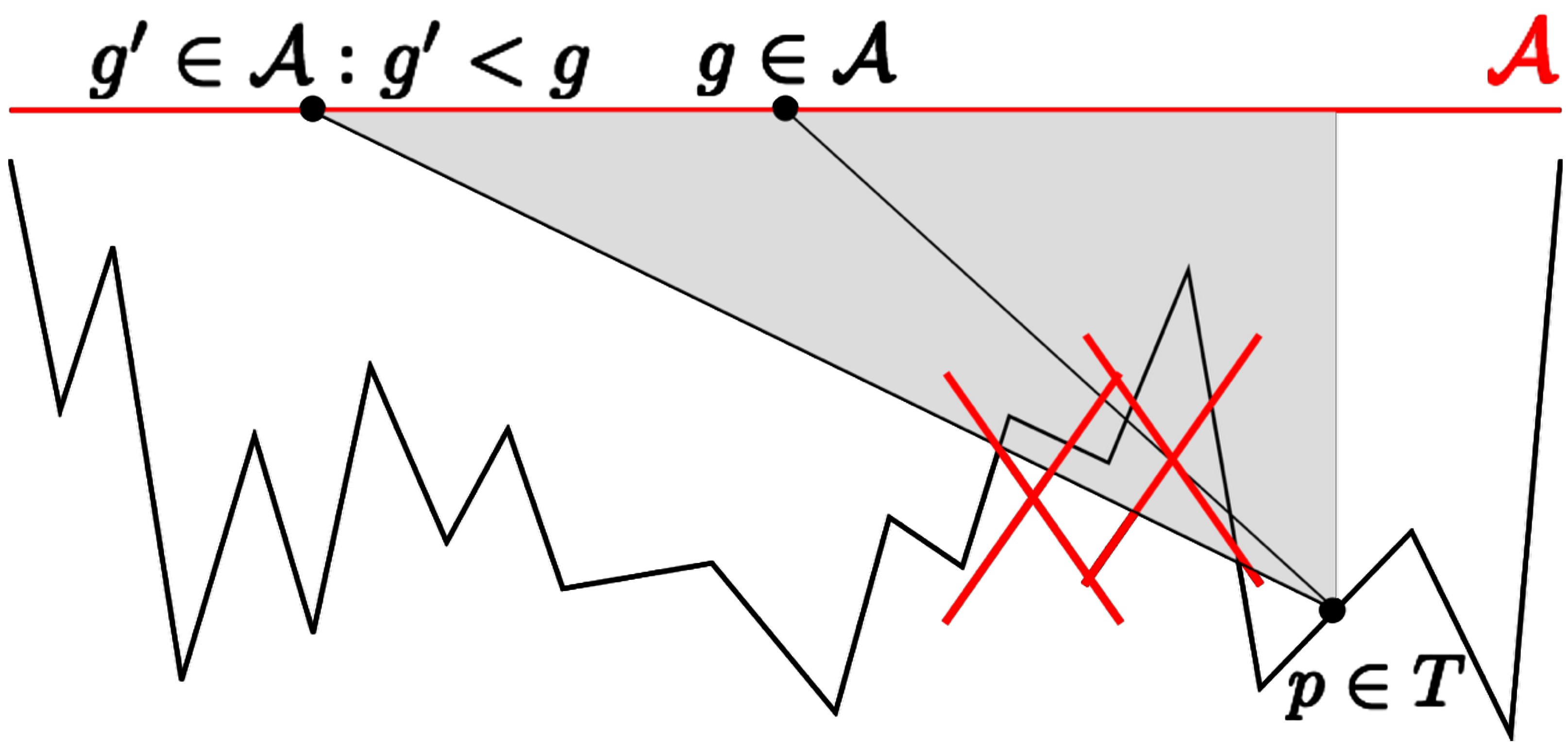
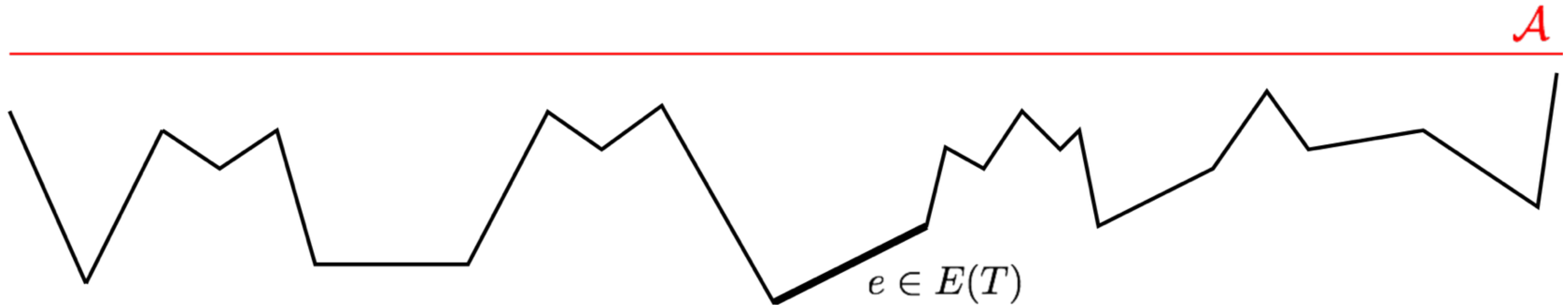


Sweep Algorithm



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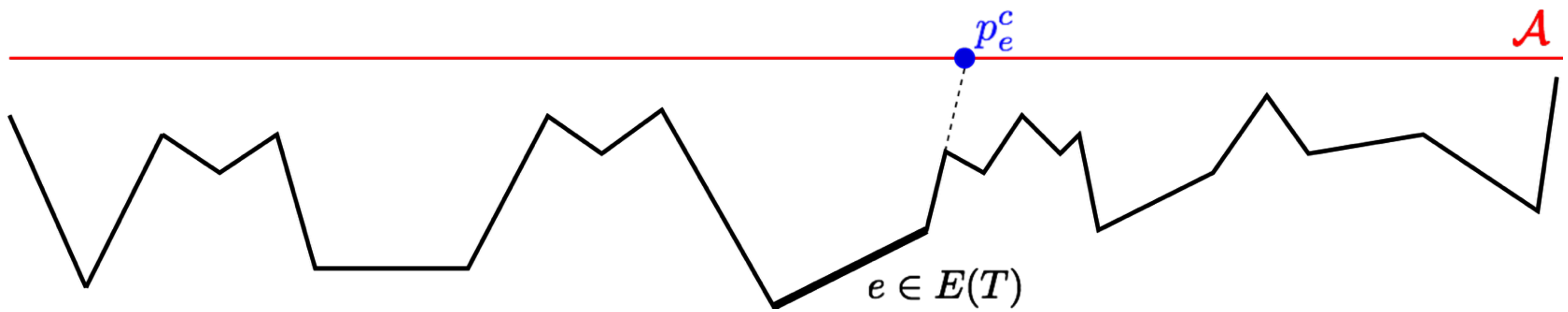

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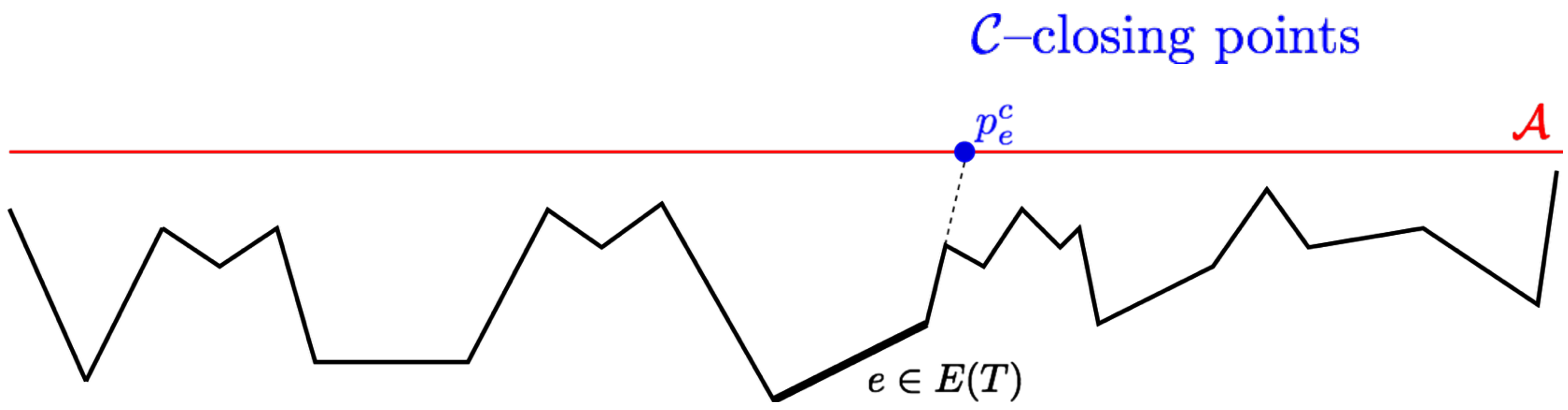

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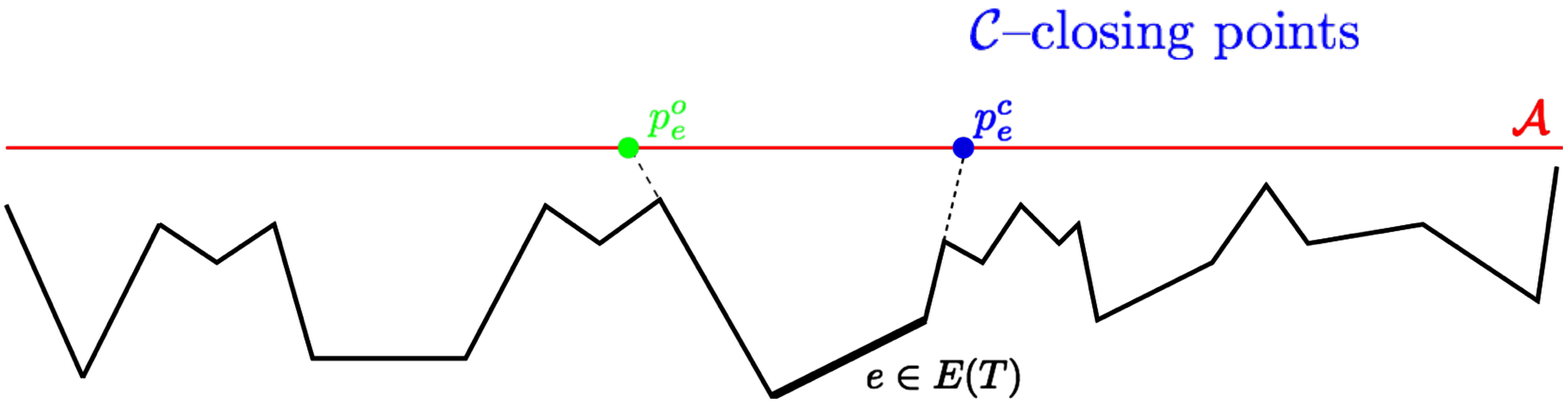

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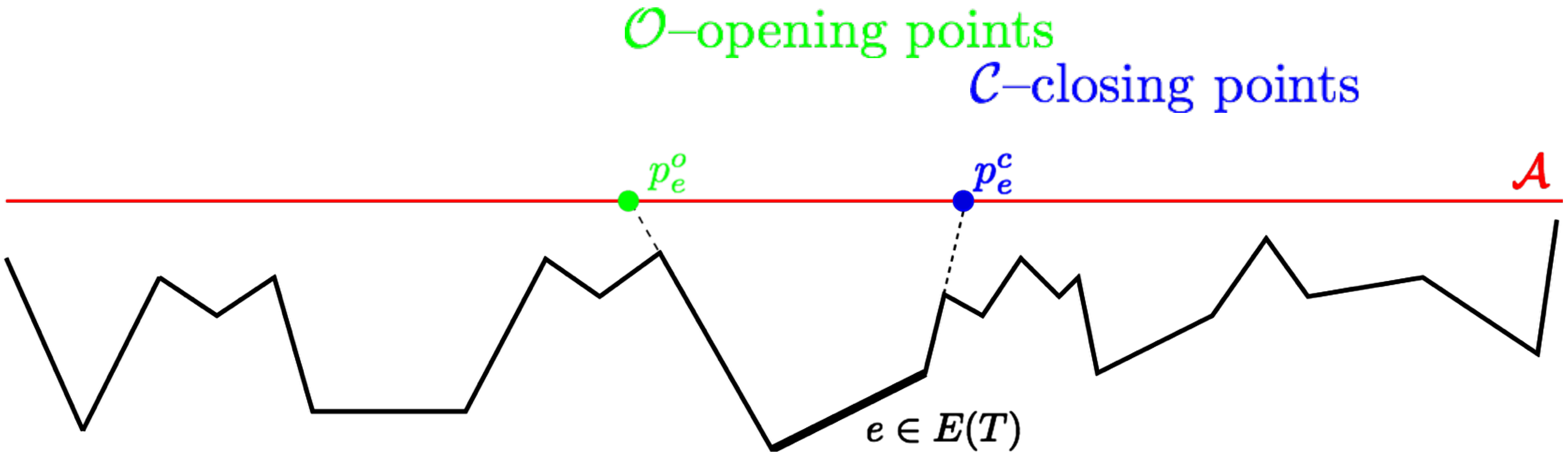

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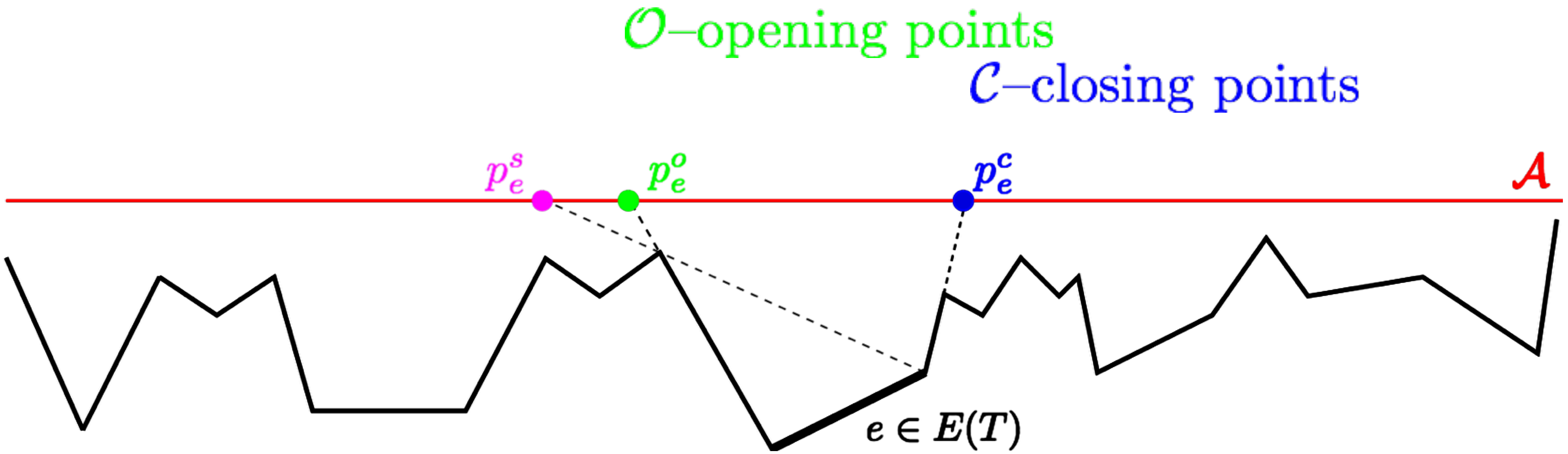

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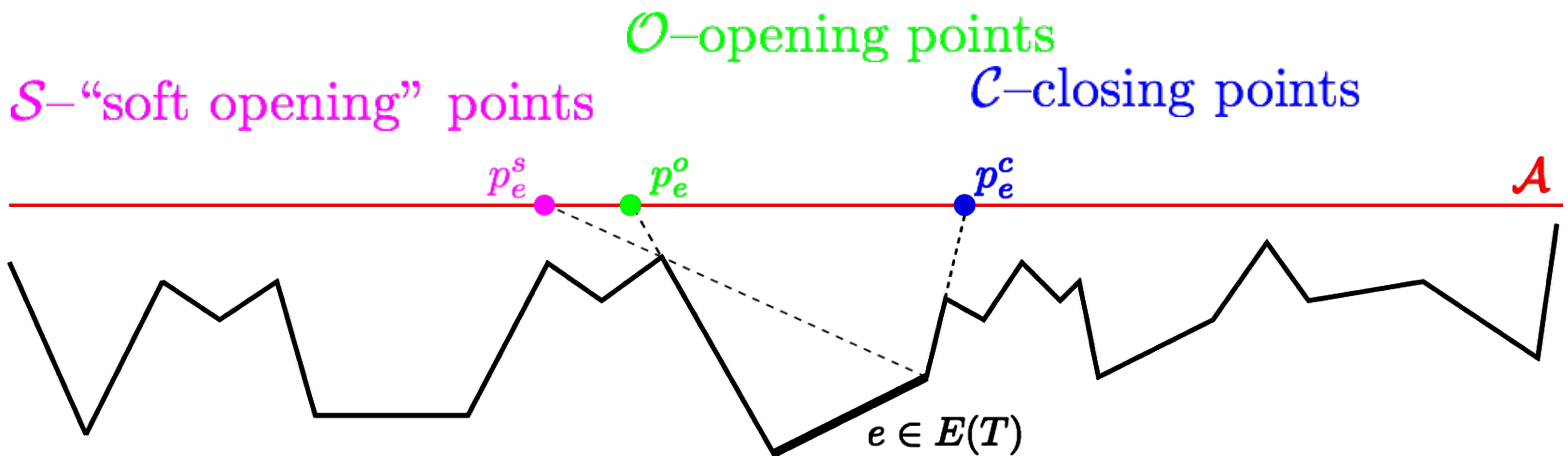

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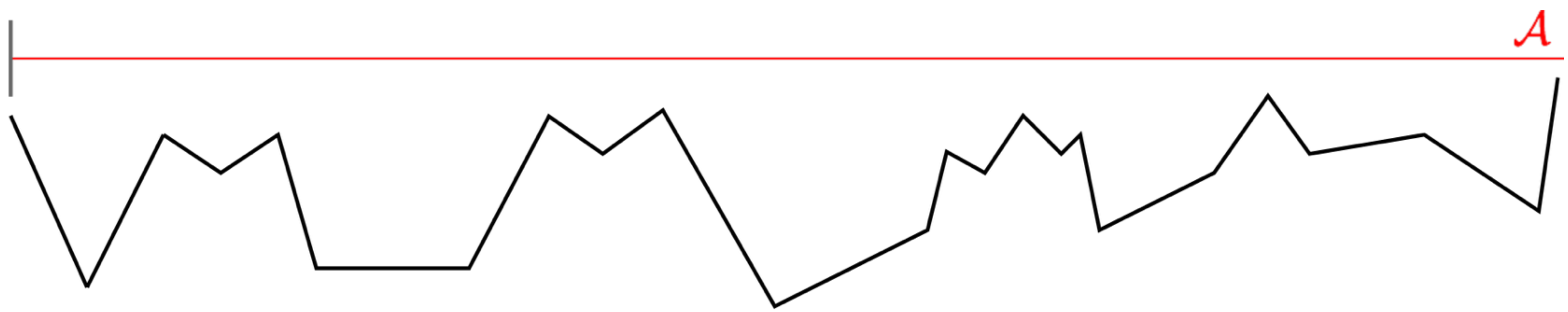

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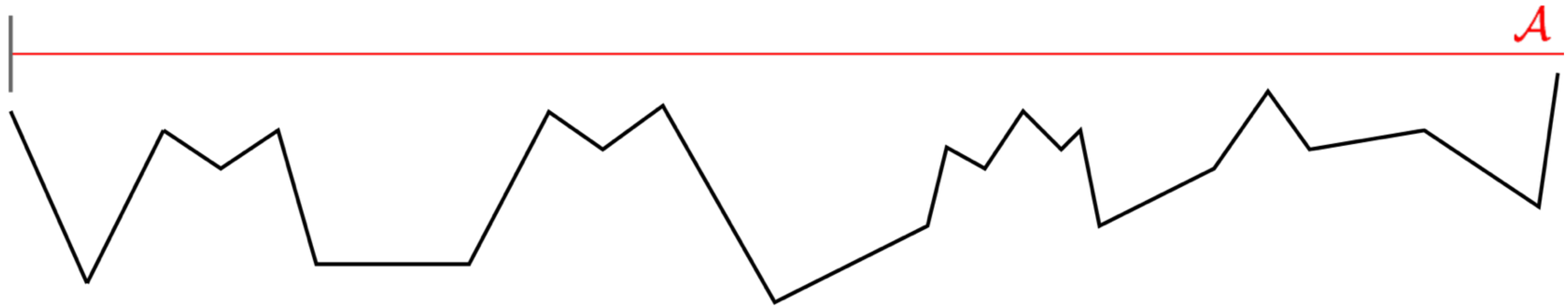

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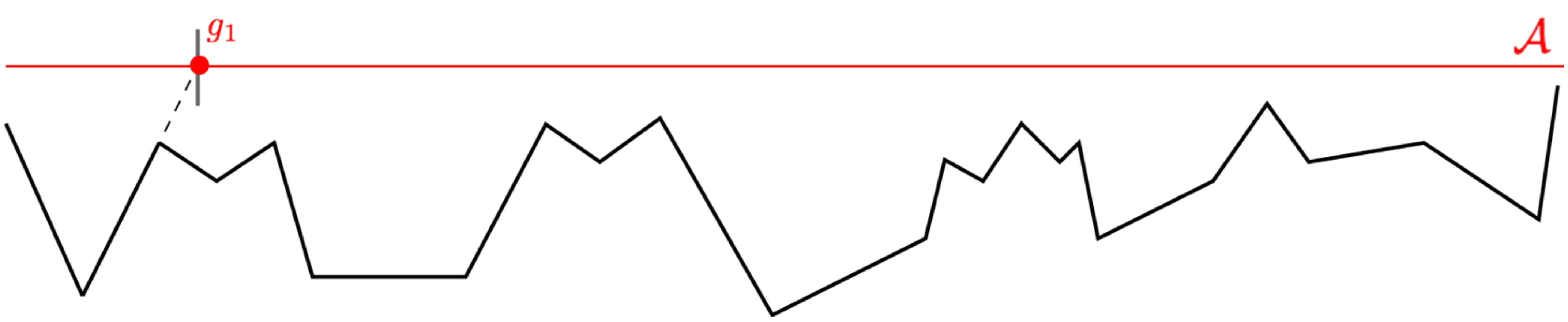

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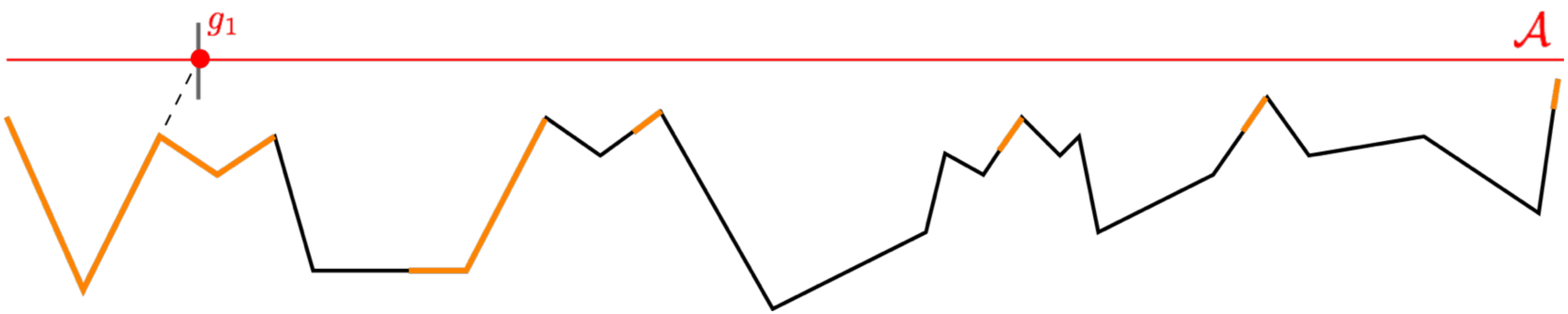

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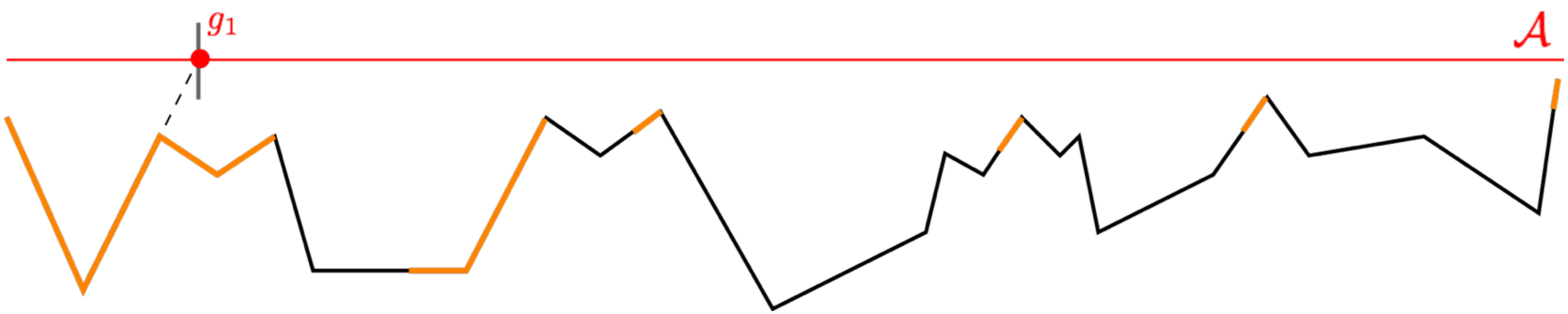

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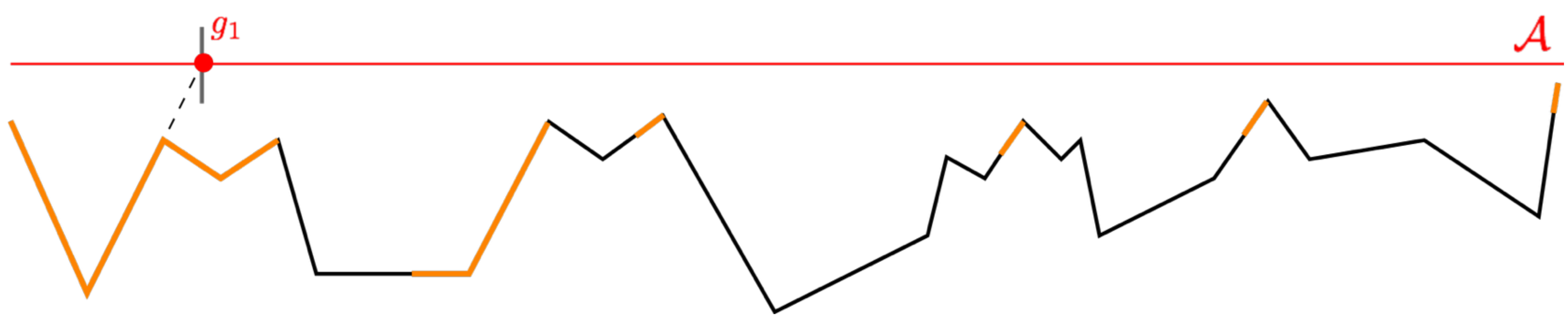

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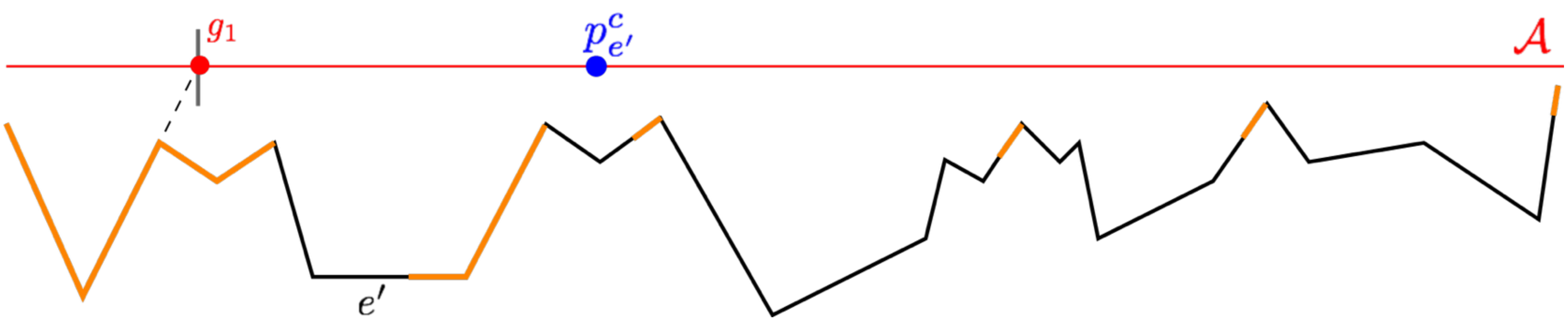

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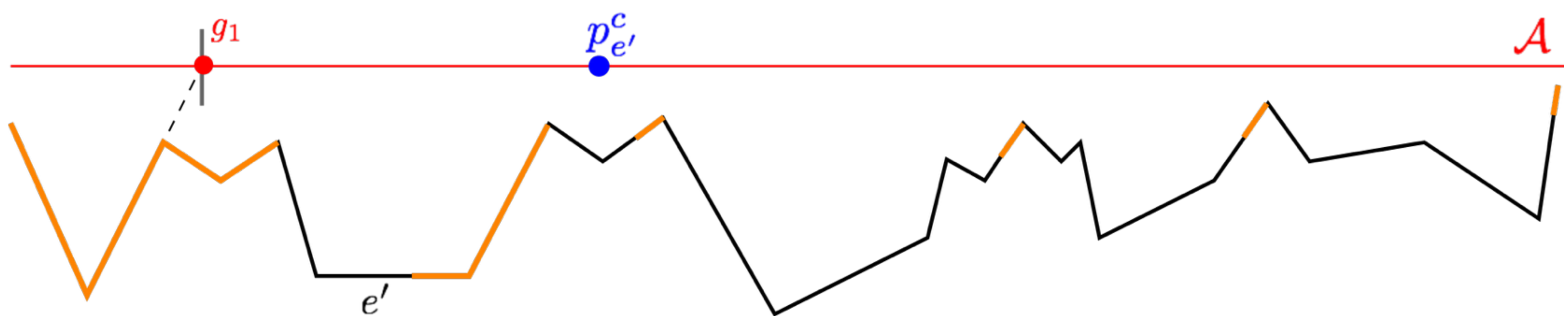

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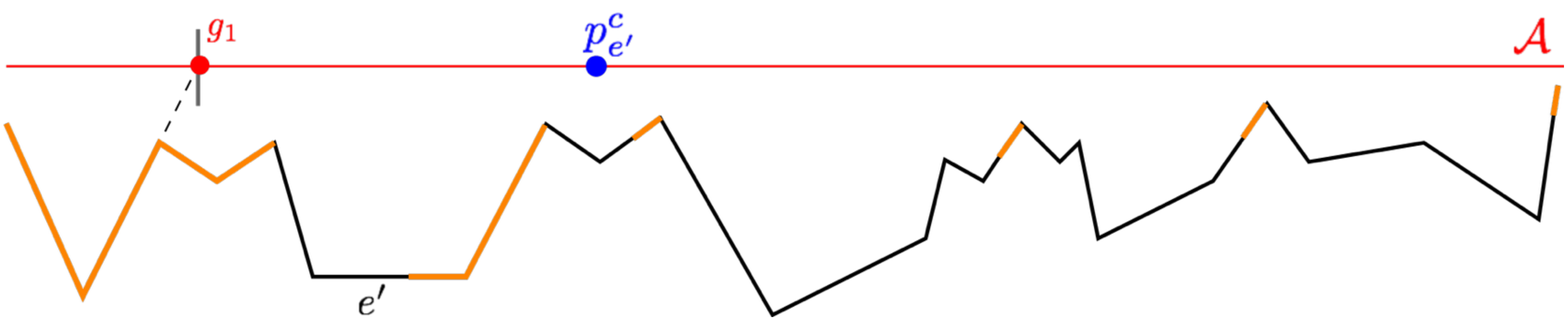

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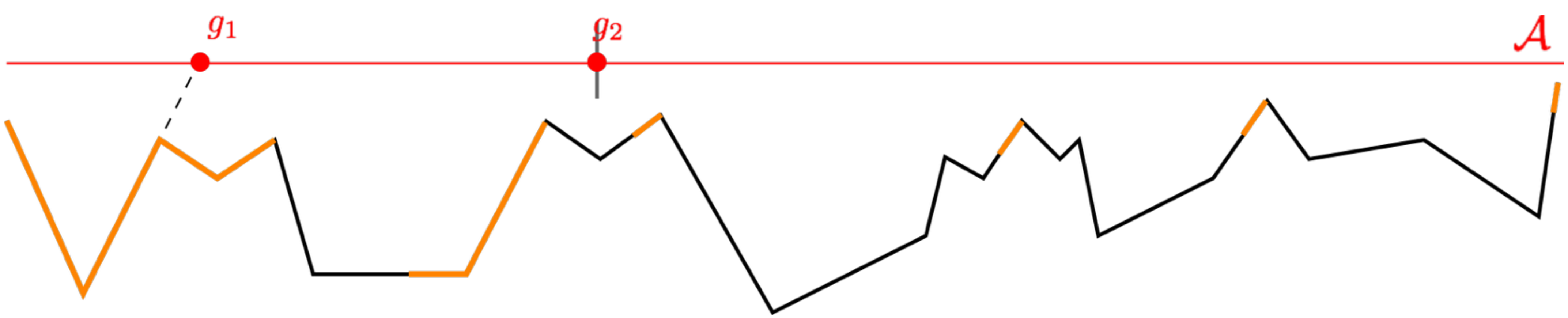

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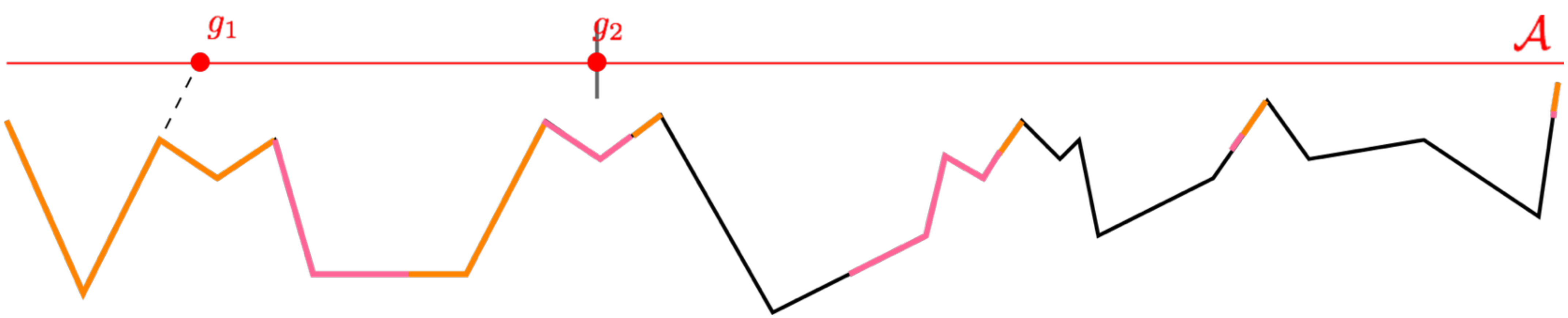

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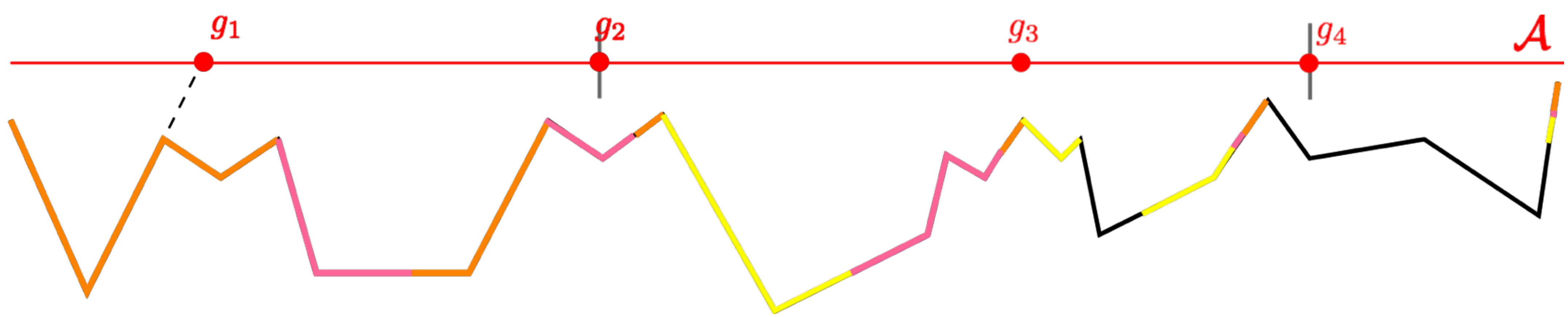

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How do we preprocess our terrain to easily identify the point on $\mathcal{A}$ that we need to add to $\mathcal{C}$ when we split an edge?

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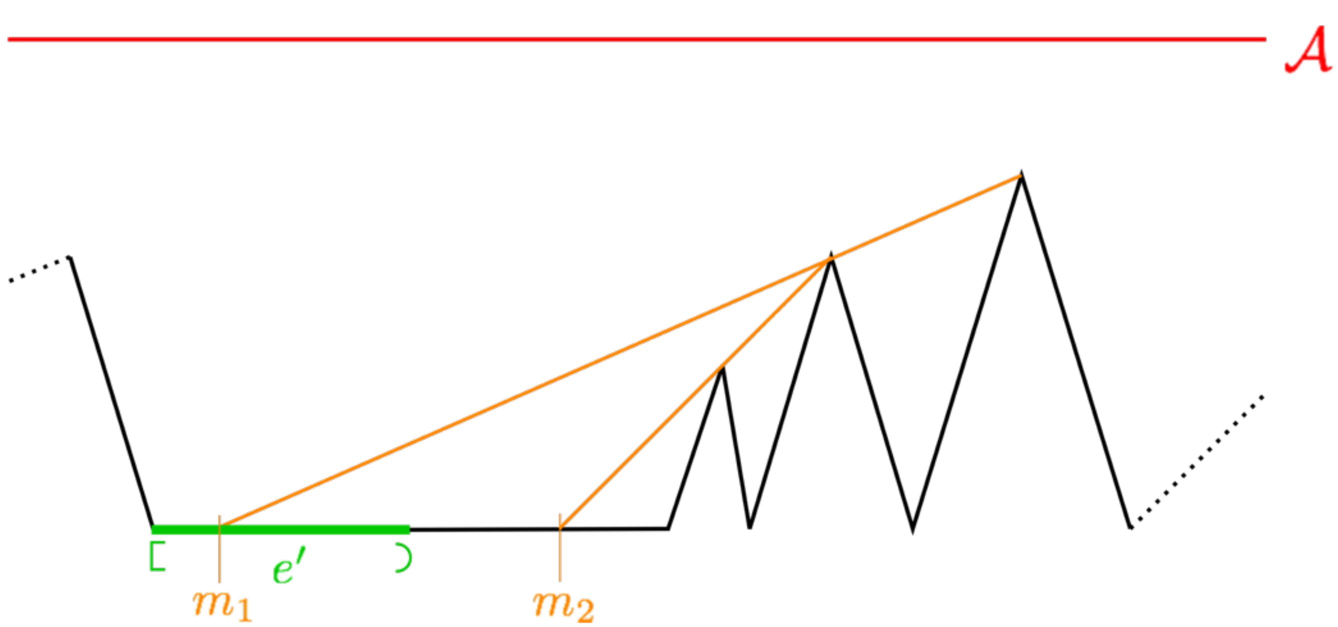

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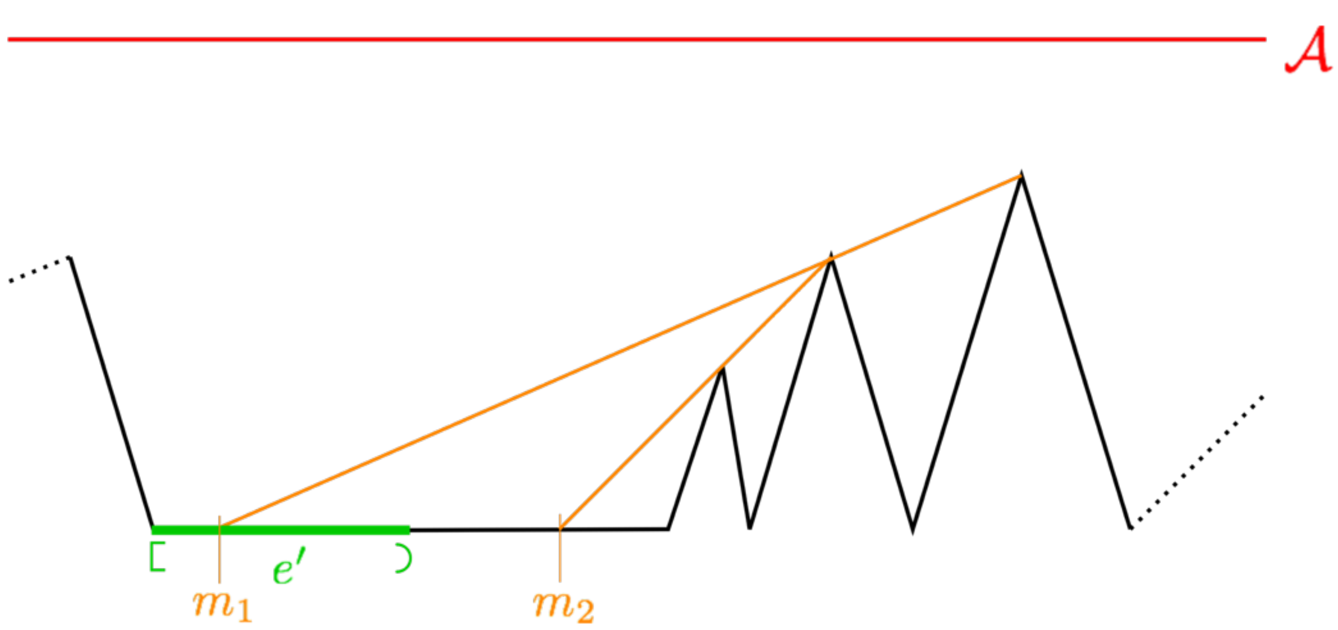

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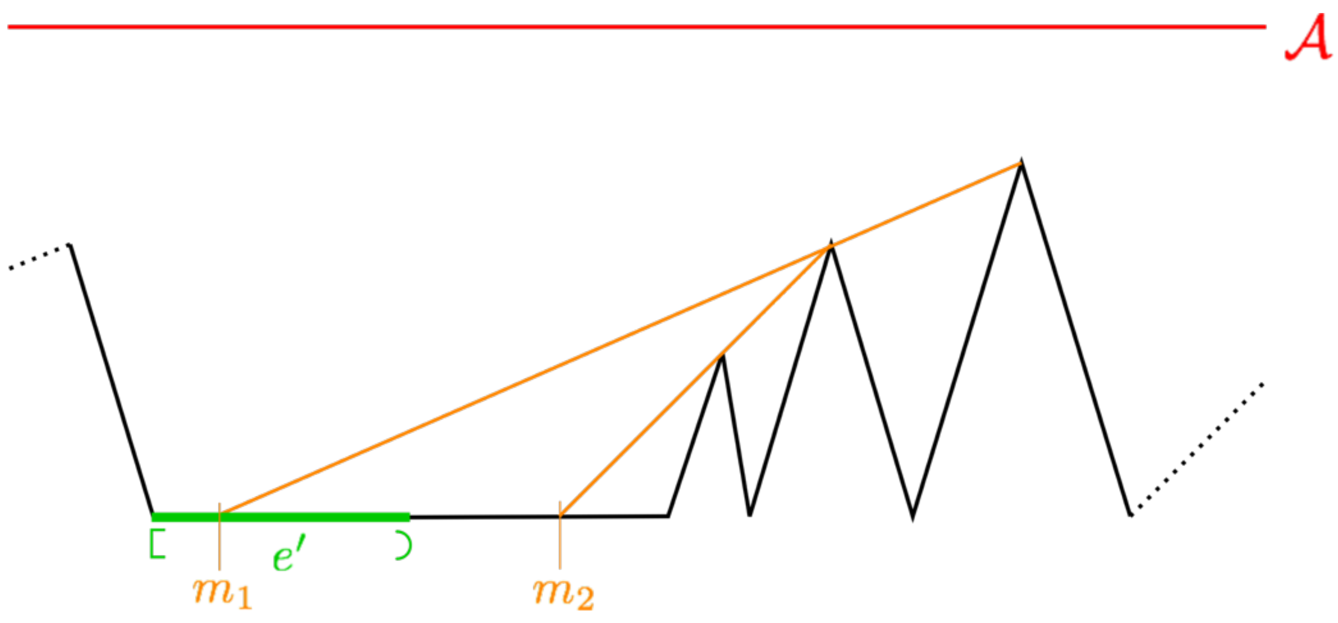

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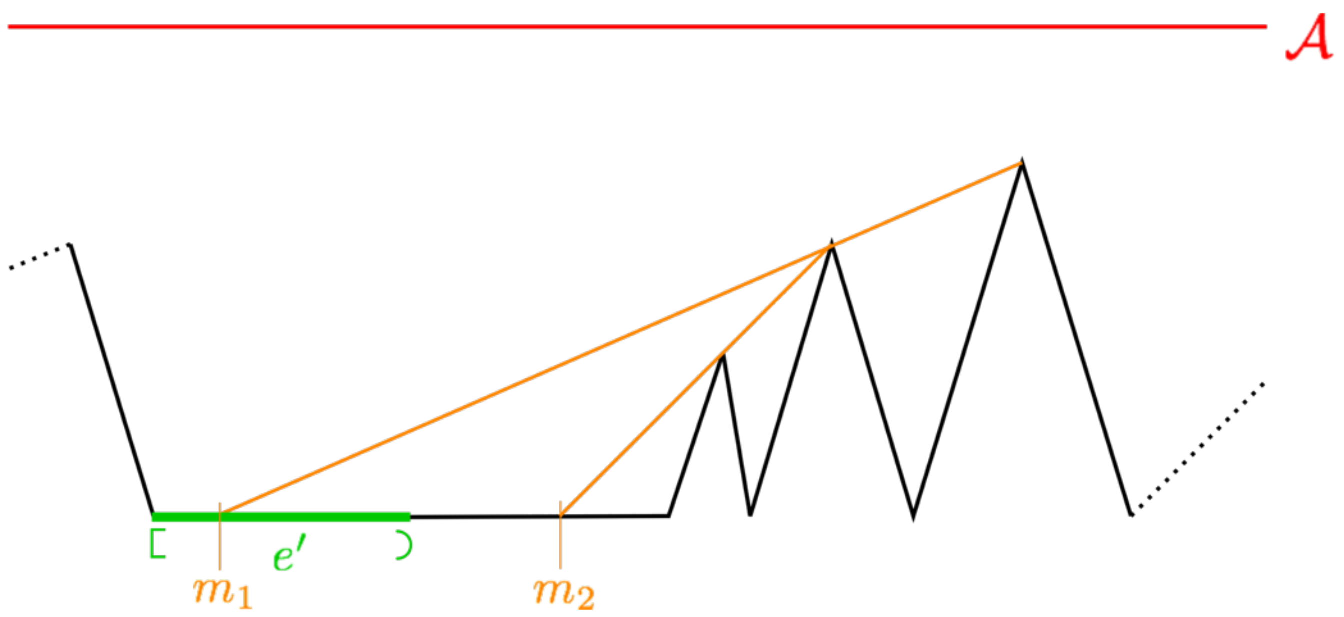

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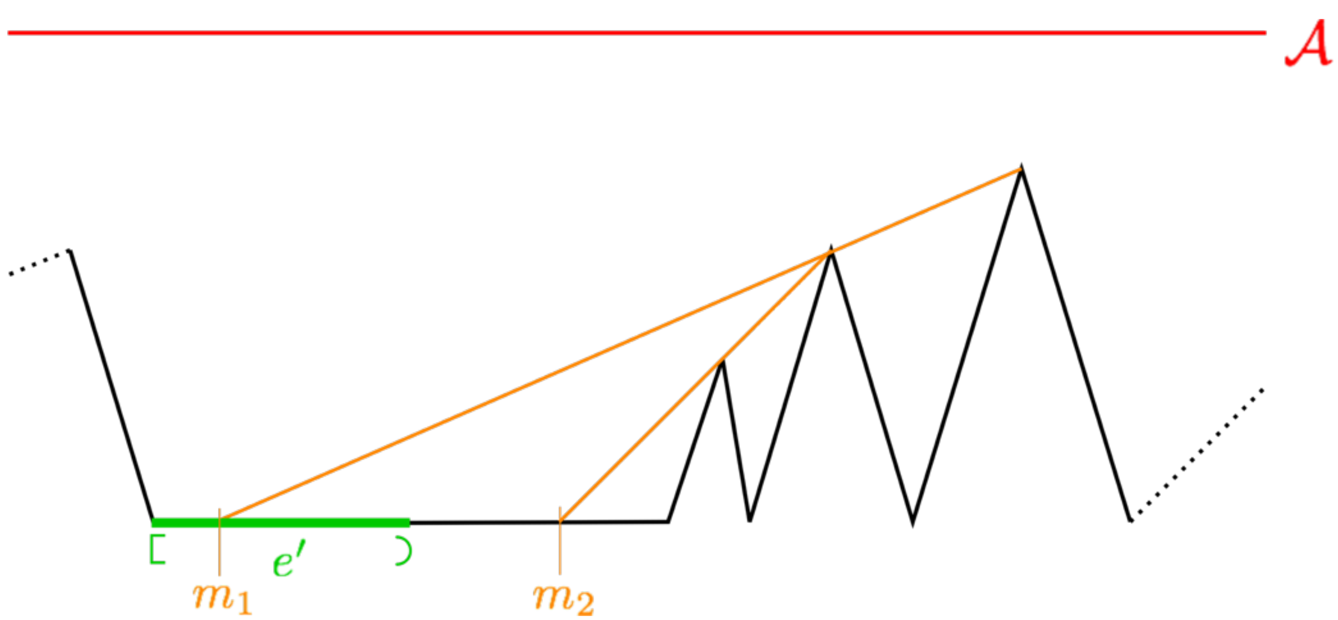
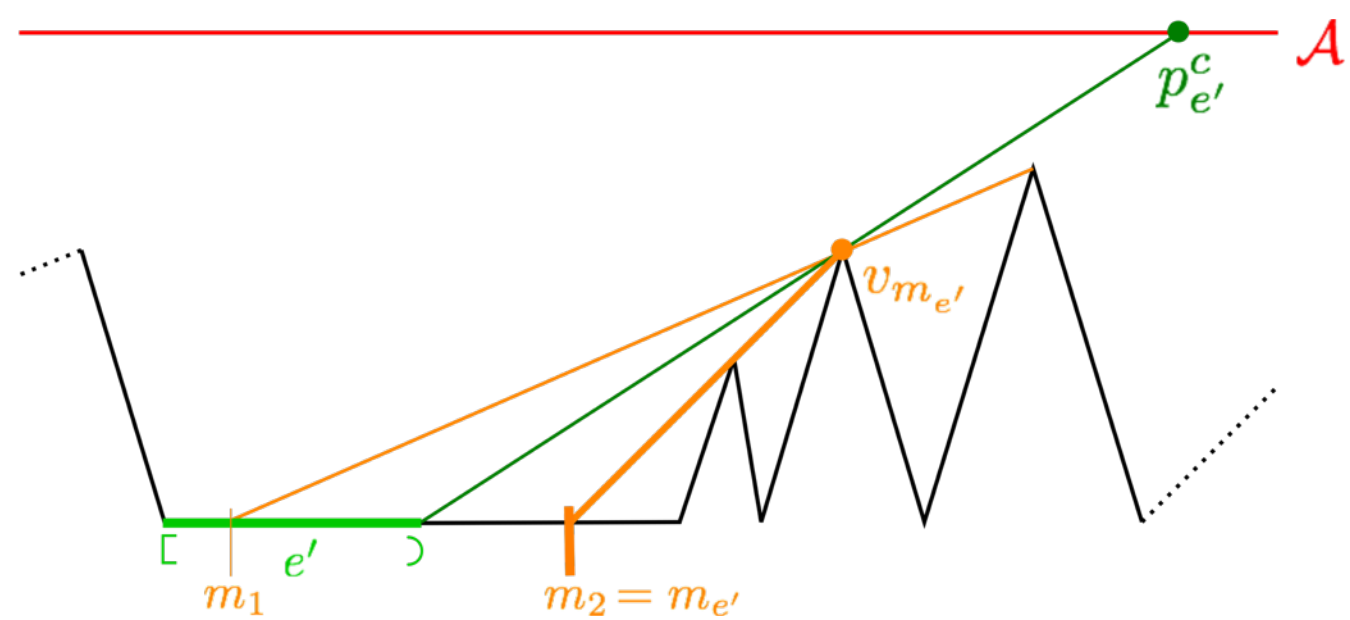
- When placing guard g splits edge e, and we are left with interval e'⊂e:





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How do we preprocess our terrain to easily identify the point on $\mathcal{A}$ that we need to add to C when we split an edge?

- Sweep rightmost to leftmost vertex
- For each vertex we shoot ray through all vertices to its left
- Where rays hit terrain: *mark points*
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- For each mark point m remember the rightmost of the two ray-vertices v_m
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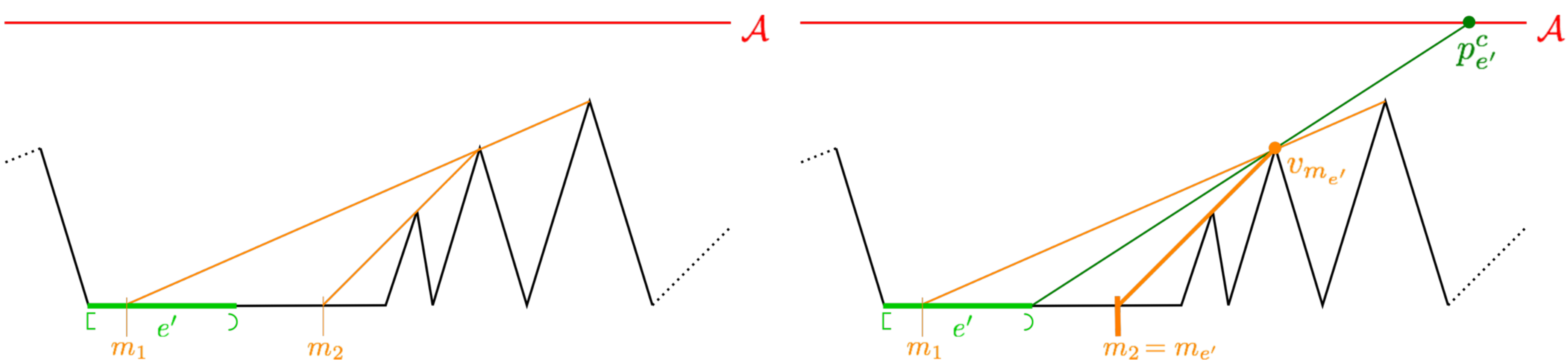

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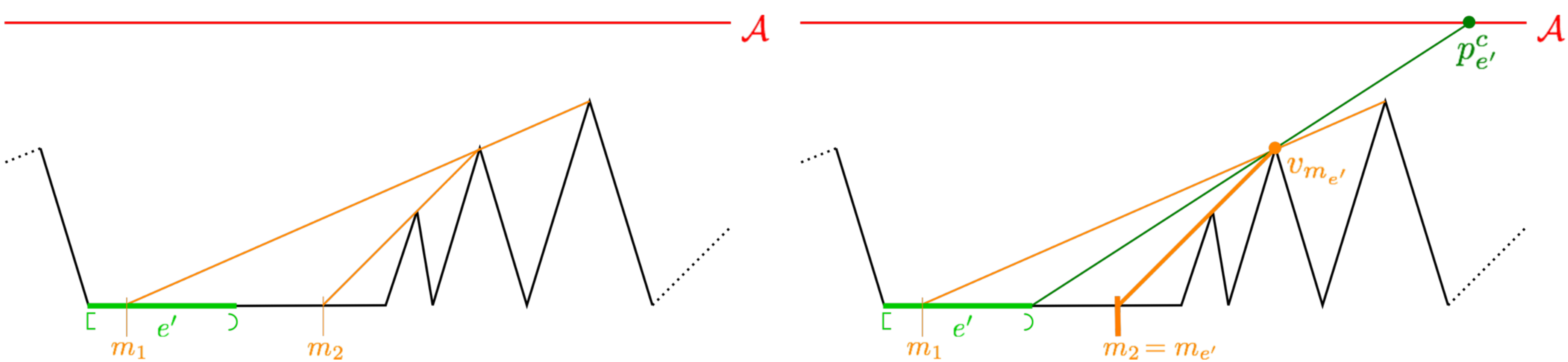

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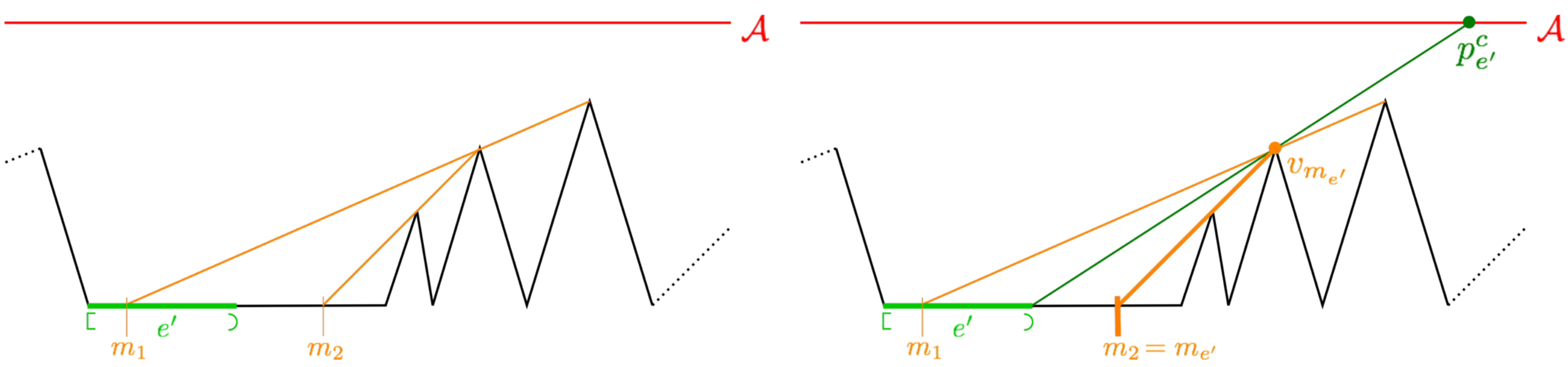

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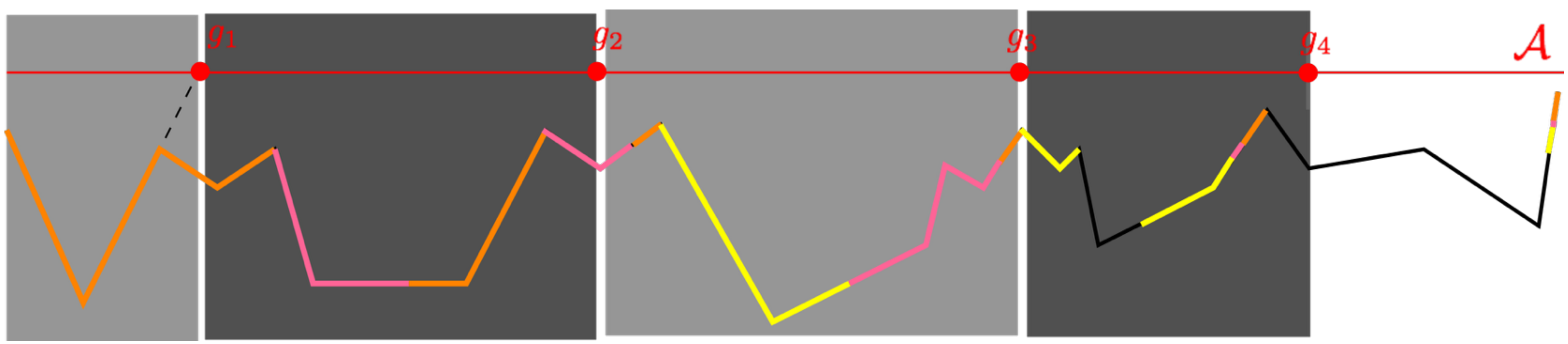

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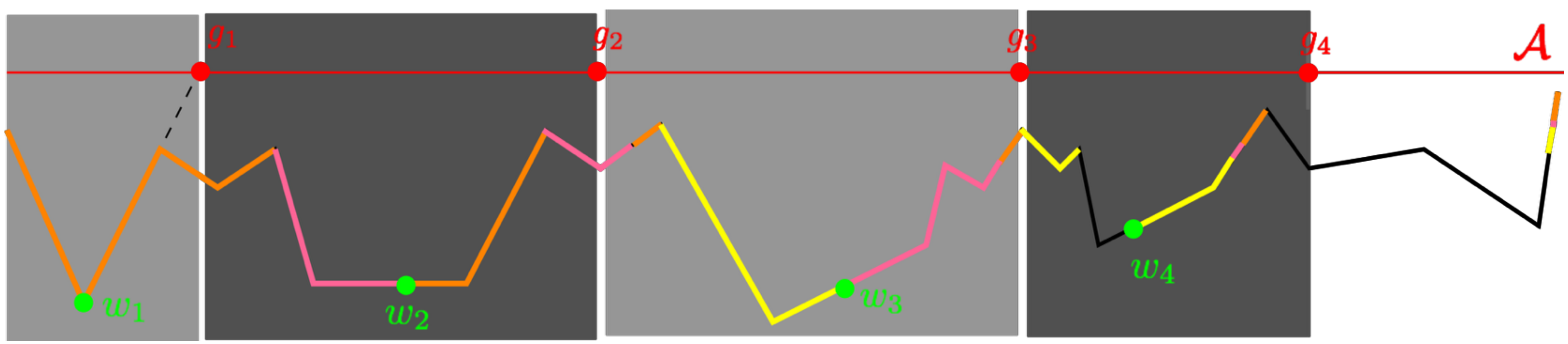

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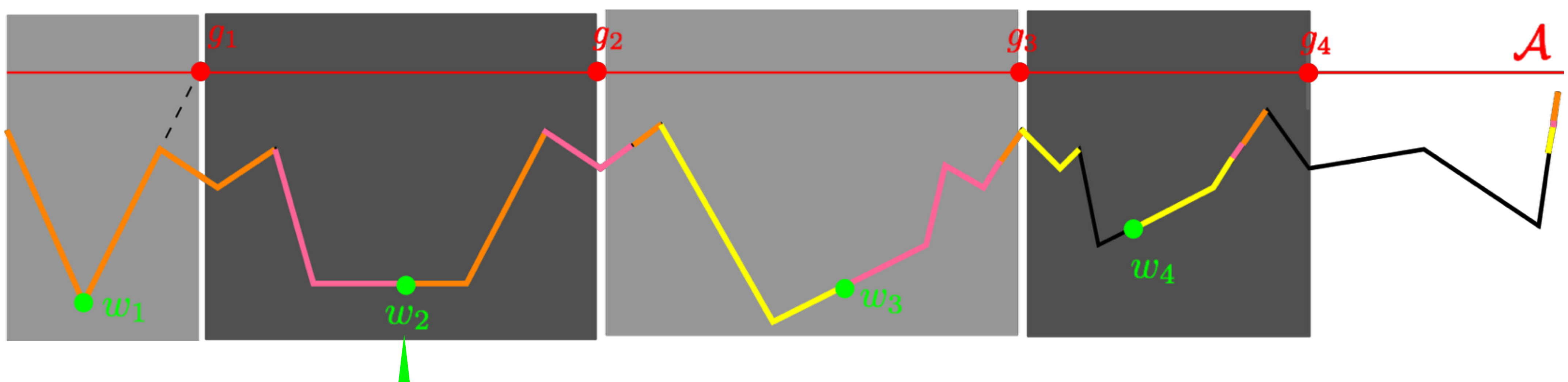

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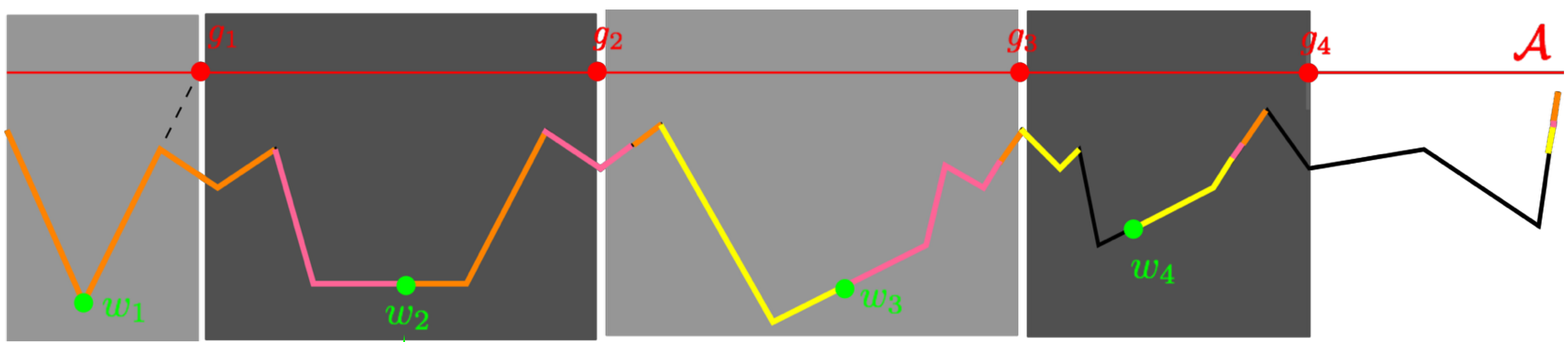

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Theorem 3: Uni-monotone polygons are perfect.

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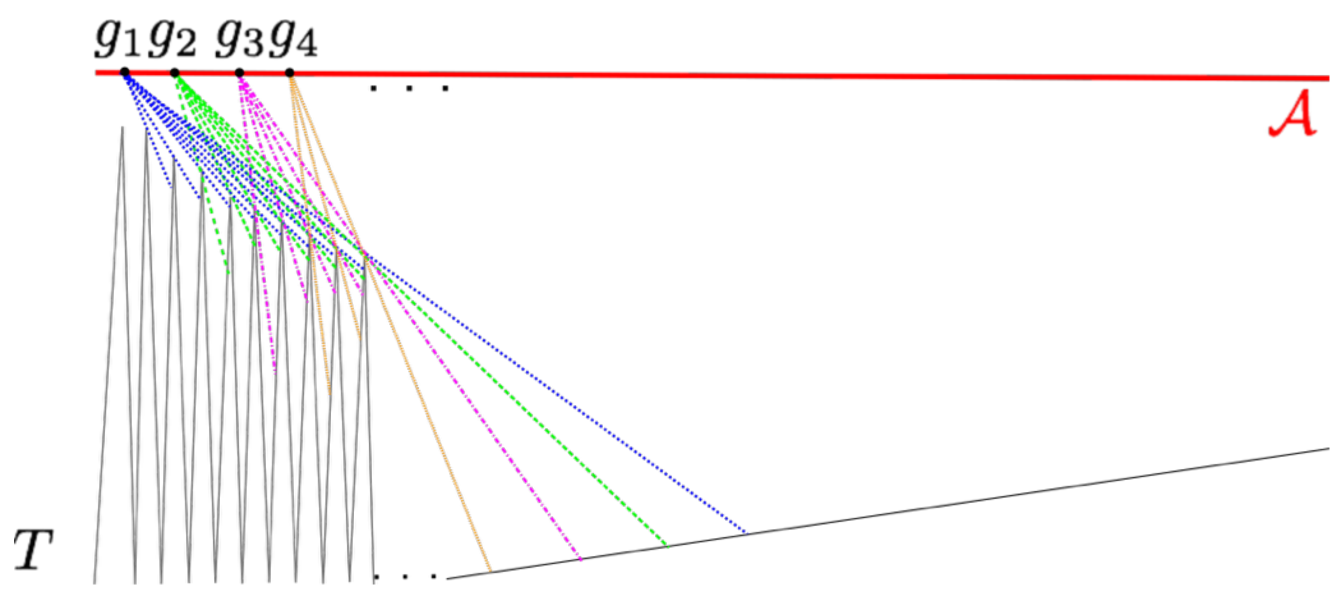

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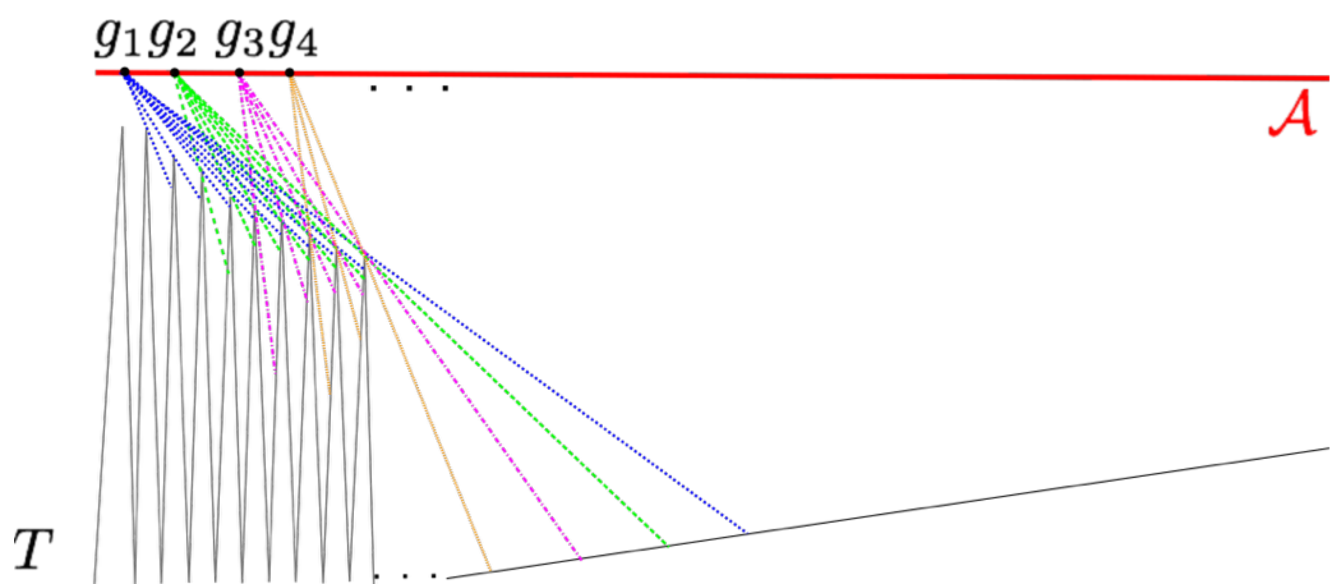

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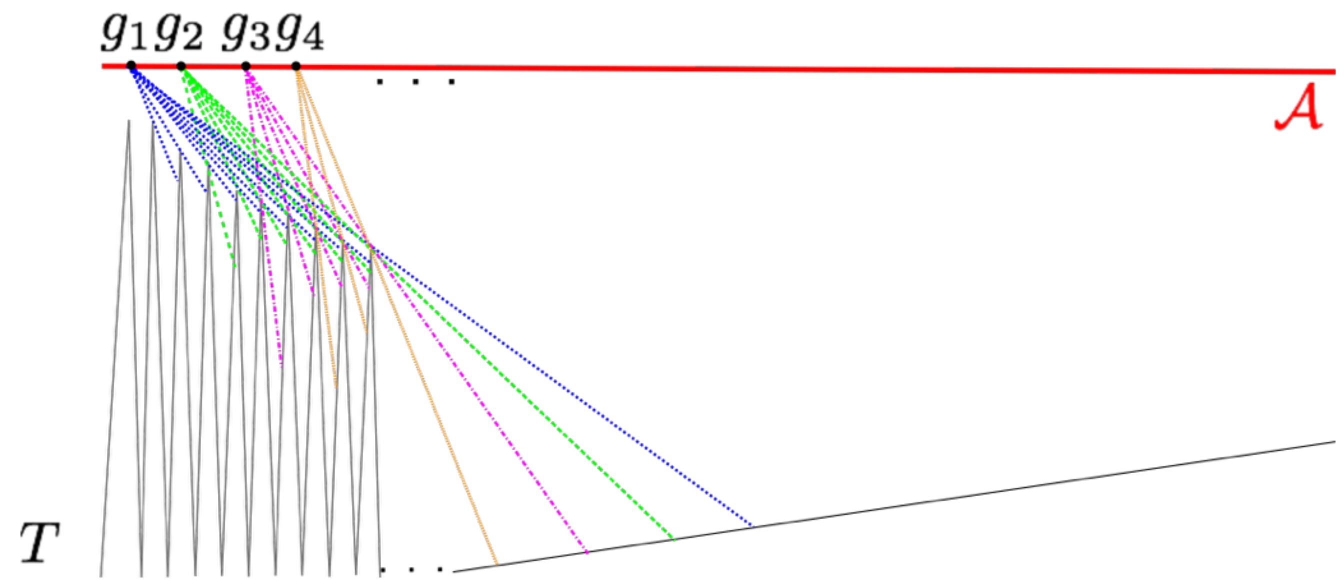

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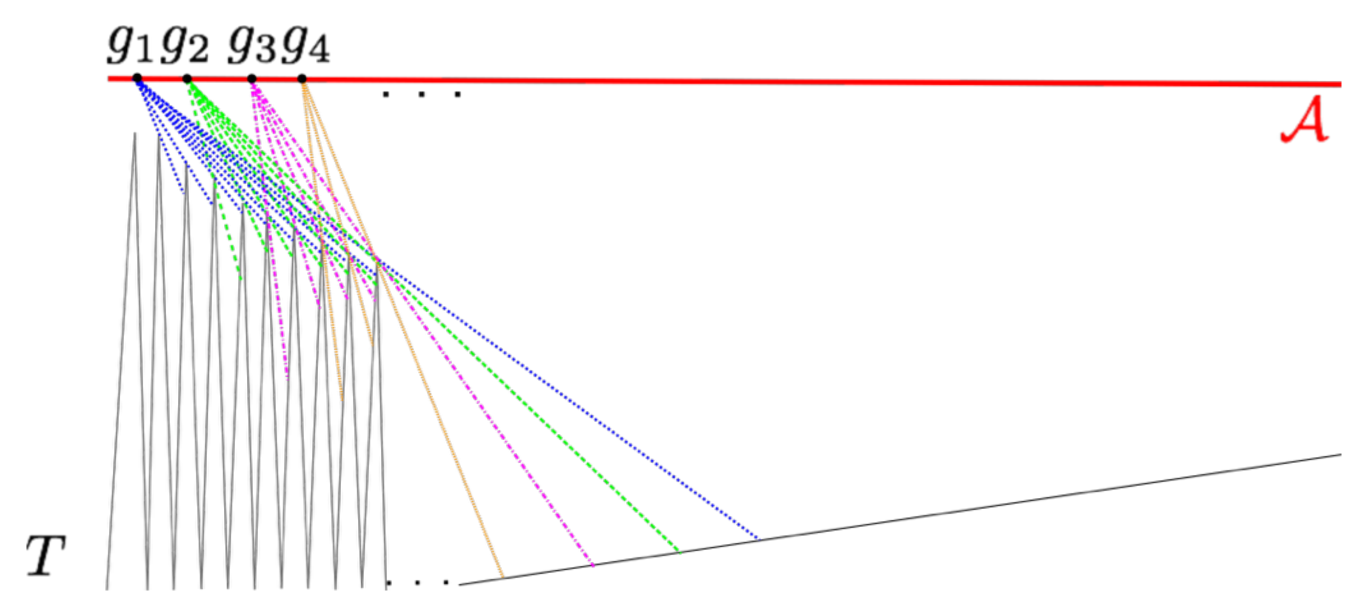

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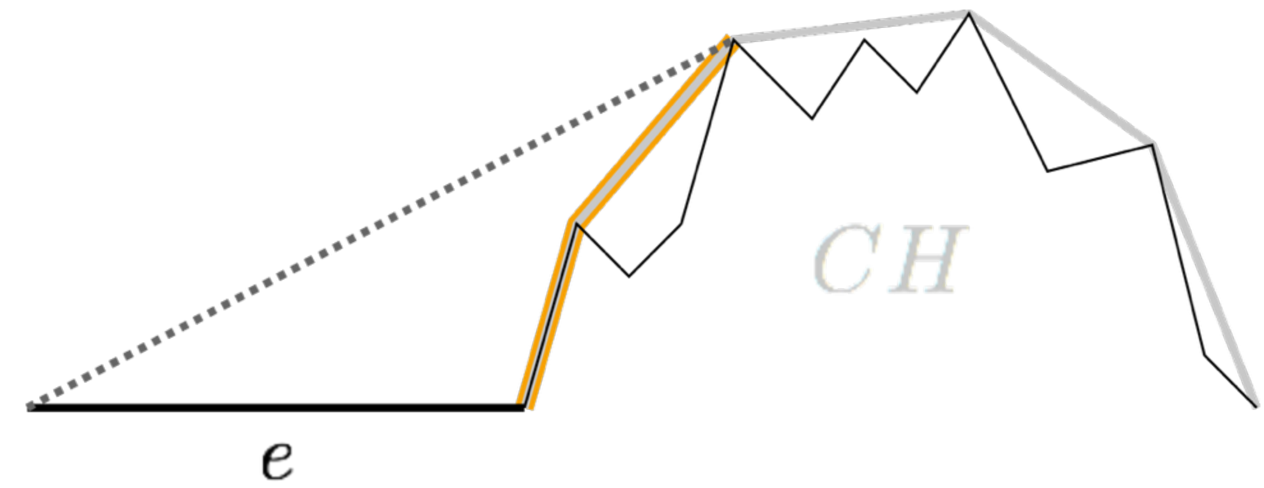

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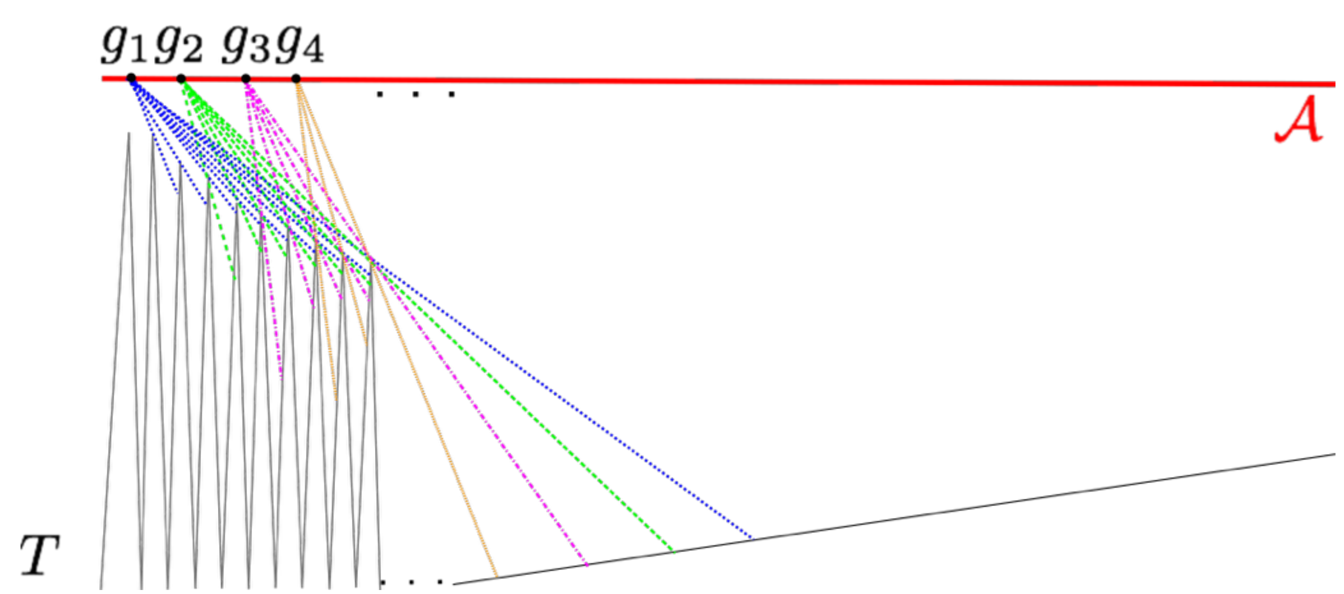

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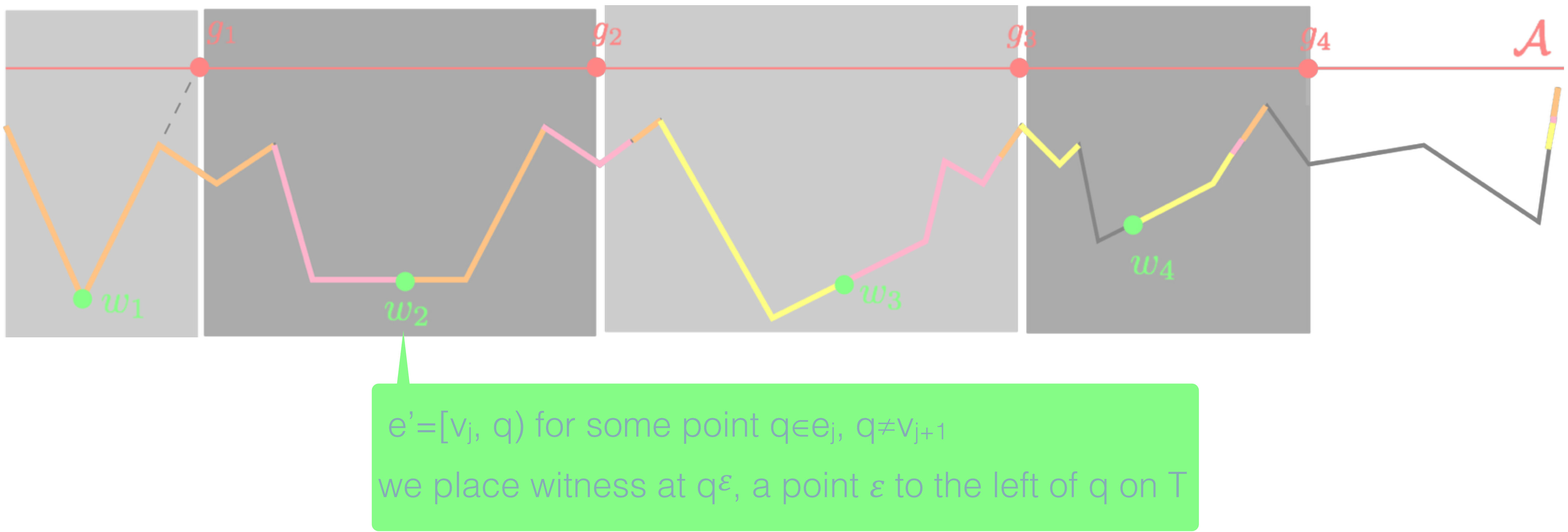

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