# A Novel MIP-based Airspace Sectorization for TMAs

Tobias Andersson Granberg, Tatiana Polishchuk, **Christiane Schmidt**

LINKÖPING UNIVERSITY

- Significant growth in air transportation

- Significant growth in air transportation
- Terminal Maneuvering Area (TMA) is particularly affected by congestion

- Significant growth in air transportation
- Terminal Maneuvering Area (TMA) is particularly affected by congestion
- Improve design of arrival and departure procedures ➜ higher throughput

- Significant growth in air transportation
- Terminal Maneuvering Area (TMA) is particularly affected by congestion
- Improve design of arrival and departure procedures ➜ higher throughput
- In Air Traffic Management (ATM): humans-in-the-loop!

- Significant growth in air transportation
- Terminal Maneuvering Area (TMA) is particularly affected by congestion
- Improve design of arrival and departure procedures ➜ higher throughput
- In Air Traffic Management (ATM): humans-in-the-loop!
    - Planes constantly monitored/guided by air traffic controllers (ATCOs)

- Significant growth in air transportation
- Terminal Maneuvering Area (TMA) is particularly affected by congestion
- Improve design of arrival and departure procedures ➜ higher throughput
- In Air Traffic Management (ATM): humans-in-the-loop!
  - Planes constantly monitored/guided by air traffic controllers (ATCOs)
  - ATCOs assigned to sectors of the airspace (TMA)

- Significant growth in air transportation
- Terminal Maneuvering Area (TMA) is particularly affected by congestion
- Improve design of arrival and departure procedures ➜ higher throughput
- In Air Traffic Management (ATM): humans-in-the-loop!
  - Planes constantly monitored/guided by air traffic controllers (ATCOs)
  - ATCOs assigned to sectors of the airspace (TMA)
  - Workload of the ATCOs should be balanced

- Significant growth in air transportation
- Terminal Maneuvering Area (TMA) is particularly affected by congestion
- Improve design of arrival and departure procedures ➜ higher throughput
- In Air Traffic Management (ATM): humans-in-the-loop!
  - Planes constantly monitored/guided by air traffic controllers (ATCOs)
  - ATCOs assigned to sectors of the airspace (TMA)
  - Workload of the ATCOs should be balanced
- Plus: geometric constraints on the sector shape

- Significant growth in air transportation
- Terminal Maneuvering Area (TMA) is particularly affected by congestion
- Improve design of arrival and departure procedures ➜ higher throughput
- In Air Traffic Management (ATM): humans-in-the-loop!
  - Planes constantly monitored/guided by air traffic controllers (ATCOs)
  - ATCOs assigned to sectors of the airspace (TMA)
  - Workload of the ATCOs should be balanced
- Plus: geometric constraints on the sector shape

Workload?

- Significant growth in air transportation
- Terminal Maneuvering Area (TMA) is particularly affected by congestion
- Improve design of arrival and departure procedures ➜ higher throughput
- In Air Traffic Management (ATM): humans-in-the-loop!
  - Planes constantly monitored/guided by air traffic controllers (ATCOs)
  - ATCOs assigned to sectors of the airspace (TMA)
  - Workload of the ATCOs should be balanced
- Plus: geometric constraints on the sector shape

Workload?
Various tasks contribute to airspace's complexity and drive ATCO's mental workload

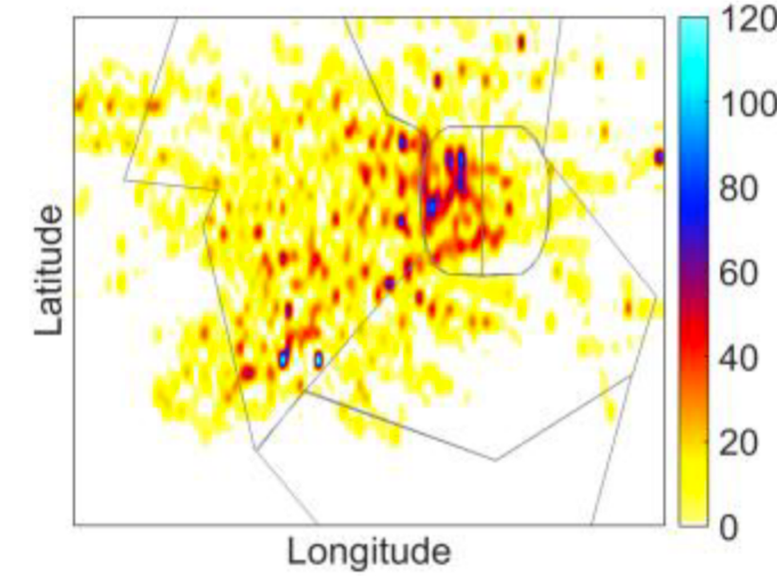
- Significant growth in air transportation
- Terminal Maneuvering Area (TMA) is particularly affected by congestion
- Improve design of arrival and departure procedures ➜ higher throughput
- In Air Traffic Management (ATM): humans-in-the-loop!
  - Planes constantly monitored/guided by air traffic controllers (ATCOs)
  - ATCOs assigned to sectors of the airspace (TMA)
  - Workload of the ATCOs should be balanced
- Plus: geometric constraints on the sector shape

Workload?

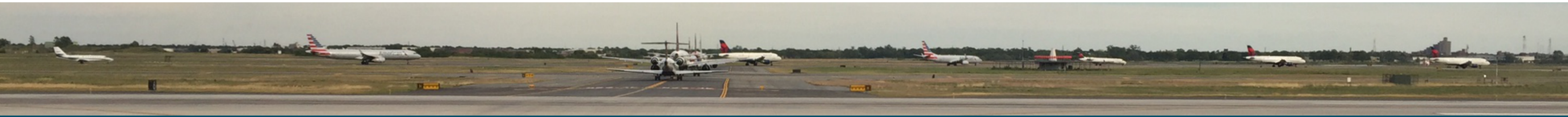
Various tasks contribute to airspace's complexity and drive ATCO's mental workload

- Taskload: objective demands of the ATCO's monitoring task

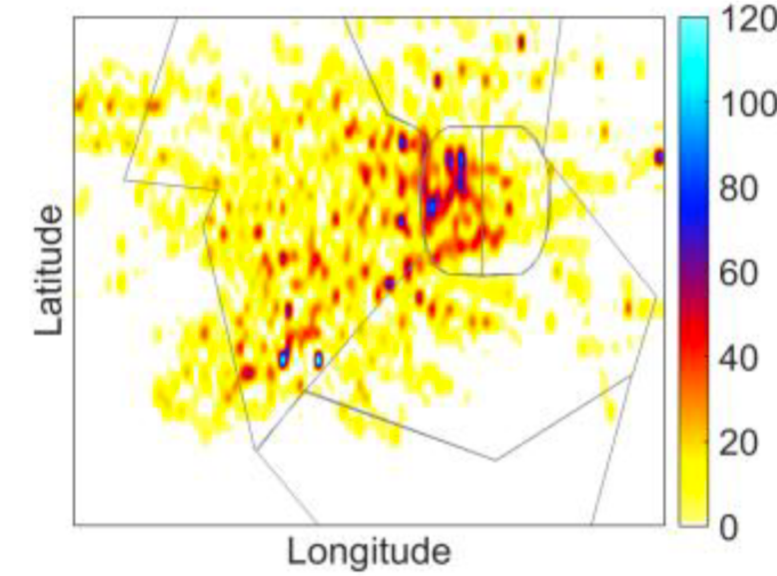
- Significant growth in air transportation
- Terminal Maneuvering Area (TMA) is particularly affected by congestion
- Improve design of arrival and departure procedures ➜ higher throughput
- In Air Traffic Management (ATM): humans-in-the-loop!
  - Planes constantly monitored/guided by air traffic controllers (ATCOs)
  - ATCOs assigned to sectors of the airspace (TMA)
  - Workload of the ATCOs should be balanced
- Plus: geometric constraints on the sector shape




Workload?
Various tasks contribute to airspace's complexity and drive ATCO's mental workload
- Taskload: objective demands of the ATCO's monitoring task
- We use heat maps of the density of weighted clicks as an input [Zohrevandi et al., 2016].



- Significant growth in air transportation
- Terminal Maneuvering Area (TMA) is particularly affected by congestion
- Improve design of arrival and departure procedures ➜ higher throughput
- In Air Traffic Management (ATM): humans-in-the-loop!
  - Planes constantly monitored/guided by air traffic controllers (ATCOs)
  - ATCOs assigned to sectors of the airspace (TMA)
  - Workload of the ATCOs should be balanced
- Plus: geometric constraints on the sector shape




Workload?

Various tasks contribute to airspace's complexity and drive ATCO's mental workload

- Taskload: objective demands of the ATCO's monitoring task
- We use heat maps of the density of weighted clicks as an input [Zohrevandi et al., 2016].
- BUT: we do not depend on specific maps.

A **sectorization** of a simple polygon P is a partition of P into k disjoint subpolygons $S_1,\ldots,S_k$ ($S_i \cap S_j = \varnothing \ \forall i \neq j$), such that $\bigcup_{i=1}^{k} S_i = P$.

A **sectorization** of a simple polygon P is a partition of P into k disjoint subpolygons $S_1,\ldots,S_k$ ($S_i \cap S_j = \varnothing\ \forall i \neq j$), such that $\bigcup_{i=1}^{k} S_i = P$.

$S_i$: **sectors**

A **sectorization** of a simple polygon P is a partition of P into k disjoint subpolygons $S_1,\ldots,S_k$ ($S_i \cap S_j = \varnothing \; \forall i \neq j$), such that $\bigcup_{i=1}^{k} S_i = P$.

$S_i$: **sectors**

**Sectorization Problem:**

A **sectorization** of a simple polygon P is a partition of P into k disjoint subpolygons $S_1,\ldots,S_k$ ($S_i \cap S_j = \emptyset\ \forall i \neq j$), such that $\cup_{i=1}^{k} S_i = P$.

$S_i$: **sectors**

**Sectorization Problem:**
Given: The coordinates of the TMA, defining a polygon P , the number of sectors |S|, and a set C of constraints on the resulting sectors.

A **sectorization** of a simple polygon P is a partition of P into k disjoint subpolygons $S_1,\dots,S_k$ ($S_i \cap S_j = \emptyset \ \forall i \neq j$), such that $\bigcup_{i=1}^{k} S_i = P$.

$S_i$: **sectors**

**Sectorization Problem:**
Given: The coordinates of the TMA, defining a polygon P , the number of sectors |S|, and a set C of constraints on the resulting sectors.
Find: A sectorization of P with k = |S| , fulfilling C.

A **sectorization** of a simple polygon P is a partition of P into k disjoint subpolygons $S_1,\ldots,S_k$ ($S_i \cap S_j = \varnothing\ \forall i \neq j$), such that $\bigcup_{i=1}^{k} S_i = P$.

$S_i$: **sectors**

**Sectorization Problem:**

Given: The coordinates of the TMA, defining a polygon P, the number of sectors |S|, and a set C of constraints on the resulting sectors.

Find: A sectorization of P with k = |S|, fulfilling C.

**(a) Balanced size**

A **sectorization** of a simple polygon P is a partition of P into k disjoint subpolygons $S_1,\ldots,S_k$ ($S_i \cap S_j = \varnothing \; \forall i \neq j$), such that $\cup_{i=1}^{k} S_i = P$.

$S_i$: **sectors**

**Sectorization Problem:**
Given: The coordinates of the TMA, defining a polygon P , the number of sectors |S|, and a set C of constraints on the resulting sectors.
Find: A sectorization of P with k = |S| , fulfilling C.
**(a) Balanced size**
**(b) Bounded taskload**

A **sectorization** of a simple polygon P is a partition of P into k disjoint subpolygons $S_1,\ldots,S_k$ ($S_i \cap S_j = \varnothing \ \forall i \neq j$), such that $\bigcup_{i=1}^{k} S_i = P$.

$S_i$: **sectors**

**Sectorization Problem:**
Given: The coordinates of the TMA, defining a polygon P, the number of sectors |S|, and a set C of constraints on the resulting sectors.
Find: A sectorization of P with k = |S|, fulfilling C.
**(a) Balanced size**
**(b) Bounded taskload**
**(c) Balanced taskload**

A **sectorization** of a simple polygon P is a partition of P into k disjoint subpolygons $S_1, \ldots, S_k$ ($S_i \cap S_j = \varnothing \; \forall i \neq j$), such that $\bigcup_{i=1}^{k} S_i = P$.

$S_i$: **sectors**

**Sectorization Problem:**
Given: The coordinates of the TMA, defining a polygon P , the number of sectors |S|, and a set C of constraints on the resulting sectors.
Find: A sectorization of P with k = |S| , fulfilling C.
**(a) Balanced size**
**(b) Bounded taskload**
**(c) Balanced taskload**
**(d) Connected sectors**

A **sectorization** of a simple polygon P is a partition of P into k disjoint subpolygons $S_1,\ldots,S_k$ ($S_i \cap S_j = \emptyset \ \forall i \neq j$), such that $\cup_{i=1}^{k} S_i = P$.

$S_i$: **sectors**

**Sectorization Problem:**

Given: The coordinates of the TMA, defining a polygon P , the number of sectors |S|, and a set C of constraints on the resulting sectors.

Find: A sectorization of P with k = |S| , fulfilling C.

**(a) Balanced size**

**(b) Bounded taskload**

**(c) Balanced taskload**

**(d) Connected sectors**

**(e) Nice shape** (smooth boundary and an easily memorable shape)

A **sectorization** of a simple polygon P is a partition of P into k disjoint subpolygons $S_1,\ldots,S_k$ ($S_i \cap S_j = \varnothing \; \forall i \neq j$), such that $\bigcup_{i=1}^{k} S_i = P$.

$S_i$: **sectors**

**Sectorization Problem:**

Given: The coordinates of the TMA, defining a polygon P , the number of sectors |S|, and a set C of constraints on the resulting sectors.

Find: A sectorization of P with k = |S| , fulfilling C.

**(a) Balanced size**

**(b) Bounded taskload**

**(c) Balanced taskload**

**(d) Connected sectors**

**(e) Nice shape** (smooth boundary and an easily memorable shape)

**(f) Convex sectors** ((straight-line) flight cannot enter and leave a convex sector multiple times)

A **sectorization** of a simple polygon P is a partition of P into k disjoint subpolygons $S_1,\ldots,S_k$ ($S_i \cap S_j = \emptyset\ \forall i \neq j$), such that $\cup_{i=1}^{k} S_i = P$.

$S_i$: **sectors**

**Sectorization Problem:**

Given: The coordinates of the TMA, defining a polygon P, the number of sectors |S|, and a set C of constraints on the resulting sectors.

Find: A sectorization of P with k = |S|, fulfilling C.

**(a) Balanced size**

**(b) Bounded taskload**

**(c) Balanced taskload**

**(d) Connected sectors**

**(e) Nice shape** (smooth boundary and an easily memorable shape)

**(f) Convex sectors** ((straight-line) flight cannot enter and leave a convex sector multiple times)

**(g) Interior conflict points** ( Points that require increased attention from ATCOs should lie in the sector's interior.)

# Grid-based IP formulation

# Grid-based IP formulation

- Square grid in the TMA

# Grid-based IP formulation

- Square grid in the TMA
- G = (V,E):
  - Every grid node connected to its 8 neighbors

# Grid-based IP formulation

- Square grid in the TMA
- G = (V,E):
  - Every grid node connected to its 8 neighbors
- N(i) = set of neighbors of i (including i)

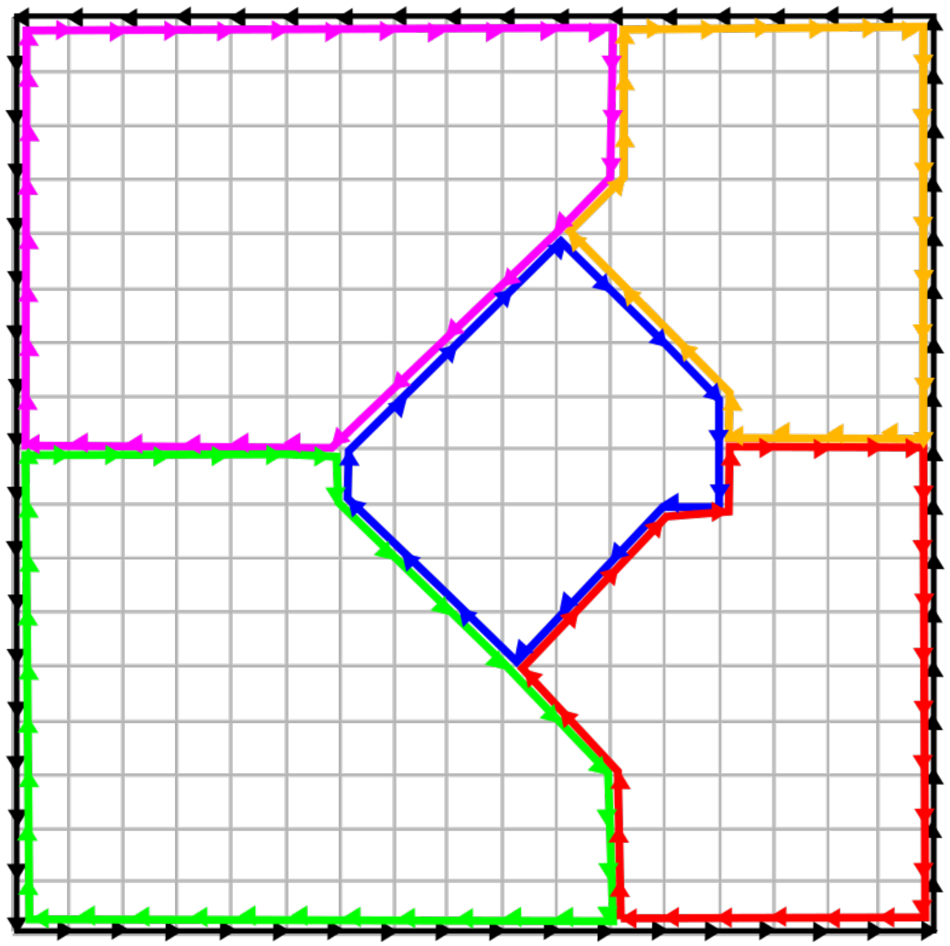
# Grid-based IP formulation

- Square grid in the TMA
- G = (V,E):
  - Every grid node connected to its 8 neighbors
- N(i) = set of neighbors of i (including i)
- $\ell_{i,j}$ length of an edge (i, j)

# Grid-based IP formulation

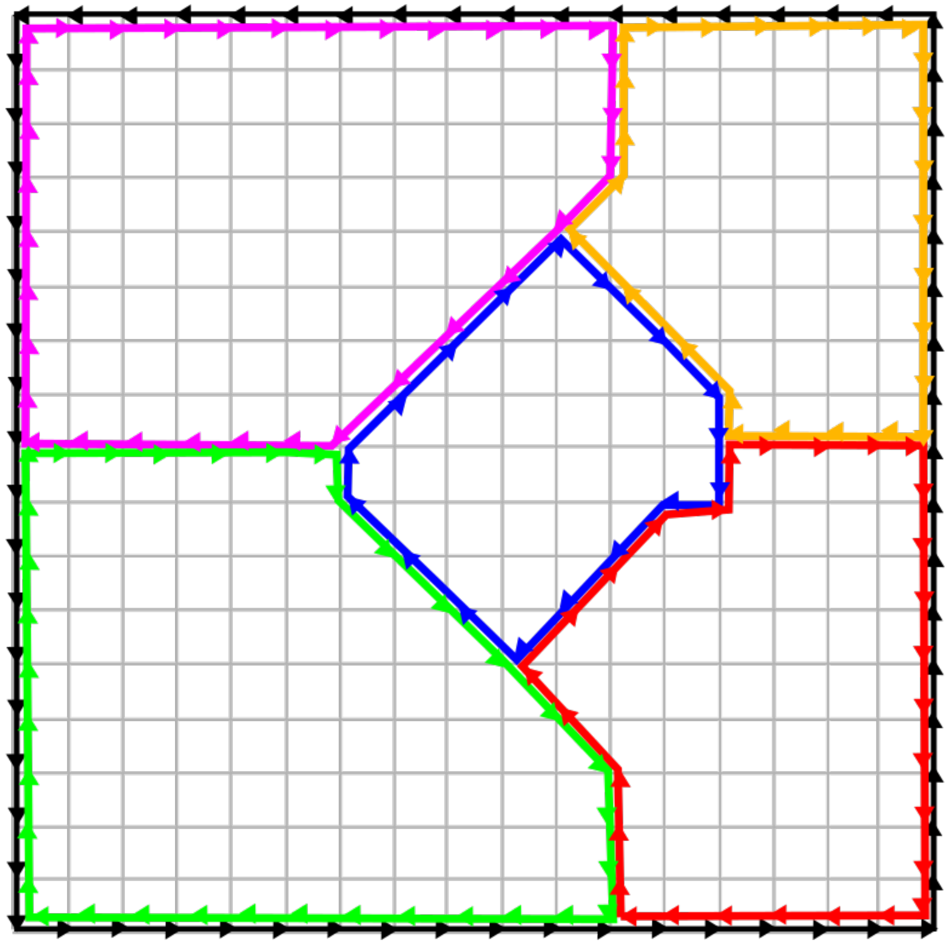
- Square grid in the TMA
- G = (V,E):
  - Every grid node connected to its 8 neighbors
- N(i) = set of neighbors of i (including i)
- $\ell_{i,j}$ length of an edge (i, j)

Main idea: use an artificial sector, $S_0$, that encompasses the complete boundary of P, using all counterclockwise (ccw) edges.



- Square grid in the TMA
- G = (V,E):
  - Every grid node connected to its 8 neighbors
- N(i) = set of neighbors of i (including i)
- $\ell_{i,j}$ length of an edge (i, j)

Main idea: use an artificial sector, $S_0$, that encompasses the complete boundary of P, using all counterclockwise (ccw) edges.
We use sectors in $S^* = S \cup S_0$ with $S = \{S_1,\dots,S_k\}$ .



# Grid-based IP formulation

$y_{i,j,s} = 1$: edge (i,j) used for sector s

$$
\begin{aligned}
y_{i,j,0} &= 1 && \forall (i,j) \in S_0 \\
\sum_{s \in \mathcal{S}^*} y_{i,j,s} - \sum_{s \in \mathcal{S}^*} y_{j,i,s} &= 0 && \forall (i,j) \in E \\
y_{i,j,s} + y_{j,i,s} &\leq 1 && \forall (i,j) \in E, \forall s \in \mathcal{S}^* \\
\sum_{s \in \mathcal{S}^*} y_{i,j,s} &\leq 1 && \forall (i,j) \in E \\
\sum_{(i,j) \in E} y_{i,j,s} &\geq 3 && \forall s \in \mathcal{S}^* \\
y_{i,j,s} &\in \{0,1\} && \forall (i,j) \in E, \forall s \in \mathcal{S}^*
\end{aligned}
$$

$$
\begin{aligned}
\sum_{l \in V:(l,i) \in E} y_{l,i,s} - \sum_{j \in V:(i,j) \in E} y_{i,j,s} &= 0 && \forall i \in V, \forall s \in \mathcal{S}^* \\
\sum_{l \in V:(l,i) \in E} y_{l,i,s} &\leq 1 && \forall i \in V, \forall s \in \mathcal{S}^*
\end{aligned}
$$

# Grid-based IP formulation

$y_{i,j,s} = 1$: edge (i,j) used for sector s

$$
\begin{aligned}
y_{i,j,0} &= 1 & \forall (i,j) \in S_0 \\
\sum_{s \in \mathcal{S}^*} y_{i,j,s} - \sum_{s \in \mathcal{S}^*} y_{j,i,s} &= 0 & \forall (i,j) \in E \\
y_{i,j,s} + y_{j,i,s} &\leq 1 & \forall (i,j) \in E, \forall s \in \mathcal{S}^* \\
\sum_{s \in \mathcal{S}^*} y_{i,j,s} &\leq 1 & \forall (i,j) \in E \\
\sum_{(i,j) \in E} y_{i,j,s} &\geq 3 & \forall s \in \mathcal{S}^* \\
y_{i,j,s} &\in \{0,1\} & \forall (i,j) \in E, \forall s \in \mathcal{S}^*
\end{aligned}
$$

All ccw boundary edges in $S_0$

$$
\begin{aligned}
\sum_{l \in V : (l,i) \in E} y_{l,i,s} - \sum_{j \in V : (i,j) \in E} y_{i,j,s} &= 0 & \forall i \in V, \forall s \in \mathcal{S}^* \\
\sum_{l \in V : (l,i) \in E} y_{l,i,s} &\leq 1 & \forall i \in V, \forall s \in \mathcal{S}^*
\end{aligned}
$$

# Grid-based IP formulation

$y_{i,j,s} = 1$: edge (i,j) used for sector s

$$y_{i,j,0} = 1 \quad \forall (i,j) \in S_0$$

All ccw boundary edges in $S_0$

$$\sum_{s \in \mathcal{S}^*} y_{i,j,s} - \sum_{s \in \mathcal{S}^*} y_{j,i,s} = 0 \quad \forall (i,j) \in E$$

If (i,j) used for some sector, (j, i) has to be used as well.

$$y_{i,j,s} + y_{j,i,s} \leq 1 \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$$

$$\sum_{s \in \mathcal{S}^*} y_{i,j,s} \leq 1 \quad \forall (i,j) \in E$$

$$\sum_{(i,j) \in E} y_{i,j,s} \geq 3 \quad \forall s \in \mathcal{S}^*$$

$$y_{i,j,s} \in \{0,1\} \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$$

$$\sum_{l \in V:(l,i) \in E} y_{l,i,s} - \sum_{j \in V:(i,j) \in E} y_{i,j,s} = 0 \quad \forall i \in V, \forall s \in \mathcal{S}^*$$

$$\sum_{l \in V:(l,i) \in E} y_{l,i,s} \leq 1 \quad \forall i \in V, \forall s \in \mathcal{S}^*$$

# Grid-based IP formulation

$y_{i,j,s} = 1$: edge (i,j) used for sector s

$$
\begin{aligned}
y_{i,j,0} &= 1 && \forall (i,j) \in S_0 \\
\sum_{s \in \mathcal{S}^*} y_{i,j,s} - \sum_{s \in \mathcal{S}^*} y_{j,i,s} &= 0 && \forall (i,j) \in E \\
y_{i,j,s} + y_{j,i,s} &\leq 1 && \forall (i,j) \in E, \forall s \in \mathcal{S}^* \\
\sum_{s \in \mathcal{S}^*} y_{i,j,s} &\leq 1 && \forall (i,j) \in E \\
\sum_{(i,j) \in E} y_{i,j,s} &\geq 3 && \forall s \in \mathcal{S}^* \\
y_{i,j,s} &\in \{0,1\} && \forall (i,j) \in E, \forall s \in \mathcal{S}^*
\end{aligned}
$$

All ccw boundary edges in $S_0$

If (i,j) used for some sector, (j, i) has to be used as well.

Sector cannot contain (i,j) and (j,i).

$$
\sum_{l \in V:(l,i) \in E} y_{l,i,s} - \sum_{j \in V:(i,j) \in E} y_{i,j,s} = 0 \quad \forall i \in V, \forall s \in \mathcal{S}^*
$$

$$
\sum_{l \in V:(l,i) \in E} y_{l,i,s} \leq 1 \quad \forall i \in V, \forall s \in \mathcal{S}^*
$$

$y_{i,j,s} = 1$: edge (i,j) used for sector s

$$
\begin{aligned}
y_{i,j,0} &= 1 && \forall (i,j) \in S_0 \\
\sum_{s \in \mathcal{S}^*} y_{i,j,s} - \sum_{s \in \mathcal{S}^*} y_{j,i,s} &= 0 && \forall (i,j) \in E \\
y_{i,j,s} + y_{j,i,s} &\leq 1 && \forall (i,j) \in E, \forall s \in \mathcal{S}^* \\
\sum_{s \in \mathcal{S}^*} y_{i,j,s} &\leq 1 && \forall (i,j) \in E \\
\sum_{(i,j) \in E} y_{i,j,s} &\geq 3 && \forall s \in \mathcal{S}^* \\
y_{i,j,s} &\in \{0,1\} && \forall (i,j) \in E, \forall s \in \mathcal{S}^*
\end{aligned}
$$

All ccw boundary edges in $S_0$

If (i,j) used for some sector, (j, i) has to be used as well.

Sector cannot contain (i,j) and (j,i).

(i,j) in $S_l$, (j,i) has to be in another sector

$$
\sum_{l \in V:(l,i) \in E} y_{l,i,s} - \sum_{j \in V:(i,j) \in E} y_{i,j,s} = 0 \quad \forall i \in V, \forall s \in \mathcal{S}^*
$$

$$
\sum_{l \in V:(l,i) \in E} y_{l,i,s} \leq 1 \quad \forall i \in V, \forall s \in \mathcal{S}^*
$$

# Grid-based IP formulation

$y_{i,j,s} = 1$: edge (i,j) used for sector s

$$y_{i,j,0} = 1 \quad \forall (i,j) \in S_0$$

All ccw boundary edges in $S_0$

$$\sum_{s \in \mathcal{S}^*} y_{i,j,s} - \sum_{s \in \mathcal{S}^*} y_{j,i,s} = 0 \quad \forall (i,j) \in E$$

If (i,j) used for some sector, (j, i) has to be used as well.

$$y_{i,j,s} + y_{j,i,s} \leq 1 \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$$

Sector cannot contain (i,j) and (j,i).

(i,j) in $S_l$, (j,i) has to be in another sector

$$\sum_{s \in \mathcal{S}^*} y_{i,j,s} \leq 1 \quad \forall (i,j) \in E$$

No edge in two sectors.

$$\sum_{(i,j) \in E} y_{i,j,s} \geq 3 \quad \forall s \in \mathcal{S}^*$$

$$y_{i,j,s} \in \{0,1\} \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$$

$$\sum_{l \in V:(l,i) \in E} y_{l,i,s} - \sum_{j \in V:(i,j) \in E} y_{i,j,s} = 0 \quad \forall i \in V, \forall s \in \mathcal{S}^*$$

$$\sum_{l \in V:(l,i) \in E} y_{l,i,s} \leq 1 \quad \forall i \in V, \forall s \in \mathcal{S}^*$$

$y_{i,j,s} = 1$: edge (i,j) used for sector s

$$y_{i,j,0} = 1 \quad \forall (i,j) \in S_0$$

All ccw boundary edges in $S_0$

$$\sum_{s \in \mathcal{S}^*} y_{i,j,s} - \sum_{s \in \mathcal{S}^*} y_{j,i,s} = 0 \quad \forall (i,j) \in E$$

If (i,j) used for some sector, (j, i) has to be used as well.

$$y_{i,j,s} + y_{j,i,s} \leq 1 \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$$

Sector cannot contain (i,j) and (j,i).

(i,j) in $S_l$, (j,i) has to be in another sector

$$\sum_{s \in \mathcal{S}^*} y_{i,j,s} \leq 1 \quad \forall (i,j) \in E$$

No edge in two sectors.

$$\sum_{(i,j) \in E} y_{i,j,s} \geq 3 \quad \forall s \in \mathcal{S}^*$$

Minimum size

$$y_{i,j,s} \in \{0,1\} \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$$

$$\sum_{l \in V:(l,i) \in E} y_{l,i,s} - \sum_{j \in V:(i,j) \in E} y_{i,j,s} = 0 \quad \forall i \in V, \forall s \in \mathcal{S}^*$$

$$\sum_{l \in V:(l,i) \in E} y_{l,i,s} \leq 1 \quad \forall i \in V, \forall s \in \mathcal{S}^*$$

# Grid-based IP formulation

$y_{i,j,s} = 1$: edge (i,j) used for sector s

$$y_{i,j,0} = 1 \quad \forall (i,j) \in S_0$$
All ccw boundary edges in $S_0$

$$\sum_{s\in\mathcal{S}^*} y_{i,j,s} - \sum_{s\in\mathcal{S}^*} y_{j,i,s} = 0 \quad \forall (i,j) \in E$$
If (i,j) used for some sector, (j, i) has to be used as well.

$$y_{i,j,s} + y_{j,i,s} \leq 1 \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$$
Sector cannot contain (i,j) and (j,i).

(i,j) in $S_l$, (j,i) has to be in another sector

$$\sum_{s\in\mathcal{S}^*} y_{i,j,s} \leq 1 \quad \forall (i,j) \in E$$
No edge in two sectors.

$$\sum_{(i,j)\in E} y_{i,j,s} \geq 3 \quad \forall s \in \mathcal{S}^*$$
Minimum size

$$y_{i,j,s} \in \{0,1\} \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$$

$$\sum_{l\in V:(l,i)\in E} y_{l,i,s} - \sum_{j\in V:(i,j)\in E} y_{i,j,s} = 0 \quad \forall i \in V, \forall s \in \mathcal{S}^*$$
Indegree=outdegree for all vertices

$$\sum_{l\in V:(l,i)\in E} y_{l,i,s} \leq 1 \quad \forall i \in V, \forall s \in \mathcal{S}^*$$

$y_{i,j,s} = 1$: edge (i,j) used for sector s

$$y_{i,j,0} = 1 \quad \forall (i,j) \in S_0$$
All ccw boundary edges in $S_0$

$$\sum_{s \in \mathcal{S}^*} y_{i,j,s} - \sum_{s \in \mathcal{S}^*} y_{j,i,s} = 0 \quad \forall (i,j) \in E$$
If (i,j) used for some sector, (j, i) has to be used as well.

$$y_{i,j,s} + y_{j,i,s} \leq 1 \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$$
Sector cannot contain (i,j) and (j,i).

(i,j) in $S_l$, (j,i) has to be in another sector

$$\sum_{s \in \mathcal{S}^*} y_{i,j,s} \leq 1 \quad \forall (i,j) \in E$$
No edge in two sectors.

$$\sum_{(i,j) \in E} y_{i,j,s} \geq 3 \quad \forall s \in \mathcal{S}^*$$
Minimum size

$$y_{i,j,s} \in \{0,1\} \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$$

$$\sum_{l \in V:(l,i) \in E} y_{l,i,s} - \sum_{j \in V:(i,j) \in E} y_{i,j,s} = 0 \quad \forall i \in V, \forall s \in \mathcal{S}^*$$
Indegree=outdegree for all vertices

$$\sum_{l \in V:(l,i) \in E} y_{l,i,s} \leq 1 \quad \forall i \in V, \forall s \in \mathcal{S}^*$$
A node has at most one ingoing edge per sector

# Grid-based IP formulation

$y_{i,j,s} = 1$: edge (i,j) used for sector s

| | | |
|---|---|---|
| $y_{i,j,0} = 1 \quad \forall (i,j) \in S_0$ | All ccw boundary edges in $S_0$ | |
| $\sum_{s \in \mathcal{S}^*} y_{i,j,s} - \sum_{s \in \mathcal{S}^*} y_{j,i,s} = 0 \quad \forall (i,j) \in E$ | If (i,j) used for some sector, (j, i) has to be used as well. | (i,j) in $S_l$, (j,i) has to be in another sector |
| $y_{i,j,s} + y_{j,i,s} \leq 1 \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$ | Sector cannot contain (i,j) and (j,i). | |
| $\sum_{s \in \mathcal{S}^*} y_{i,j,s} \leq 1 \quad \forall (i,j) \in E$ | No edge in two sectors. | |
| $\sum_{(i,j) \in E} y_{i,j,s} \geq 3 \quad \forall s \in \mathcal{S}^*$ | Minimum size | |
| $y_{i,j,s} \in \{0,1\} \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$ | | |
| $\sum_{l \in V:(l,i) \in E} y_{l,i,s} - \sum_{j \in V:(i,j) \in E} y_{i,j,s} = 0 \quad \forall i \in V, \forall s \in \mathcal{S}^*$ | Indegree=outdegree for all vertices | |
| $\sum_{l \in V:(l,i) \in E} y_{l,i,s} \leq 1 \quad \forall i \in V, \forall s \in \mathcal{S}^*$ | A node has at most one ingoing edge per sector | |

$\Longrightarrow$ Union of the |S| sectors completely covers the TMA.

## (a) Balanced size

**(a) Balanced size**

We need to assign area to sector selected by boundary edges!

**(a) Balanced size**

We need to assign area to sector selected by boundary edges!

Area of polygon $P$ with rational vertices and can be computed efficiently [Fekete et al., 2015]:

**(a) Balanced size**

We need to assign area to sector selected by boundary edges!

Area of polygon *P* with rational vertices and can be computed efficiently [Fekete et al., 2015]:

- We introduce reference point *r*.

LINKÖPING UNIVERSITY

**(a) Balanced size**

We need to assign area to sector selected by boundary edges!

Area of polygon *P* with rational vertices and can be computed efficiently [Fekete et al., 2015]:

- We introduce reference point *r*.
- We compute the area of the triangle of each directed edge *e* of *P*.

LINKÖPING UNIVERSITY

**(a) Balanced size**

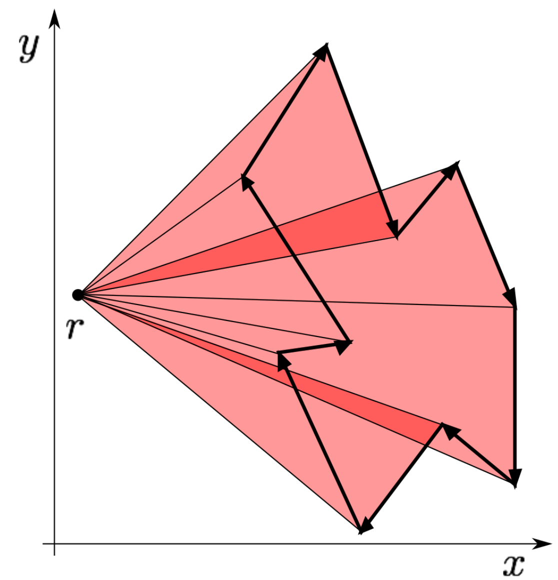
We need to assign area to sector selected by boundary edges!
Area of polygon *P* with rational vertices and can be computed efficiently [Fekete et al., 2015]:

- We introduce reference point *r*.
- We compute the area of the triangle of each directed edge *e* of *P*.
- We sum up the triangle area for all edges of *P*:

### (a) Balanced size

We need to assign area to sector selected by boundary edges!
Area of polygon *P* with rational vertices and can be computed efficiently [Fekete et al., 2015]:

- We introduce reference point *r*.
- We compute the area of the triangle of each directed edge *e* of *P*.
- We sum up the triangle area for all edges of *P*:
  - cw triangles contribute positive

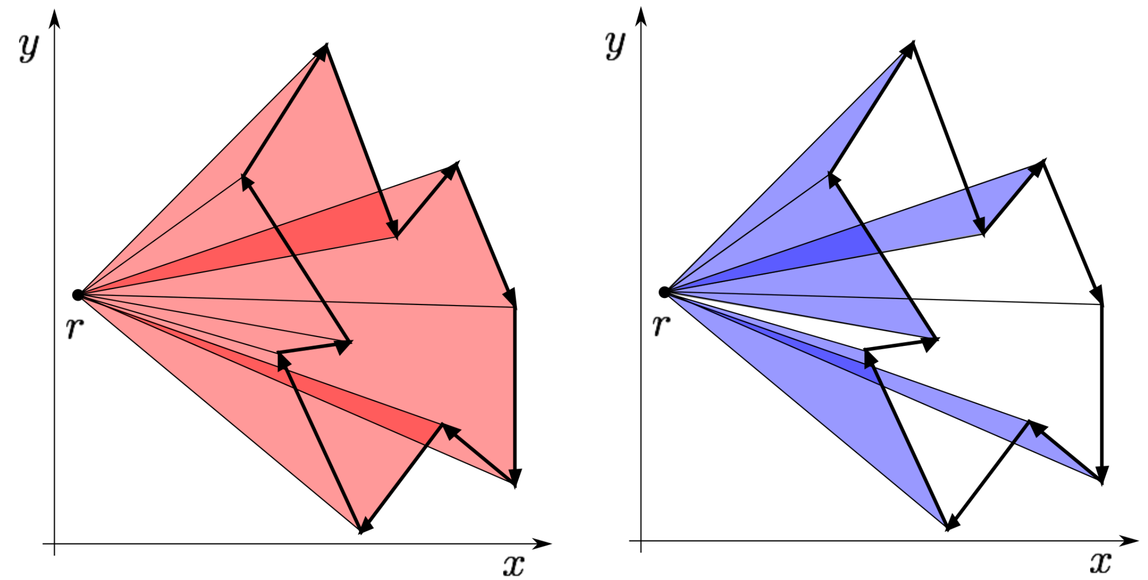


## (a) Balanced size

We need to assign area to sector selected by boundary edges!

Area of polygon *P* with rational vertices and can be computed efficiently [Fekete et al., 2015]:

- We introduce reference point *r*.
- We compute the area of the triangle of each directed edge *e* of *P*.
- We sum up the triangle area for all edges of *P*:
  - cw triangles contribute positive
  - ccw triangles contribute negative



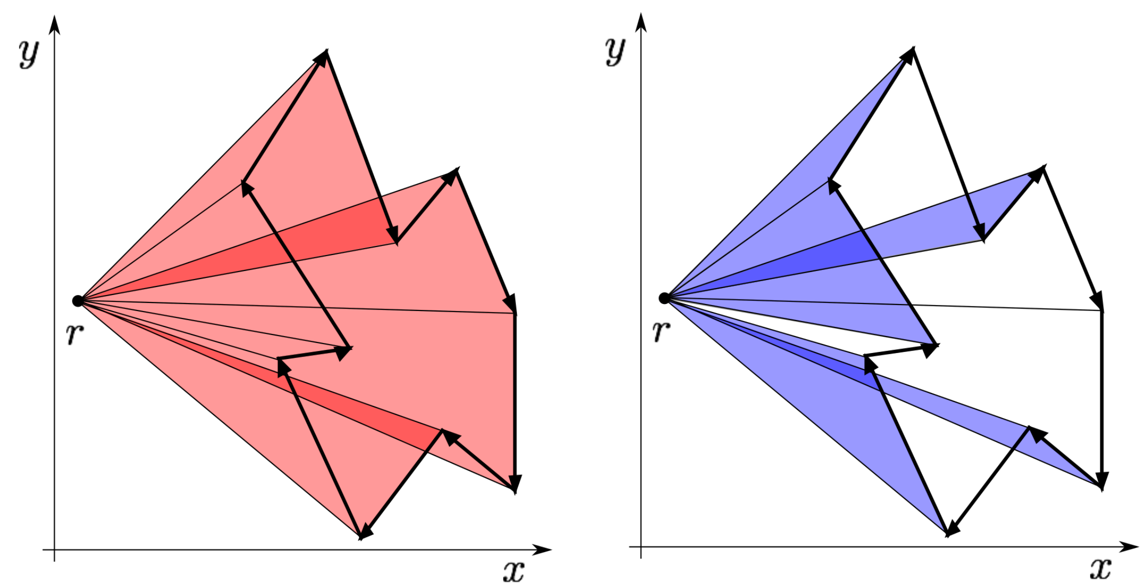
LINKÖPING UNIVERSITY

**(a) Balanced size**

We need to assign area to sector selected by boundary edges!

Area of polygon *P* with rational vertices and can be computed efficiently [Fekete et al., 2015]:

- We introduce reference point *r*.
- We compute the area of the triangle of each directed edge *e* of *P*.
- We sum up the triangle area for all edges of *P*:
  - cw triangles contribute positive
  - ccw triangles contribute negative
- $f_{i,j}$: signed area of the triangle (i,j) and *r*

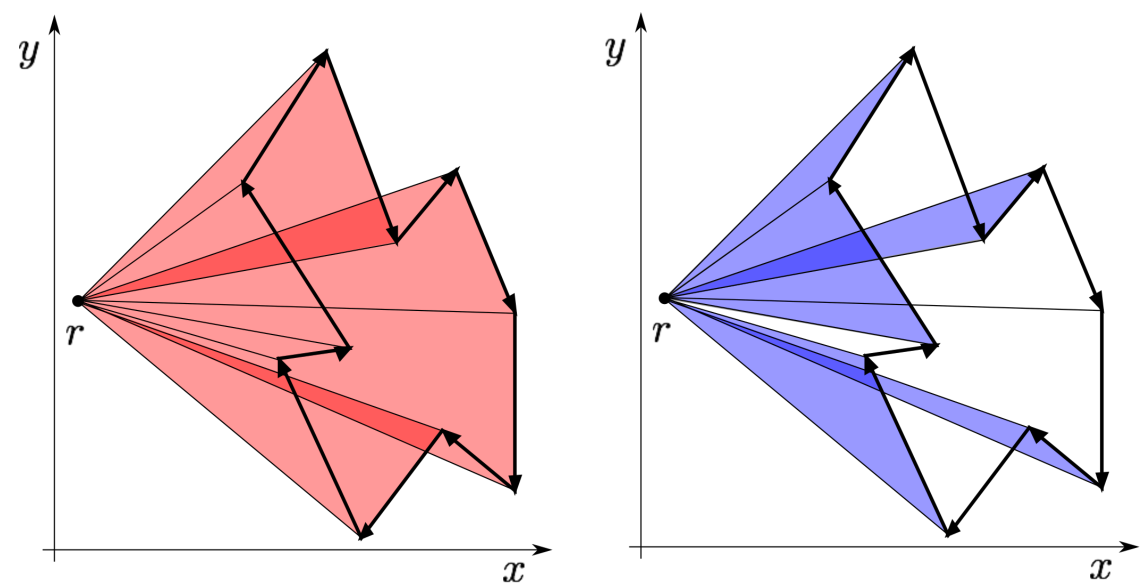


LINKÖPING UNIVERSITY

### (a) Balanced size

We need to assign area to sector selected by boundary edges!
Area of polygon *P* with rational vertices and can be computed efficiently [Fekete et al., 2015]:

- We introduce reference point *r*.
- We compute the area of the triangle of each directed edge *e* of *P*.
- We sum up the triangle area for all edges of *P*:
  - cw triangles contribute positive
  - ccw triangles contribute negative
- $f_{i,j}$: signed area of the triangle (i,j) and *r*




$$\sum_{(i,j)\in E} f_{i,j}\, y_{i,j,s} - a_s = 0 \qquad \forall s \in \mathcal{S}^*$$

Assigns area of sector s to $a_s$

$$\sum_{s\in\mathcal{S}} a_s = a_0$$

$$a_s \geq a_{LB} \qquad \forall s \in \mathcal{S}$$

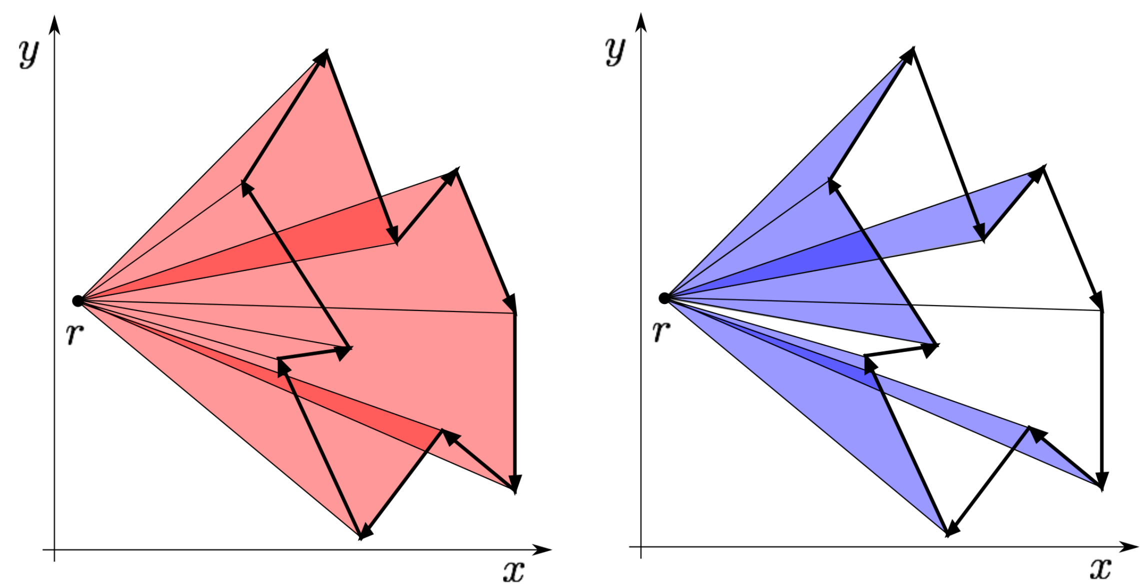
$$a_{LB} = c_1 \cdot a_0/|\mathcal{S}| \ \text{, with , e.g.,} c_1 = 0.9$$

LINKÖPING UNIVERSITY

**(a) Balanced size**

We need to assign area to sector selected by boundary edges!
Area of polygon *P* with rational vertices and can be computed efficiently [Fekete et al., 2015]:

- We introduce reference point *r*.
- We compute the area of the triangle of each directed edge *e* of *P*.
- We sum up the triangle area for all edges of *P*:
  - cw triangles contribute positive
  - ccw triangles contribute negative
- $f_{i,j}$: signed area of the triangle (i,j) and *r*




$$\sum_{(i,j)\in E} f_{i,j}\, y_{i,j,s} - a_s = 0 \qquad \forall s \in \mathcal{S}^*$$

Assigns area of sector s to $a_s$

$$\sum_{s\in\mathcal{S}} a_s = a_0$$

Sum of areas = area of $S_0$

$$a_s \geq a_{LB} \qquad \forall s \in \mathcal{S}$$

$$a_{LB} = c_1 \cdot a_0/|\mathcal{S}| \text{ , with , e.g.,} c_1 = 0.9$$

## (b) Bounded taskload/ (c)Balanced taskload

LINKÖPING UNIVERSITY

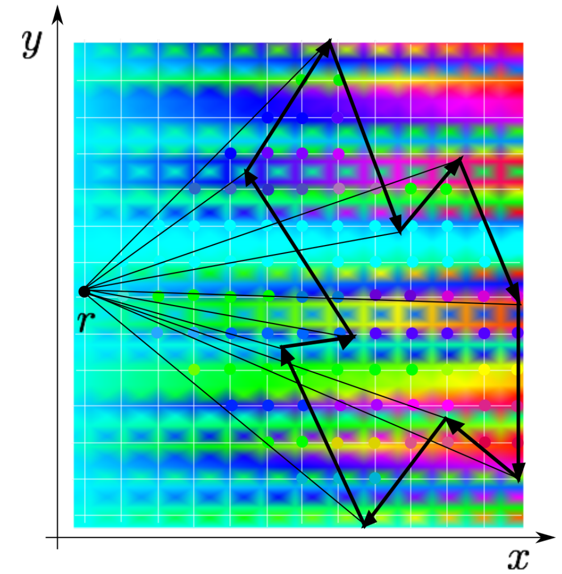
**(b) Bounded taskload/ (c)Balanced taskload**

We need to associate task load with a sector.

**(b) Bounded taskload/ (c)Balanced taskload**

We need to associate task load with a sector.

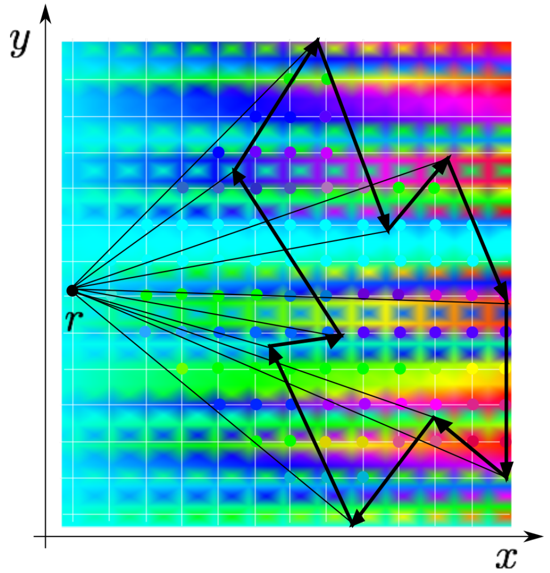
- Overlay heat map with a grid.

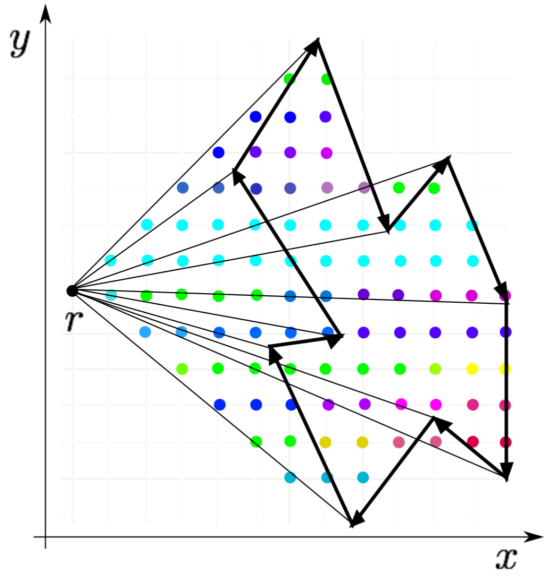


**(b) Bounded taskload/ (c)Balanced taskload**

We need to associate task load with a sector.

- Overlay heat map with a grid.
- Extract values at the grid points.

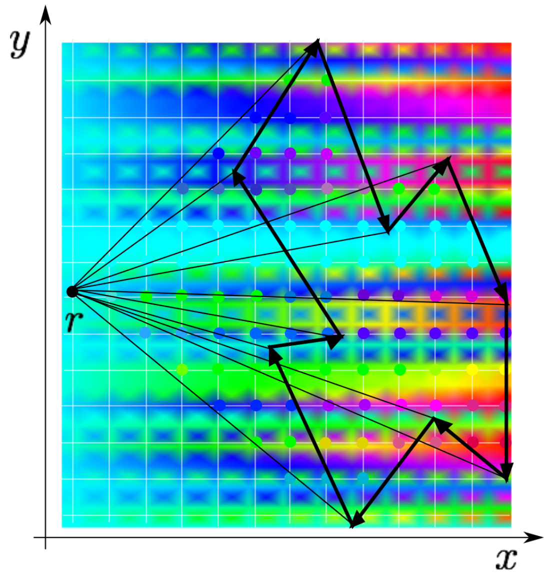


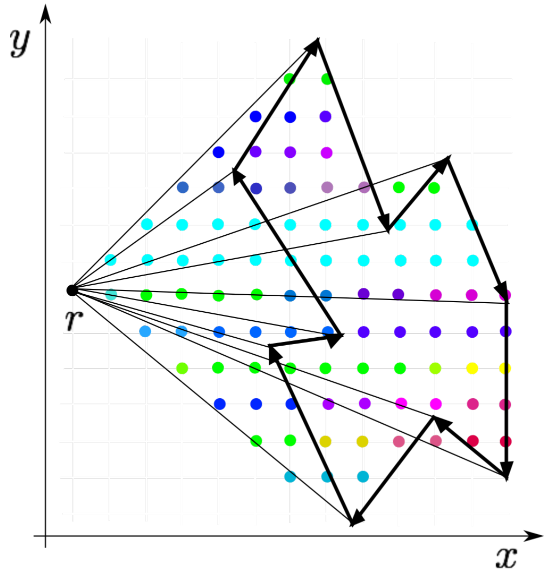



**(b) Bounded taskload/ (c)Balanced taskload**

We need to associate task load with a sector.

- Overlay heat map with a grid.
- Extract values at the grid points.
- Use discretized heat map.

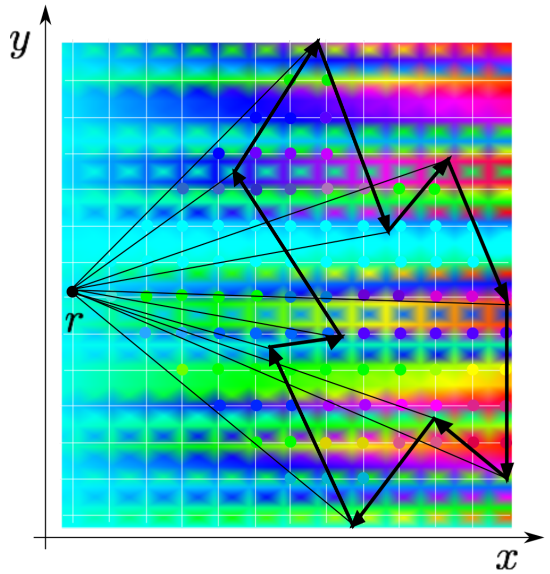


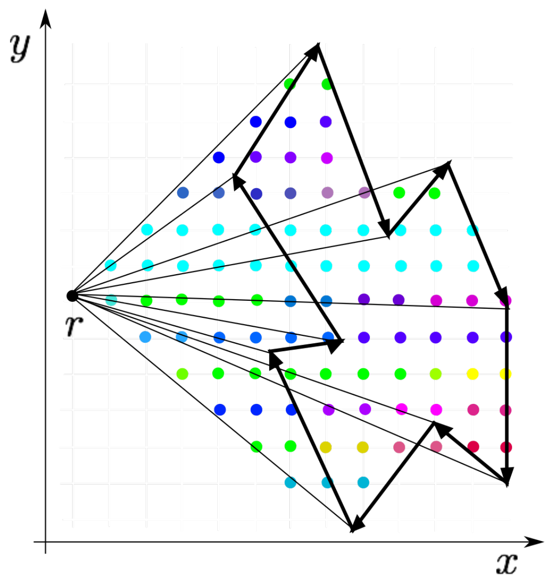



## (b) Bounded taskload/ (c)Balanced taskload

We need to associate task load with a sector.

- Overlay heat map with a grid.
- Extract values at the grid points.
- Use discretized heat map.
- Each discrete heat map point q: “heat value” $h_q$

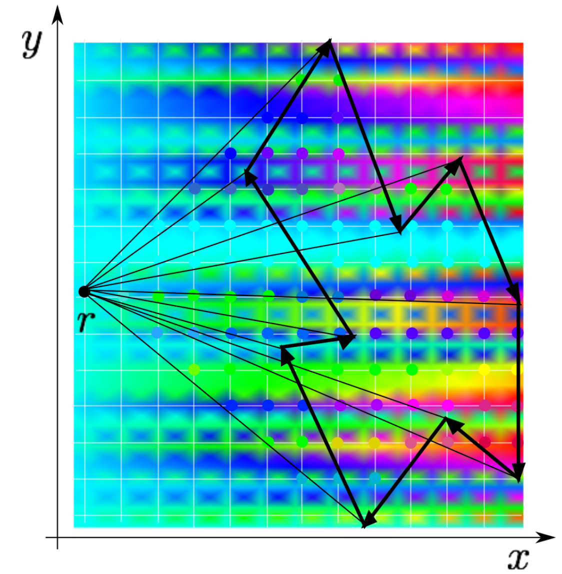


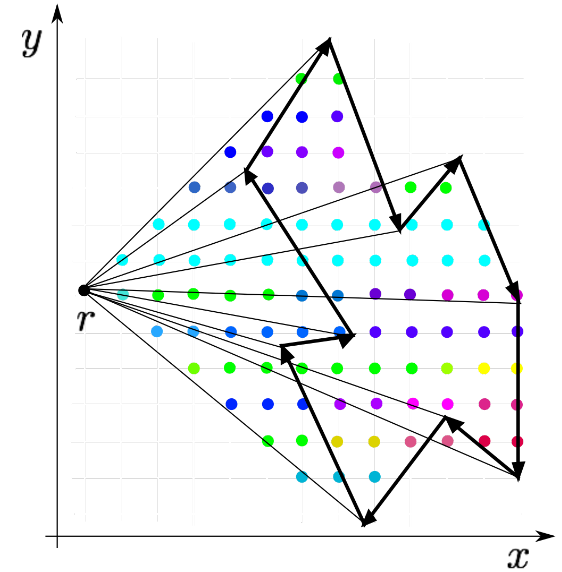



**(b) Bounded taskload/ (c)Balanced taskload**

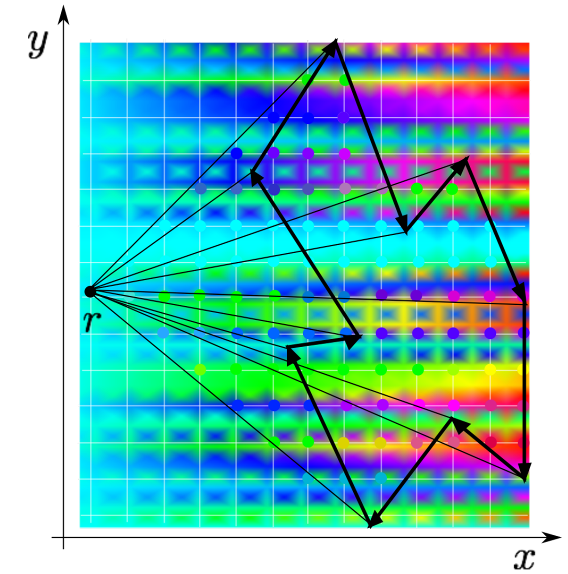
We need to associate task load with a sector.

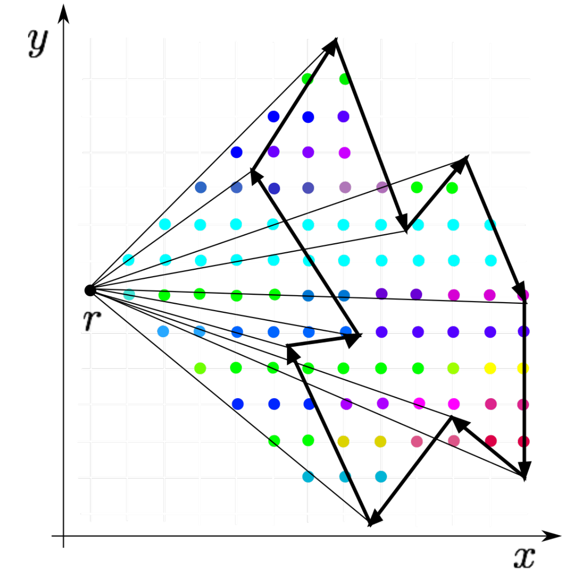
- Overlay heat map with a grid.
- Extract values at the grid points.
- Use discretized heat map.
- Each discrete heat map point q: “heat value” $h_q$
- Let the sign of $f_{i,j}$ be $p_{i,j}$



LINKÖPING UNIVERSITY




## (b) Bounded taskload/ (c)Balanced taskload

We need to associate task load with a sector.

- Overlay heat map with a grid.
- Extract values at the grid points.
- Use discretized heat map.
- Each discrete heat map point q: "heat value" $h_q$
- Let the sign of $f_{i,j}$ be $p_{i,j}$

$$h_{i,j} = p_{i,j} \sum_{q \in \Delta(i,j,r)} h_q$$

$$\begin{aligned}
\sum_{(i,j)\in E} h_{i,j}\, y_{i,j,s} - t_s &= 0 && \forall s \in \mathcal{S} \\
t_s &\geq t_{LB} && \forall s \in \mathcal{S} \\
t_s &\leq t_{UB} && \forall s \in \mathcal{S} \\
t_{LB} &= c_2 \cdot t_0/|\mathcal{S}| && \text{with, e.g., } c_2 = 0.9
\end{aligned}$$

**(b) Bounded taskload/ (c)Balanced taskload**

We need to associate task load with a sector.

- Overlay heat map with a grid.
- Extract values at the grid points.
- Use discretized heat map.
- Each discrete heat map point q: “heat value” $h_q$
- Let the sign of $f_{i,j}$ be $p_{i,j}$

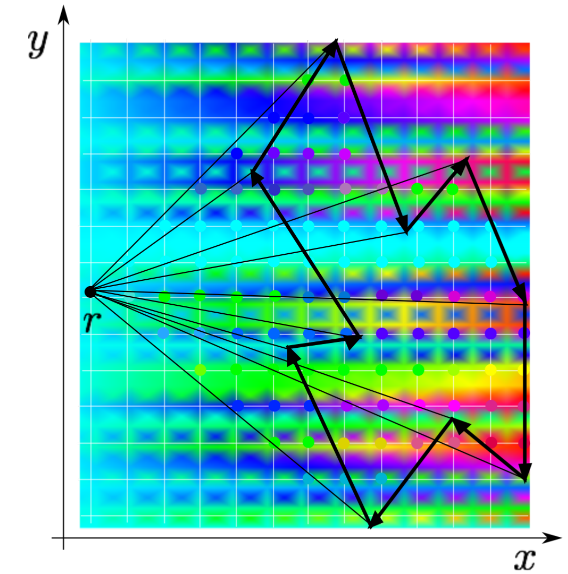
$$h_{i,j} = p_{i,j} \sum_{q \in \Delta(i,j,r)} h_q$$

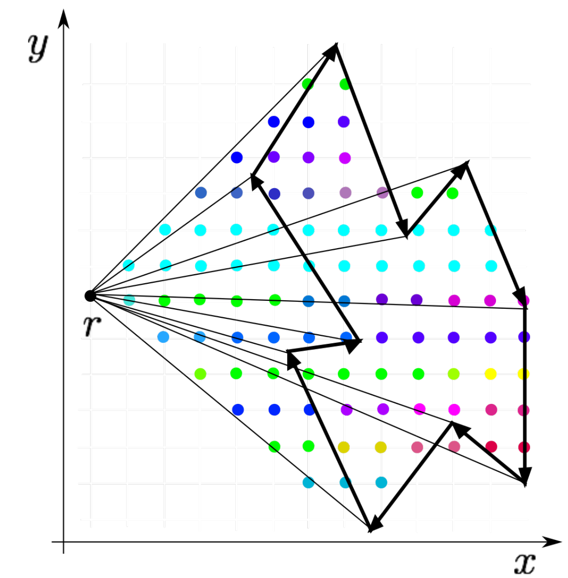
$$\sum_{(i,j)\in E} h_{i,j}\, y_{i,j,s} - t_s = 0 \qquad \forall s \in \mathcal{S}$$

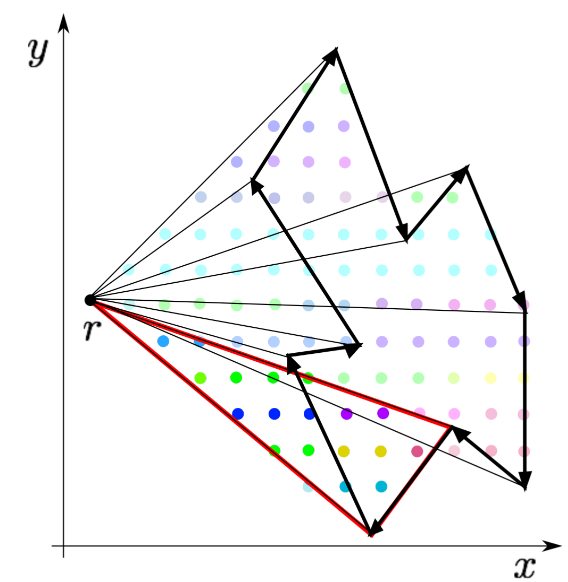
$$t_s \geq t_{LB} \qquad \forall s \in \mathcal{S}$$

$$t_s \leq t_{UB} \qquad \forall s \in \mathcal{S}$$

$$t_{LB} = c_2 \cdot t_0 / |\mathcal{S}| \quad \text{with, e.g., } c_2 = 0.9$$



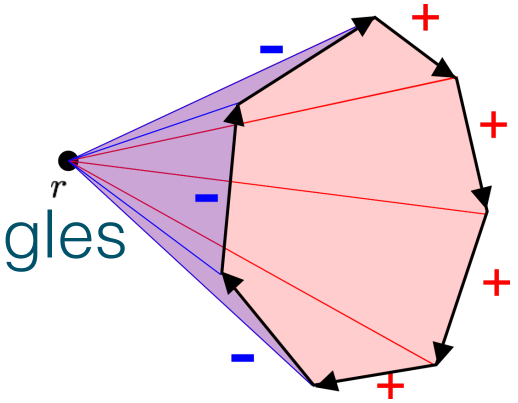
## (f) Convex sectors

## (f) Convex sectors

- Convex sector:

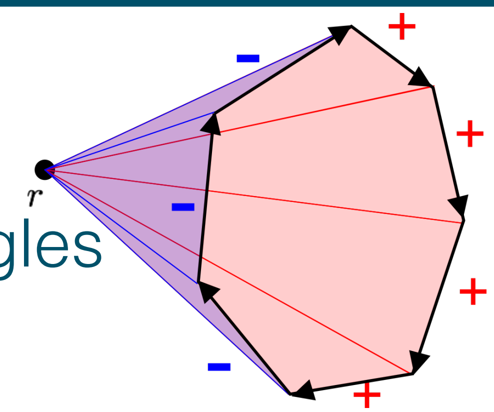
## (f) Convex sectors

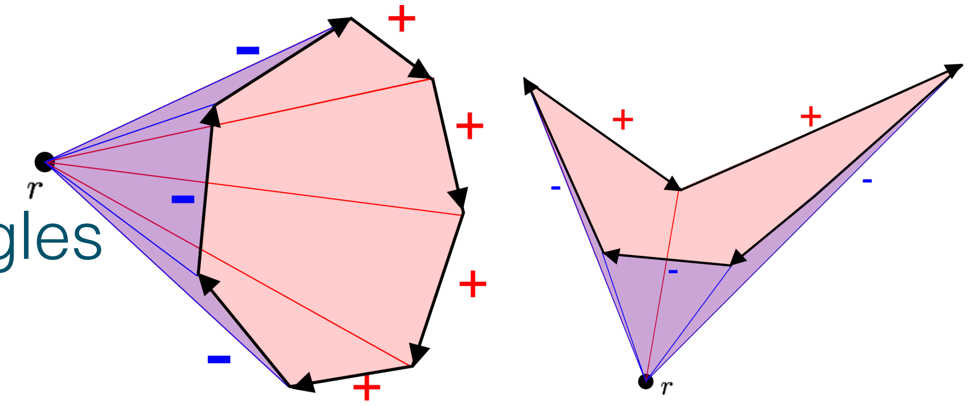
- Convex sector:
  - only one connected chain of edges with cw triangles



## (f) Convex sectors

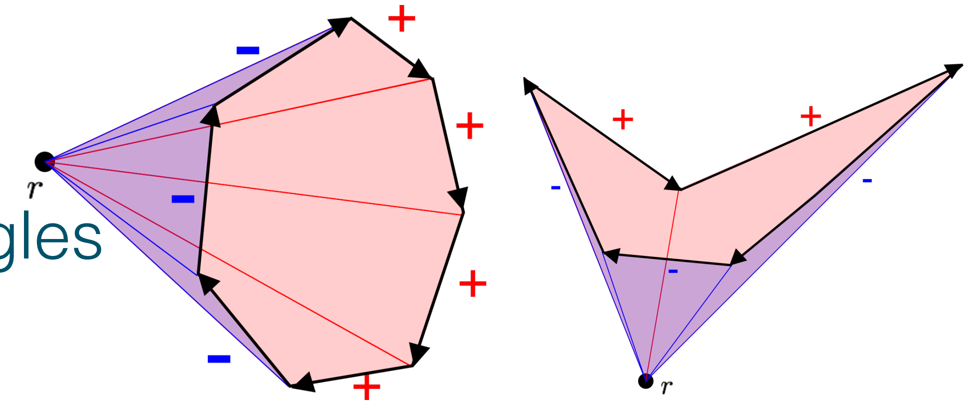
- Convex sector:
  - only one connected chain of edges with cw triangles
  - one connected chain of edges with ccw triangles






## (f) Convex sectors

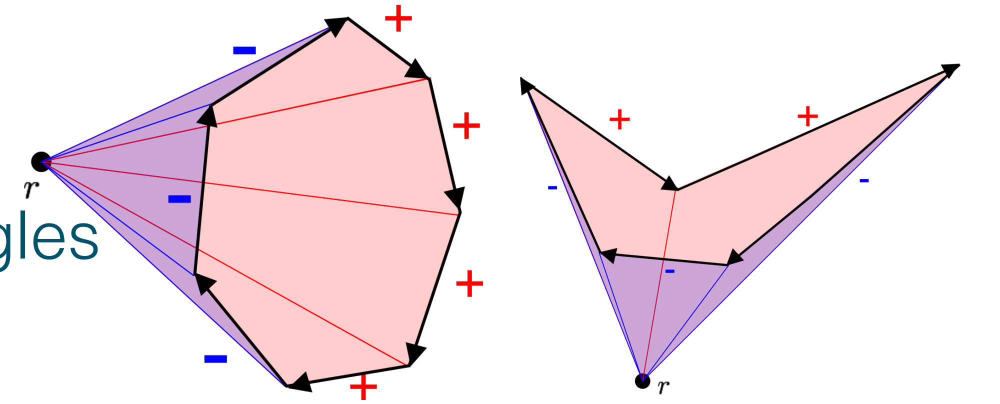
- Convex sector:
  - only one connected chain of edges with cw triangles
  - one connected chain of edges with ccw triangles
- Only-if-part of that statement is not true



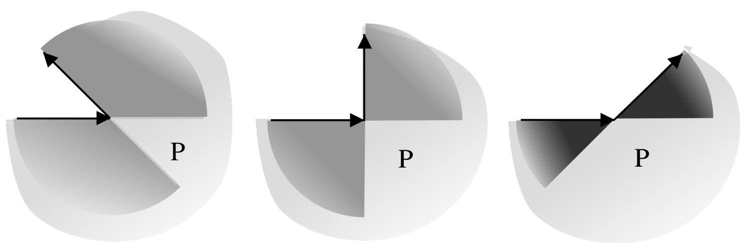

## (f) Convex sectors

- Convex sector:
  - only one connected chain of edges with cw triangles
  - one connected chain of edges with ccw triangles
- Only-if-part of that statement is not true
- BUT: we have only eight edge directions

## (f) Convex sectors

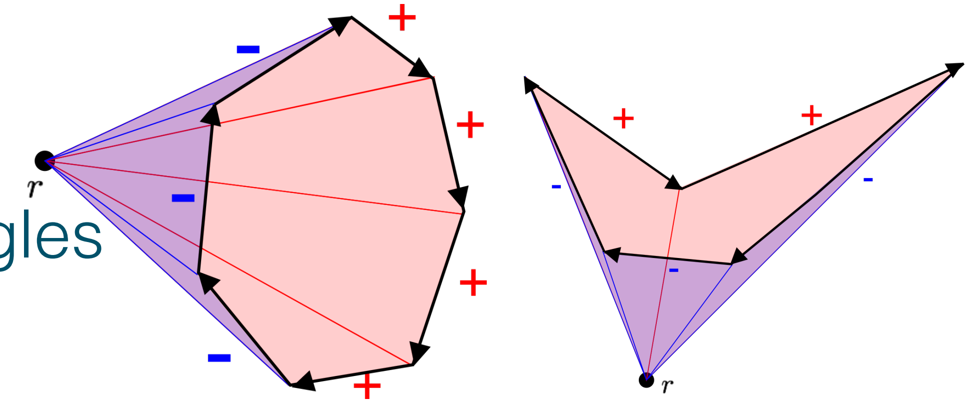


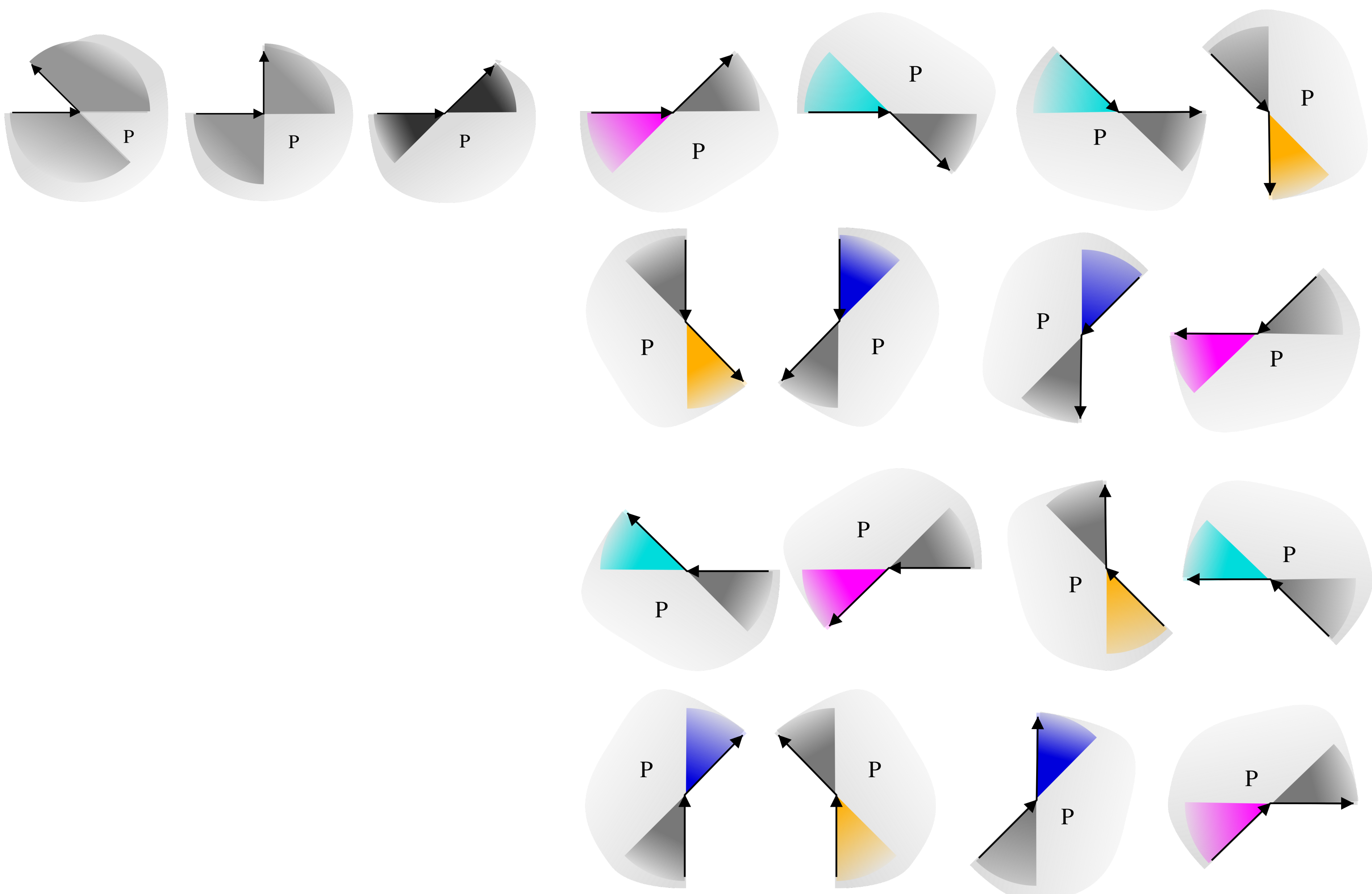

- Convex sector:
  - only one connected chain of edges with cw triangles
  - one connected chain of edges with ccw triangles
- Only-if-part of that statement is not true
- BUT: we have only eight edge directions



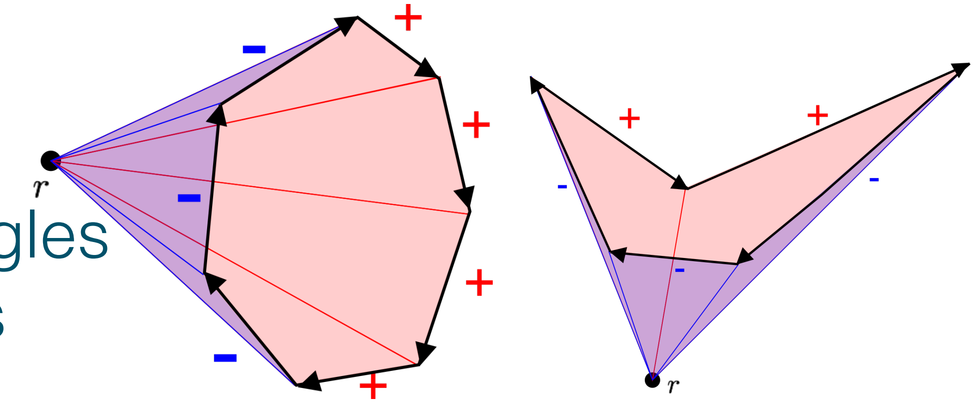
## (f) Convex sectors

- Convex sector:
  - only one connected chain of edges with cw triangles
  - one connected chain of edges with ccw triangles
- Only-if-part of that statement is not true
- BUT: we have only eight edge directions

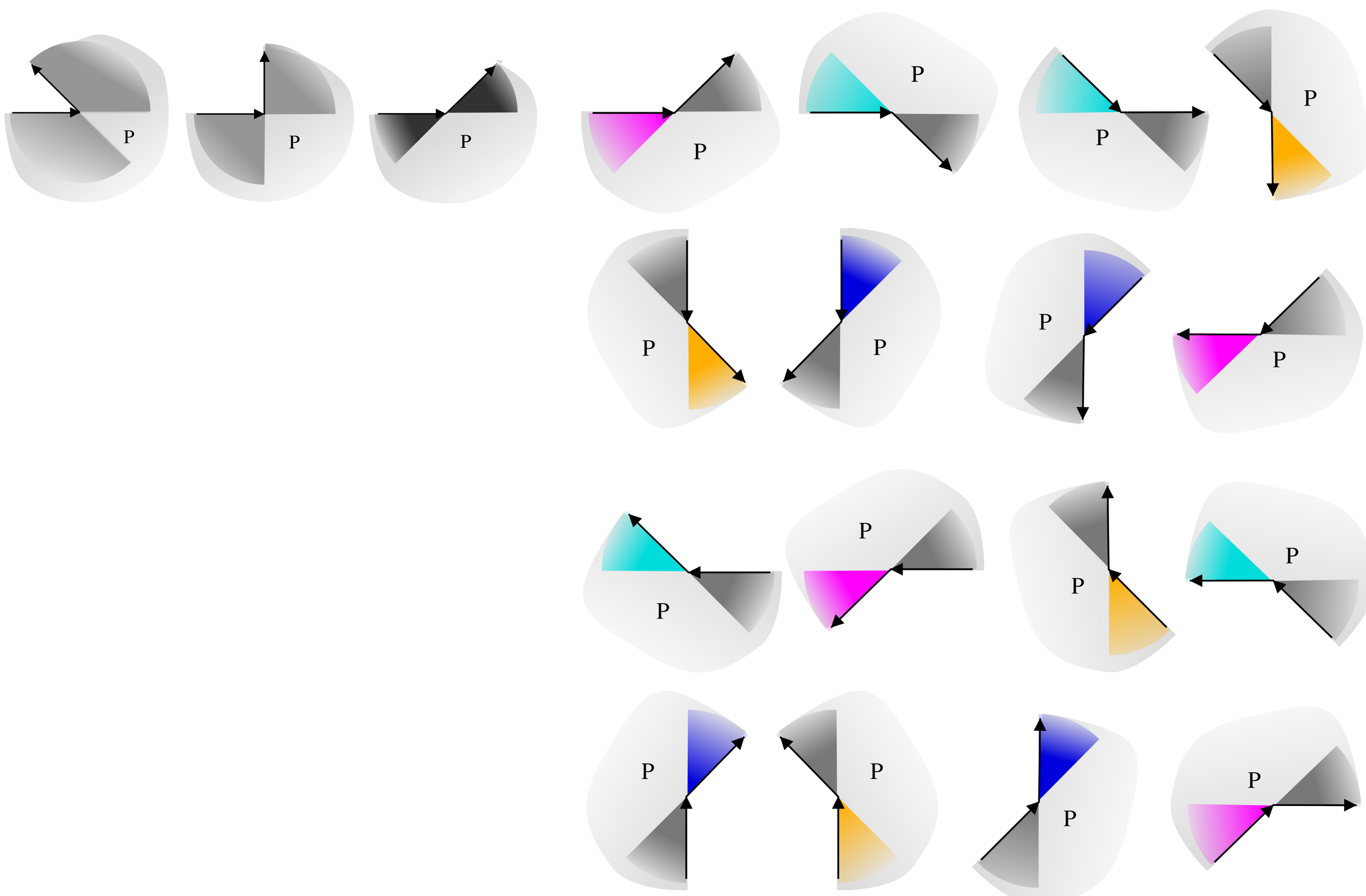


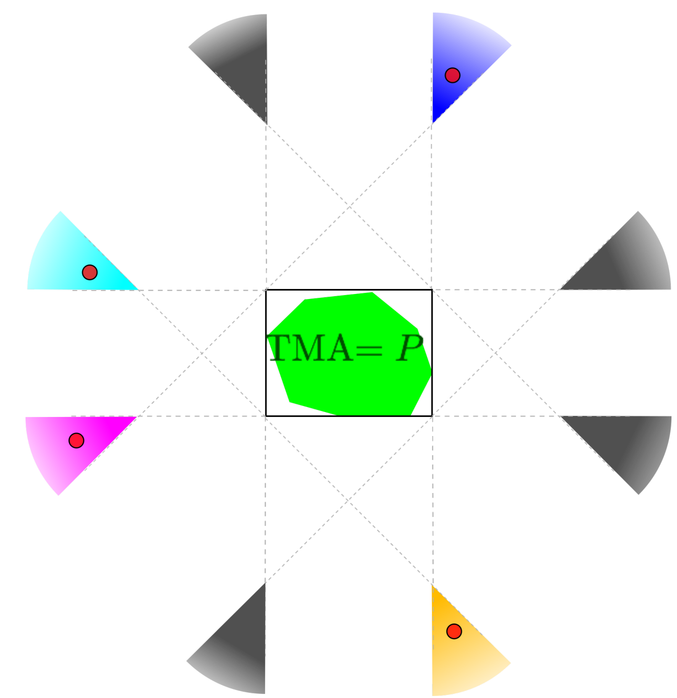



## (f) Convex sectors



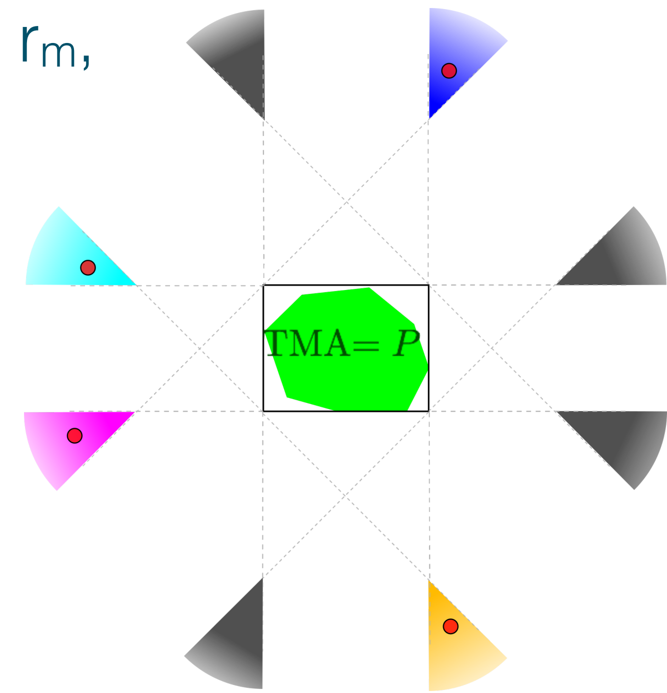

- Convex sector:
  - only one connected chain of edges with cw triangles
  - one connected chain of edges with ccw triangles
- Only-if-part of that statement is not true
- BUT: we have only eight edge directions



# Grid-based IP formulation

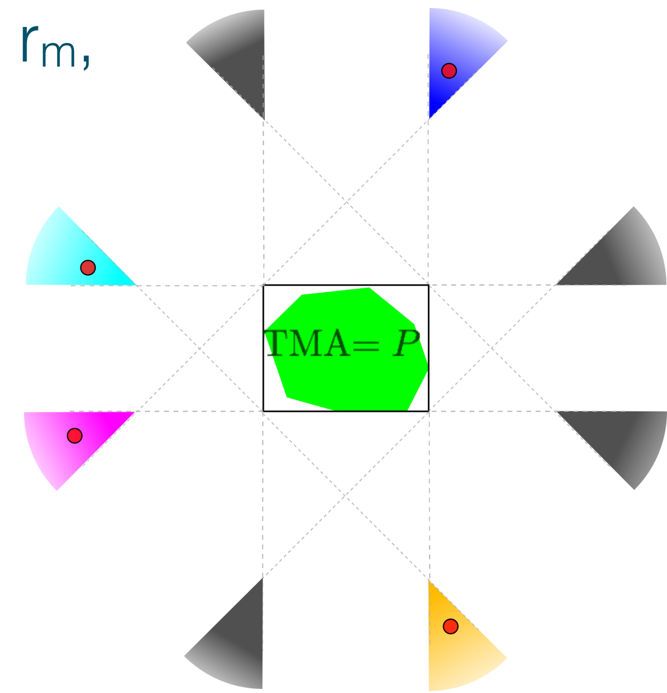
- One reference point in each of the four colored cones: $r_1,\dots,r_4$ ($r = r_m$, for some $m \in M=\{1,2,3,4\}$




$$
\begin{aligned}
q^s_{j,m} &= \frac{1}{2}\left(\sum_{i:(i,j)\in E} p_{i,j,m}\, y_{i,j,s} - \sum_{l:(j,l)\in E} p_{j,l,m}\, y_{j,l,s}\right) && \forall s \in \mathcal{S},\ \forall j \in V,\ \forall m \in \mathcal{M}\\
qabs^s_{j,m} &\geq q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}\\
qabs^s_{j,m} &\geq -q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}\\
\sum_{i\in V}\sum_{j\in V} y_{i,j,s} \cdot qabs^s_{j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
0 &\leq z^s_{i,j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\leq qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\leq y_{i,j,s} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\geq y_{i,j,s} - 1 + qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
\sum_{i\in V}\sum_{j\in V} z^s_{i,j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}
\end{aligned}
$$

# Grid-based IP formulation

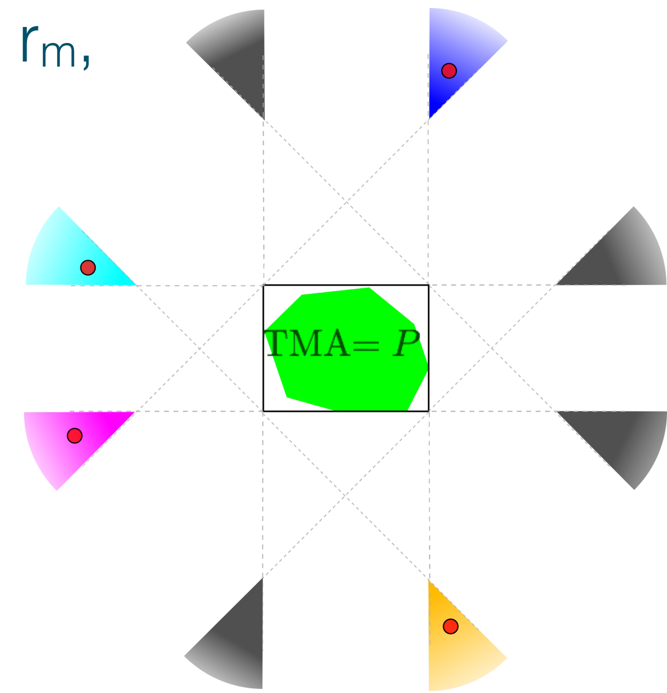
- One reference point in each of the four colored cones: $r_1, \ldots, r_4$ ($r = r_m$, for some $m \in M = \{1,2,3,4\}$
- At least one of the $r_m$ will result in a cw/ccw switch for non-convex polygons.




$$
\begin{aligned}
q^s_{j,m} &= \frac{1}{2}\left(\sum_{i:(i,j)\in E} p_{i,j,m}\, y_{i,j,s} - \sum_{l:(j,l)\in E} p_{j,l,m}\, y_{j,l,s}\right) && \forall s \in \mathcal{S},\ \forall j \in V,\ \forall m \in \mathcal{M}\\
qabs^s_{j,m} &\geq q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}\\
qabs^s_{j,m} &\geq -q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}\\
\sum_{i\in V}\sum_{j\in V} y_{i,j,s} \cdot qabs^s_{j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
0 &\leq z^s_{i,j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\leq qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\leq y_{i,j,s} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\geq y_{i,j,s} - 1 + qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
\sum_{i\in V}\sum_{j\in V} z^s_{i,j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}
\end{aligned}
$$

# Grid-based IP formulation

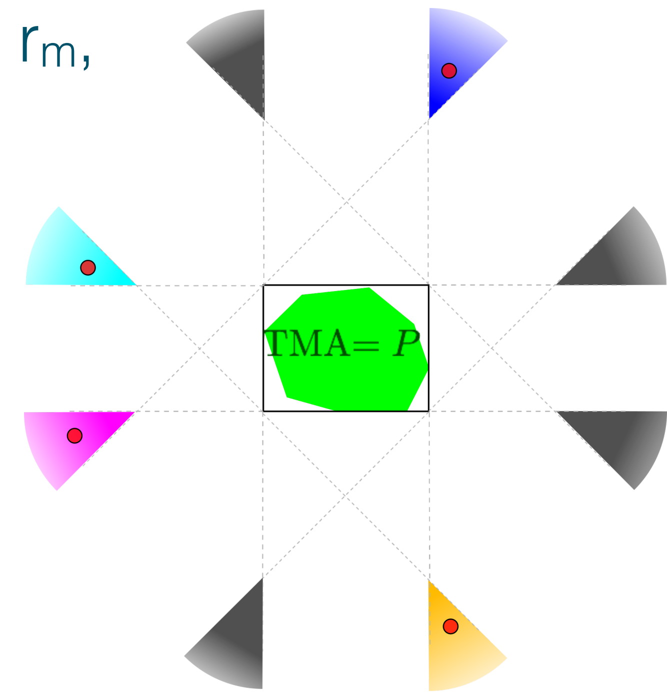
- One reference point in each of the four colored cones: $r_1, \ldots, r_4$ ($r = r_m$, for some $m \in M = \{1,2,3,4\}$
- At least one of the $r_m$ will result in a cw/ccw switch for non-convex polygons.
- $p_{i,j,m}$: sign of the triangle (i,j) and $r_m$




$$
\begin{aligned}
q^s_{j,m} &= \frac{1}{2}\left(\sum_{i:(i,j)\in E} p_{i,j,m}\, y_{i,j,s} - \sum_{l:(j,l)\in E} p_{j,l,m}\, y_{j,l,s}\right) && \forall s \in \mathcal{S},\ \forall j \in V,\ \forall m \in \mathcal{M}\\
qabs^s_{j,m} &\geq q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}\\
qabs^s_{j,m} &\geq -q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}\\
\sum_{i\in V}\sum_{j\in V} y_{i,j,s}\cdot qabs^s_{j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
0 &\leq z^s_{i,j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\leq qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\leq y_{i,j,s} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\geq y_{i,j,s} - 1 + qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
\sum_{i\in V}\sum_{j\in V} z^s_{i,j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}
\end{aligned}
$$

# Grid-based IP formulation

- One reference point in each of the four colored cones: $r_1,\ldots,r_4$ ($r = r_m$, for some $m \in M=\{1,2,3,4\}$
- At least one of the $r_m$ will result in a cw/ccw switch for non-convex polygons.
- $p_{i,j,m}$: sign of the triangle (i,j) and $r_m$




Assigns, for each sector, a value of -1,0,1 to each vertex.

$$q^s_{j,m} = \frac{1}{2}\left(\sum_{i:(i,j)\in E} p_{i,j,m}\, y_{i,j,s} - \sum_{l:(j,l)\in E} p_{j,l,m}\, y_{j,l,s}\right) \quad \forall s \in \mathcal{S},\ \forall j \in V,\ \forall m \in \mathcal{M}$$

$$qabs^s_{j,m} \geq q^s_{j,m} \quad \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}$$

$$qabs^s_{j,m} \geq -q^s_{j,m} \quad \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}$$

$$\sum_{i\in V}\sum_{j\in V} y_{i,j,s} \cdot qabs^s_{j,m} = 2 \quad \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}$$

$$0 \leq z^s_{i,j,m} \quad \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}$$

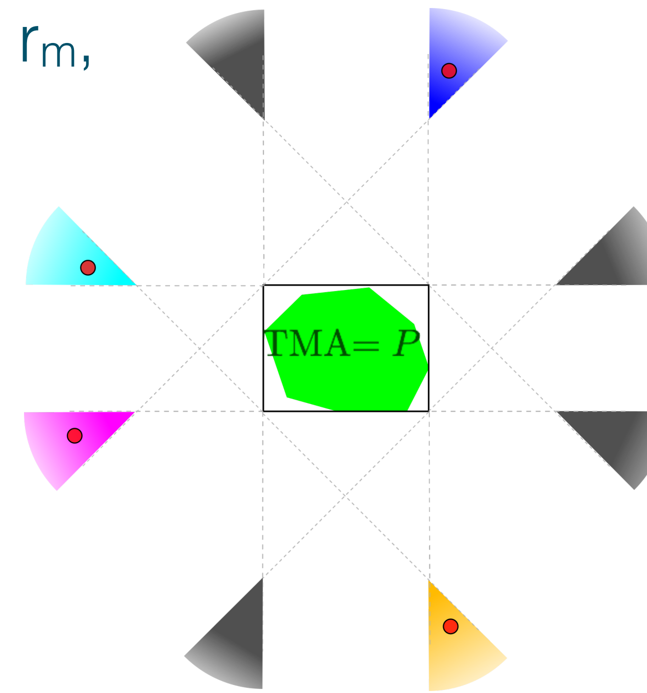
$$z^s_{i,j,m} \leq qabs^s_{j,m} \quad \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}$$

$$z^s_{i,j,m} \leq y_{i,j,s} \quad \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}$$

$$z^s_{i,j,m} \geq y_{i,j,s} - 1 + qabs^s_{j,m} \quad \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}$$

$$\sum_{i\in V}\sum_{j\in V} z^s_{i,j,m} = 2 \quad \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}$$

- One reference point in each of the four colored cones: $r_1, \ldots, r_4$ ($r = r_m$, for some $m \in M = \{1,2,3,4\}$)
- At least one of the $r_m$ will result in a cw/ccw switch for non-convex polygons.
- $p_{i,j,m}$: sign of the triangle (i,j) and $r_m$

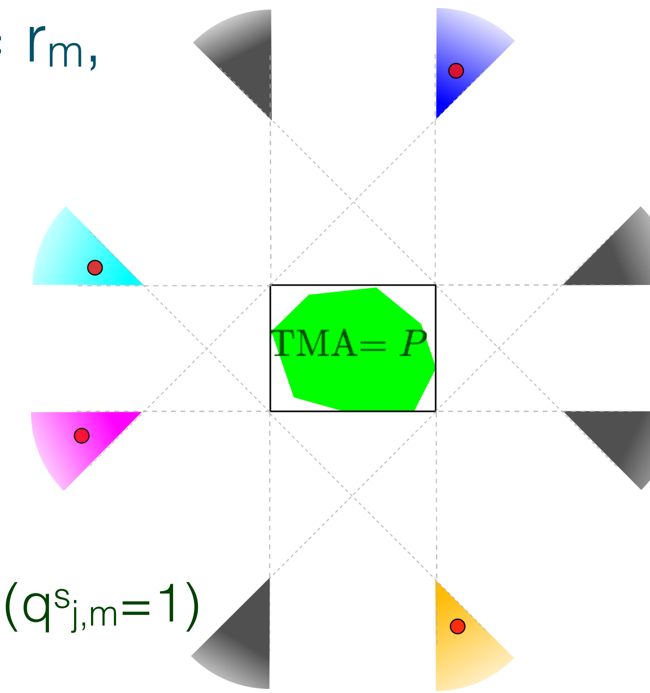



Assigns, for each sector, a value of -1,0,1 to each vertex.
Interior vertex of chain of cw /ccw triangles has $q^s_{j,m}=0$

$$
\begin{aligned}
q^s_{j,m} &= \frac{1}{2}\left(\sum_{i:(i,j)\in E} p_{i,j,m}\, y_{i,j,s} - \sum_{l:(j,l)\in E} p_{j,l,m}\, y_{j,l,s}\right) && \forall s \in \mathcal{S},\ \forall j \in V,\ \forall m \in \mathcal{M} \\
qabs^s_{j,m} &\geq q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M} \\
qabs^s_{j,m} &\geq -q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M} \\
\sum_{i\in V}\sum_{j\in V} y_{i,j,s} \cdot qabs^s_{j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M} \\
0 &\leq z^s_{i,j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M} \\
z^s_{i,j,m} &\leq qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M} \\
z^s_{i,j,m} &\leq y_{i,j,s} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M} \\
z^s_{i,j,m} &\geq y_{i,j,s} - 1 + qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M} \\
\sum_{i\in V}\sum_{j\in V} z^s_{i,j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}
\end{aligned}
$$

# Grid-based IP formulation

- One reference point in each of the four colored cones: $r_1, \ldots, r_4$ ($r = r_m$, for some $m \in M = \{1,2,3,4\}$)
- At least one of the $r_m$ will result in a cw/ccw switch for non-convex polygons.
- $p_{i,j,m}$: sign of the triangle $(i,j)$ and $r_m$

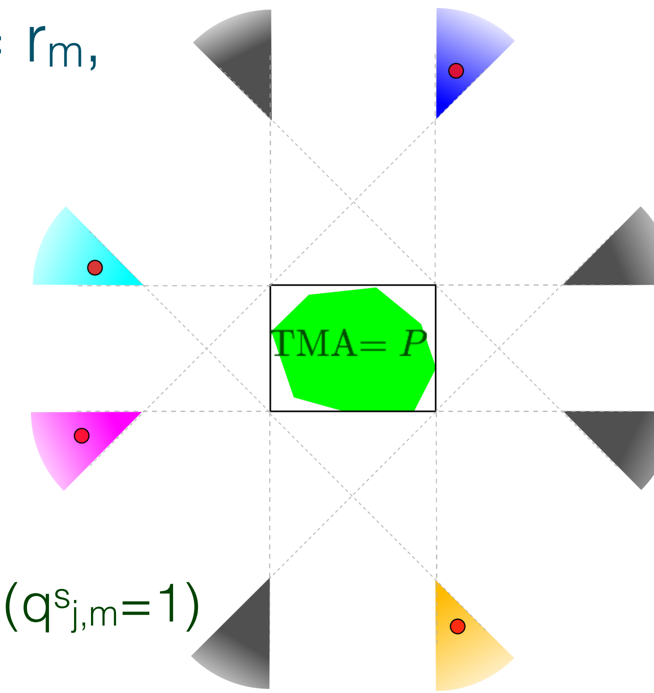



Assigns, for each sector, a value of -1,0,1 to each vertex.
Interior vertex of chain of cw /ccw triangles has $q^s_{j,m}=0$
At j a chain with ccw (cw) triangles switches to a chain of cw (ccw) triangles $q^s_{j,m}=-1$ ($q^s_{j,m}=1$)

$$
\begin{aligned}
q^s_{j,m} &= \frac{1}{2}\left(\sum_{i:(i,j)\in E} p_{i,j,m}\, y_{i,j,s} - \sum_{l:(j,l)\in E} p_{j,l,m}\, y_{j,l,s}\right) && \forall s \in \mathcal{S},\ \forall j \in V,\ \forall m \in \mathcal{M}\\
qabs^s_{j,m} &\geq q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}\\
qabs^s_{j,m} &\geq -q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}\\
\sum_{i\in V}\sum_{j\in V} y_{i,j,s}\cdot qabs^s_{j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
0 &\leq z^s_{i,j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\leq qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\leq y_{i,j,s} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\geq y_{i,j,s} - 1 + qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
\sum_{i\in V}\sum_{j\in V} z^s_{i,j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}
\end{aligned}
$$

# Grid-based IP formulation

- One reference point in each of the four colored cones: $r_1,\ldots,r_4$ ($r = r_m$, for some $m \in M=\{1,2,3,4\}$
- At least one of the $r_m$ will result in a cw/ccw switch for non-convex polygons.
- $p_{i,j,m}$: sign of the triangle (i,j) and $r_m$




Assigns, for each sector, a value of -1,0,1 to each vertex.
Interior vertex of chain of cw /ccw triangles has $q^s_{j,m}=0$
At j a chain with ccw (cw) triangles switches to a chain of cw (ccw) triangles $q^s_{j,m}=-1$ ($q^s_{j,m}=1$)
For a convex sector: sum of the $|q^s_{j,m}|=2$ for all reference points

$$q^s_{j,m} = \frac{1}{2}\left(\sum_{i:(i,j)\in E} p_{i,j,m}\, y_{i,j,s} - \sum_{l:(j,l)\in E} p_{j,l,m}\, y_{j,l,s}\right) \quad \forall s\in\mathcal{S},\ \forall j\in V,\ \forall m\in\mathcal{M}$$

$$qabs^s_{j,m} \geq q^s_{j,m} \quad \forall s\in\mathcal{S}, \forall j\in V,\ \forall m\in\mathcal{M}$$

$$qabs^s_{j,m} \geq -q^s_{j,m} \quad \forall s\in\mathcal{S}, \forall j\in V,\ \forall m\in\mathcal{M}$$

$$\sum_{i\in V}\sum_{j\in V} y_{i,j,s}\cdot qabs^s_{j,m} = 2 \quad \forall s\in\mathcal{S},\ \forall m\in\mathcal{M}$$

$$0 \leq z^s_{i,j,m} \quad \forall i,j\in V\ \forall s\in\mathcal{S},\ \forall m\in\mathcal{M}$$

$$z^s_{i,j,m} \leq qabs^s_{j,m} \quad \forall i,j\in V\ \forall s\in\mathcal{S},\ \forall m\in\mathcal{M}$$

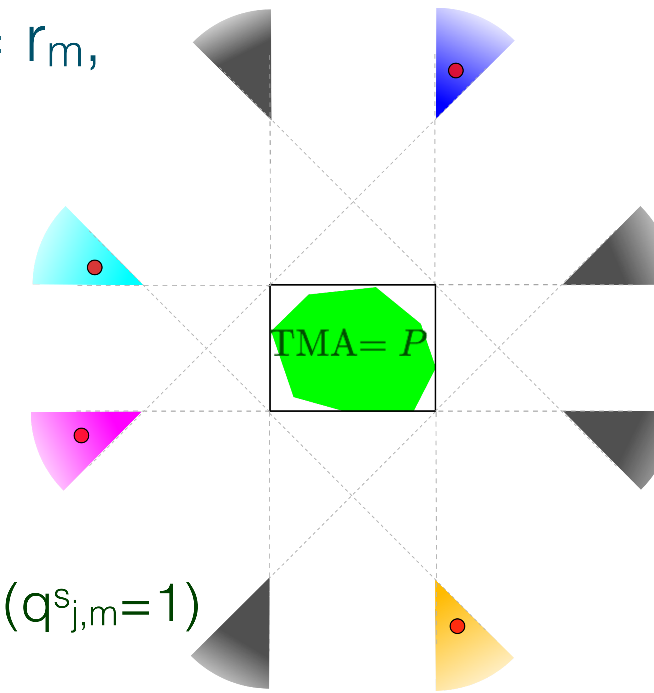
$$z^s_{i,j,m} \leq y_{i,j,s} \quad \forall i,j\in V\ \forall s\in\mathcal{S},\ \forall m\in\mathcal{M}$$

$$z^s_{i,j,m} \geq y_{i,j,s} - 1 + qabs^s_{j,m} \quad \forall i,j\in V\ \forall s\in\mathcal{S},\ \forall m\in\mathcal{M}$$

$$\sum_{i\in V}\sum_{j\in V} z^s_{i,j,m} = 2 \quad \forall s\in\mathcal{S},\ \forall m\in\mathcal{M}$$

# Grid-based IP formulation

- One reference point in each of the four colored cones: $r_1, \ldots, r_4$ ($r = r_m$, for some $m \in M=\{1,2,3,4\}$
- At least one of the $r_m$ will result in a cw/ccw switch for non-convex polygons.
- $p_{i,j,m}$: sign of the triangle (i,j) and $r_m$




Assigns, for each sector, a value of -1,0,1 to each vertex.
Interior vertex of chain of cw /ccw triangles has $q^s_{j,m}=0$
At j a chain with ccw (cw) triangles switches to a chain of cw (ccw) triangles $q^s_{j,m}=-1$ ($q^s_{j,m}=1$)
For a convex sector: sum of the $|q^s_{j,m}|=2$ for all reference points

$$
\begin{aligned}
q^s_{j,m} &= \frac{1}{2}\left(\sum_{i:(i,j)\in E} p_{i,j,m}\, y_{i,j,s} - \sum_{l:(j,l)\in E} p_{j,l,m}\, y_{j,l,s}\right) && \forall s \in \mathcal{S},\ \forall j \in V,\ \forall m \in \mathcal{M}\\
qabs^s_{j,m} &\geq q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}\\
qabs^s_{j,m} &\geq -q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}\\
\sum_{i\in V}\sum_{j\in V} y_{i,j,s}\cdot qabs^s_{j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
0 &\leq z^s_{i,j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\leq qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\leq y_{i,j,s} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\geq y_{i,j,s} - 1 + qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
\sum_{i\in V}\sum_{j\in V} z^s_{i,j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}
\end{aligned}
$$

**Multiplication of two variables ➜ define $z^s_{i,j,m} = y_{i,j,s} * qabs^s_{j,m}$.**

# Grid-based IP formulation

## Objective Function

### Objective Function

- Choice not obvious.

**Objective Function**

- Choice not obvious.
- Used in literature:
    - Taskload imbalance(constraint c)
    - Number of sectors (input)

**Objective Function**

- Choice not obvious.
- Used in literature:
    - Taskload imbalance(constraint c)
    - Number of sectors (input)

$$\min \sum_{s \in \mathcal{S}} \sum_{(i,j) \in E} \ell_{i,j} y_{i,j,s}$$

**Objective Function**

$$\min \sum_{s \in \mathcal{S}} \sum_{(i,j) \in E} \ell_{i,j} y_{i,j,s}$$

- Choice not obvious.
- Used in literature:
  - Taskload imbalance(constraint c)
  - Number of sectors (input)
- If we want to balance the area of the sectors, but are not interested in the sector taskload, this objective function ensures that sectors are connected (constraint d)

**Objective Function**

$$\min \sum_{s \in \mathcal{S}} \sum_{(i,j) \in E} \ell_{i,j} y_{i,j,s}$$

- Choice not obvious.
- Used in literature:
    - Taskload imbalance(constraint c)
    - Number of sectors (input)
- If we want to balance the area of the sectors, but are not interested in the sector taskload, this objective function ensures that sectors are connected (constraint d)
- With taskload: only connected sectors if $c_2$ allows it:

**Objective Function**

$$\min \sum_{s \in \mathcal{S}} \sum_{(i,j) \in E} \ell_{i,j} y_{i,j,s}$$

- Choice not obvious.
- Used in literature:
  - Taskload imbalance(constraint c)
  - Number of sectors (input)
- If we want to balance the area of the sectors, but are not interested in the sector taskload, this objective function ensures that sectors are connected (constraint d)
- With taskload: only connected sectors if $c_2$ allows it:

Given the current complexity map: user must allow larger imbalances between controller's taskload, if having connected sectors is a necessary condition.

**Objective Function**

$$\min \sum_{s\in\mathcal{S}} \sum_{(i,j)\in E} \ell_{i,j} y_{i,j,s}$$

- Choice not obvious.
- Used in literature:
  - Taskload imbalance(constraint c)
  - Number of sectors (input)
- If we want to balance the area of the sectors, but are not interested in the sector taskload, this objective function ensures that sectors are connected (constraint d)
- With taskload: only connected sectors if $c_2$ allows it:

Given the current complexity map: user must allow larger imbalances between controller's taskload, if having connected sectors is a necessary condition.

- With constraint (g), interior conflict points:

$$\min \sum_{s\in\mathcal{S}} \sum_{(i,j)\in E} (\gamma \ell_{i,j} + (1-\gamma) w_{i,j})\, y_{i,j,s}, \quad 0 \le \gamma < 1$$

$$w_{i,j} = h_i + h_j$$

$$w_{i,j} = \sum_{k\in N(i)} h_k + \sum_{l\in N(j)} h_l$$

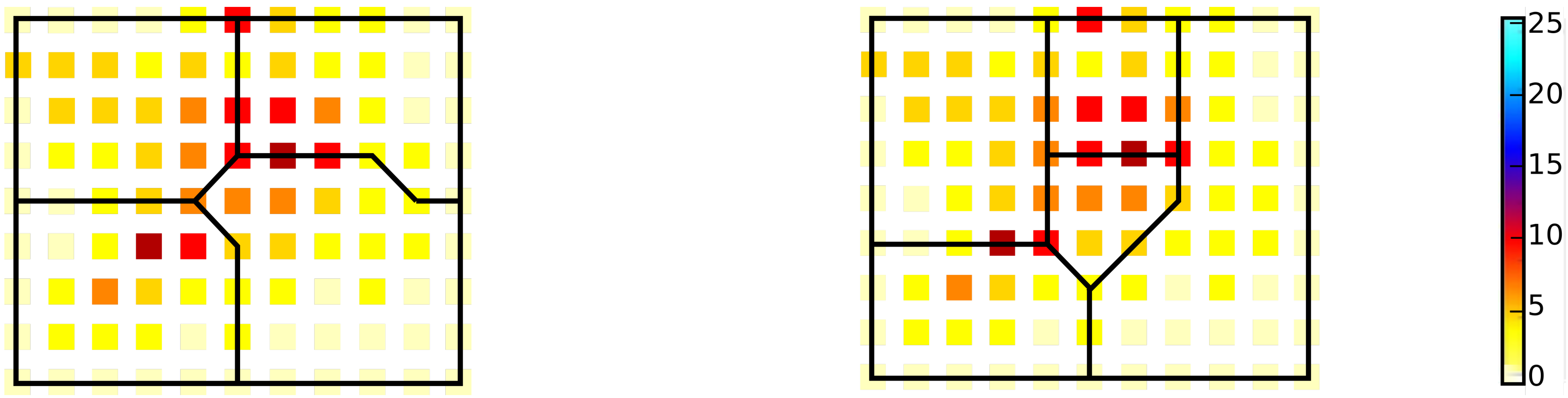
# Experimental Study: Arlanda Airport

# Experimental Study: Arlanda Airport

**(c)Balanced task load, (d) Connected sectors, (e) Nice shape** (we use preprocessing)

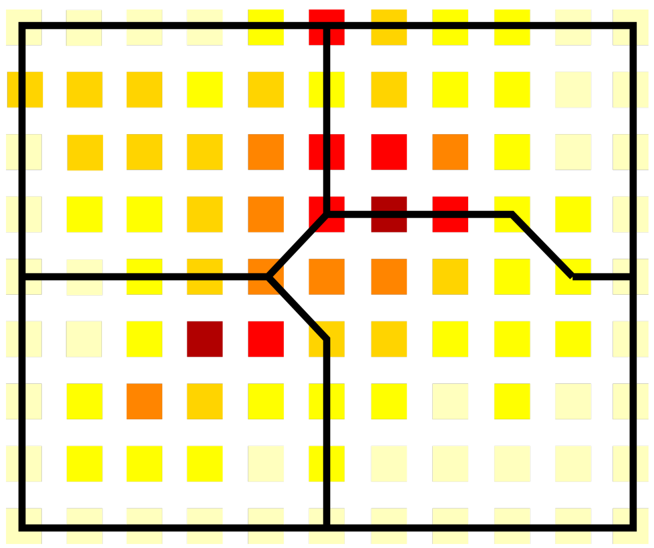
AMPL and CPLEX 12.6 on a single server with 24GB RAM and four kernels running on Linux. Each instance was run until a solution with less than 1% gap had not been found, or for a maximum of one CPU-hour. No instance finished with an optimality gap of more than 6%.

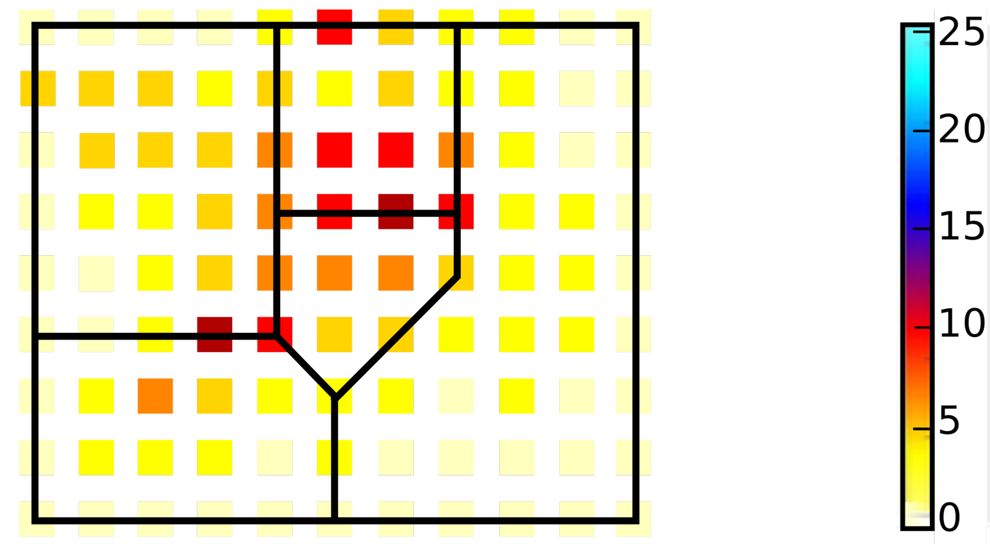
**(c)Balanced task load, (d) Connected sectors, (e) Nice shape** (we use preprocessing)



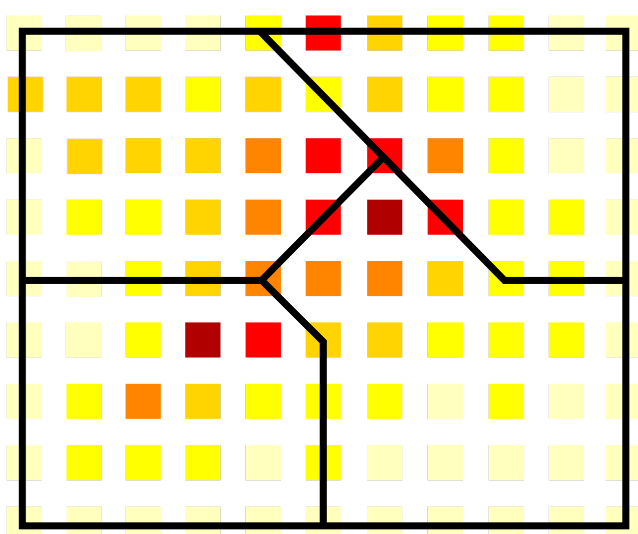

AMPL and CPLEX 12.6 on a single server with 24GB RAM and four kernels running on Linux. Each instance was run until a solution with less than 1% gap had not been found, or for a maximum of one CPU-hour. No instance finished with an optimality gap of more than 6%.

**(c)Balanced task load, (d) Connected sectors, (e) Nice shape** (we use preprocessing)



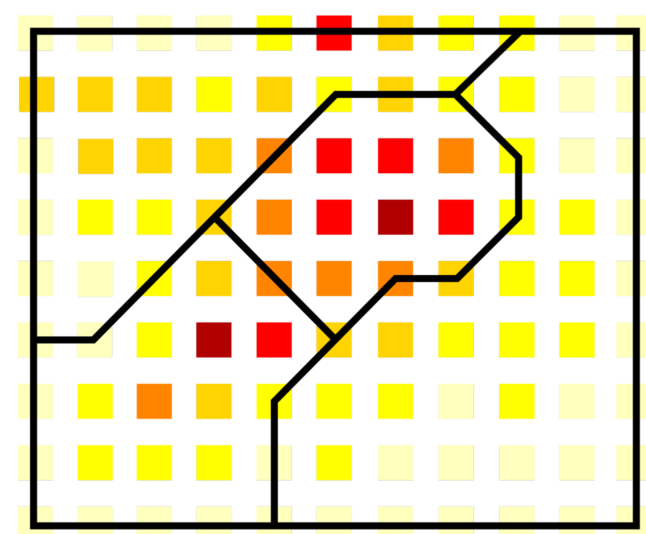



**(c)Balanced task load, (d) Connected sectors, (e) Nice shape, (g) Interior Conflict Points**



$w_{i,j} = h_i + h_j$.

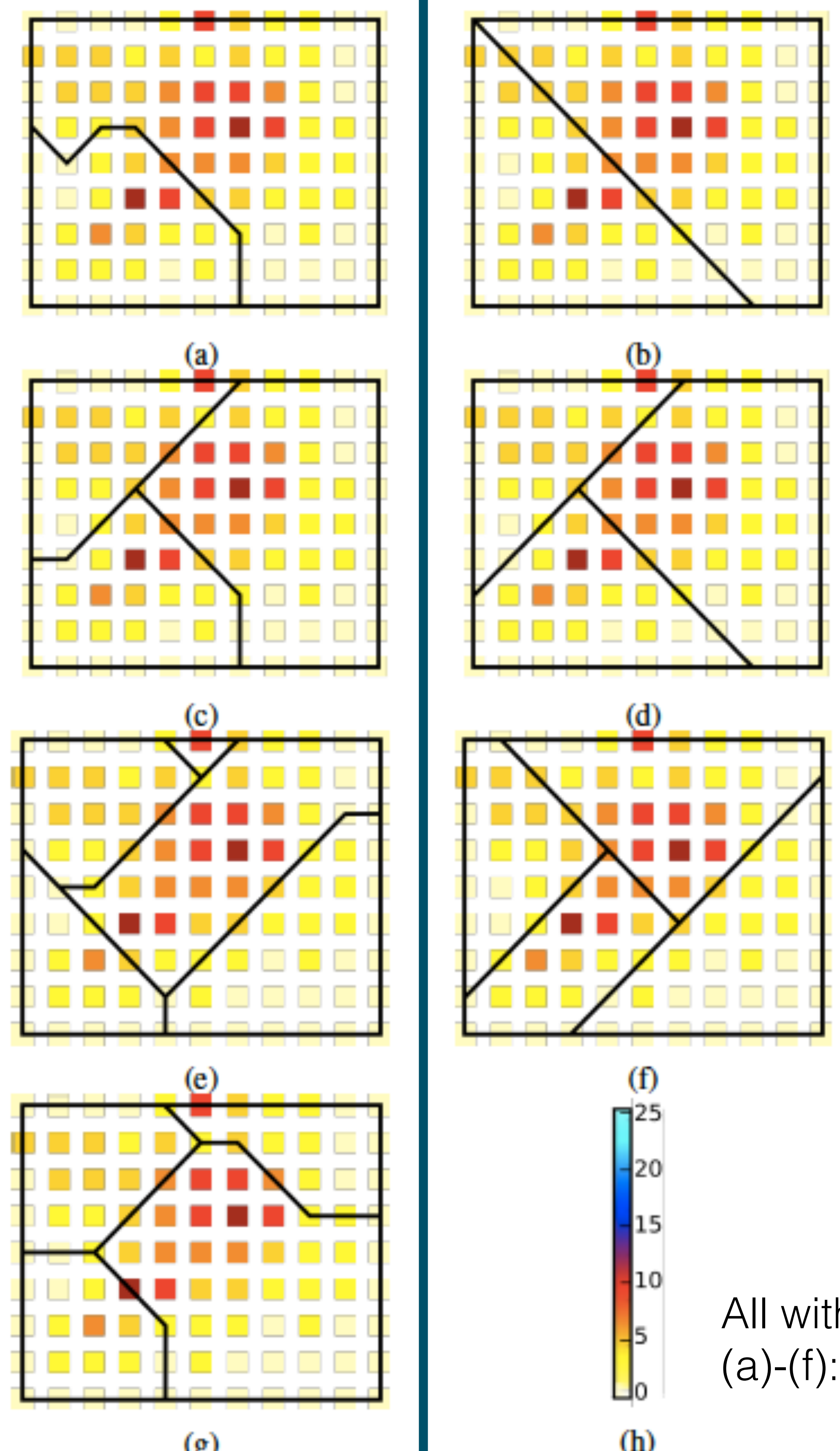
←$\gamma$ = 0.9 $\gamma$= 0.5→.



AMPL and CPLEX 12.6 on a single server with 24GB RAM and four kernels running on Linux. Each instance was run until a solution with less than 1% gap had not been found, or for a maximum of one CPU-hour. No instance finished with an optimality gap of more than 6%.

# Experimental Study: Arlanda Airport

**(c) Balanced task load**
**(d) Connected sectors**
**(e) Nice shape**
**(g) Interior conflict points**



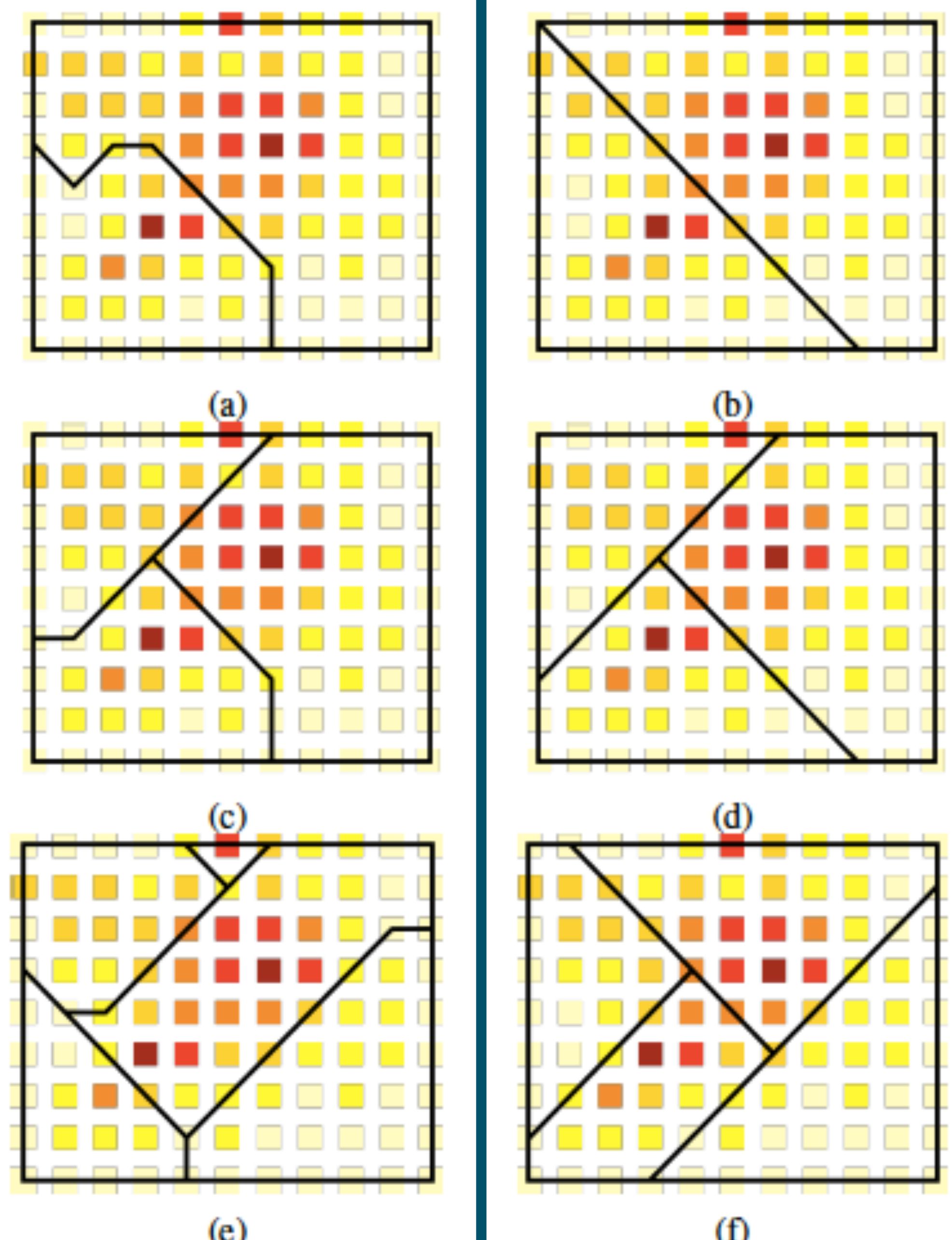

**(c) Balanced task load**
**(d) Connected sectors**
**(e) Nice shape**
**(f) Convex sectors**
**(g) Interior conflict points**

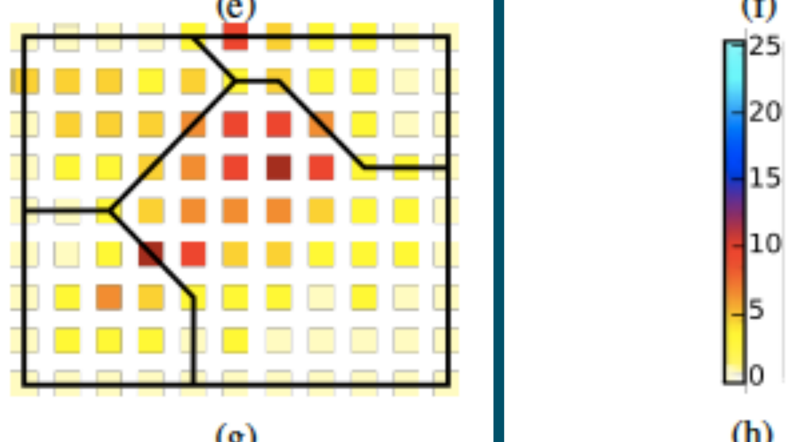
All with $c_2$=0.6 and $w_{i,j} = h_i + h_j$.
(a)-(f): $\gamma$ = 0:2, (g): $\gamma$= 0:8.

# Experimental Study: Arlanda Airport

**(c) Balanced task load**
**(d) Connected sectors**
**(e) Nice shape**
**(g) Interior conflict points**

Disconnected sector ➜




**(c) Balanced task load**
**(d) Connected sectors**
**(e) Nice shape**
**(f) Convex sectors**
**(g) Interior conflict points**

All with $c_2$=0.6 and $w_{i,j} = h_i + h_j$.
(a)-(f): $\gamma$ = 0:2, (g): $\gamma$= 0:8.

**Influence of choosing $w_{i,j}$ :**

**Influence of choosing $w_{i,j}$ :**
**|S|=2 ➜ One cut through rectangle**

# Experimental Study: Arlanda Airport

LINKÖPING UNIVERSITY

**Influence of choosing $w_{i,j}$ :**

**ISI=2 ➜ One cut through rectangle**

**(c)Balanced task load, (d) Connected sectors, (e) Nice shape, (g) Interior Conflict Points**

# Experimental Study: Arlanda Airport

**Influence of choosing $w_{i,j}$ :**

**|S|=2 ➜ One cut through rectangle**

**(c)Balanced task load, (d) Connected sectors, (e) Nice shape, (g) Interior Conflict Points**

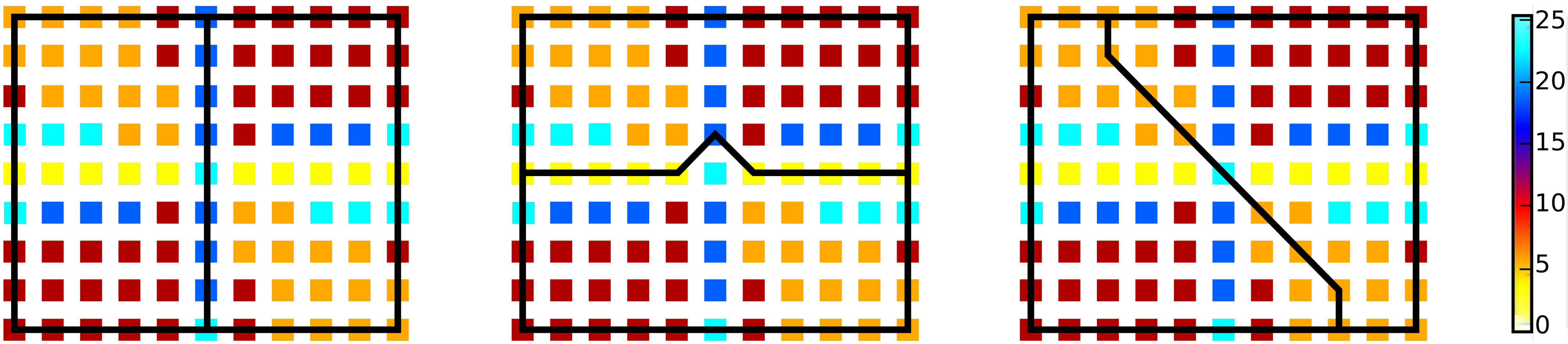
$$\min \sum_{s\in\mathcal{S}} \sum_{(i,j)\in E} (\gamma \ell_{i,j} + (1-\gamma) w_{i,j})\, y_{i,j,s}, \quad 0 \leq \gamma < 1$$

**Influence of choosing $w_{i,j}$ :**

**|S|=2 ➜ One cut through rectangle**

**(c)Balanced task load, (d) Connected sectors, (e) Nice shape, (g) Interior Conflict Points**

$$\min \sum_{s\in\mathcal{S}} \sum_{(i,j)\in E} (\gamma\ell_{i,j} + (1-\gamma)w_{i,j})\, y_{i,j,s}, \quad 0 \le \gamma < 1$$




$\gamma$=1

$\gamma$=0.5 and $w_{i,j} = h_i + h_j$

$\gamma$=0.5 and $w_{i,j} = \sum_{k\in N(i)} h_k + \sum_{l\in N(j)} h_l$

LINKÖPING UNIVERSITY

# Conclusion/Outlook

# Conclusion/Outlook

- Allow usage of a few reflex vertices

LINKÖPING UNIVERSITY

# Conclusion/Outlook

- Allow usage of a few reflex vertices

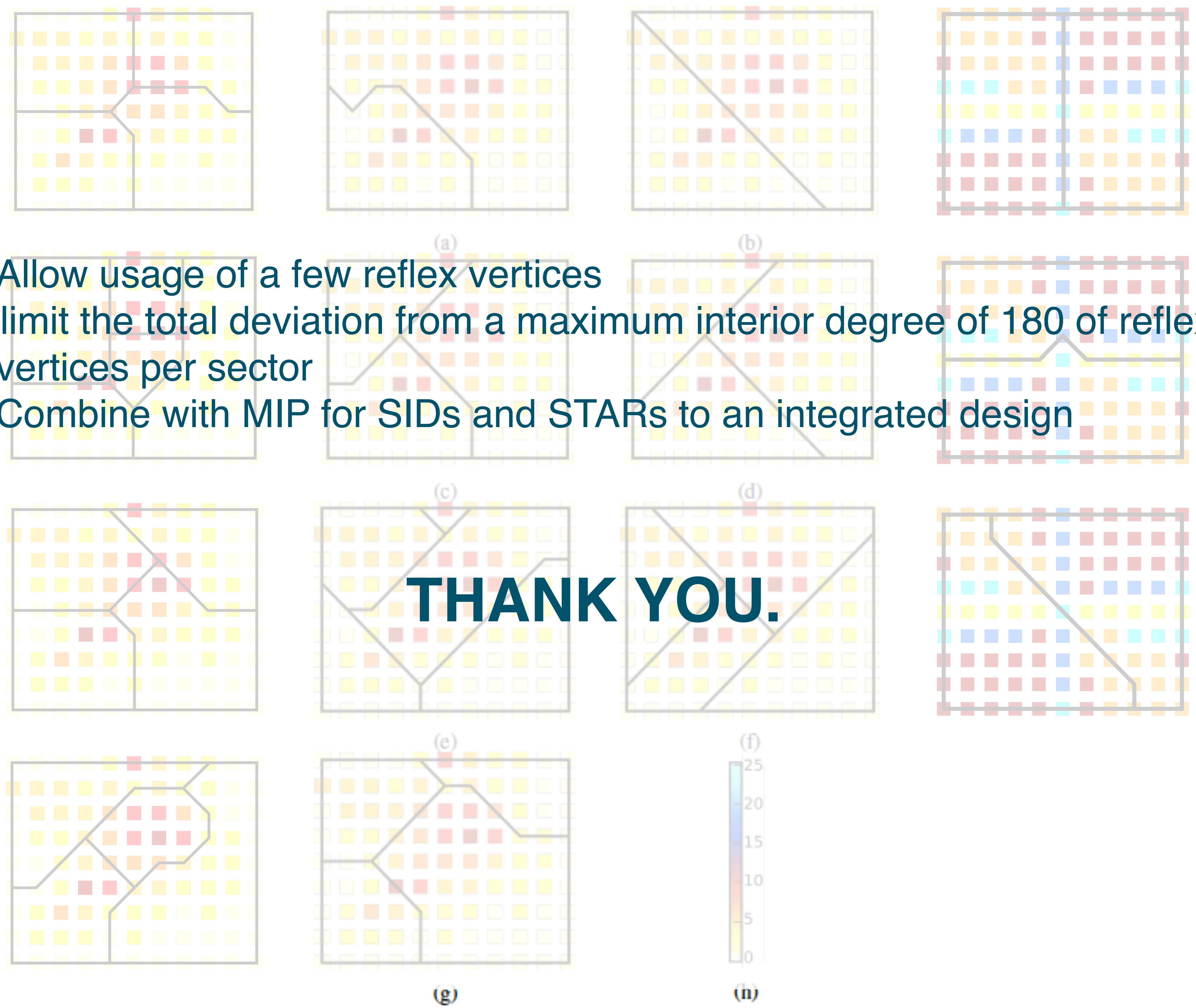
➡limit the total deviation from a maximum interior degree of 180 of reflex vertices per sector

# Conclusion/Outlook

- Allow usage of a few reflex vertices

➡limit the total deviation from a maximum interior degree of 180 of reflex vertices per sector

- Combine with MIP for SIDs and STARs to an integrated design




- Allow usage of a few reflex vertices

➡limit the total deviation from a maximum interior degree of 180 of reflex vertices per sector

- Combine with MIP for SIDs and STARs to an integrated design

# THANK YOU.