Convex Sectorization—A Novel Integer Programming Approach

Christiane Schmidt, Tobias Andersson Granberg, Tatiana Polishchuk, Valentin Polishchuk

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Introduction: Air transportation, Workload/Taskload, Sectorization

Review Grid-based IP formulation

Integration of Convexity Constraint in the Grid-based IP formulation

Enumeration of Topologies

Experimental Study: Arlanda Airport

Conclusion/Outlook

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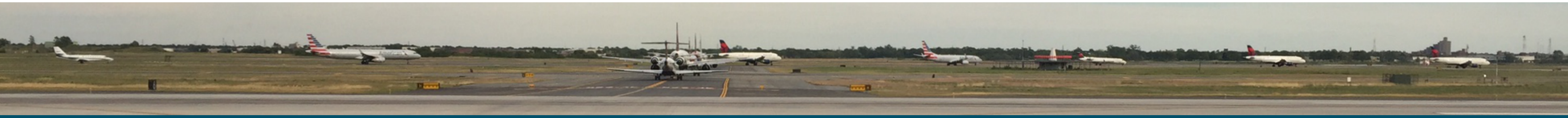

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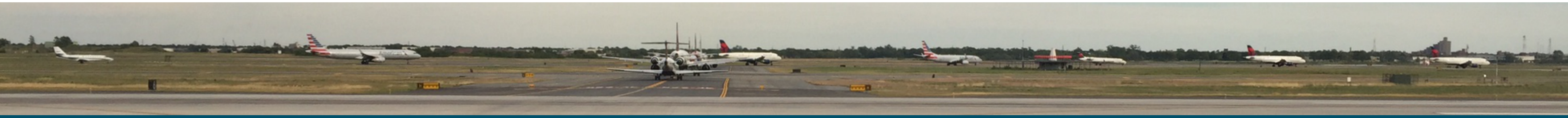
        - We can directly enforce convexity in our approach!



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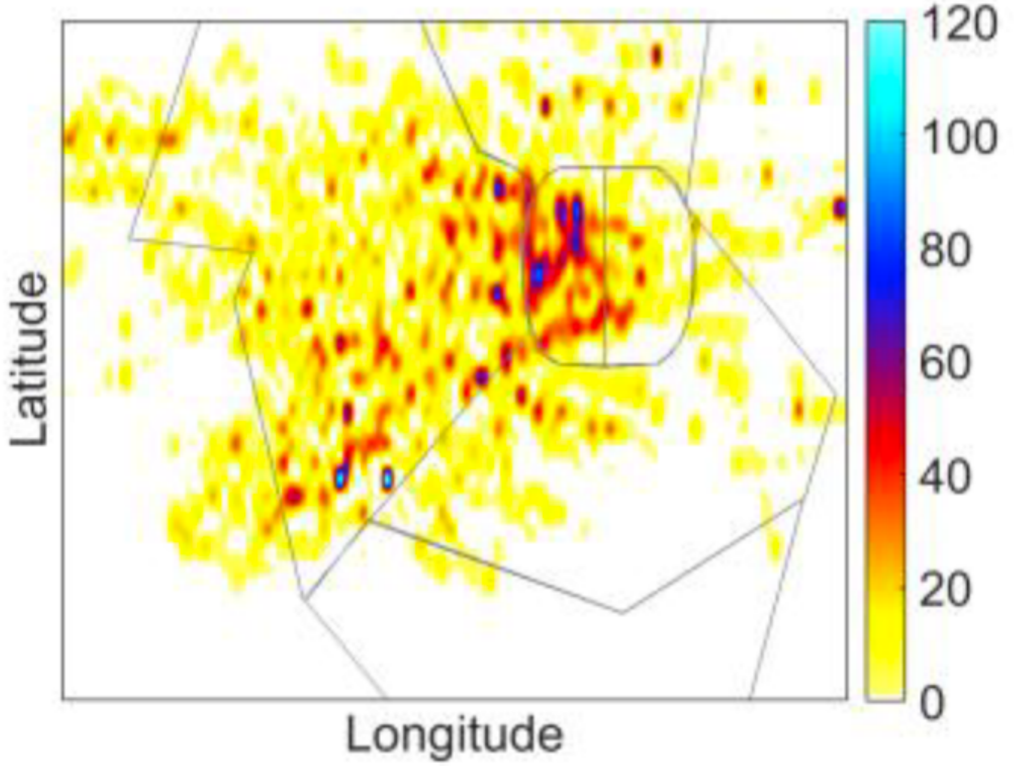

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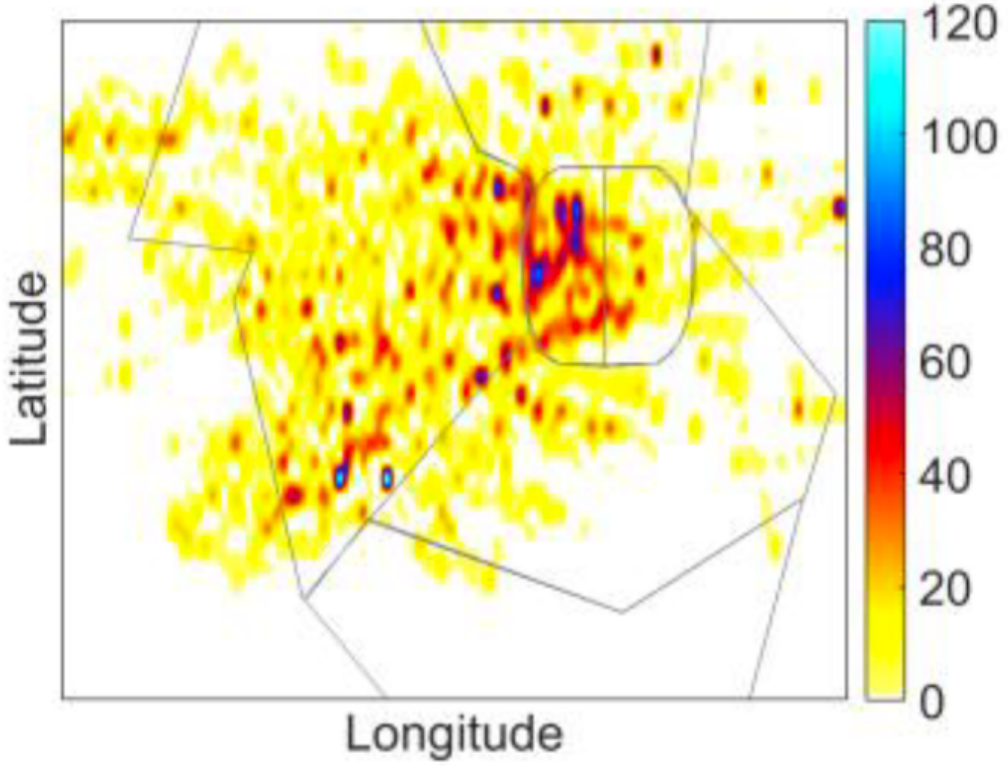
BUT: we do not depend on specific maps.



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(e) Interior conflict points (Points that require increased attention from ATCOs should lie in the sector's interior.)

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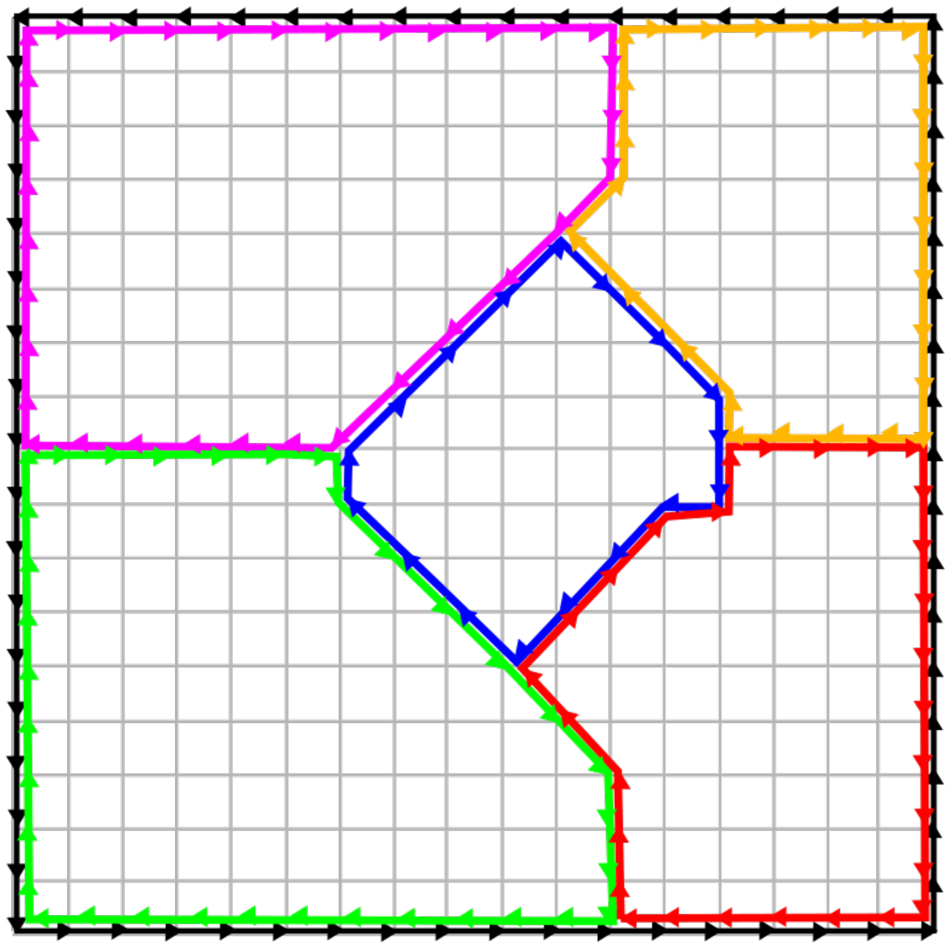
Main idea: use an artificial sector, S_0, that encompasses the complete boundary of P, using all counterclockwise (ccw) edges.



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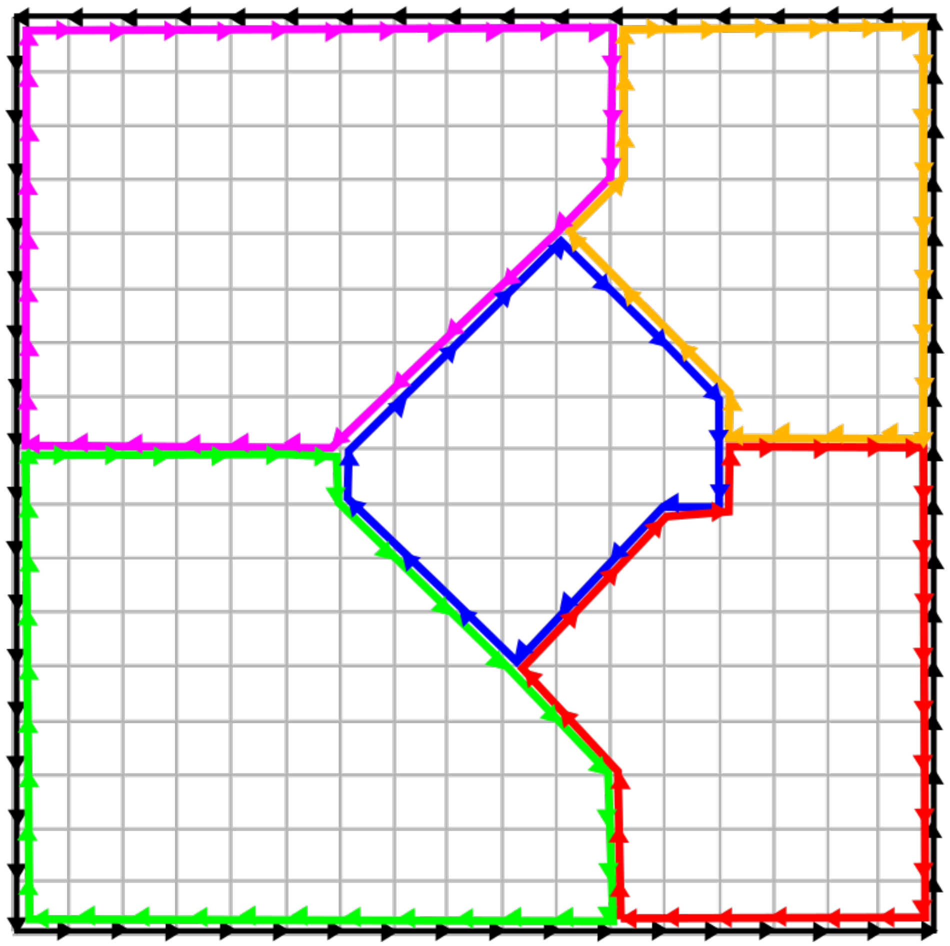
We use sectors in $S^* = S \cup S_0$ with $S = \{S_1,\dots,S_k\}$.



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$y_{i,j,s} = 1$: edge (i,j) used for sector s

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Minimum size

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- All ccw boundary edges in S_0
- If (i,j) used for some sector, (j, i) has to be used as well.
- Sector cannot contain (i,j) and (j,i).
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$y_{i,j,s} + y_{j,i,s} \leq 1$	$\forall (i,j) \in E, \forall s \in \mathcal{S}^*$	Sector cannot contain (i,j) and (j,i).
$\sum_{s\in\mathcal{S}^*} y_{i,j,s} \leq 1$	$\forall (i,j) \in E$	No edge in two sectors.
$\sum_{(i,j)\in E} y_{i,j,s} \geq 3$	$\forall s \in \mathcal{S}^*$	Minimum size
$y_{i,j,s} \in \{0,1\}$	$\forall (i,j) \in E, \forall s \in \mathcal{S}^*$	
$\sum_{l\in V:(l,i)\in E} y_{l,i,s} - \sum_{j\in V:(i,j)\in E} y_{i,j,s} = 0$	$\forall i \in V, \forall s \in \mathcal{S}^*$	Indegree=outdegree for all vertices
$\sum_{l\in V:(l,i)\in E} y_{l,i,s} \leq 1$	$\forall i \in V, \forall s \in \mathcal{S}^*$	A node has at most one ingoing edge per sector

(i,j) in S_l, (j,i) has to be in another sector

Review: Grid-based IP formulation

$y_{i,j,s} = 1$: edge (i,j) used for sector s

$$y_{i,j,0} = 1 \quad \forall (i,j) \in S_0$$
All ccw boundary edges in S_0

$$\sum_{s \in \mathcal{S}^*} y_{i,j,s} - \sum_{s \in \mathcal{S}^*} y_{j,i,s} = 0 \quad \forall (i,j) \in E$$
If (i,j) used for some sector, (j, i) has to be used as well.

$$y_{i,j,s} + y_{j,i,s} \leq 1 \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$$
Sector cannot contain (i,j) and (j,i).

(i,j) in S_l, (j,i) has to be in another sector

$$\sum_{s \in \mathcal{S}^*} y_{i,j,s} \leq 1 \quad \forall (i,j) \in E$$
No edge in two sectors.

$$\sum_{(i,j) \in E} y_{i,j,s} \geq 3 \quad \forall s \in \mathcal{S}^*$$
Minimum size

$$y_{i,j,s} \in \{0,1\} \quad \forall (i,j) \in E, \forall s \in \mathcal{S}^*$$

$$\sum_{l \in V:(l,i) \in E} y_{l,i,s} - \sum_{j \in V:(i,j) \in E} y_{i,j,s} = 0 \quad \forall i \in V, \forall s \in \mathcal{S}^*$$
Indegree=outdegree for all vertices

$$\sum_{l \in V:(l,i) \in E} y_{l,i,s} \leq 1 \quad \forall i \in V, \forall s \in \mathcal{S}^*$$
A node has at most one ingoing edge per sector

$\Longrightarrow$ Union of the |S| sectors completely covers the TMA.

(a) Balanced taskload

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First step: We need to assign area to sector selected by boundary edges!

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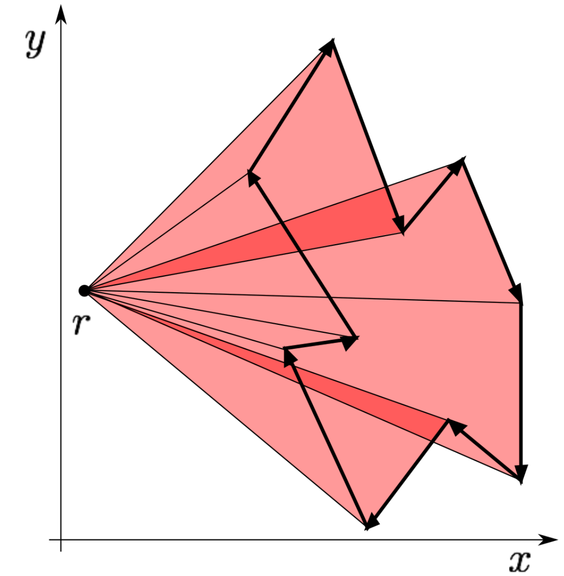
  - cw triangles contribute positive



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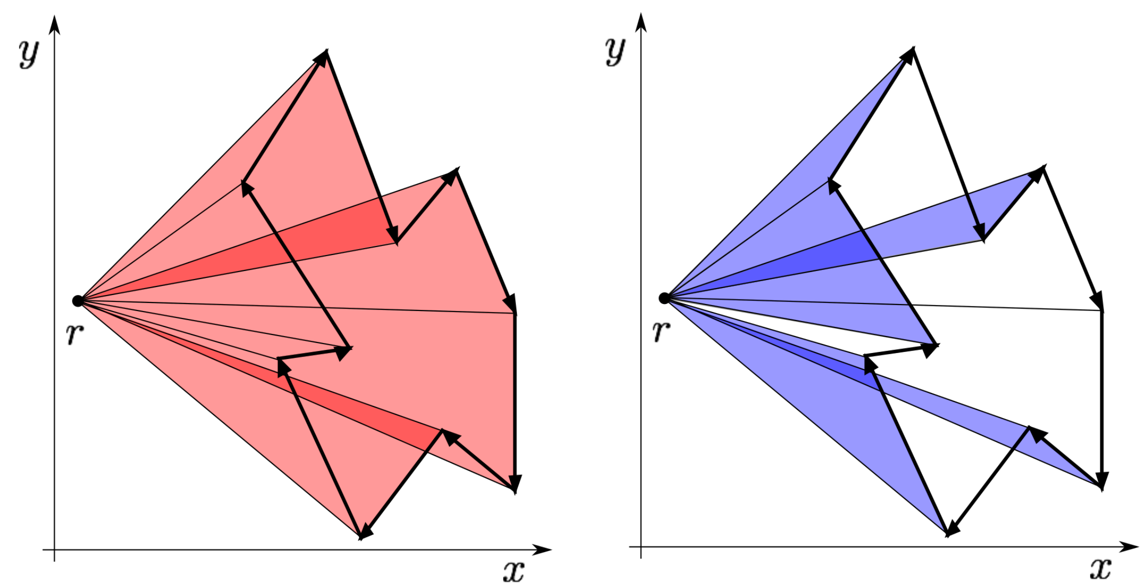

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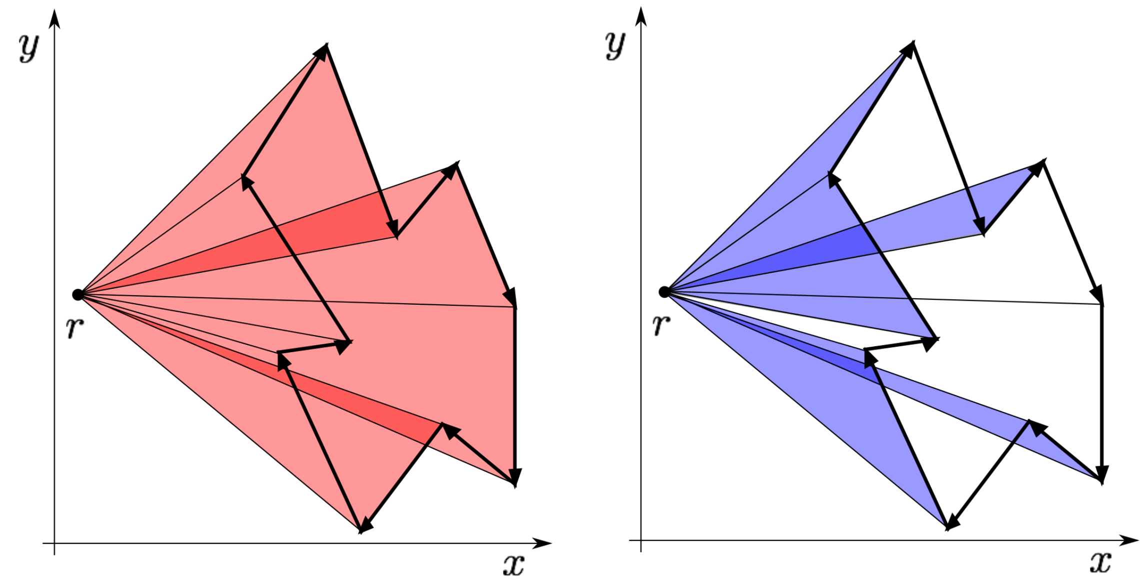

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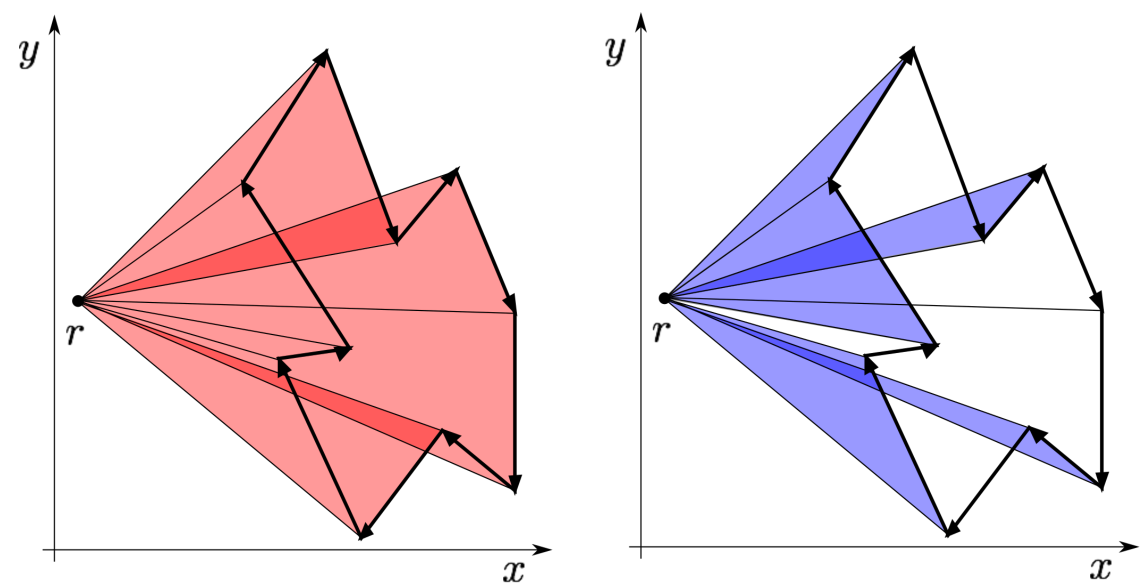

$$\sum_{(i,j)\in E} f_{i,j}\, y_{i,j,s} - a_s = 0 \;\; \forall s \in \mathcal{S}^*$$

$$\sum_{s\in\mathcal{S}} a_s = a_0$$

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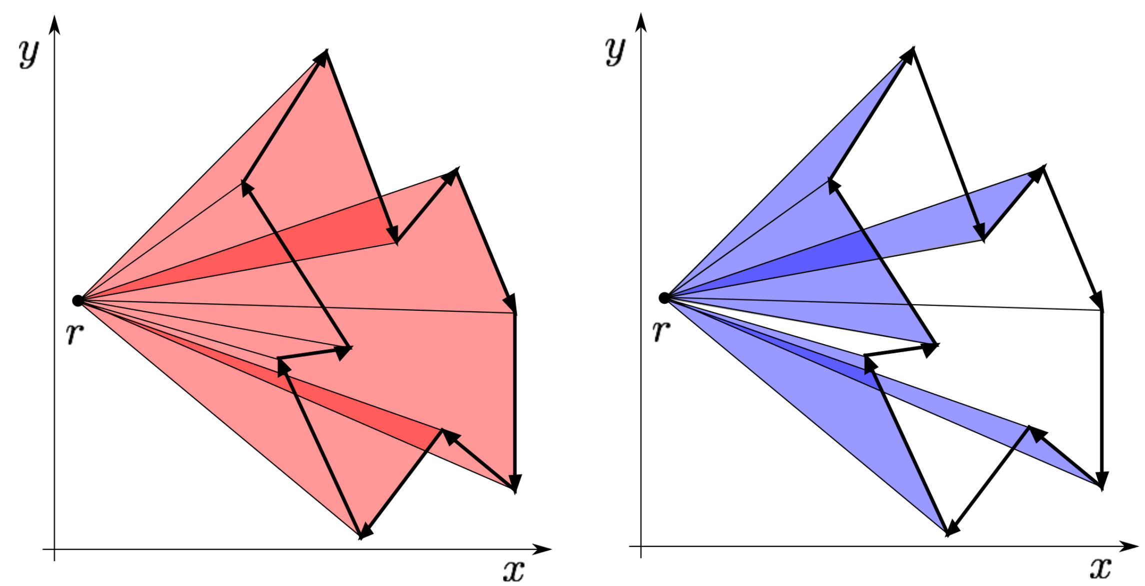

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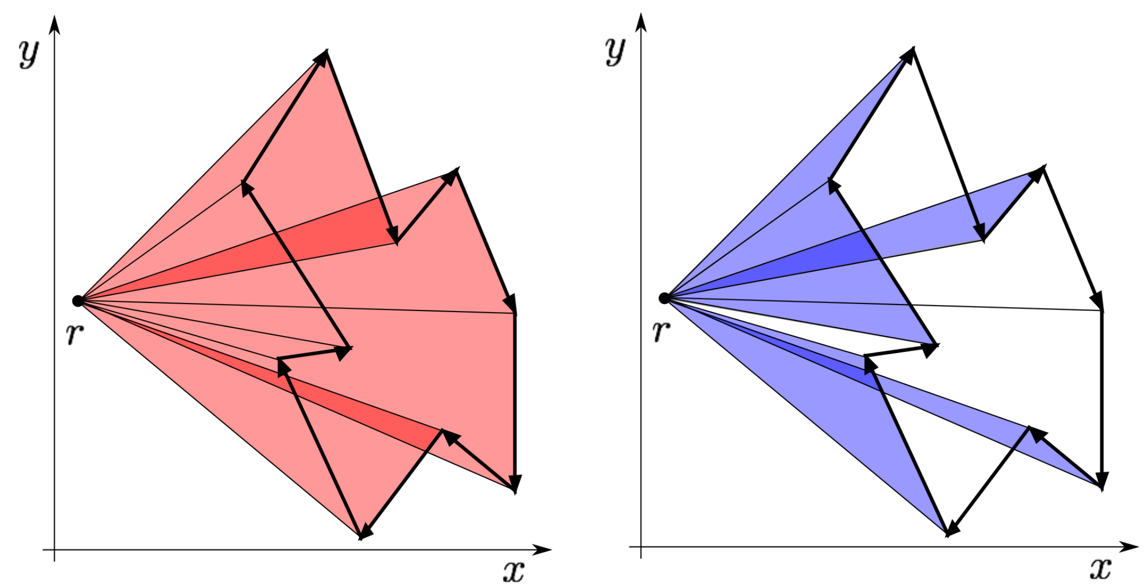

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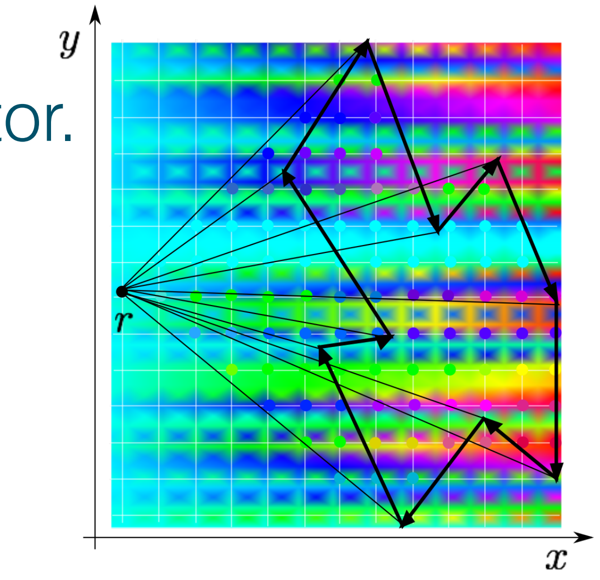
Sum of areas = area of S_0

Second step: We need to associate task load with a sector.

Review: Grid-based IP formulation

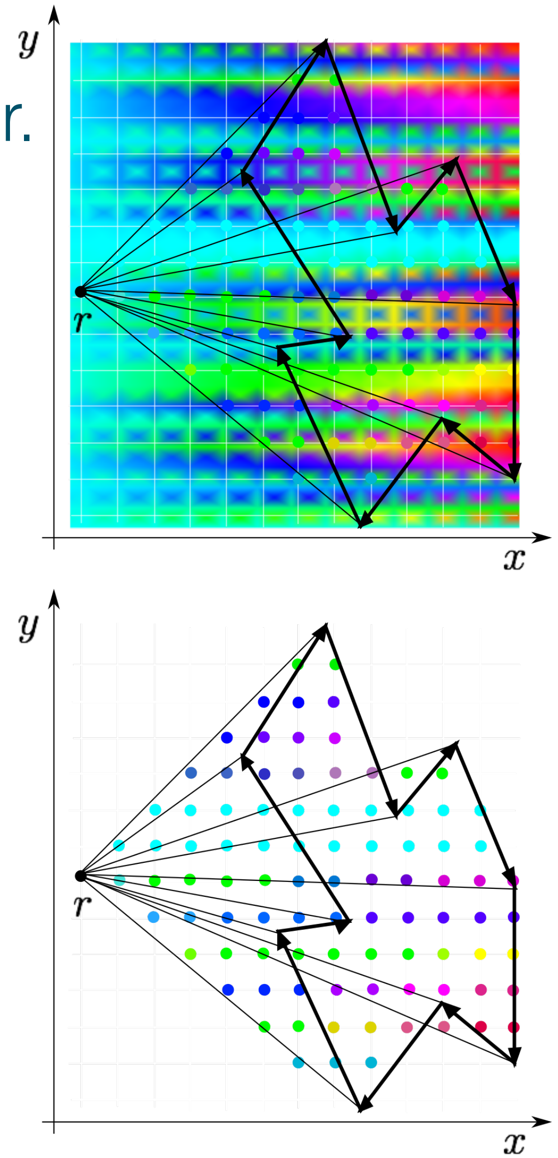
Second step: We need to associate task load with a sector.

- Overlay heat map with a grid.



Second step: We need to associate task load with a sector.

- Overlay heat map with a grid.
- Extract values at the grid points.



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- Overlay heat map with a grid.
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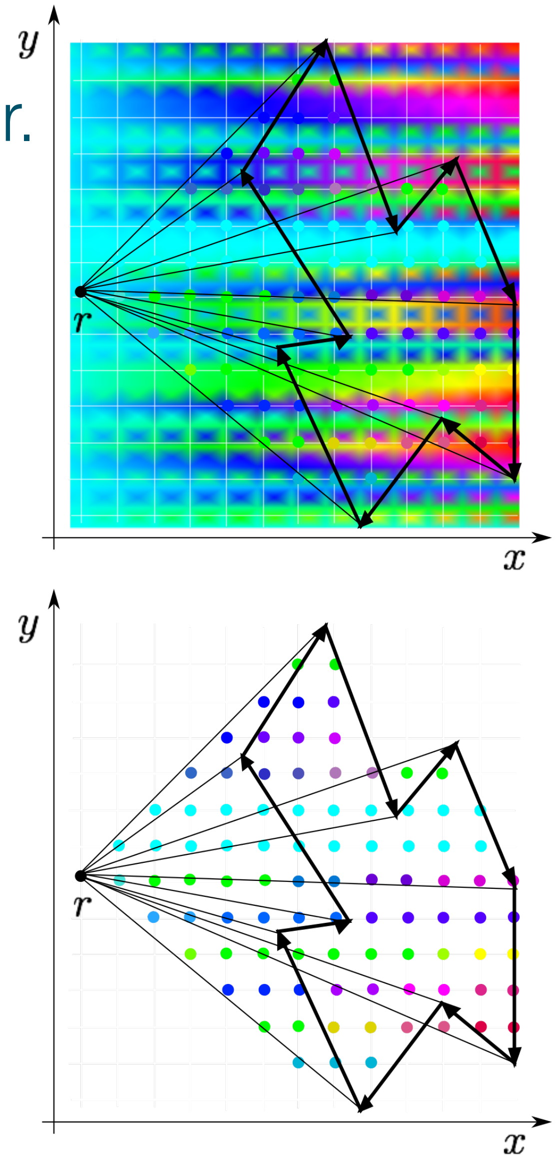

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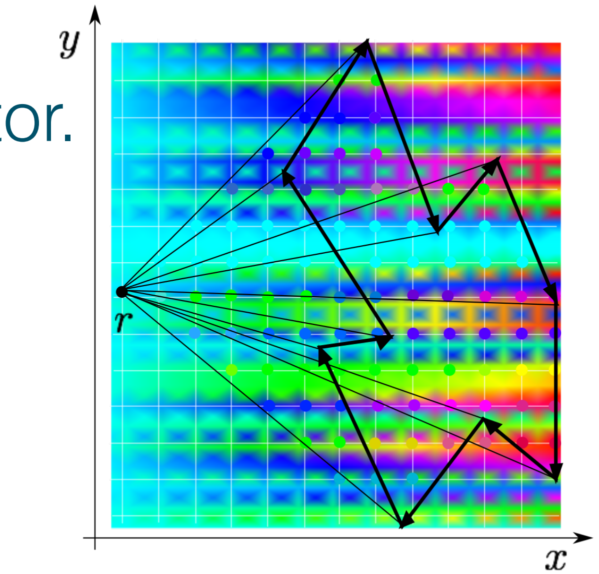

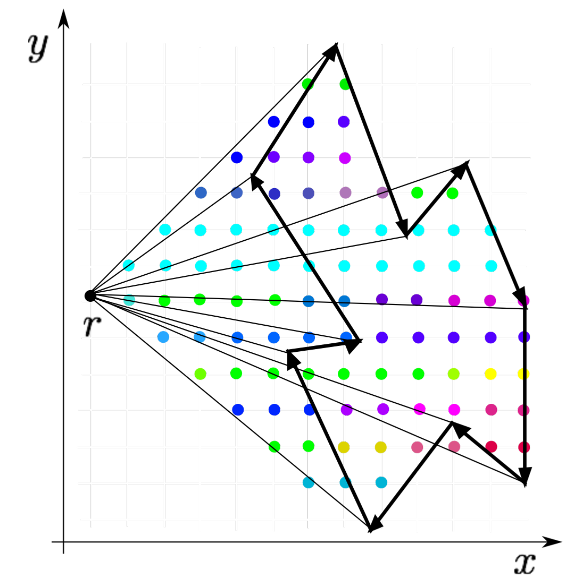

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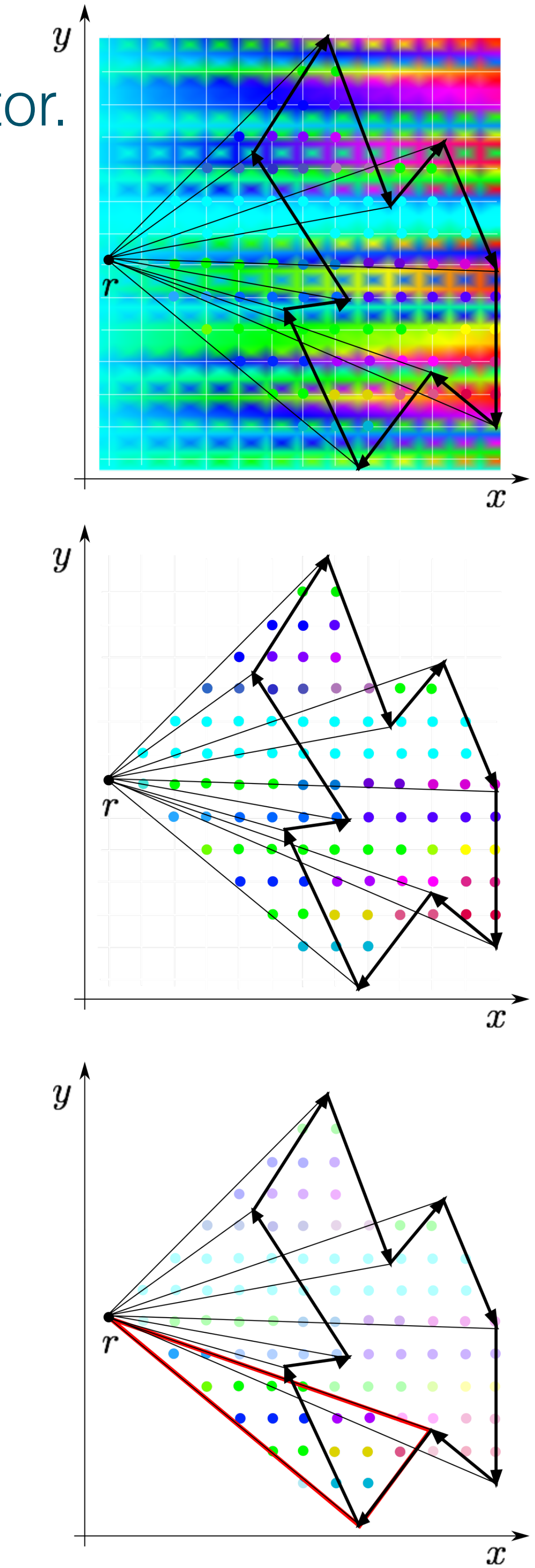
- Let the sign of $f_{i,j}$ be $p_{i,j}$



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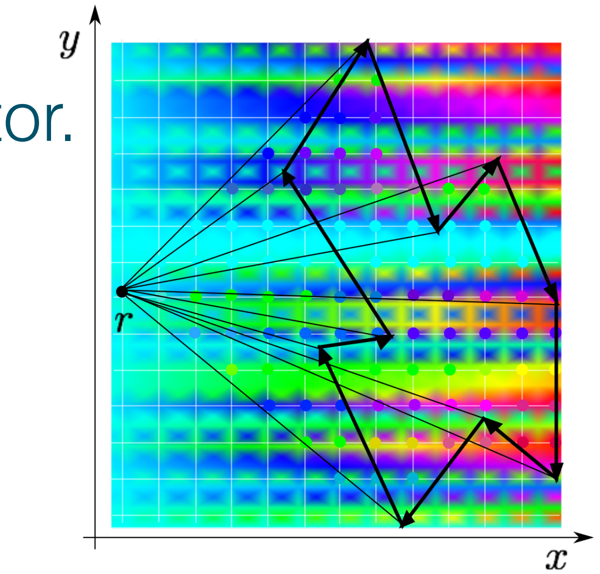

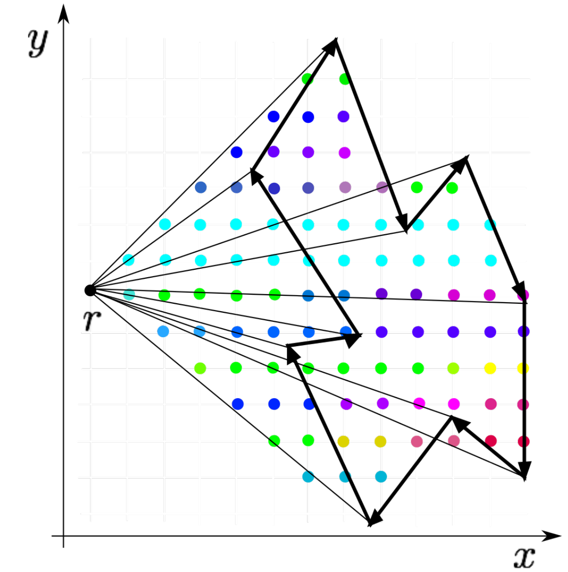

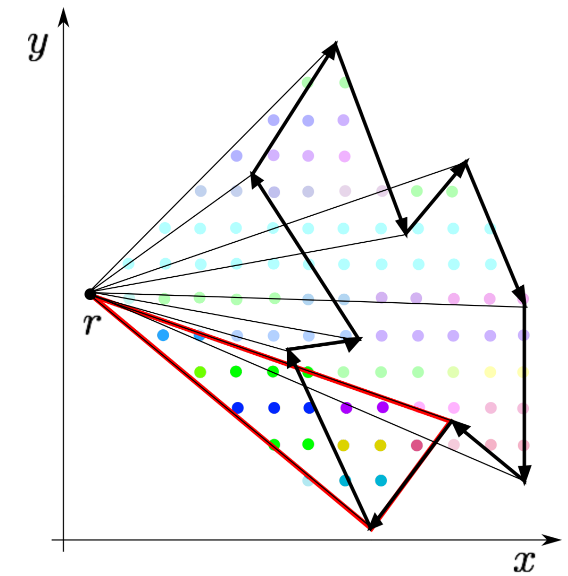

$$
\begin{aligned}
h_{i,j} =& p_{i,j} \sum_{q \in \Delta(i,j,r)} h_q & & \\
\sum_{(i,j)\in E} h_{i,j}\, y_{i,j,s} - t_s =& \quad 0 & & \forall s \in \mathcal{S} \\
t_s \geq& \quad t_{LB} & & \forall s \in \mathcal{S} \\
t_s \leq& \quad t_{UB} & & \forall s \in \mathcal{S} \\
t_{LB} =& \quad c_2 \cdot t_0 / |\mathcal{S}| & & \text{with, e.g., } c_2 = 0.9
\end{aligned}
$$

Review: Grid-based IP formulation

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- With constraint (e), interior conflict points:

$$\min \sum_{s\in\mathcal{S}} \sum_{(i,j)\in E} (\gamma \ell_{i,j} + (1-\gamma) w_{i,j})\, y_{i,j,s}, \quad 0 \le \gamma < 1$$

$$w_{i,j} = h_i + h_j$$

$$w_{i,j} = \sum_{k\in N(i)} h_k + \sum_{l\in N(j)} h_l$$

Integration of Convexity Constraint in the Grid-based IP formulation

(d) Convex sectors

- Convex sector:

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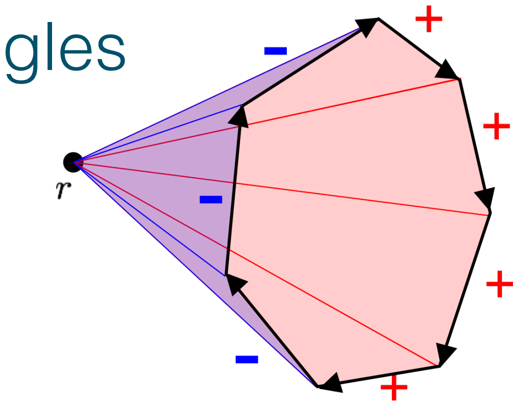
- Convex sector:
  - only one connected chain of edges with cw triangles



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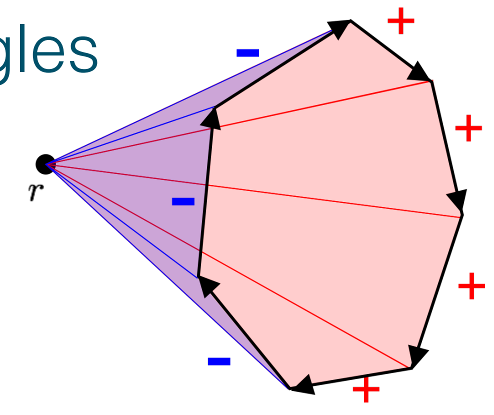

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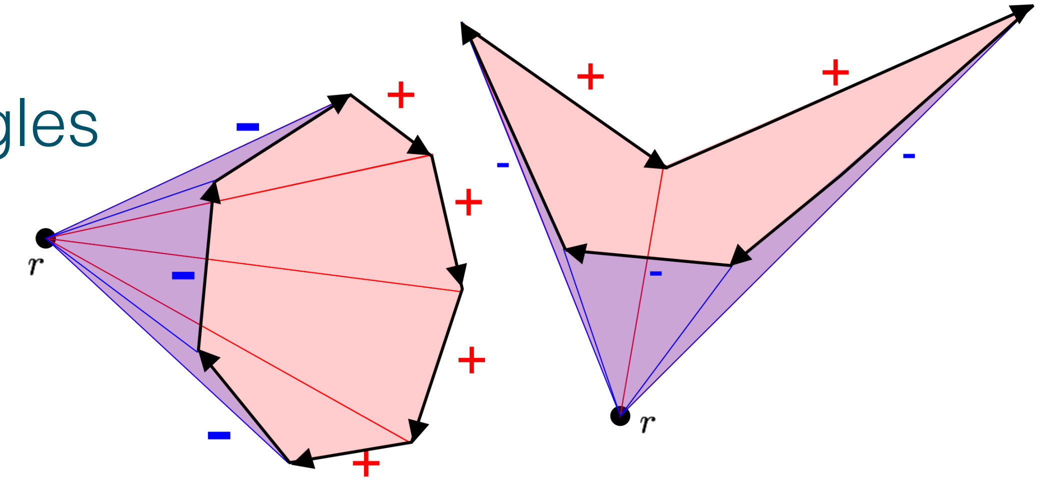
- Only-if-part of that statement is not true



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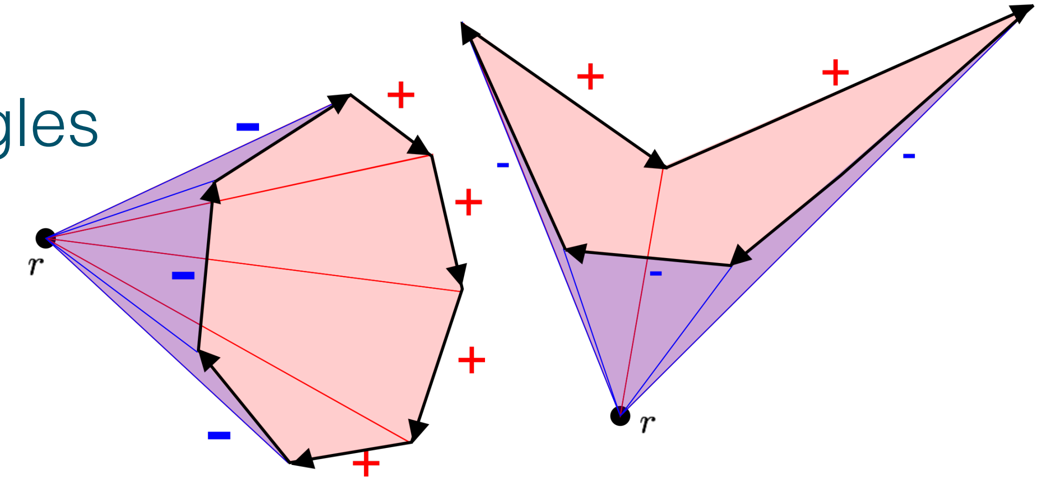

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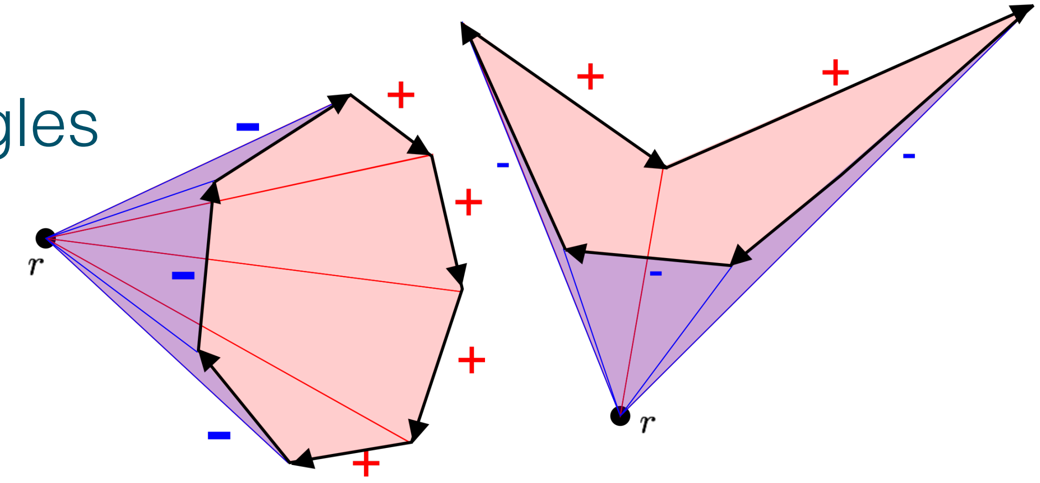

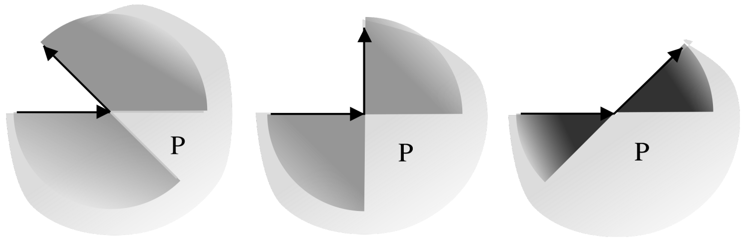

Three outgoing edge directions yield a non-convex polygon

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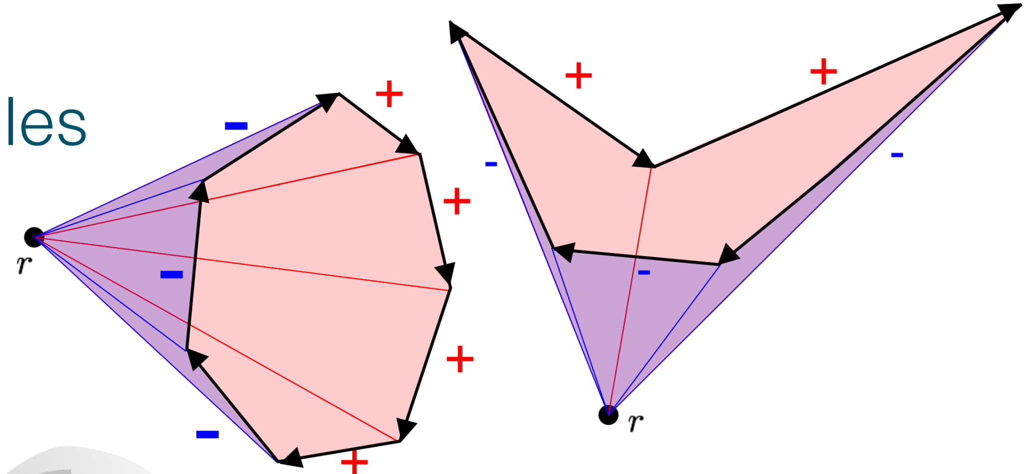

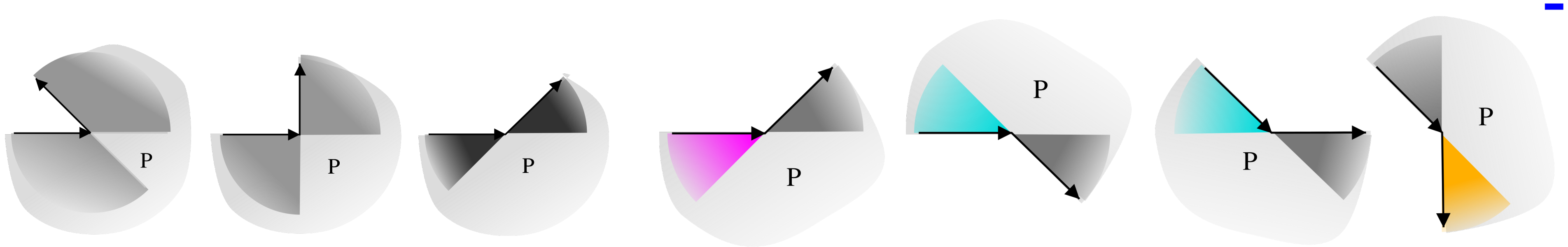

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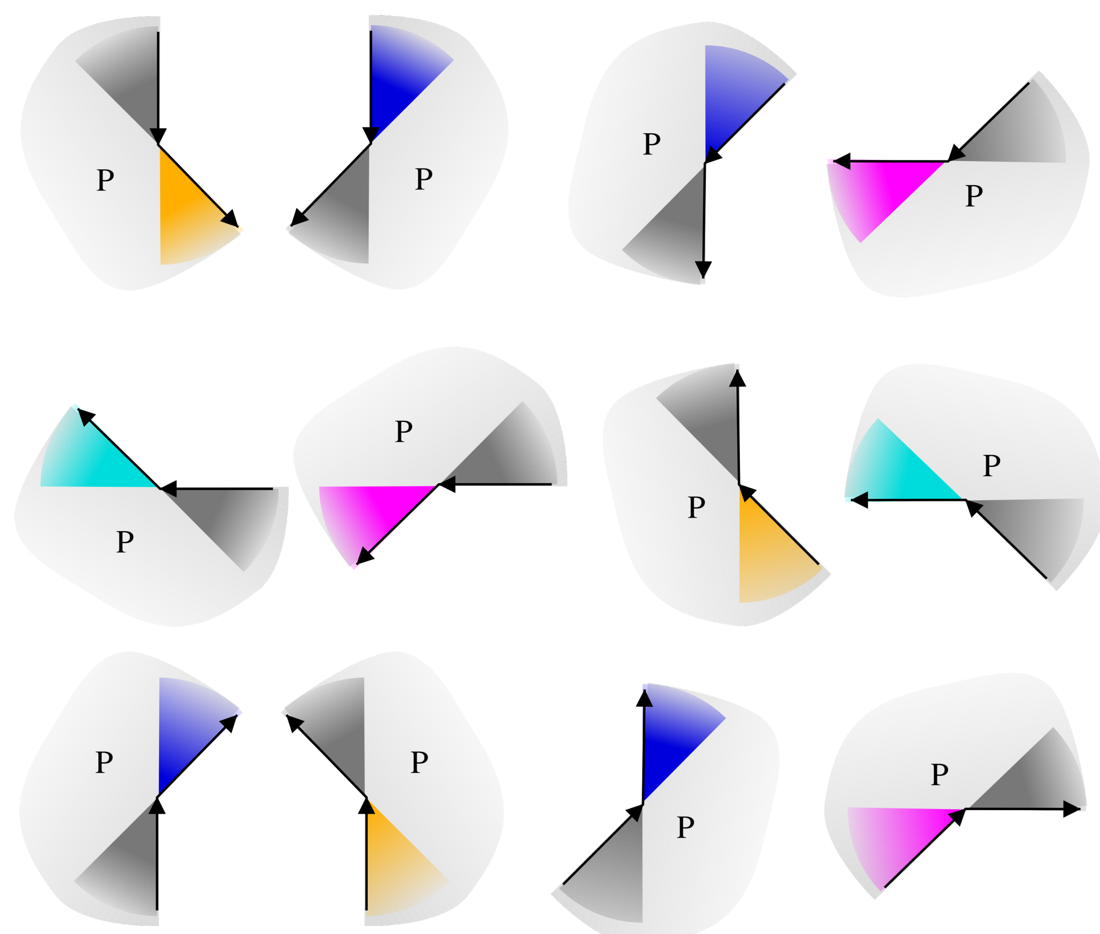

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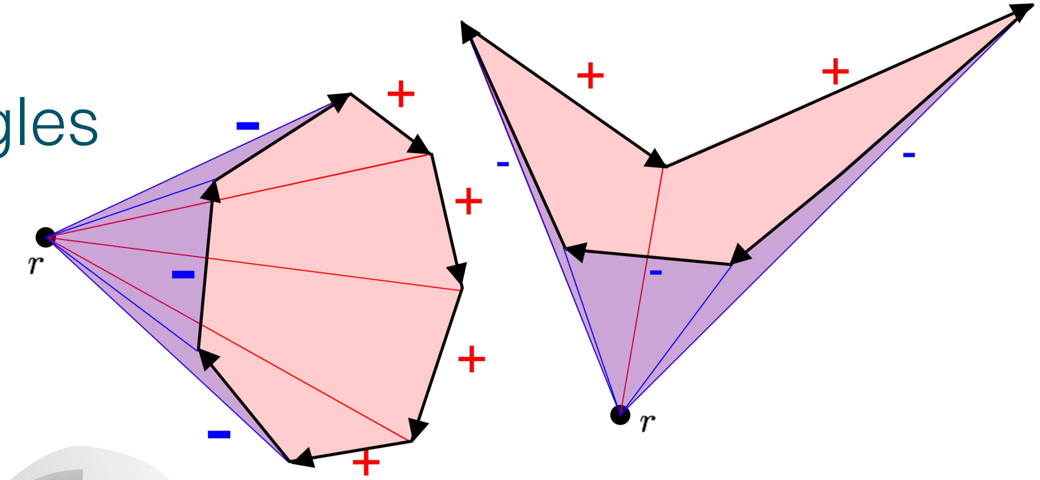

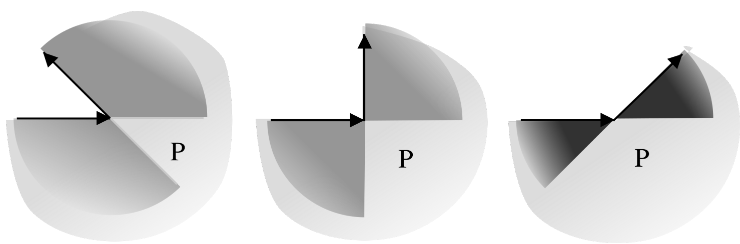

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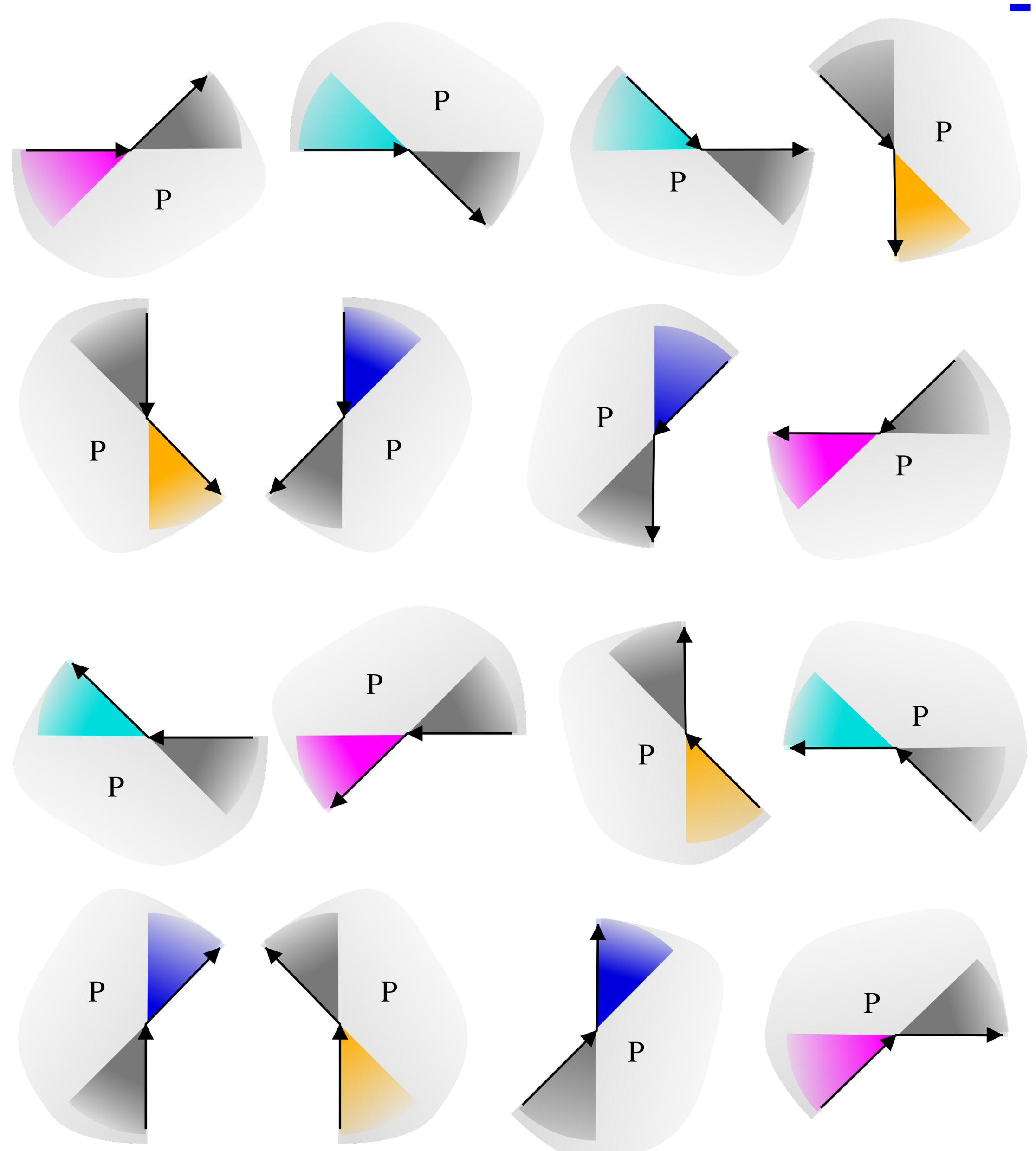
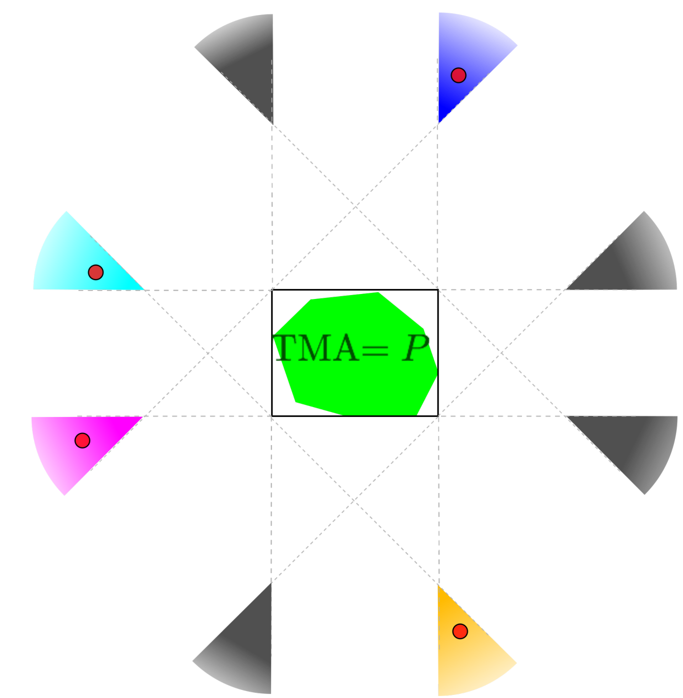
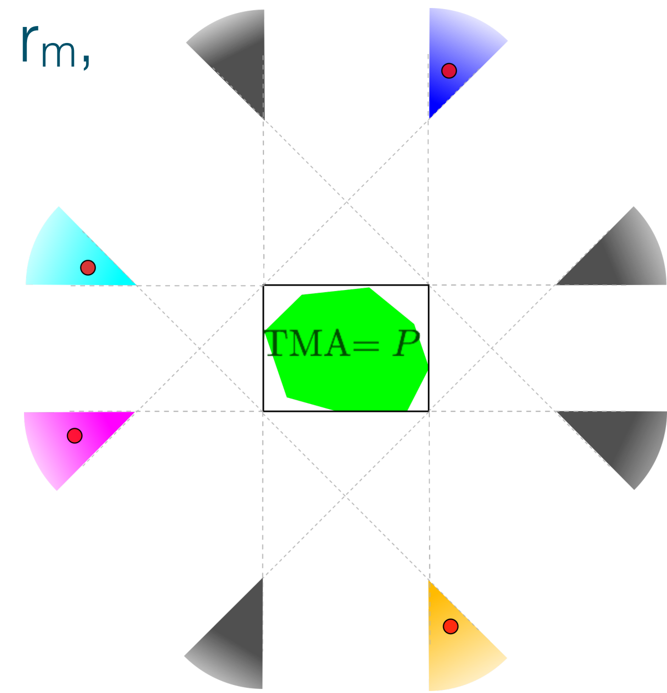




Our reference points

Convexity Constraints

- One reference point in each of the four colored cones: $r_1,\ldots,r_4$ ($r = r_m$, for some $m \in M=\{1,2,3,4\}$



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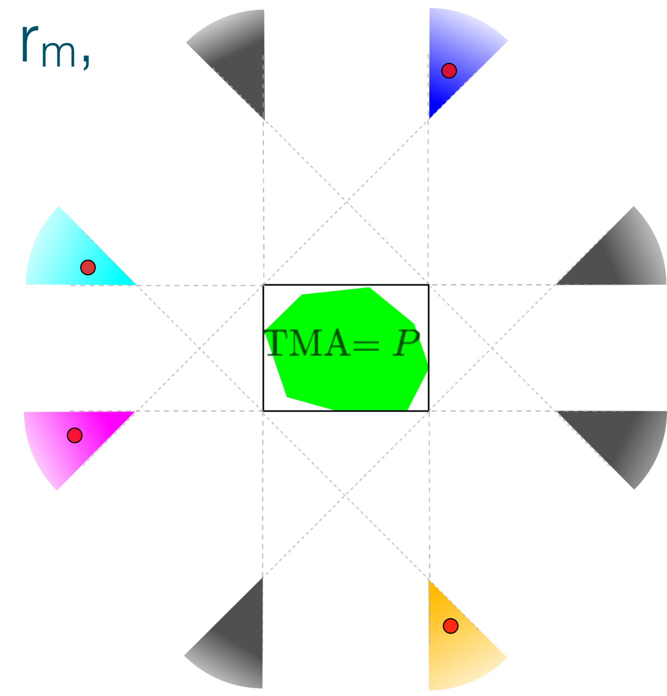

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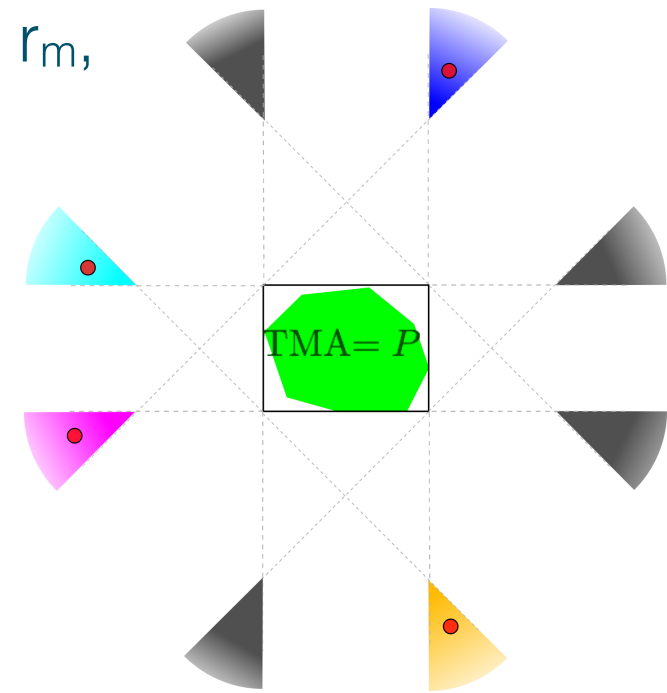

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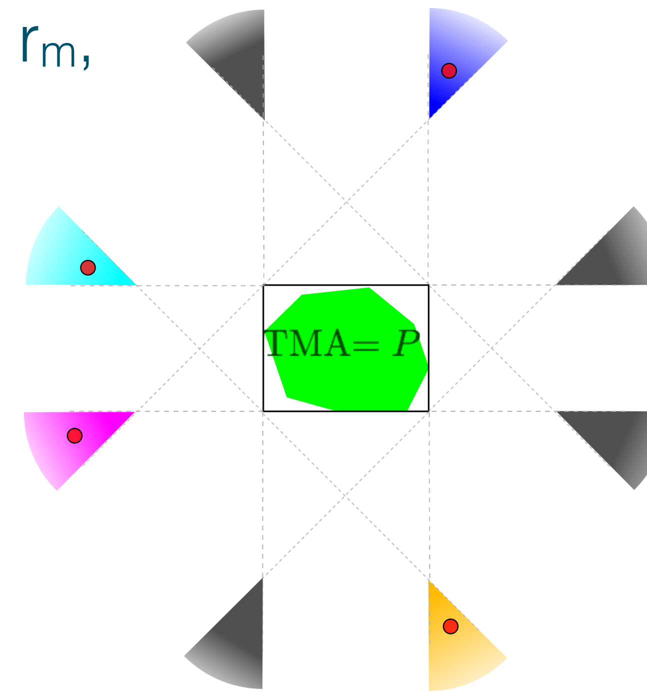

$$
\begin{aligned}
q^s_{j,m} &= \frac{1}{2}\left(\sum_{i:(i,j)\in E} p_{i,j,m}\, y_{i,j,s} - \sum_{l:(j,l)\in E} p_{j,l,m}\, y_{j,l,s}\right) && \forall s \in \mathcal{S},\ \forall j \in V,\ \forall m \in \mathcal{M}\\
qabs^s_{j,m} &\geq q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}\\
qabs^s_{j,m} &\geq -q^s_{j,m} && \forall s \in \mathcal{S}, \forall j \in V,\ \forall m \in \mathcal{M}\\
\sum_{i\in V}\sum_{j\in V} y_{i,j,s} \cdot qabs^s_{j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
0 &\leq z^s_{i,j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\leq qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\leq y_{i,j,s} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
z^s_{i,j,m} &\geq y_{i,j,s} - 1 + qabs^s_{j,m} && \forall i,j \in V\ \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}\\
\sum_{i\in V}\sum_{j\in V} z^s_{i,j,m} &= 2 && \forall s \in \mathcal{S},\ \forall m \in \mathcal{M}
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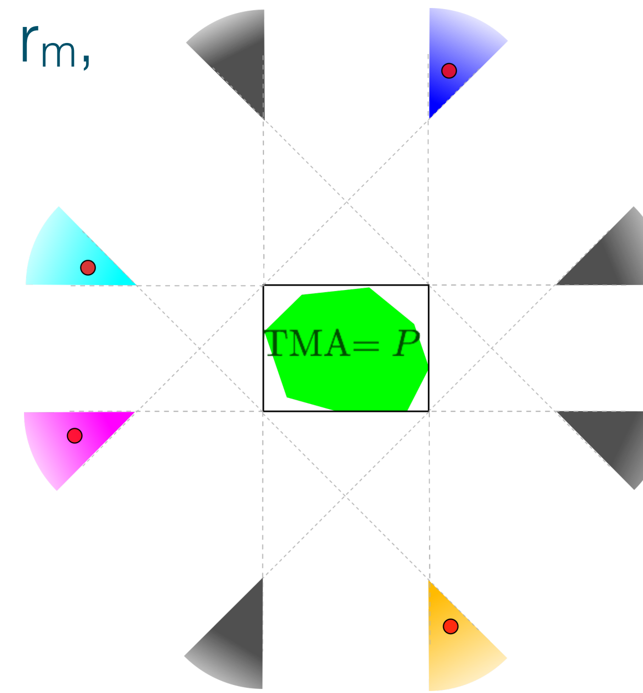

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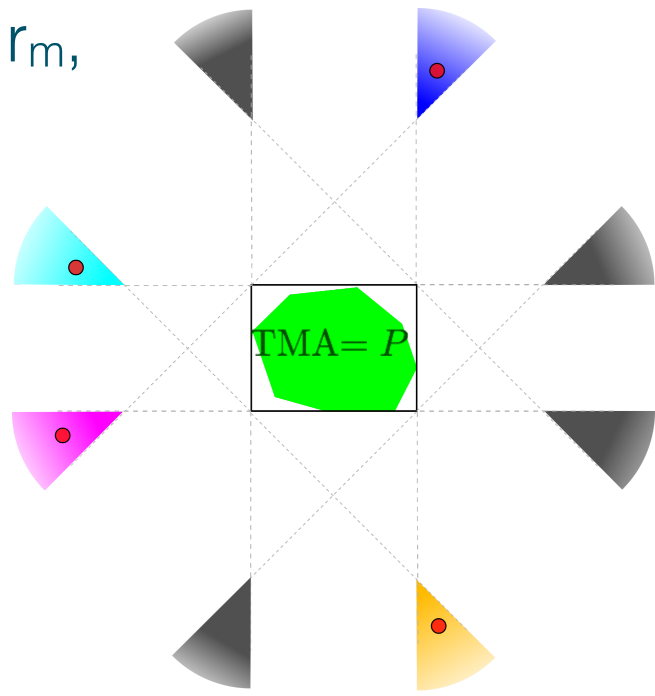

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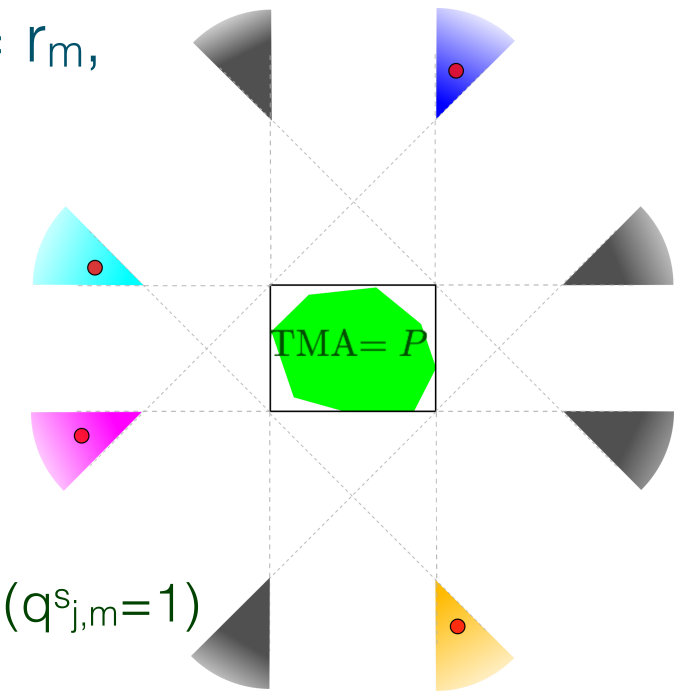

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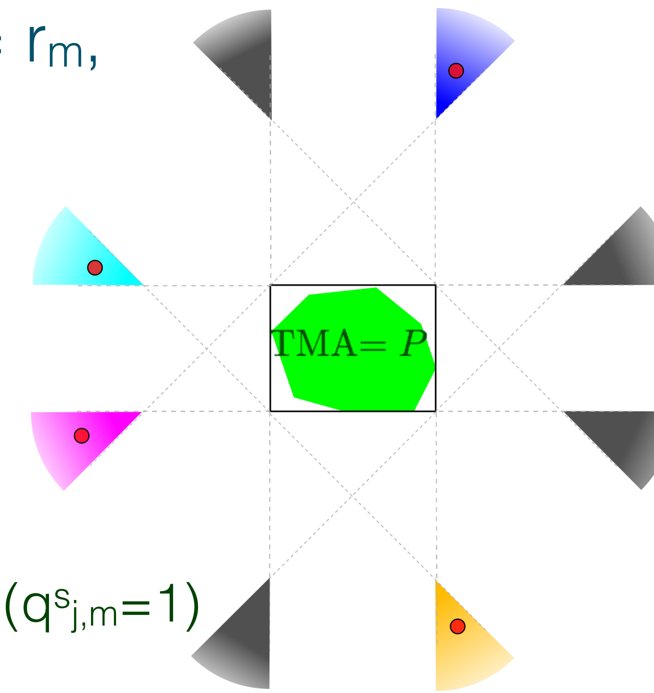

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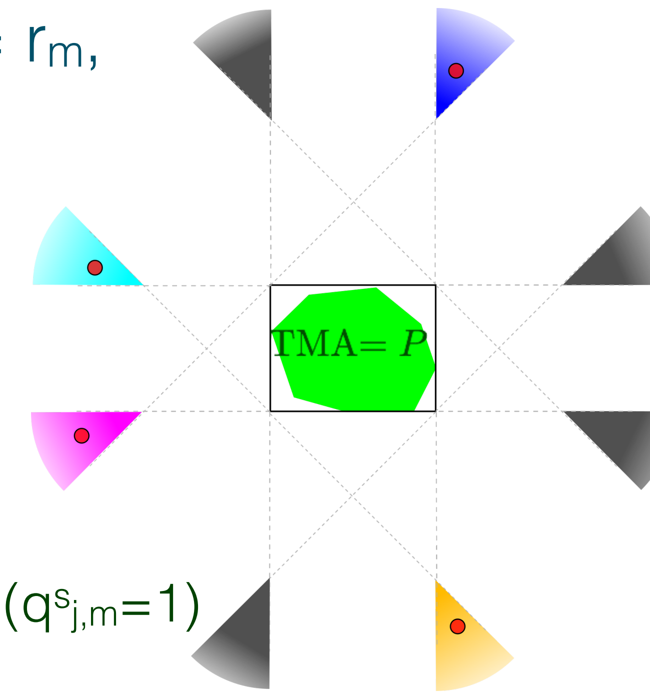

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Multiplication of two variables ➜ define $z^s_{i,j,m} = y_{i,j,s} * qabs^s_{j,m}$.

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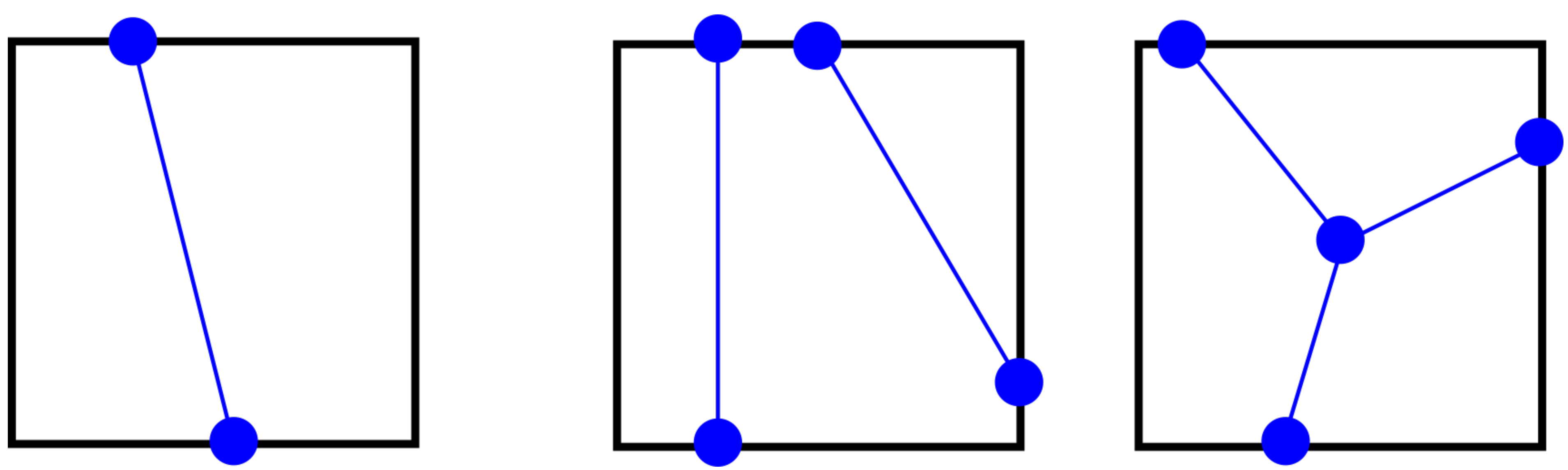

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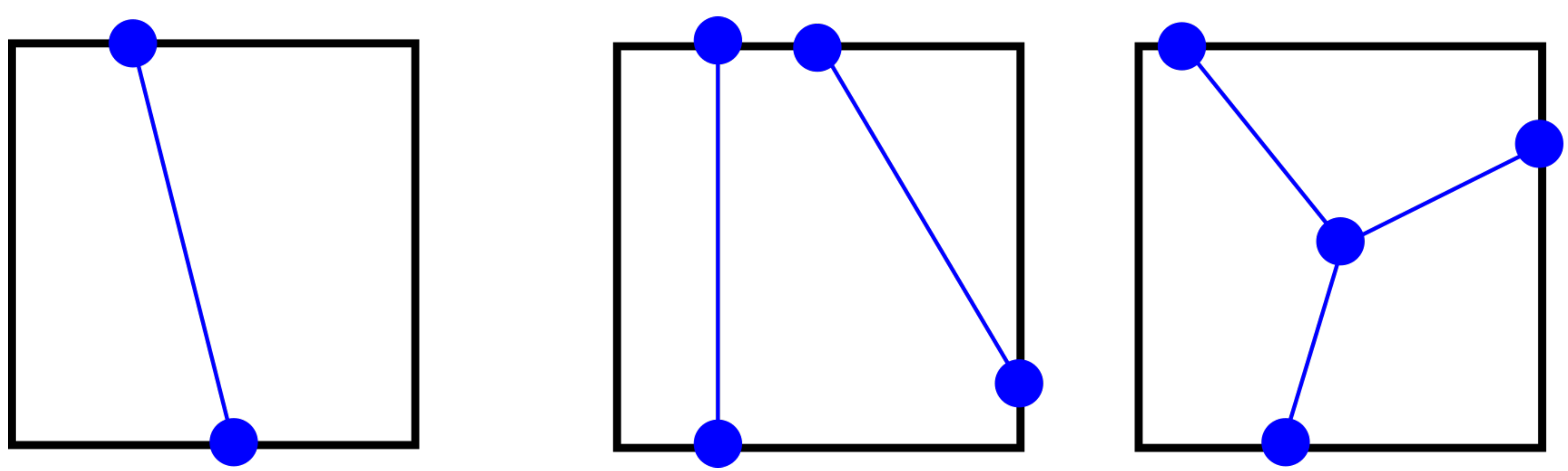

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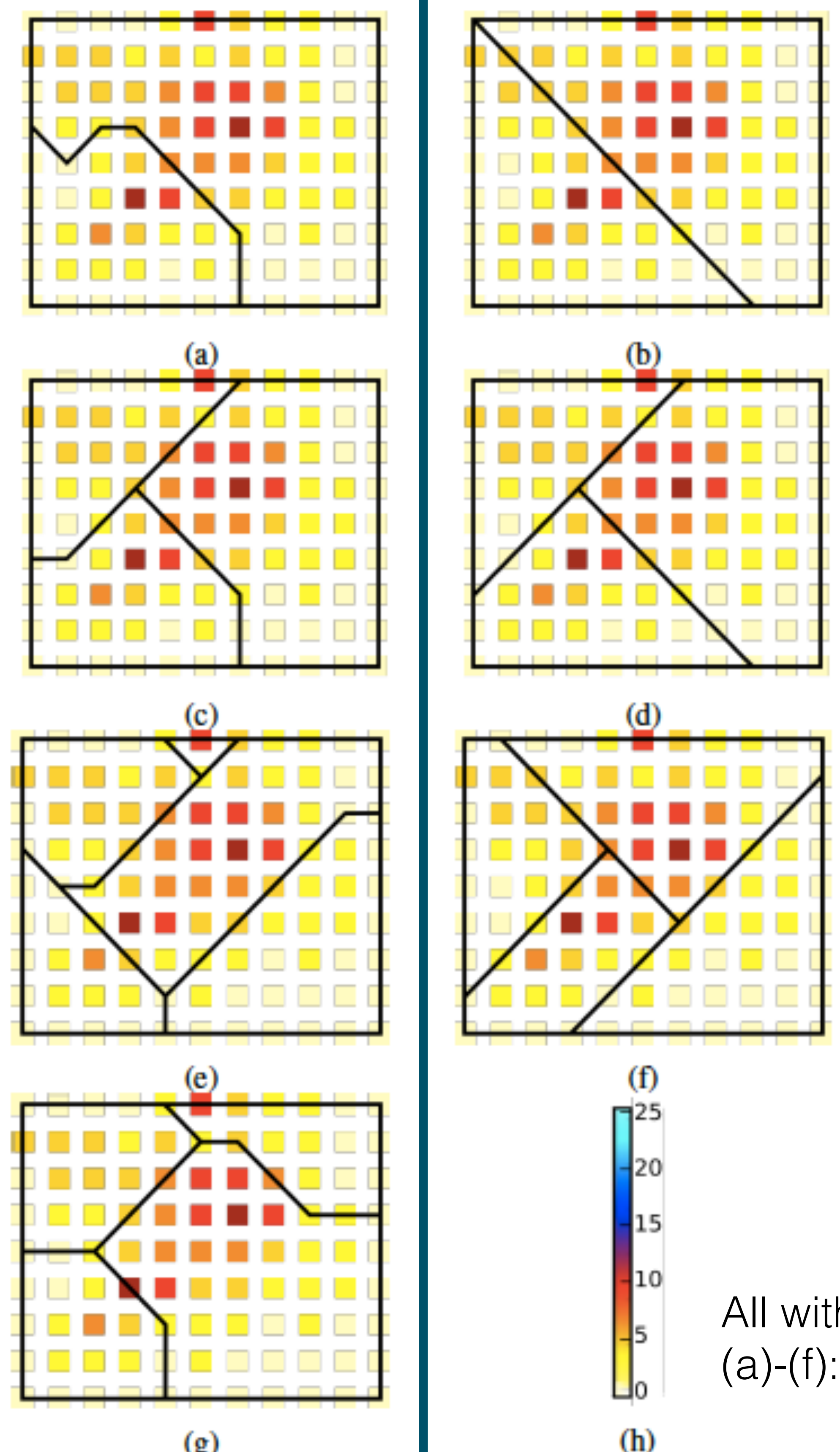
(a) Balanced task load
(b) Connected sectors
(c) Nice shape
(e) Interior conflict points



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All with c_2=0.6 and $w_{i,j} = h_i + h_j$.
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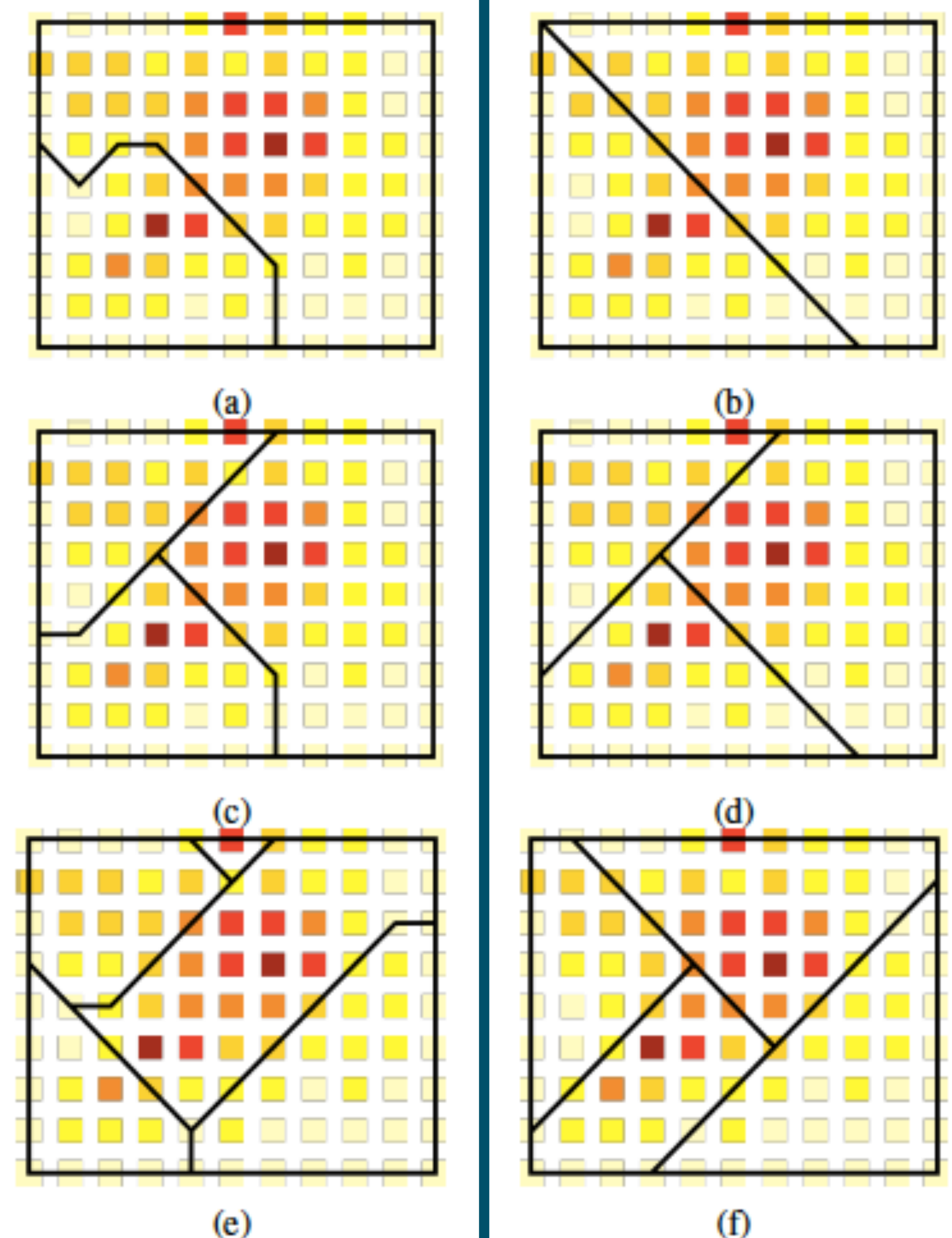
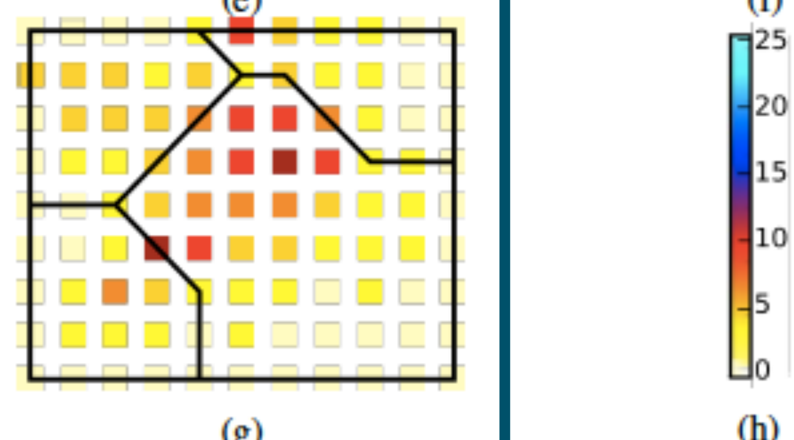
Disconnected sector ➜





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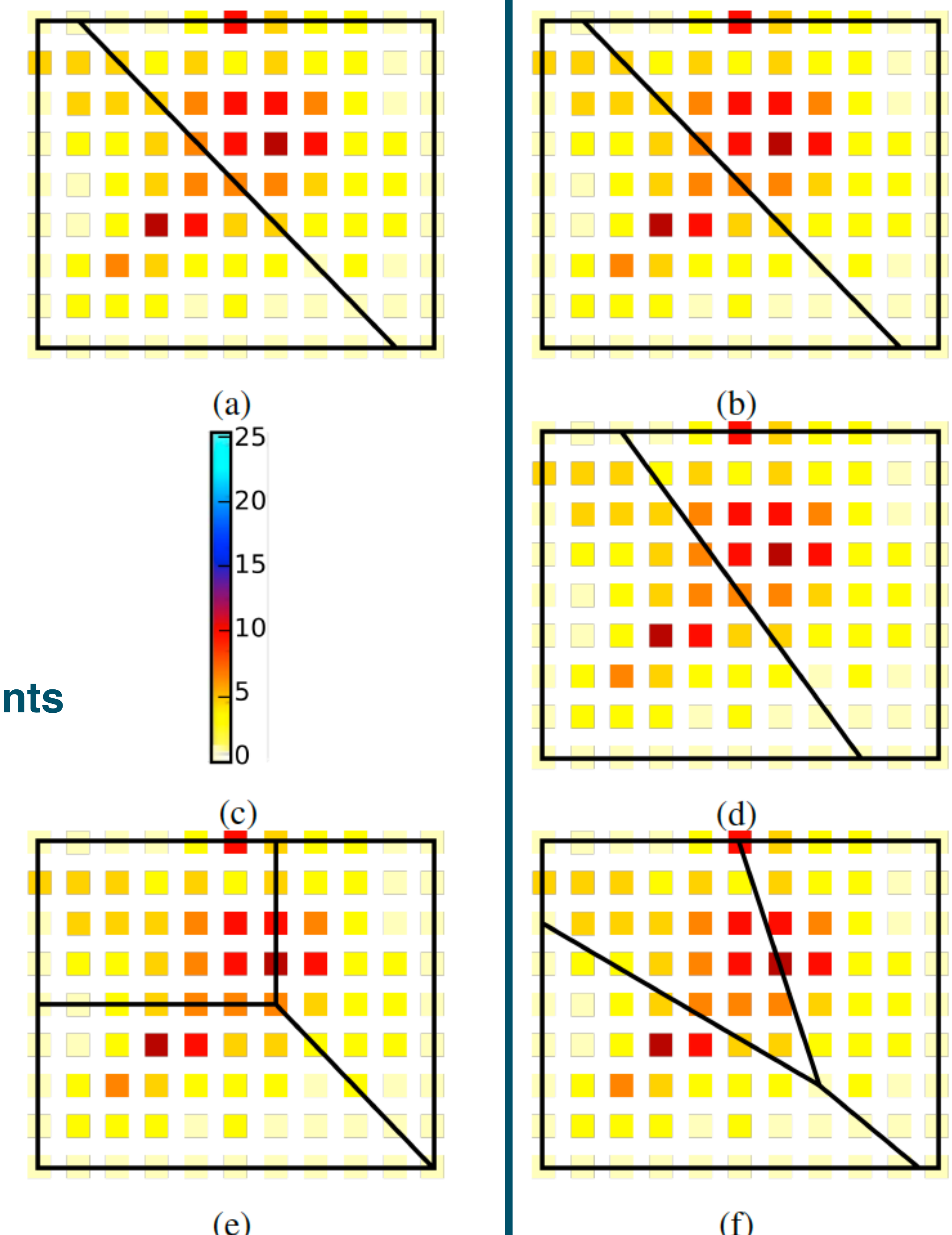

$c_2=0.95$

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All perfect taskload balance.

Conclusion/Outlook

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Outlook

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- Combine with MIP for SIDs and STARs to an integrated design

THANK YOU.

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