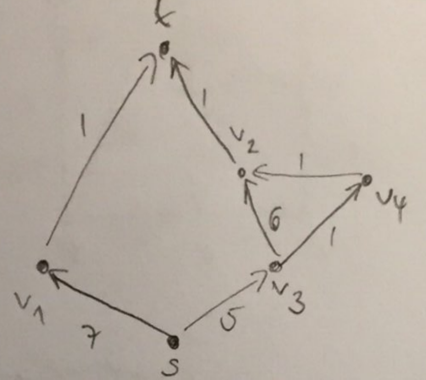
BLA exam, 24.10.2017

<u>Problem 1:</u>



1. $R=\emptyset$, $\ell(t)=\ell(v_1)=\ell(v_2)=\ell(v_3)=\ell(v_4)=\infty$, $\ell(s)=0$
2. Choose s
3. $R=\{s\}$
4. • $(s,v_1)\in E$, $v_1\notin R$: $\infty=\ell(v_1)>\ell(s)+c((s,v_1))=0+7$
   $\Rightarrow \ell(v_1)=7$, $pred(v_1)=s$
   • $(s,v_3)\in E$, $v_3\notin R$: $\infty=\ell(v_3)>\ell(s)+c((s,v_3))=0+5$
   $\Rightarrow \ell(v_3)=5$, $pred(v_3)=s$

5. $R\neq V(G)$
2. Choose v_3
3. $R=\{s,v_3\}$
4. • $(v_3,v_2)\in E$, $v_2\notin R$: $\infty=\ell(v_2)>\ell(v_3)+c((v_3,v_2))=5+6$
   $\Rightarrow \ell(v_2)=11$, $pred(v_2)=v_3$
   • $(v_3,v_4)\in E$, $v_4\notin R$: $\infty=\ell(v_4)>\ell(v_3)+c((v_3,v_4))=5+1$
   $\Rightarrow \ell(v_4)=6$, $pred(v_4)=v_3$

5. $R\neq V(G)$
2. Choose v_4
3. $R=\{s,v_3,v_4\}$
4. $(v_4,v_2)\in E$, $v_2\notin R$: $11=\ell(v_2)>\ell(v_4)+c((v_4,v_2))=6+1$
   $\Rightarrow \ell(v_2)=7$, $pred(v_2)=v_4$

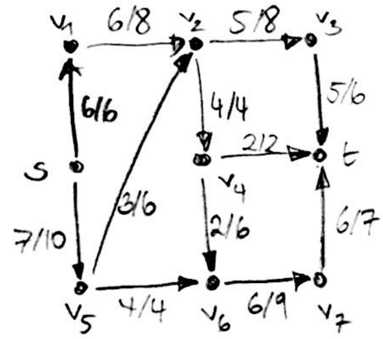
5. $R\neq V(G)$
2. Choose v_1 ; 3. $R=\{s,v_1,v_3,v_4\}$
4. $(v_1,t)\in E$, $t\notin R$: $\infty=\ell(t)>\ell(v_1)+c((v_1,t))=7+1$
   $\Rightarrow \ell(t)=8$, $pred(t)=v_1$

5. $R\neq V(G)$
2. Choose v_2
3. $R=\{s,v_1,v_2,v_3,v_4\}$
4. $(v_2,t)\in E$, $t\notin R$: $8=\ell(t)\not>\ell(v_2)+c((v_2,t))=7+1$
5. $R\neq V(G)$
2. Choose t ; 3. $R=\{s,t,v_1,v_2,v_3,v_4\}$; 4. /
5. $R=V(G)$ STOP

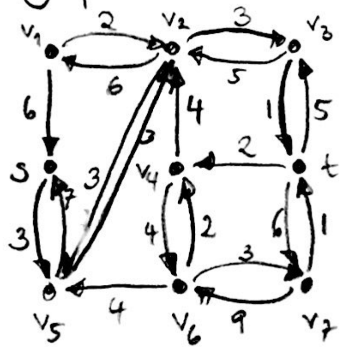
The shortest path from s to t has length 8, it is $s-v_1-t$.

Scanned by CamScanner

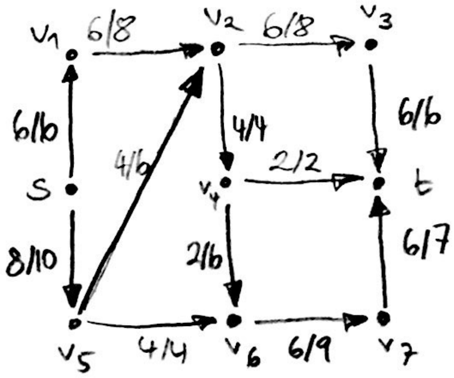
Problem 2:



(a) residual graph

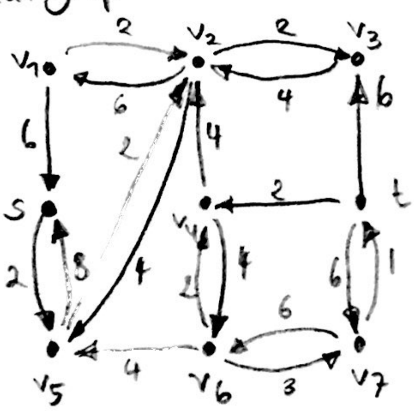


(b) f-augmenting path $s - v_5 - v_2 - v_3 - t$, $\gamma = 1$



(c) optimal?

residual graph



There exist no s-t-path in the residual graph (only vertices v_5, v_2, v_1, and v_3 can be reached from s)

$\Rightarrow$ the flow is optimal

value of the flow: 14

Problem 3:

Adam	Hannah	Emilie	Frida	Greta
Bert	Frida	Greta	Emilie	Hannah
Charles	Frida	Hannah	Greta	Emilie
David	Greta	Emilie	Hannah	Frida

Emilie	David	Adam	Charles	Bert
Frida	Adam	Charles	Bert	David
Greta	Adam	Bert	Charles	David
Hannah	David	Adam	Charles	Bert

(a) Gale-Shapley with women proposing

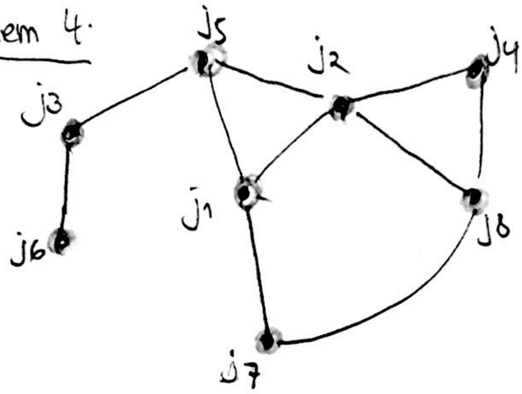
- Emilie unmatched (w = Emilie)
  m = David
  m is unmatched → (Emilie, David) engaged
- Frida unmatched (w = Frida)
  m = Adam
  m is unmatched → (Frida, Adam) engaged
- Greta unmatched (w = Greta)
  m = Adam
  m does not prefer w to his current match w^c = Frida
  ↳ Adam rejects Greta
  m = Bert
  m is unmatched → (Greta, Bert) engaged
- Hannah unmatched (w = Hannah)
  m = David
  m does not prefer w to his current match w^c = Emilie
  ↳ David rejects Hannah
  m = Adam
  m prefers w (Hannah) to his current match w^c = Frida
  ↳ (Hannah, Adam) engaged
  Frida unmatched
- Frida unmatched (w = Frida)
  m = Charles
  m is unmatched → (Frida, Charles) engaged

We obtain the pairs:
Frida, Charles
Hannah, Adam
Greta, Bert
Emilie, David

(b) Hannah suggested this matching. It is not stable, because Emilie + David build a blocking pair, that is, by changing from their current partner to this pair, they could both improve, and wouldn't stick w. the suggested assignment

Problem 4.

1)



We want to determine a coloring. Each color is an IS, and represents jobs that can be scheduled on the same day

$\Rightarrow$ # colors = # days

We use algorithm 6.9, it guarantees that we need at most 5 colors. ($\hat{=}$ 5 days).

1. $c(j_i) = \infty \ \forall i = 1, -, 8$
2.
   - j_1: $c(j_1) = \min(\mathbb{N}) = 1$
   - j_2: $c(j_2) = \min(\mathbb{N} \setminus \{1\}) = 2$
   - j_3: $c(j_3) = \min(\mathbb{N}) = 1$
   - j_4: $c(j_4) = \min(\mathbb{N} \setminus \{2\}) = 1$
   - j_5: $c(j_5) = \min(\mathbb{N} \setminus \{1, 2\}) = 3$
   - j_6: $c(j_6) = \min(\mathbb{N} \setminus 1) = 2$
   - j_7: $c(j_7) = \min(\mathbb{N} \setminus 1) = 2$
   - j_8: $c(j_8) = \min(\mathbb{N} \setminus \{1, 2\}) = 3$

$\hookrightarrow$ we need 3 colors, that is we can schedule for 3 days.

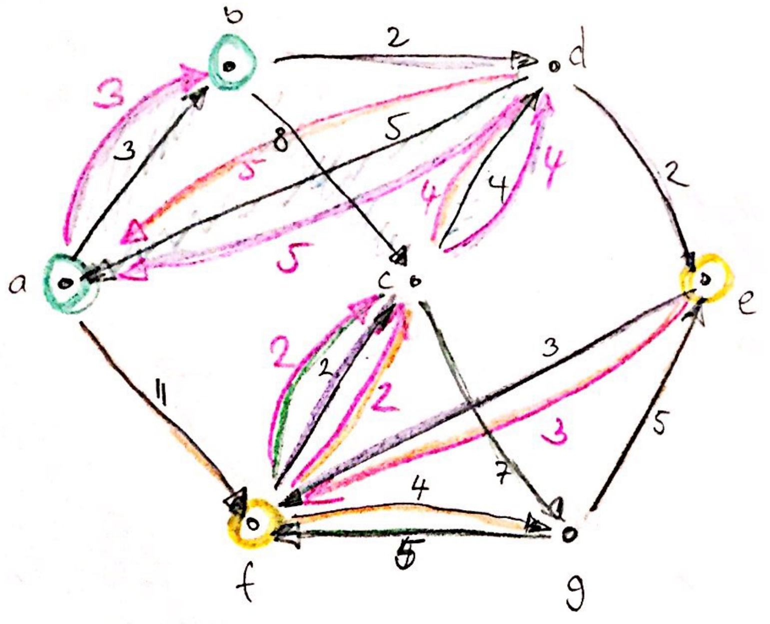
Problem 5:

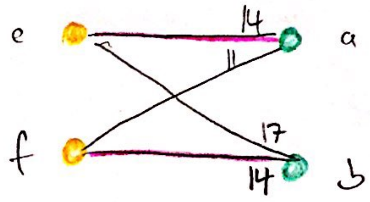
(a) outdegree ≠ indegree for a, b, e, f ⇒ does not exist

(b) Alg. 7.7

too few outgoing edges

too few incoming edges





→ all outdegree = indegree → use Hierholzer algorithm

1. Choose a, C = a, b, c, d, a
2. not a Eulerian cycle
3. Delete edges (a,b), (b,c), (c,d), (d,a)

   Choose b ∈ C, C' = b, d, e, f, c, d, a, b

   C = a, b, d, e, f, c, d, a, b, c, d, a

2. not a Eulerian cycle
3. Delete edges from C

   Choose d ∈ C, C' = d, a, f, g, e, f, c, d

   C = a, b, d, a, f, g, e, f, c, d, e, f, c, d, a, b, c, d, a

2. not a Eulerian cycle
3. Delete edges from C

   Choose c ∈ C, C' = c, g, f, c

   C = a, b, d, a, f, g, e, f, c, g, f, c, d, e, f, c, d, a, b, d, a

2. C is Eulerian cycle, STOP

length: 28 (added edges) + (3+2+8+5+4+2+11+2+7+3+4+5+5) (original edges) = 89

Problem 6:

```
k=0
FOR n=1 TO 10
    FOR m=1 TO 50
        k=k+1
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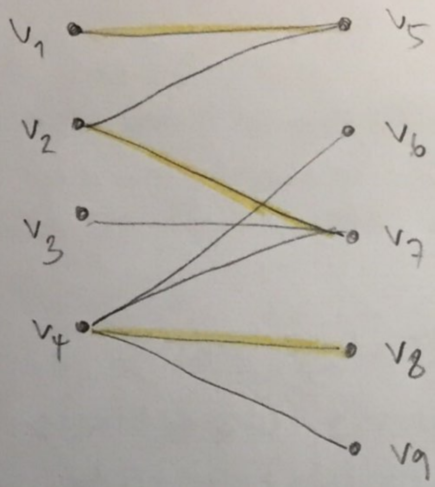
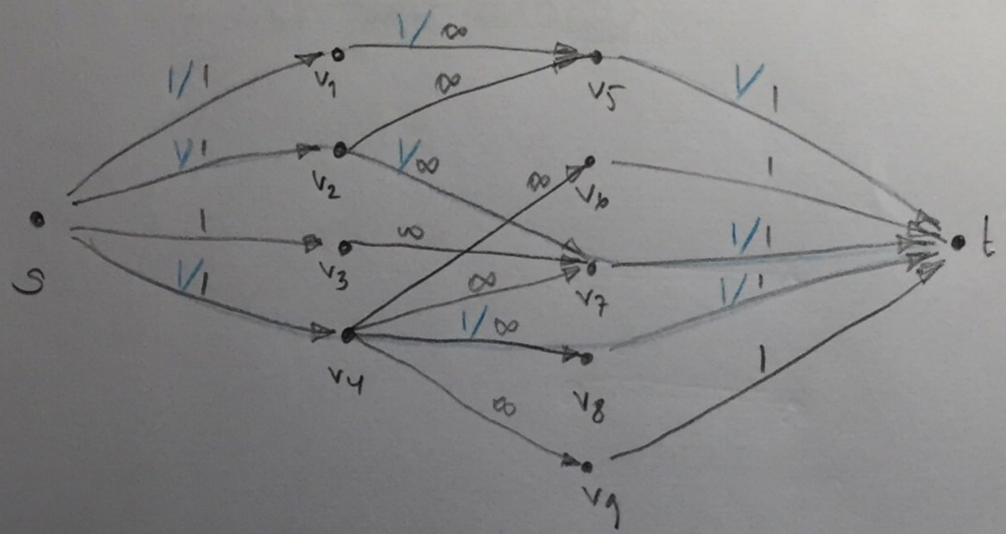
is called 500 times, each time started with k=0

⇒ k=500 is the final value of k

Problem 7:

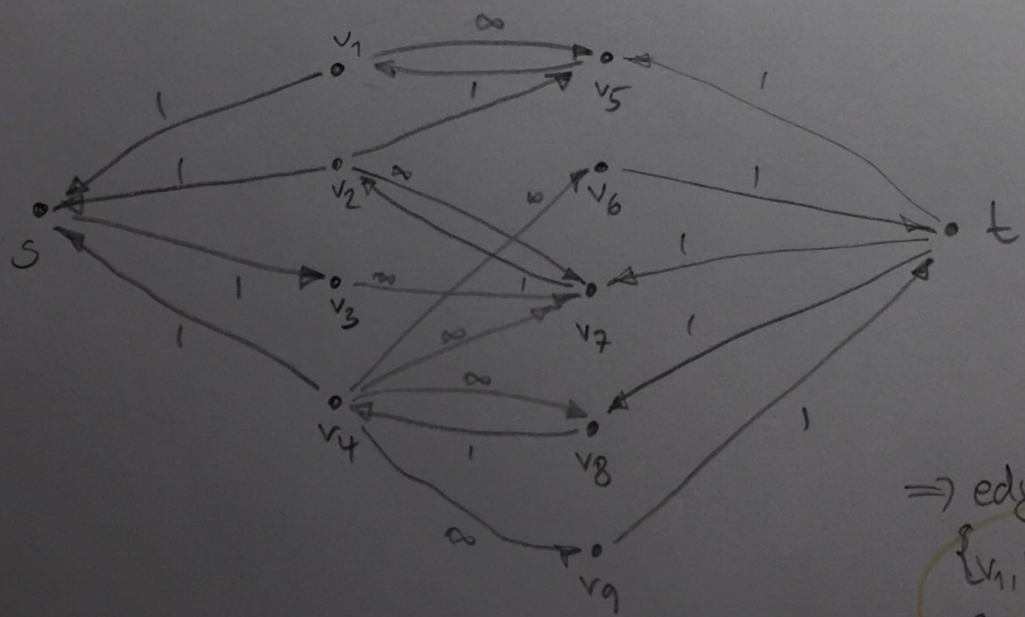


(a)



(b) Flow of value 3

resulting residual graph



no s-t-path in the residual graph → flow of 3 optimal

⇒ edges $\{v_1, v_5\}$, $\{v_2, v_7\}$, $\{v_4, v_8\}$ are matching edges in max. matching

Problem 8:

max
$b(v)$: bottleneck of a s-v-path, (1)
pred(v) as before

We adapt Dijkstra's algorithm:

very small, we want to find paths with larger and larger bottleneck

1. $b(v) = 0 \ \forall v \in V \setminus \{s\}$, $b(s) = \infty$, $R = \emptyset$
2. Find a vertex $v \in V(G) \setminus R$ with $b(v) = \max$
3. $R = R \cup \{v\}$
4. FOR $\forall w \in V(G) \setminus R$ with $(v,w) \in E(G)$ DO
   IF $(b(w) < \min(b(v), c((v,w)))$
   Set $b(w) = \min(b(v), c((v,w)))$
   $pred(w) = v$
5. IF $R \neq V(G)$ THEN GOTO 2.

if we go via v, we have to cover the path to v (with bottleneck $b(v)$) and the edge (v,w), as this edge could be the new bottleneck, we have to build the minimum of the two