Communications and Transport Systems
Department of Science and Technology **Fall 2021**
Linköping University

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Design and Analysis of Algorithms Part 1 - Mathematical Tools and Network Problems homework 2, 24.11.2021

Problem 1 (Trees - moved from Homework set 1):

(a) Prove Theorem 1.61 from the lecture.

(b) Prove Theorem 1.62 from the lecture.

(c) Prove Corollary 1.64 from the lecture.

Problem 2 (Directed cycles and directed cuts - moved from Homework set 1):
Show:
In a digraph G, each edge belongs either to a (directed) cycle or to a directed cut. Moreover, the following statements are equivalent:

(a) G ist strongly connected.

(b) G contains no directed cut.

(c) G is connected and each edge of G belongs to a cycle.

(Hint: Take a look at the statements you proved in Problem 2.)

Problem 3 (Heap Sort):
Prepare a 10 minute presentation of heapsort: the algorithm, its correctness and runnning time.

Problem 4 (Best-case running time for quicksort):
Proof Lemma 3.12 from the lecture, that is, the best-case running time for quicksort

Problem 5 (Merge sort):

Sort the sequence $(33, 14, 7, 9, 2, 11, 45, 21)$ using merge sort. Give the intermediate steps in appropriate form.

Problem 6 (Mastertheorem):

a) Determine the asymptotic growth of the following recursion using the master theorem

$U(n) = 4 \cdot U(\frac{n}{3}) + 17 \cdot n^2 + 20 \cdot U(\frac{n}{6})$.

Determine the value of all parameters used in the master theorem.

b) Determine the asymptotic growth of the following recursion using the master theorem

$V(n) = 14 \cdot V(\frac{n}{36}) + 23n + 12 \cdot V(\frac{n}{24}) + V(\frac{n}{10})$.

Determine the value of all parameters used in the master theorem.

c) Determine the asymptotic growth of the following recursion using the master theorem

$T(n) = 49 \cdot T(\frac{n}{7}) + 42n$.

Determine the value of all parameters used in the master theorem.

Problem 7 (Quicksort):

Sort the numbers in the following array using the algorithm quicksort presented in the lecture.

$$A[1] = 14 \qquad A[2] = 3 \qquad A[3] = 7 \qquad A[4] = 1 \qquad A[5] = 2$$

The reference element should be chosen as in the lecture (that is, $A[r]$). Give the array after **each** swap operation. Give the intermediate steps from Quicksort- and Partition calls.

Problem 8 (The Kevin Bacon oracle):
The *Kevn Bacon oracle* is based on the actor graph G: actors are given as vertices. Two actor vertices are connected by an edge if they appeared in a movie together. The vertex of Kevin Bacon has value 0; the *Kevin-Bacon number* (KBN) of another actor is the length of a shortest path in G. (Tom Hanks played with Kevin Bacon in Apollo 13, thus, he has Kevin-Bacon number 1.)
The oracle is available here: `http://oracleofbacon.org/`. The movie data it is based on is taken from the *Internet Movie Database*: `http://www.imdb.com`.
Our questions:

(a) Describe a strategy to definitely find an actor with a KBN as high as possible in G, even if you've never heard of Hollywood. On which graph algorithm is this strategy based?

(b) Find a vertex with KBN at least 4.

Problem 9 (Eulerian Path):

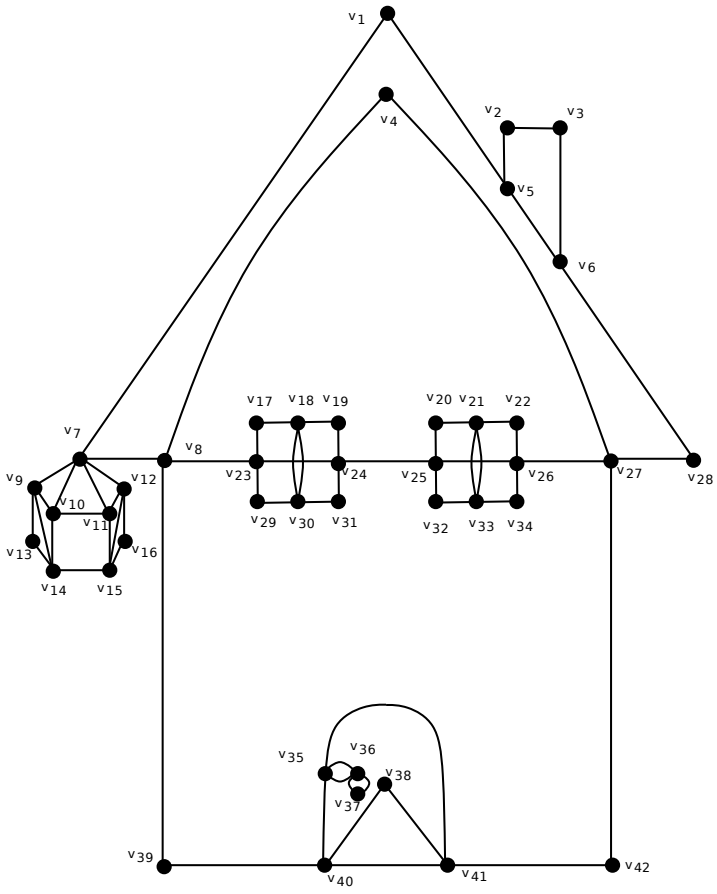

Abbildung 1: Euler on his way home!

Find a Eulerian path in the graph from Figure 1 or show that none exists.