Communications and Transport Systems
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Design and Analysis of Algorithms Part 2 - Approximation and Online Algorithms Homework 5, 03.03.23

Problem 1 (First-Fit-Decreasing for Bin Packing):
Show that the First-Fit-Decreasing Algorithm for Bin Packing presented in class has an approximation factor of 3/2.

Problem 2 (Bin Packing II):
Consider another algorithm for MIN BIN PACKING: the next fit algorithm. At each step there is exactly one open bin B_j. The next item is packed into B_j if it fits, otherwise, a new bin B_{j+1} gets opened, B_j gets closed and will never be opened again.

(a) Show that the next fit algorithm has an approximation factor of 2.

(b) Show that the bound from (a) cannot be improved.

Problem 3 (Greedy Set Cover Algorithm):
Apply the Greedy Set Cover Algorithm (Algorithm 5.5 from the lecture) to the following Set Cover instance:
$c(S_i) = |S_i| + 1$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$, and
$S_1 = \{1, 2, 3, 4\}$
$S_2 = \{5, 6, 7, 8\}$
$S_3 = \{9, 10, 11, 12\}$
$S_4 = \{13, 14, 15, 16\}$
$S_5 = \{17, 18, 19, 20\}$
$S_6 = \{1, 2, 3, 5, 6, 7, 9, 10, 11\}$
$S_7 = \{14, 15, 16, 17, 18, 19\}$
$S_8 = \{12, 13, 14, 15\}$
$S_9 = \{4, 5, 6\}$
$S_{10} = \{7, 8, 9\}$
$S_{11} = \{18, 19, 20\}$.

In case the maximum in step 2 is not uniquely defined, choose set S_i with minimum index.
What is the value of the computed set cover?
Can you give a better set cover?

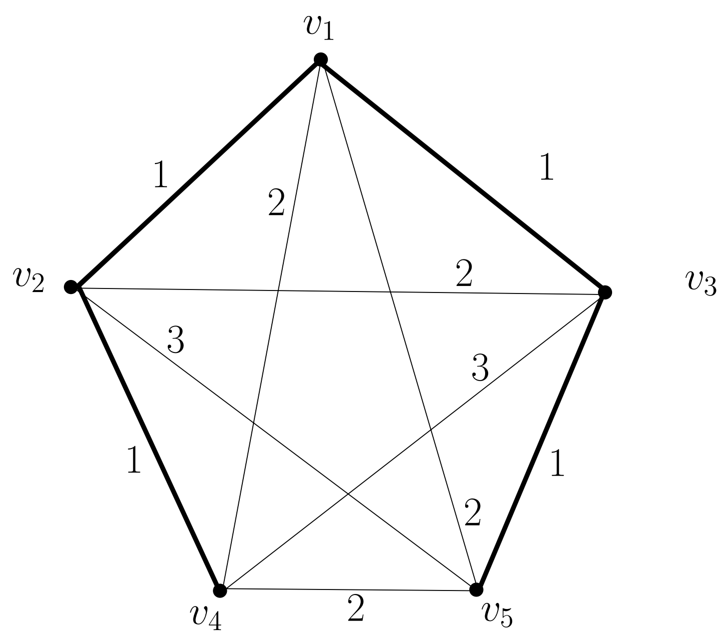

Figure 1: Graph H. An MST rooted in v_1 is shown in bold.

Problem 4 (4/3-approximation for $(1,2)$-TSP):
Consider a complete undirected graph G in which all edges have length either 1 or 2 (G satisfies the triangle inequality!). Give a 4/3-approximation for this special TSP variant.
Hint: Start with a minimum 2-matching in G. A 2-matching is a subset M_2 of edges so that every vertex in G is incident to exactly two edges in M_2. Note: a 2-matching can be computed in polynomial time.

Problem 5 (Bottleneck TSP):
Take a graph G with edge costs that satisfy the triangle inequality. We want to find a Hamiltonian cycle C for which the maximum cost edge in C is minimized.

(a) Give a 3-approximation algorithm for this problem.
Hints: (i) Consider the MST of G. (ii) Think about "appropriate" shortcuts.

(b) Apply your algorithm to the graph H from Figure 1, using the given MST.