Flight schedule

- SAS 2014
  - 807 flights/day
  - 125 destinations
  - 138 a/c (aircraft)
  - 5 a/c types
- Large = hard problem
- Exercise:
  - Determine which aircraft should fly which flight every day during a season
- How?
  - Split up the problem

Discuss: How can the problem be decomposed and solved?

Flight schedule - problem decomposition

- Geographical
  - Domestic – international
    - Advantage at airport because of different terminals and gates
    - Schengen or not
    - Simplifies crew planning
  - Must take maintenance and crew into account
- Different companies
- Cargo or passengers
- Aircraft types
  - Partition fleet in subgroups of interchangeable aircraft
  - Assign flights to subfleets without determining how and when which aircraft should fly **fleet assignment**
  - Create routes for fleets **aircraft routing**

Four successive aircraft and crew schedule problems

1 Timetable planning

2 Fleet assignment

3 Aircraft routing

4 Crew pairing

Fleet assignment and aircraft routing

TGAI Chapter 7.33

FA=Fleet assignment=Assign aircraft types to flight legs

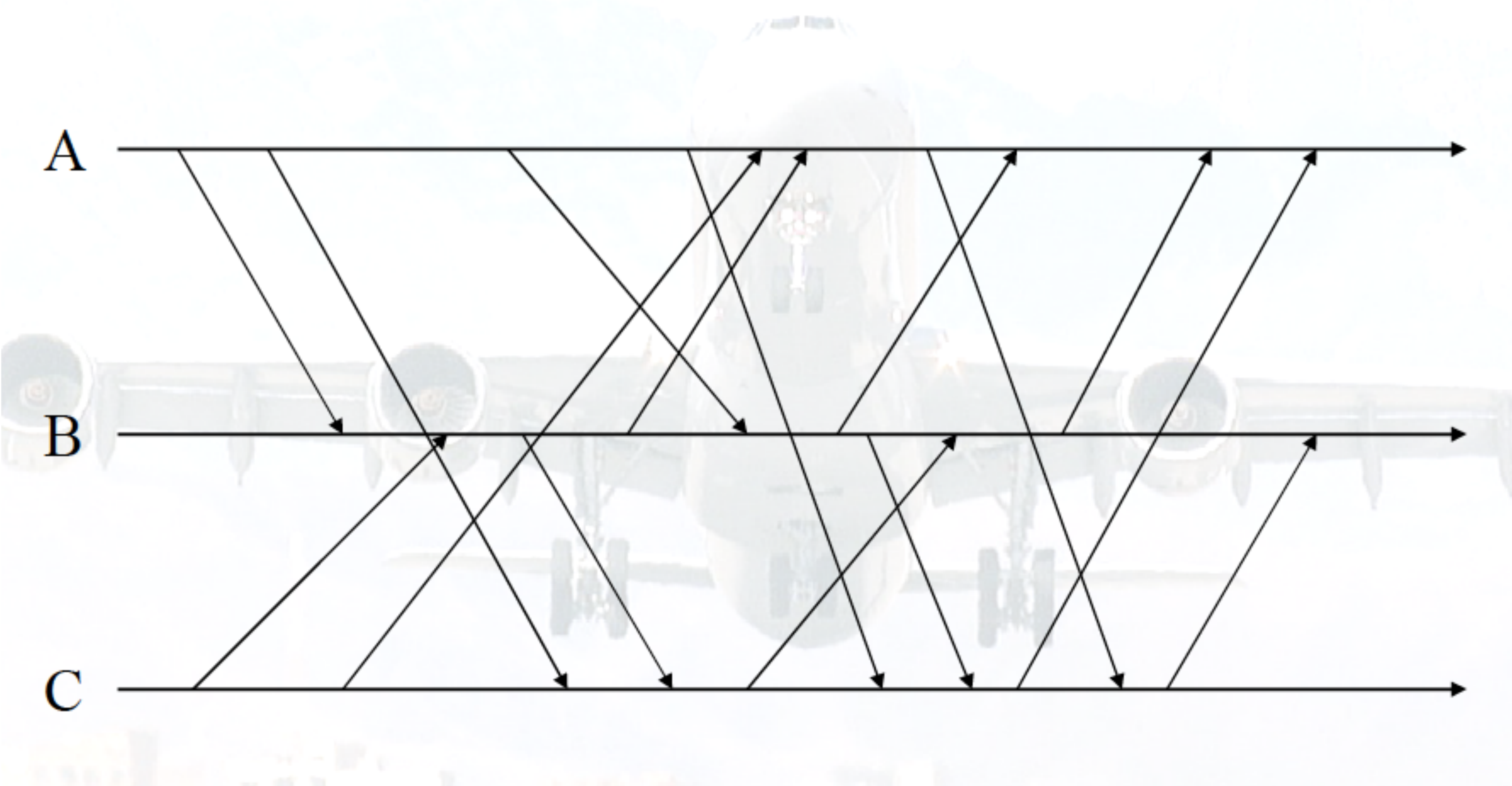
- Input: A schedule, flights cost (depending on demand and airplane type), fleet sizes
- Output: A fleeting
  - Goals of FA:
    - Max pax
    - Min costs
    - Robustness
  - Requirements:
    - Balance
    - Airport Limitations
    - Maintenance
    - Crew
  - Routing problem for each subfleet

TGAI Chapter 8.2

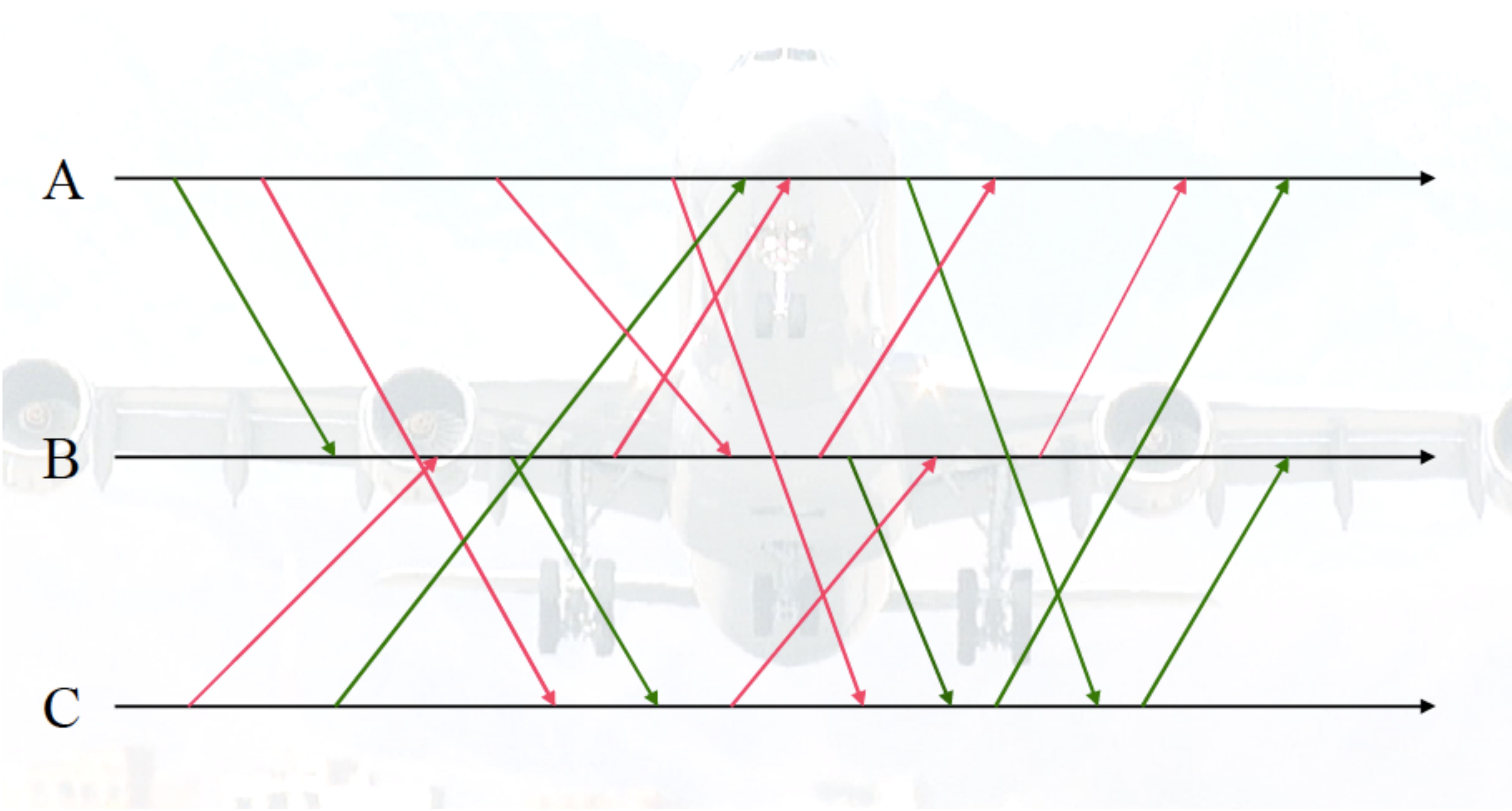
Aircraft routing: determining a route for each aircraft

- Input: A one-fleet schedule, maintenance constraints, border conditions
- Output: A feasible routing
  - Goals of FR:
    - Feasibility
    - Robustness
  - Requirements:
    - Maintenance
      - Timer
  - Routes for each subfleet

Fleet assignment: example



Fleet assignment: example



Aircraft routing

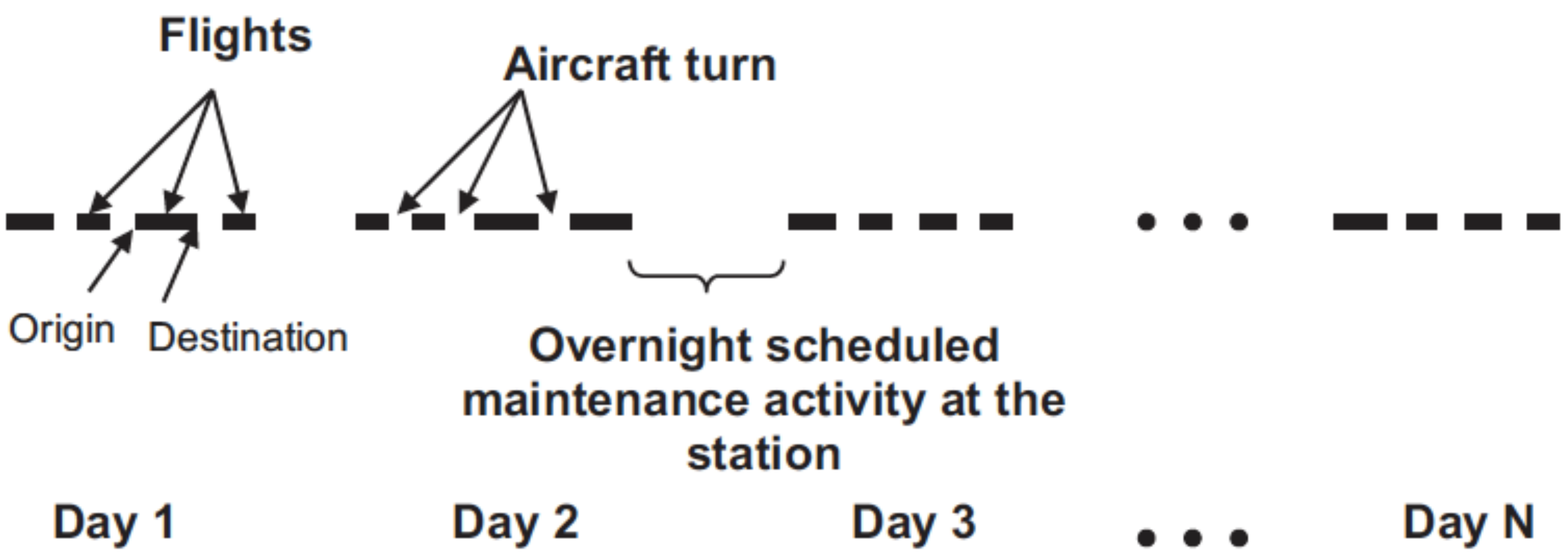


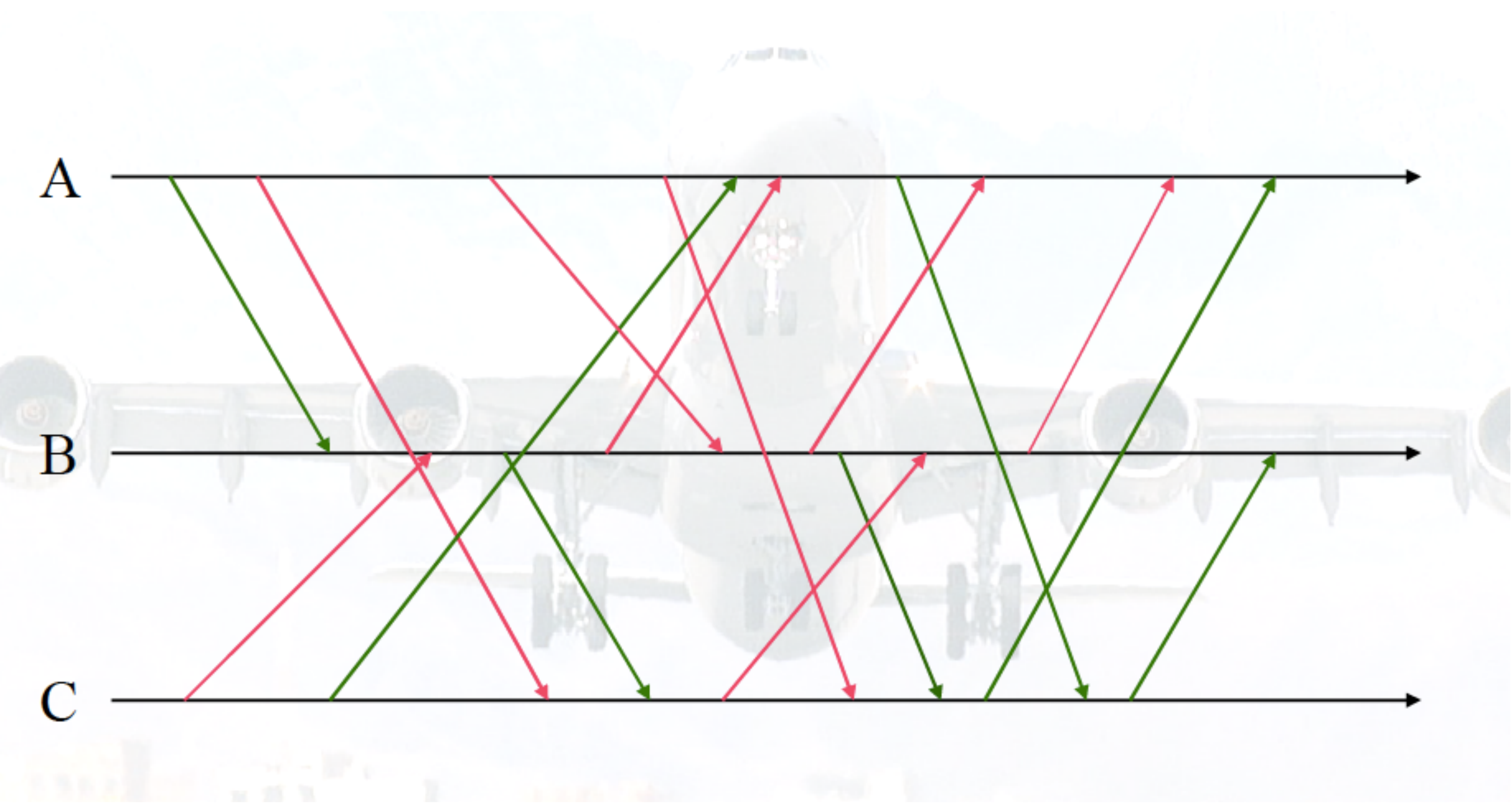
Figure 1.8 Example of an aircraft route

Abdelghany&Abdelghany, 2010

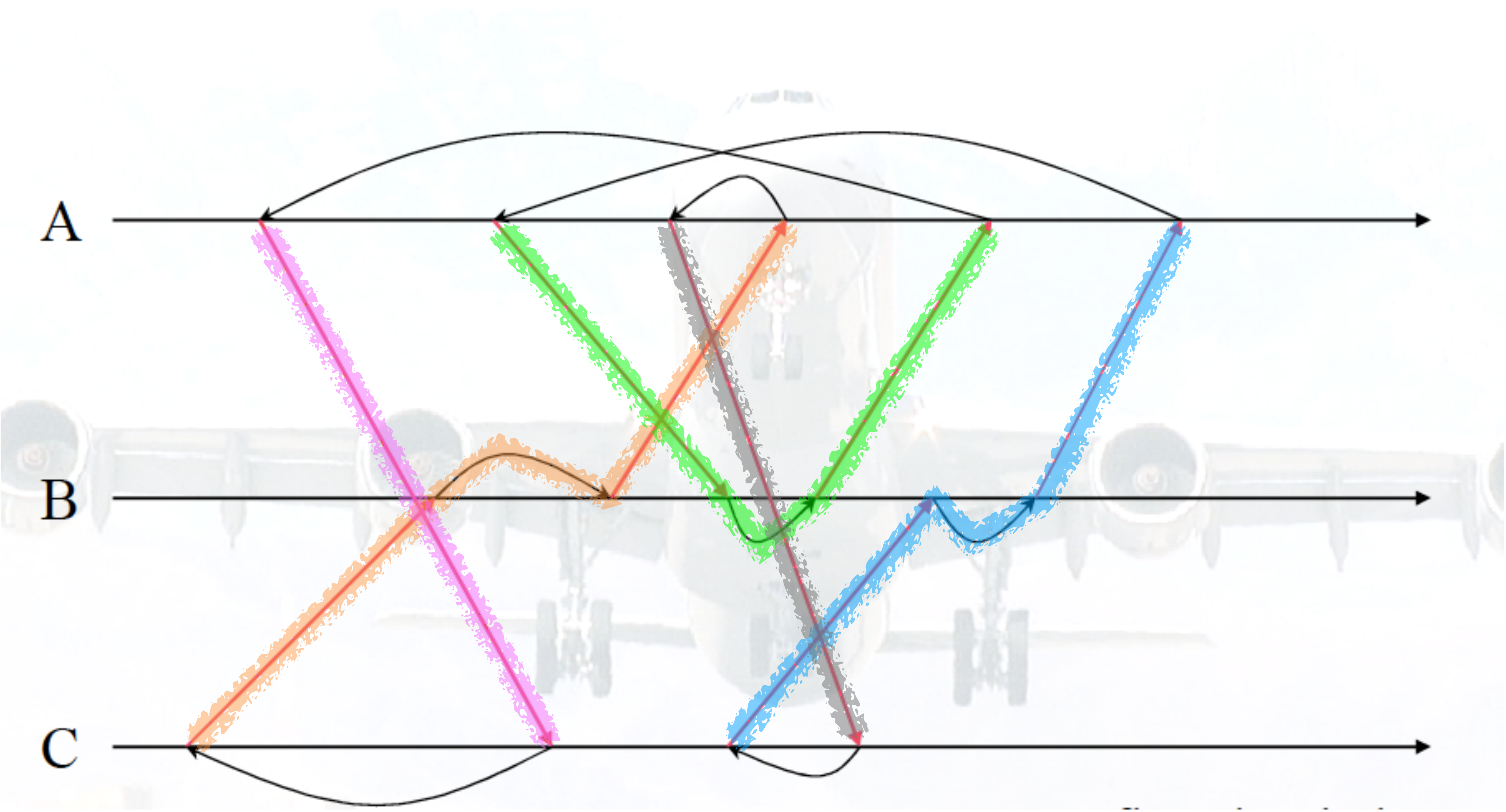
- Route of aircraft consists of:
  - Sequence of flights
  - Maintenance activities
- Extend over a few days
- Flights are selected to ensure enough time between them to complete aircraft turn or maintenance activity

Aircraft turn: time difference between arrival time of a flight and departure time of the next flight.

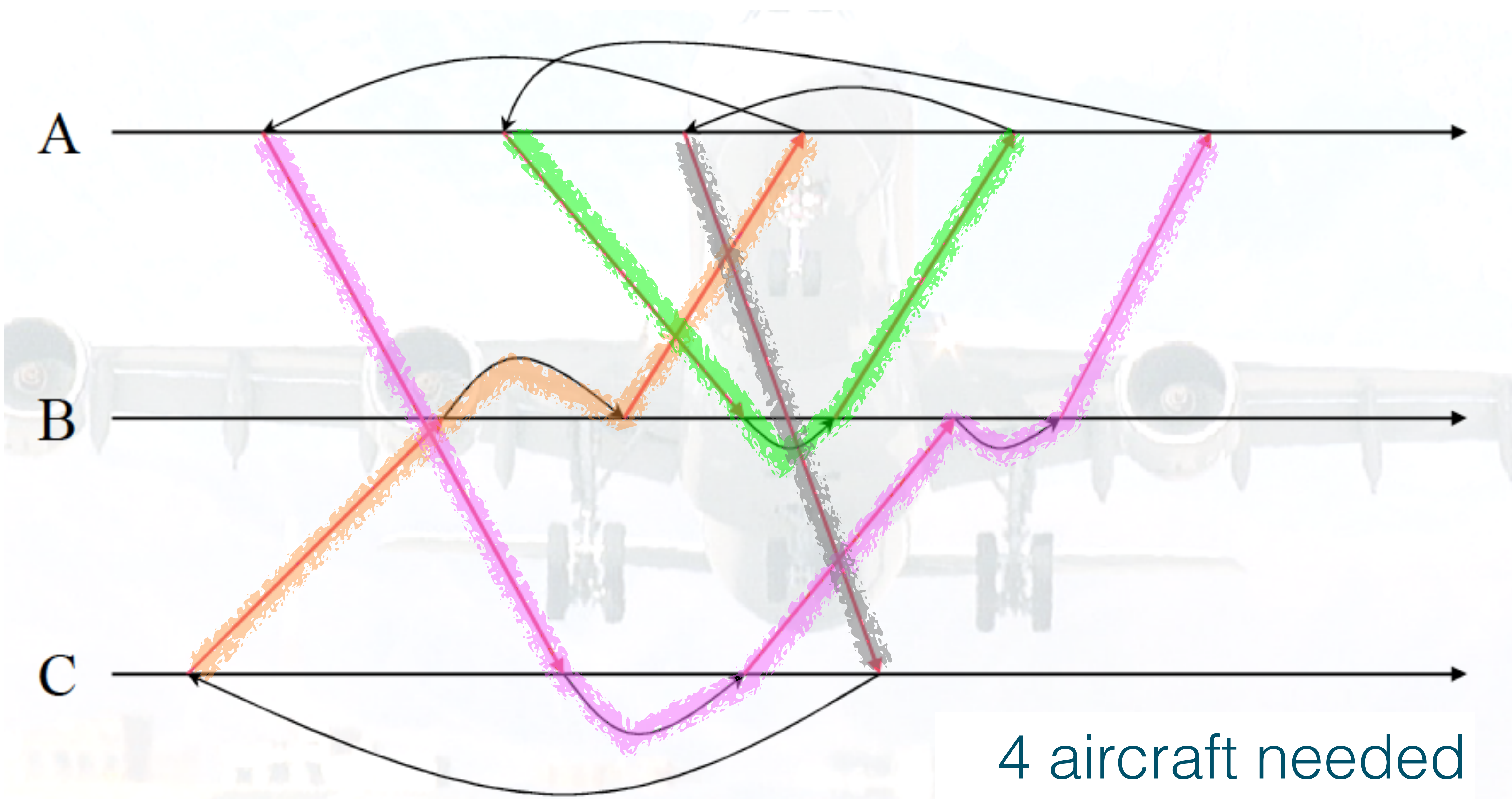
Fleet assignment: example



Aircraft routing: an example



5 aircraft needed



4 aircraft needed
better routing?

Maintenance

- Regular check and service
- Requirement from civil aviation authorities (CAA): FAA, EASA,…
- Usually: each airline develops own CAA-approved maintenance program
- Executed at:
    - Maintenance base (largest, most versatile, best-equipped facility)
    - Major station (incl. large hub cities, substantial inventory of spare parts, extensive facilities)
    - Service station (large stations, not at major hub cities, well equipped and staffed, less than major stations)

Maintenance

Maintenance types:

- Visual inspection
  - Prior to flight (sometimes called " walk-around")
  - Ensure no obvious problems: leaks, missing rivets, cracks
- Overnight maintenance
  - End of working day
  - Ad hoc repairs
  - 1 – 1.5 hours
- A-check
  - Appx. every 125 flight hours (2 – 3 weeks)
  - Amplified visual inspection, easily reachable parts
- B-check
  - Appx. every 750 flight hours
  - Exterior wash, engine oil spectro-analysis, oil filters re ame carefully examined
  - Incorporates A-check
- C-check
  - appx. every 3000 flight hours or 15 months
  - Incorporates both A- and B-check
  - Plus: components repaired, flight controls chanisms tested, …
- D-check
  - Most intensive form
  - Every 6-8 years/appx. every 20000 flight hours
  - Cabin interiors removed —> careful structural inspections
  - 15-30 days

"line" maintenance:
at airport
usually overnight

"heavy" maintenance:
special facilities
extensive downtime

Maintenance

Maintenance types:

- Non routine Maintenance
  - Unforeseen event (accident, random occurrence)
  - Response to AD (Airworthiness Directive)

Maintenance

Planning:

- Timers used, e.g., A-timer
- If the check is not performed in time the aircraft can be grounded
- Maintenance must be carefully included in flight schedule

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Dichotomy of Demand and Supply

You are working for a large, international airline. In conversation with a representative of a large dairy company at a conference, said representative asks you to quantify demand and supply on the route Arlanda-Newark. He is surprised to hear that you cannot easily quantify the demand and supply, as he easily can for, for example, milk with 3,25% fat in Stockholm in January. Give the dairy representative a detailed explanation on dichotomy of demand and supply in the airline industry.

The assignment should be done individually.

Homework #1, question 2

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Planning of aircraft routes

A small Swedish airline focusing on domestic traffic has the following timetable:

Flightnr	Dep time	Arr time	Dep AP	Arr AP	E[Pax]	R
1	450	900	A	L	16	500
2	1000	1230	A	G	18	300
3	1020	1410	A	L	25	500
4	1810	2200	A	L	49	500
5	510	840	L	G	12	400
6	1030	1225	L	U	21	350
7	1510	1810	L	G	55	400
8	2020	2350	L	A	24	400
9	615	800	U	A	21	200
10	1545	1740	U	A	23	200
11	1745	1930	U	L	19	250
12	2000	2310	U	G	17	500
13	430	710	G	A	12	400
14	920	1250	G	U	24	500
15	1330	1640	G	U	53	500
16	1920	2250	G	U	11	500

The timetable is cyclic, with a cycle time of one day. This means that each flight in the table should be own once each day (including weekends).

Fleet. The aircraft fleet consists of two types of aircraft, two Jetstream 31 (J31) and four Fokker 50 (F50). The F50 has a capacity for 50 passengers and requires 50 minutes from landing until it can start again (i.e. turn-around time). The J31 can take 18 passengers and needs 30 minutes of turn around time. The airline approximates the operating cost as 1000 per hour in for the J31 and 1500 for the F50 aircraft.
Maintenance. The same rules for maintenance applies to both aircraft types. After a maximum of 30 hours in flight, a maintenance check has to be performed. This takes five hours. The maintenance base for the J31 is located at airport A, while the base for the F50 fleet is located at airport L.
Assignment. Your assignment is to create a feasible aircraft schedule for the next summer season (5 months, May-Sept). The objective is to maximise profit.

Write a simple report describing how you solved the problem, presenting your solution, and discussing advantages and disadvantages with the schedule.

The assignment should be done in groups of 2-3 students.

Airlines #2
Fleet Assignment

Fleet assignment and aircraft routing

TGAI Chapter 7.33

Reminder

FA=Fleet assignment=Assign aircraft types to flight legs

- Input: A schedule, flights cost (depending on demand and airplane type), fleet sizes
- Output: A fleeting
  - Goals of FA:
    - Max pax
    - Min costs
    - Robustness
  - Requirements:
    - Balance
    - Airport Limitations
    - Maintenance
    - Crew
  - Routing problem for each subfleet

TGAI Chapter 8.2

Aircraft routing: determining a route for each aircraft

- Input: A one-fleet schedule, maintenance constraints, border conditions
- Output: A feasible routing
  - Goals of FR:
    - Feasibility
    - Robustness
  - Requirements:
    - Maintenance
      - Timer
  - Routes for each subfleet

Problem formulation

Input

- Schedule

For each flight:

  - O, D
  - departure time
  - ...
- Fleet
  - number of different-type aircraft
    - e.g.: 2 A310, 1 A340, 3 B747

Output

- assignment of aircraft *types* to flights

Problem Definition

Given:

- Flight Schedule
  - Each flight covered exactly once by one fleet type
- Number of Aircraft by Equipment Type
  - Can't assign more aircraft than are available, for each type
- Turn Times by Fleet Type at each Station
- Other Restrictions: Maintenance, Gate, Noise, Runway, etc.
- Operating Costs, Spill and Recapture Costs, Total Potential Revenue of Flights, by Fleet Type

Problem Objective

Find:

- Cost minimizing
- NB: sometimes a max problem (profit maximizing)
- assignment of aircraft fleets to scheduled flights such that maintenance requirements are satisfied, conservation of flow (balance) of aircraft is achieved, and the number of aircraft used does not exceed the number available (in each fleet type)

Fleet (=color) assignment

Input

- Schedule

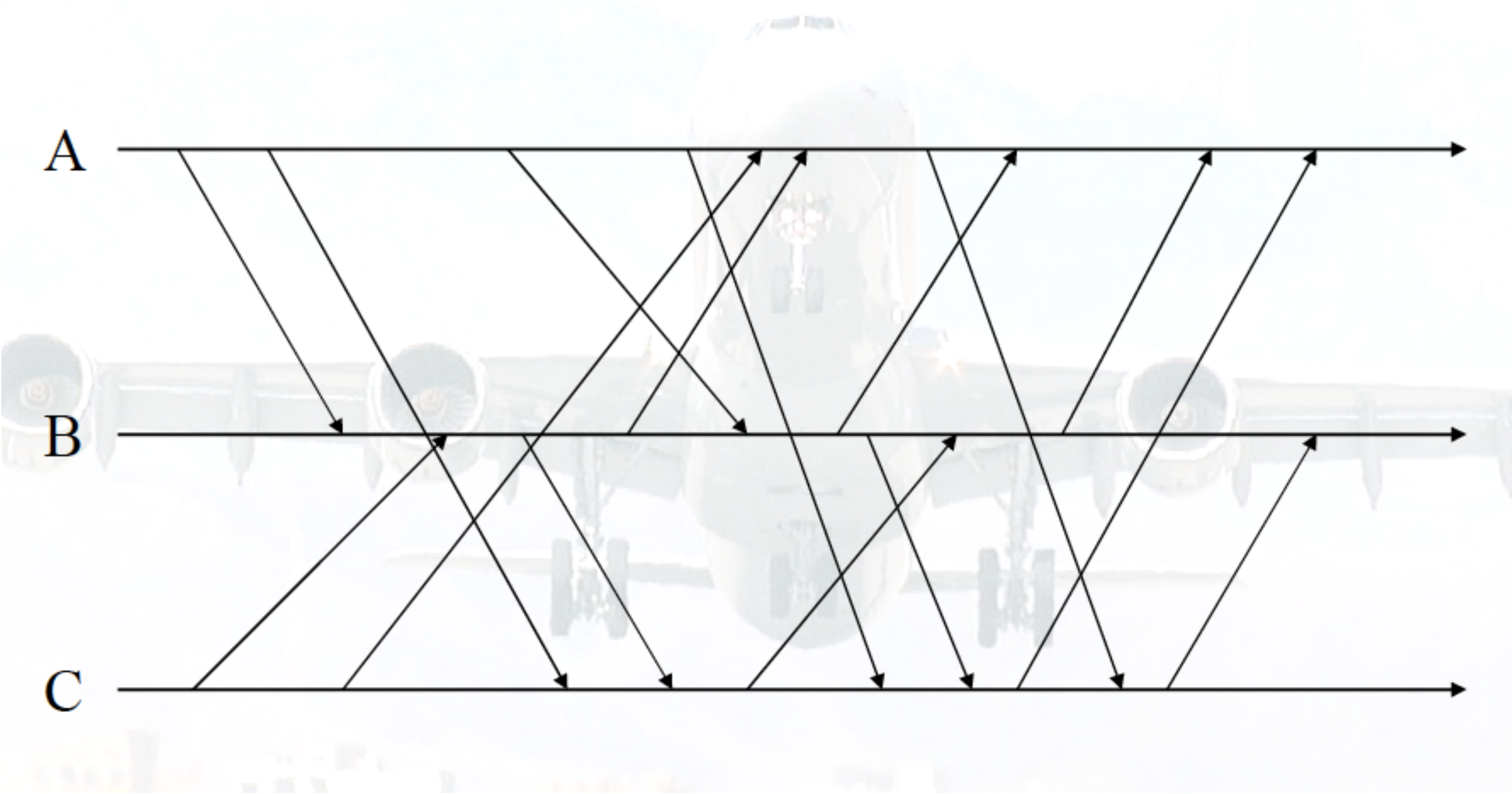
For each flight:

  - O, D
  - departure time
  - # of pass
- Fleet
  - number of different-type aircraft
    - e.g.: **2 A310**, **1 A340**, **3 B747**

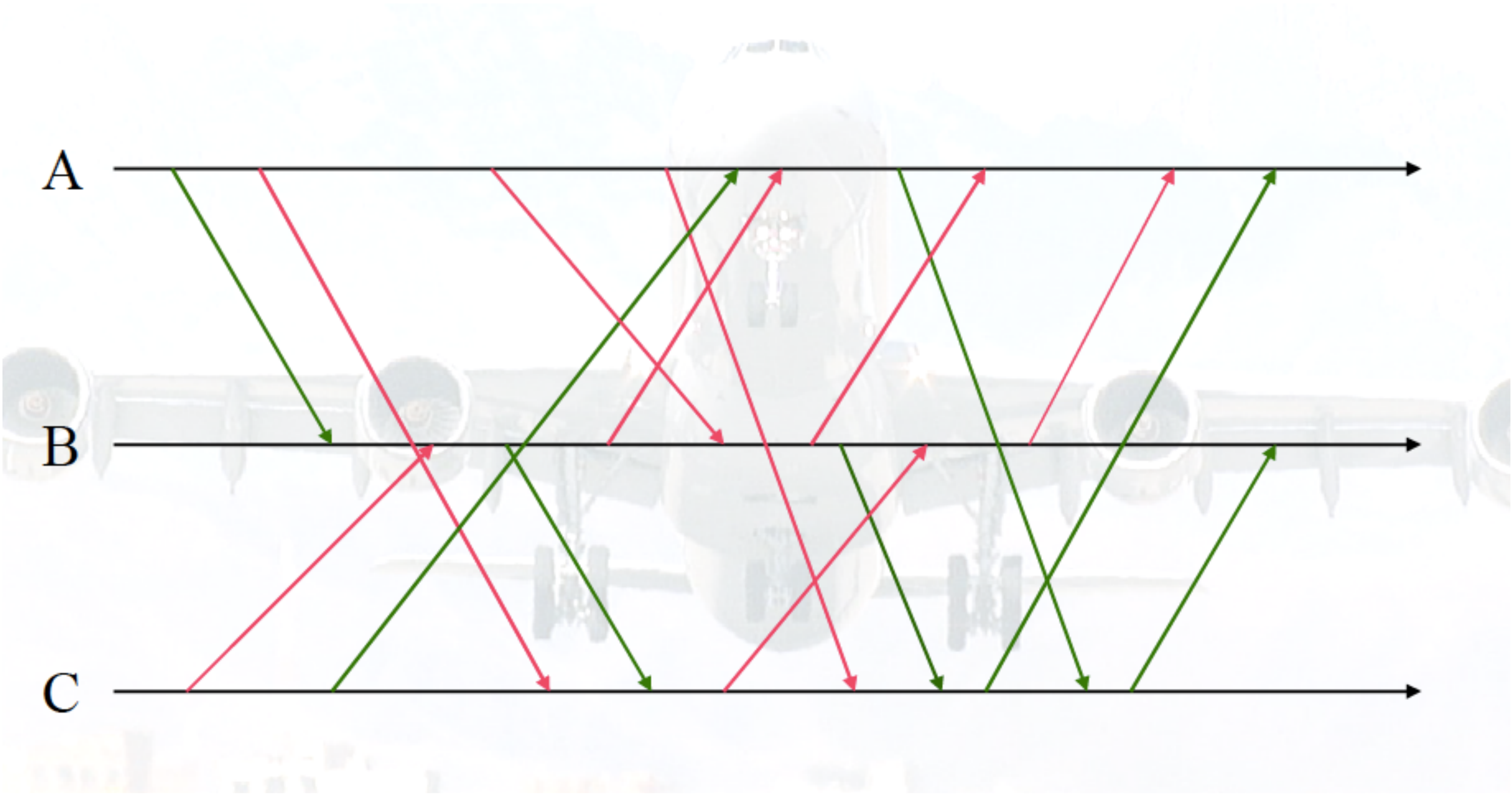
Output

- assignment of **colors** to flights

Fleet assignment: example



Fleet assignment: example



Simple case:
Airline with Single Fleet Type
(= single color)

Super simple case:
Airline with a single aircraft

Fleet (=color) assignment

Problem void?

Input

- Schedule

For each flight:

  - O, D
  - departure time
  - # of pass

- Fleet
  - number of different-type aircraft
    - e.g. 1 B747

Solution: all flights served by the same (one and only) aircraft (type)

Output

- assignment of plane types (**colors**) to flights

Still many interesting questions!

Feasibility

- Is the given schedule realizable?
  = Given the available plane,
  is it enough to implement the schedule?

Optimization

- Maximize the number of flights flown
  - assuming not all flights must be flown

Schedule

turn-around time = 30min

Flight number	Departure time	Arrival time	Departure airport	Arrival airport
1	0550	0750	B	C
2	0930	1125	C	A
3	1400	1600	A	B
4	1700	1915	B	A
5	2100	2300	A	B

Schedule

turn-around time = 30min

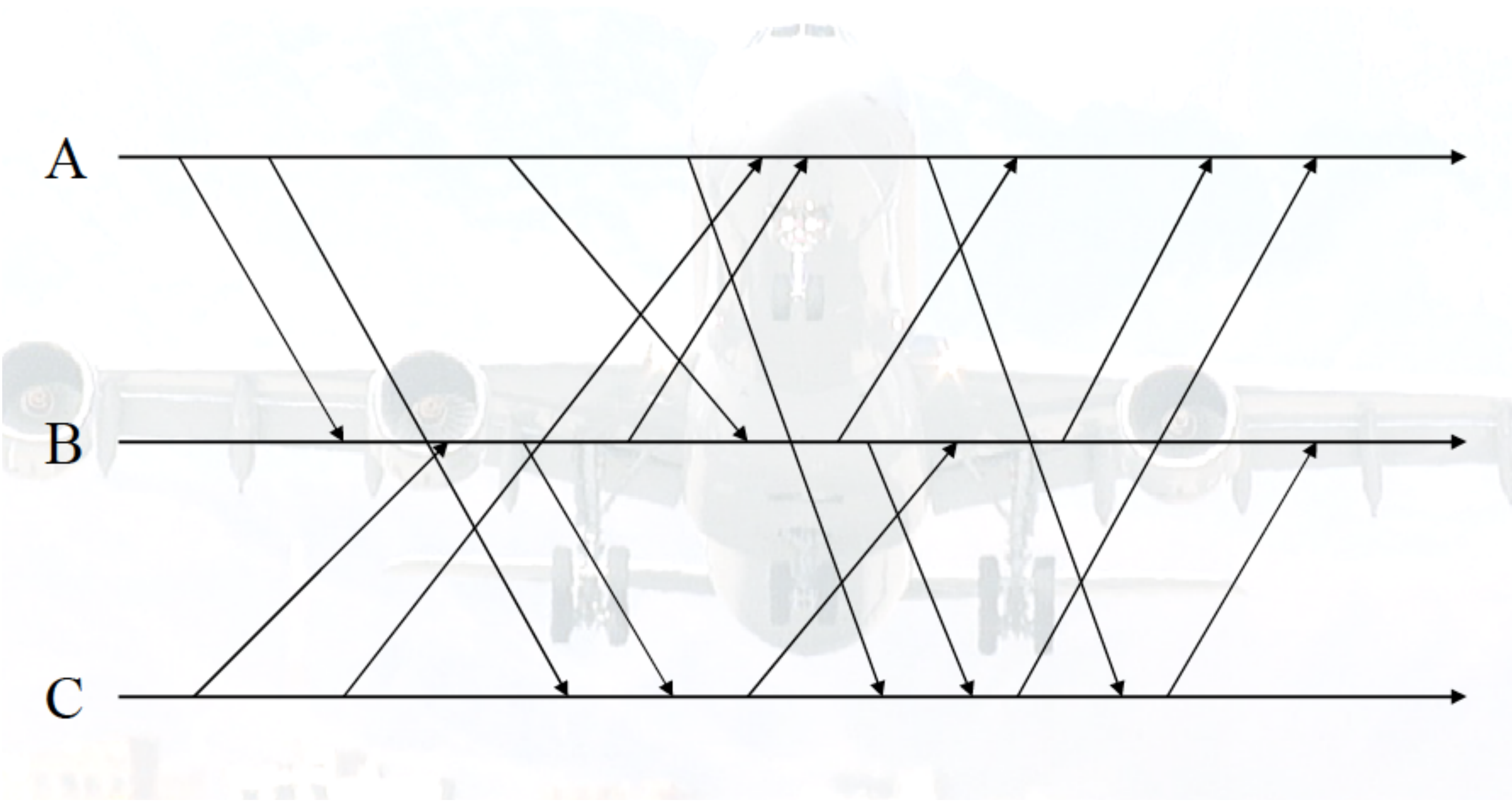
Flight number	Departure time	Arrival time	Departure airport	Arrival airport
1	0550	0750	B	C
2	0930	1125	C	A
3	1400	1600	A	B
4	1700	1915	B	A
5	2100	2300	A	B

Time-expanded network

time-staged network, time-line network

Arc per flight

Fleet assignment: example



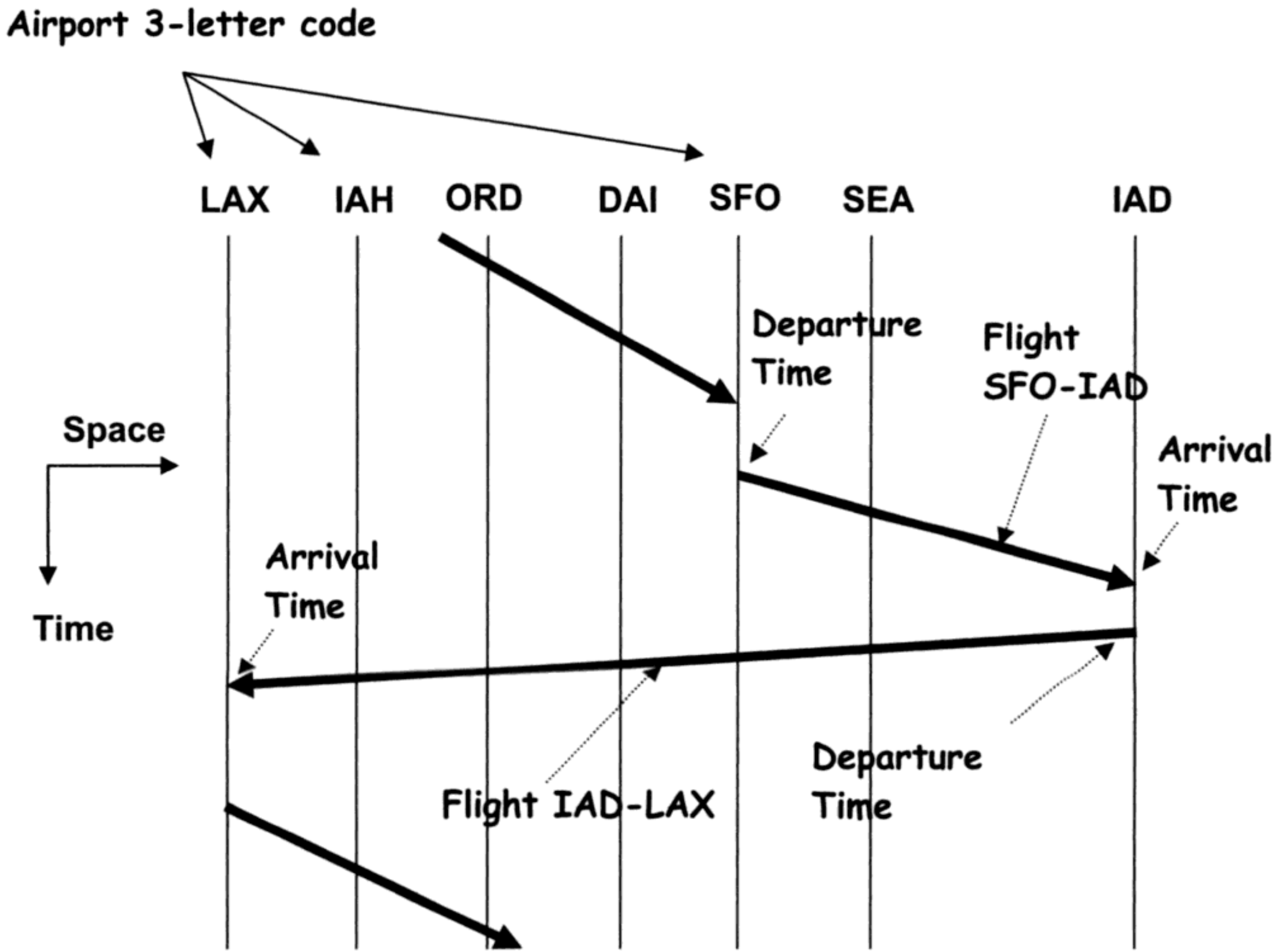
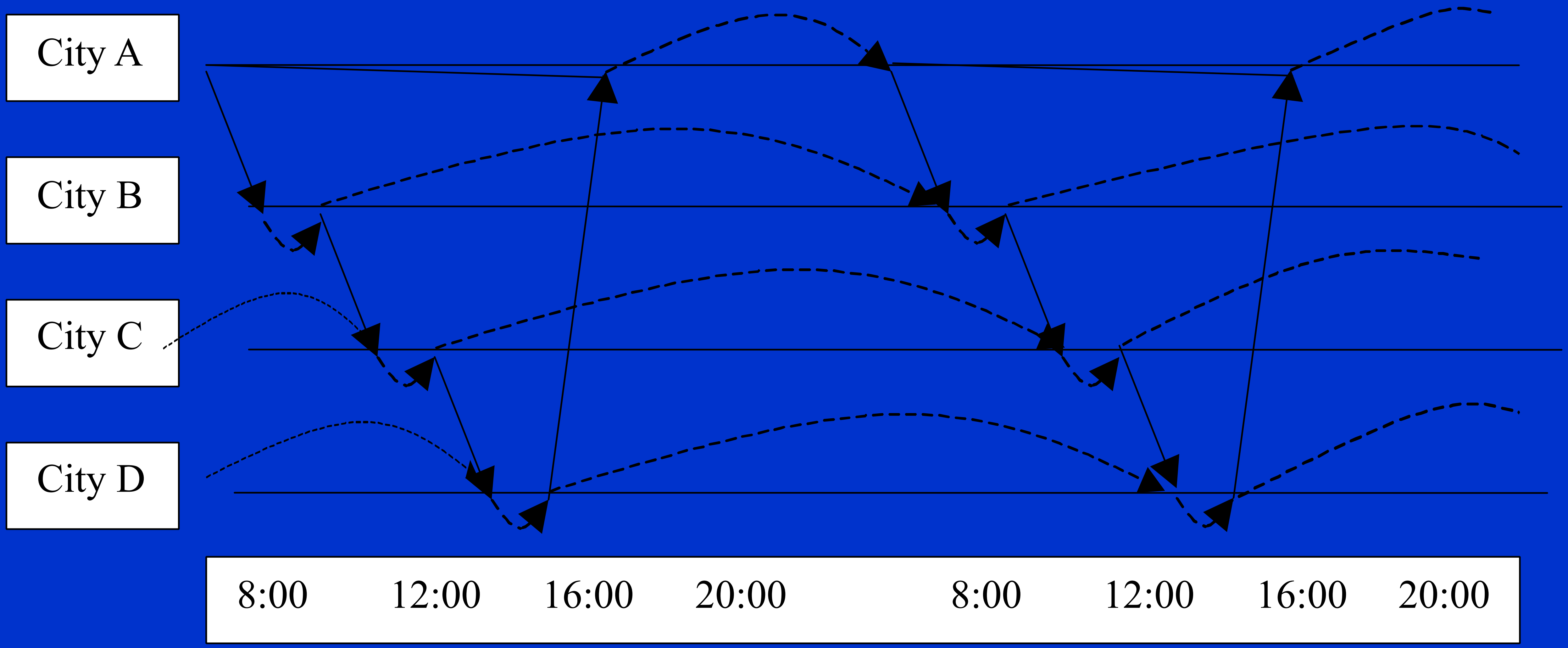


Figure 4.1 Time-staged flight network

Time-Line Network

- Ground arcs



xyt

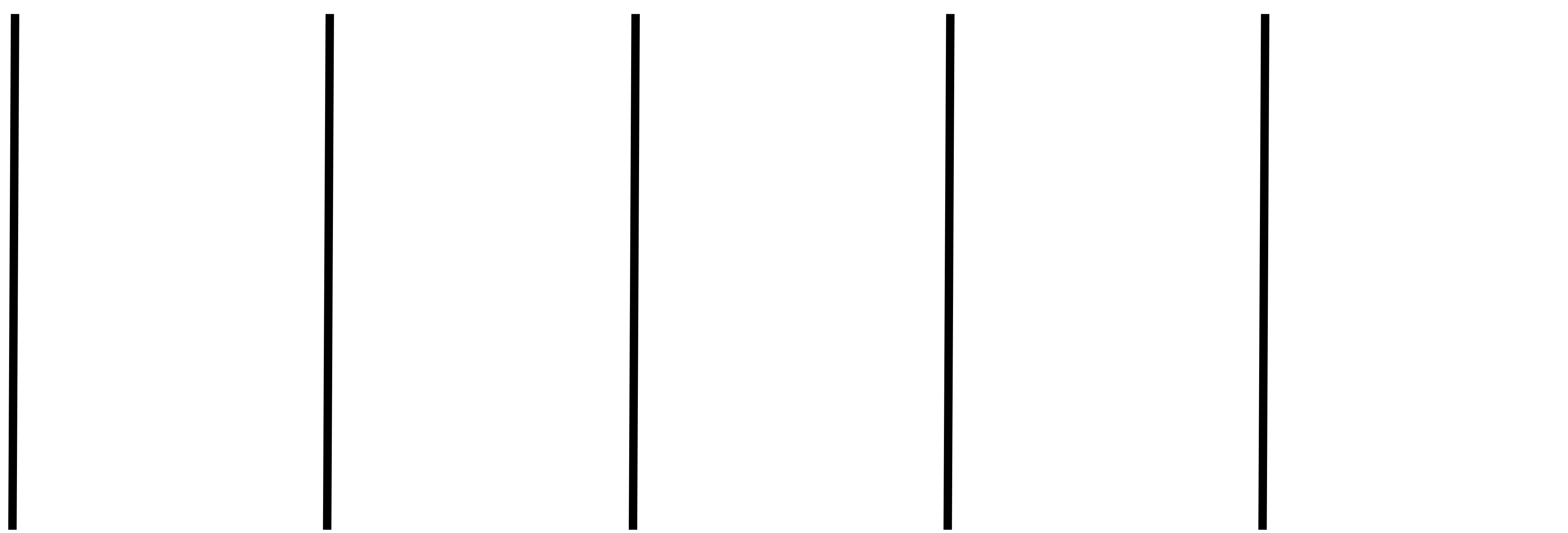
Line per airport

Time line

- time-line network

This course:

- Vertical line
- Time goes down



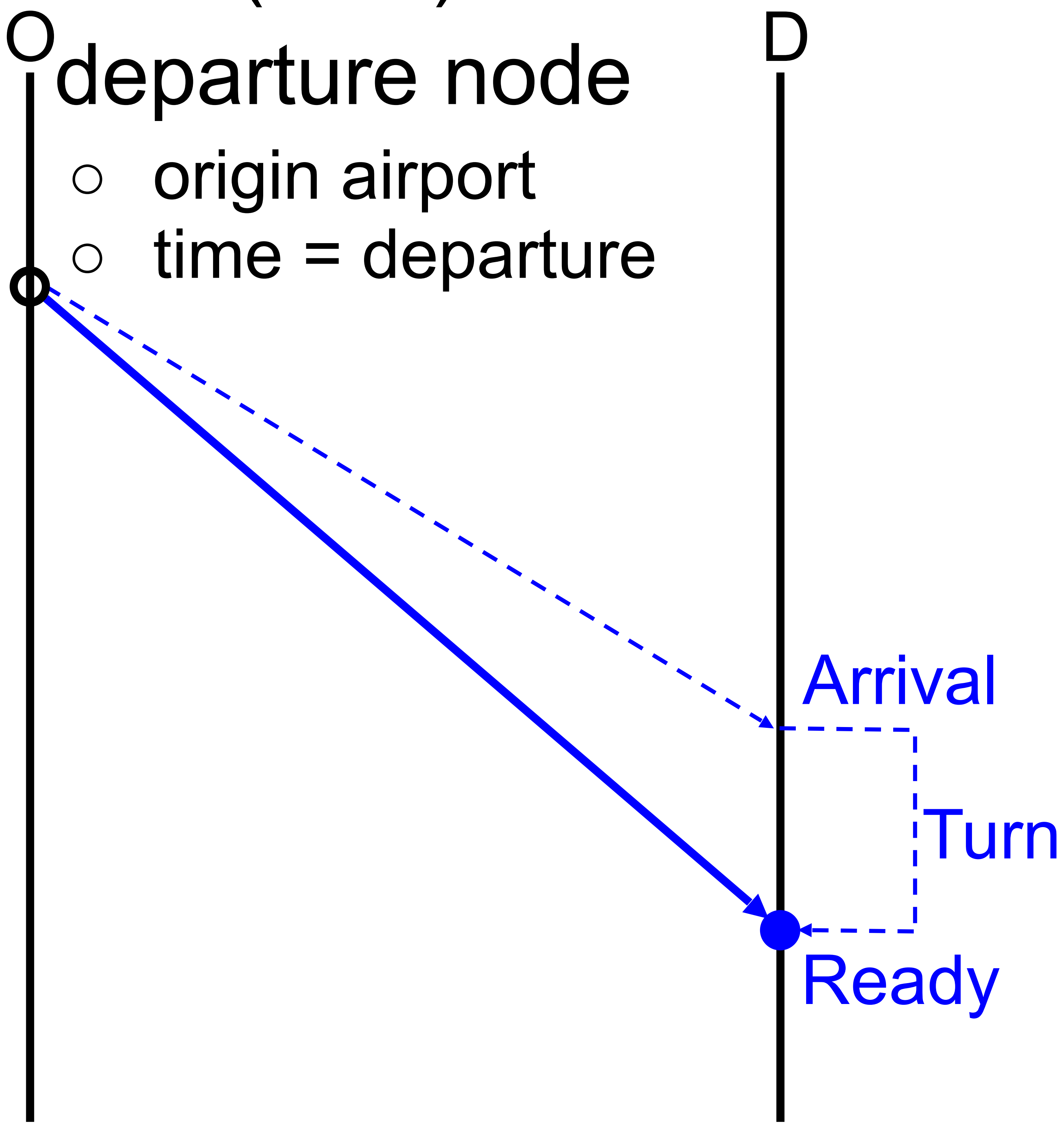
flight arcs representing flight legs

Directed arc (arrow) per flight

Tail (start):

departure node

- origin airport
- time = departure



Head (end):

"arrival" node

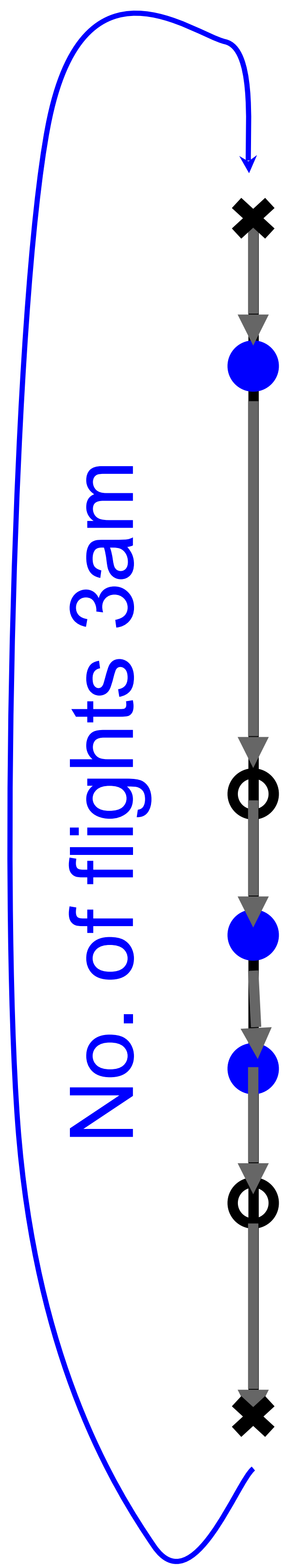
- destination airport
- time = ready time = arrival time + turn around

ready node

Ready-time network

representing aircraft staying at the same station for a given period of time

RON arcs



Ground arcs

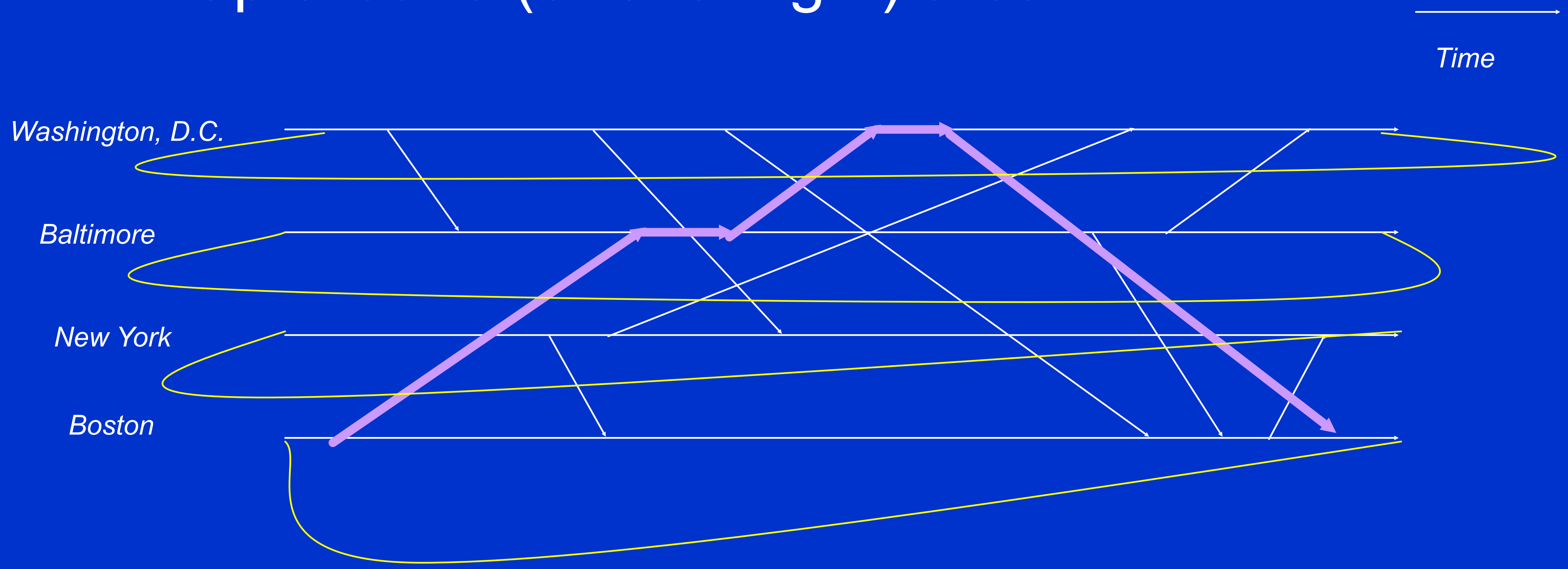
Not in this course
(minimal non-trivial model)

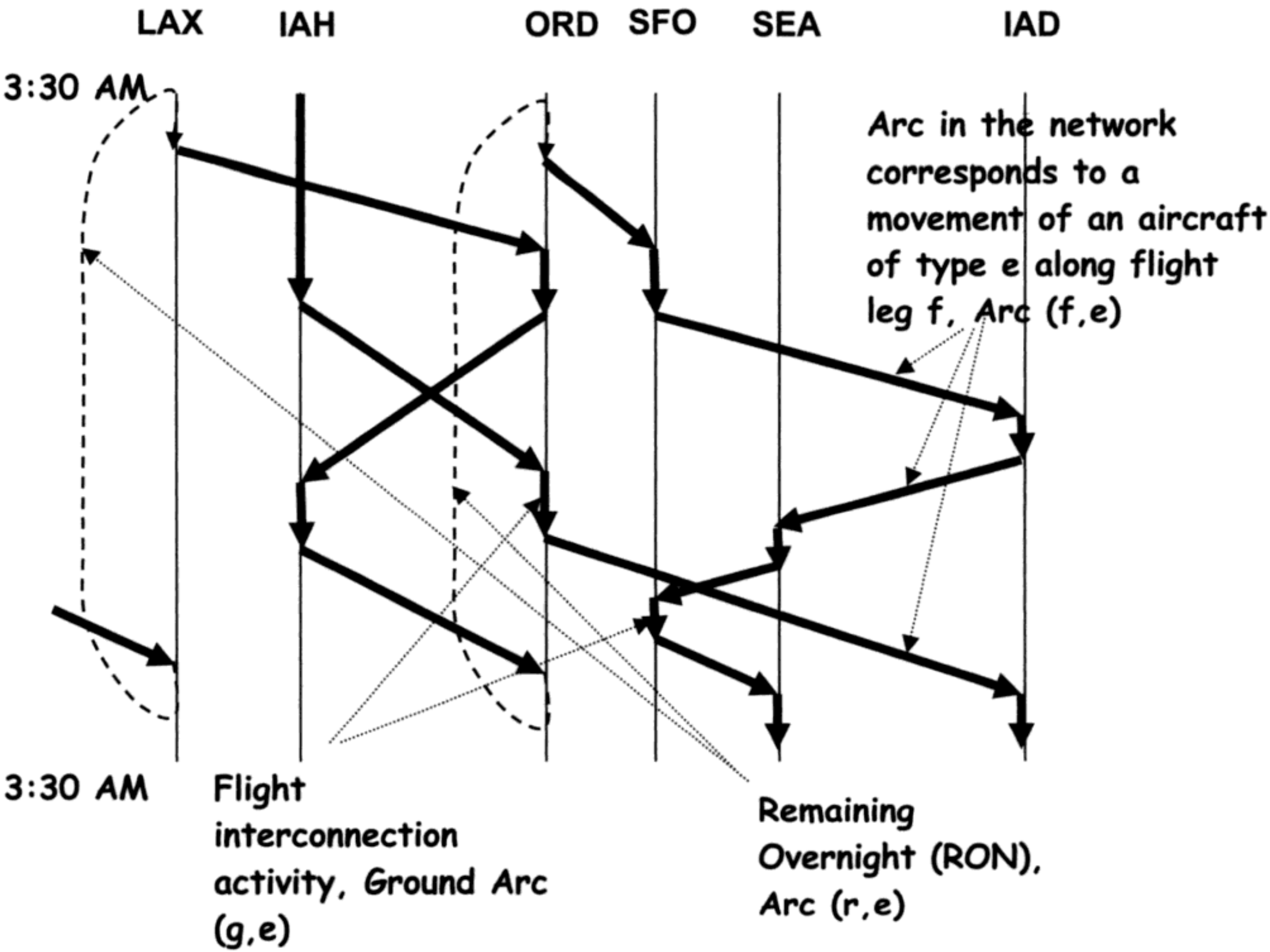
Many extras
interconnection nodes
etc.

Remaining over night:
connecting last events of the day with the first events of the day, which (same daily schedule) replicate the first events of the following day. This “wrap-around” ensures continuity of the fleet assignment every day

Time-Line Network

- "Daily" problem
  - Wrap-around (or overnight) arcs




LAX
IAH
ORD
SFO
SEA
IAD
3:30 AM
Arc in the network corresponds to a movement of an aircraft of type e along flight leg f, Arc (f,e)
3:30 AM
Flight interconnection activity, Ground Arc (g,e)
Remaining Overnight (RON), Arc (r,e)

Interconnection nodes for conservation

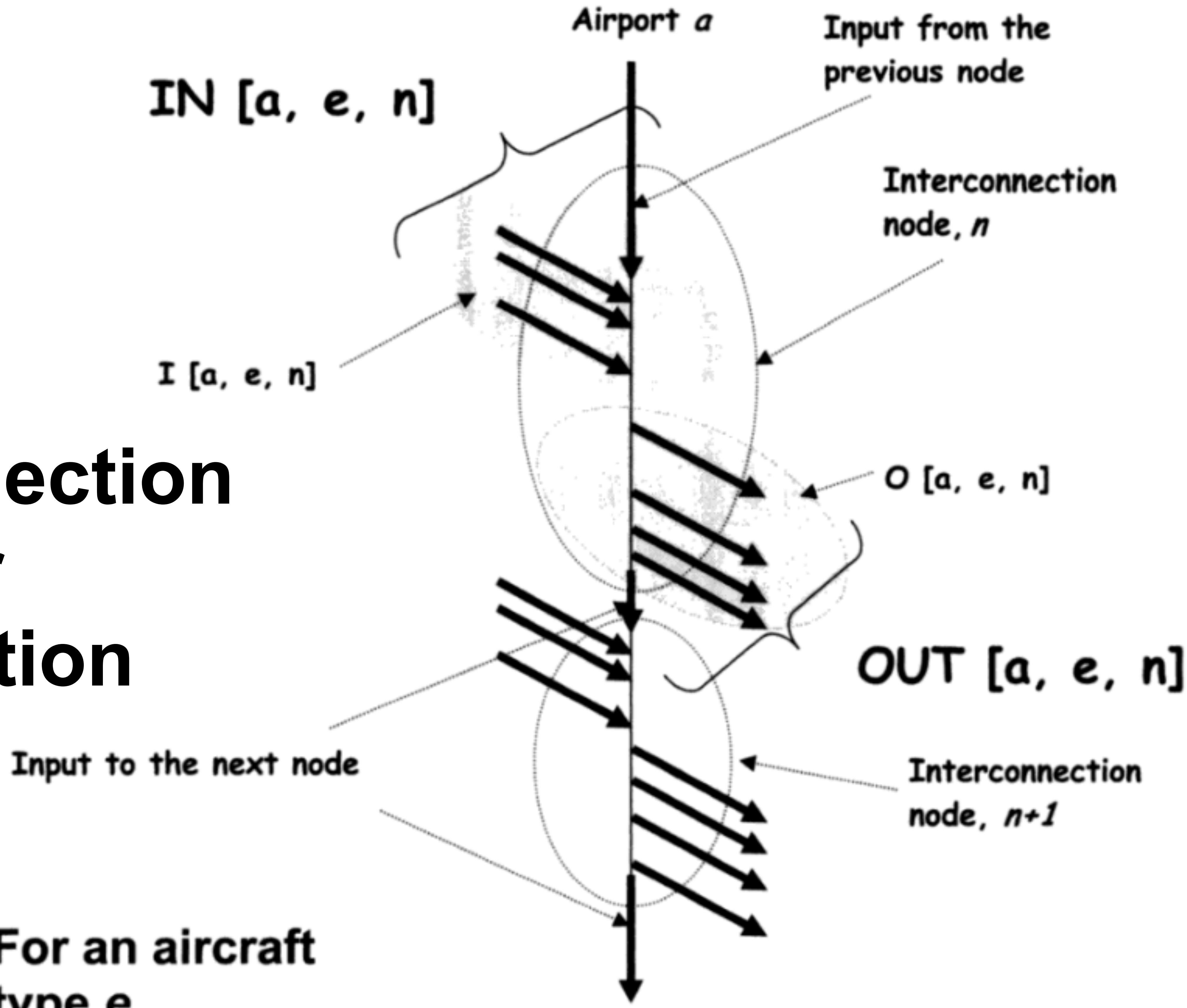
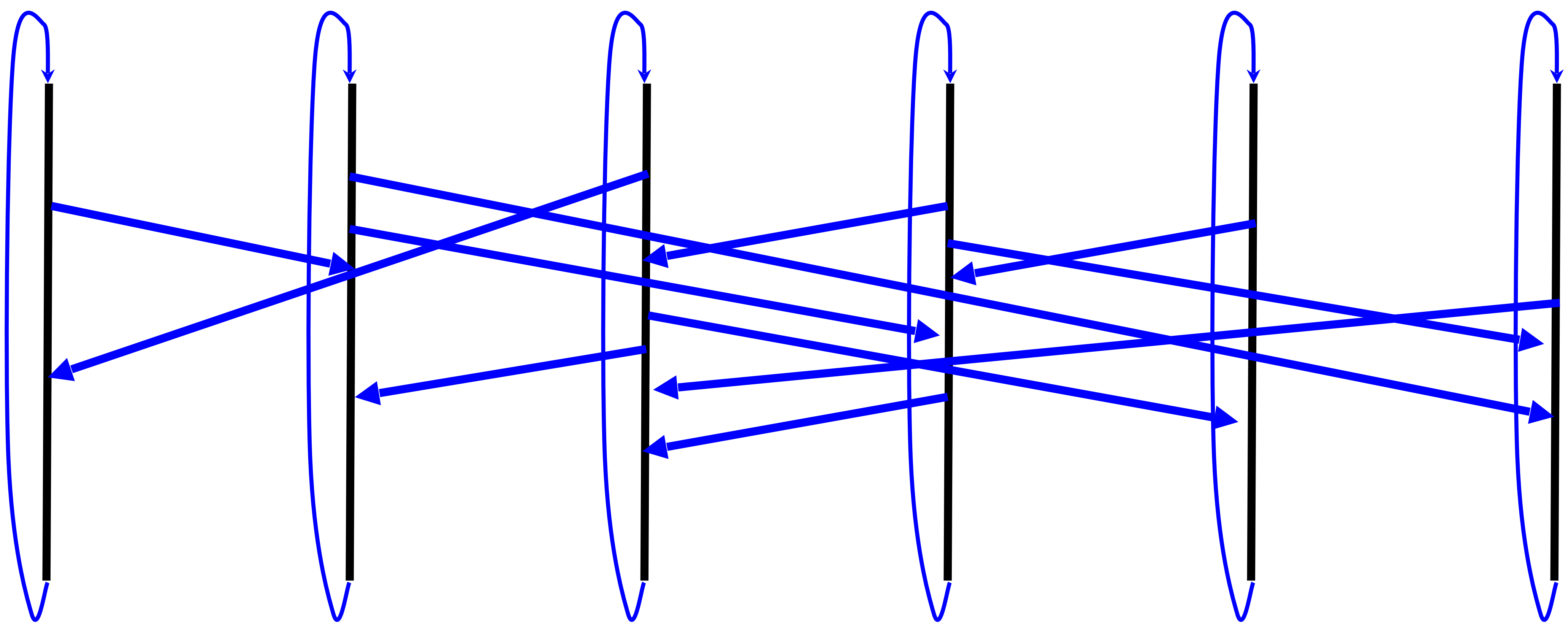


Figure 4.4 Illustration of an interconnection node at a station

Time-expanded (-line, -staged) Network
Ready-time Network



Assumption: One operation per time

If ready = departure, ready is earlier

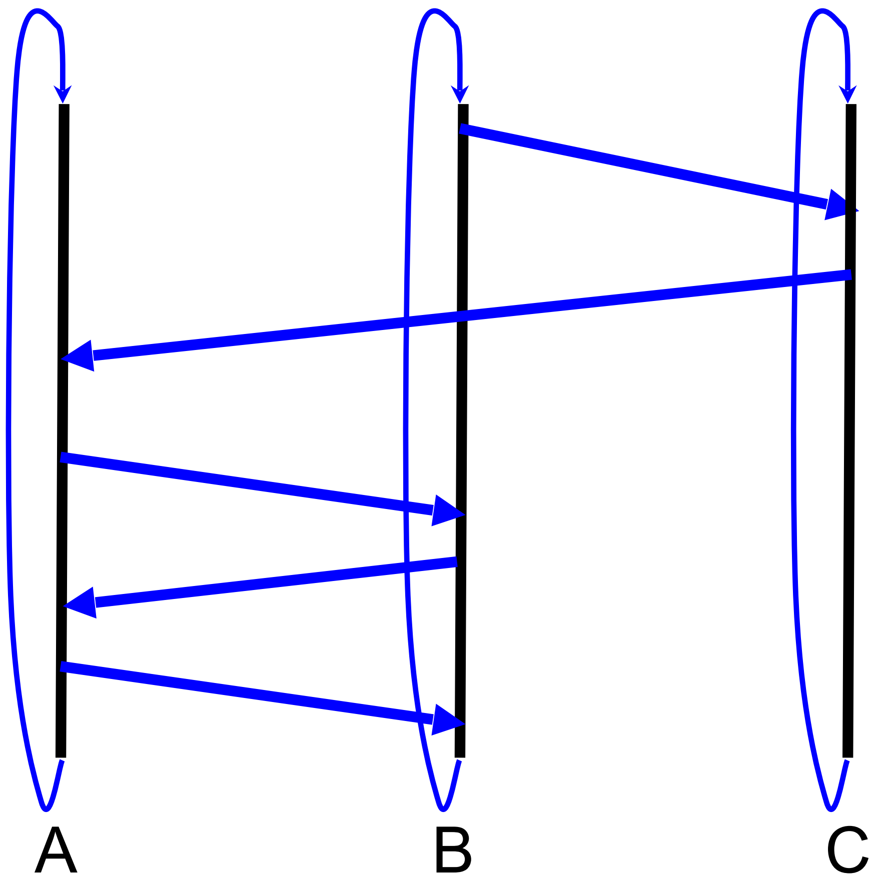
To implement, slightly shorten turn time

30min -> 29min 59sec

Schedule

turn-around time = 30min

Flight number	Departure time	Arrival time	Departure airport	Arrival airport
1	0550	0750	B	C
2	0930	1125	C	A
3	1400	1600	A	B
4	1700	1915	B	A
5	2100	2300	A	B



Earliest departure first

Enough toys!

Mathematical Program

Not needed for single aircraft (current topic)

- can solve from schedule
- can solve with network

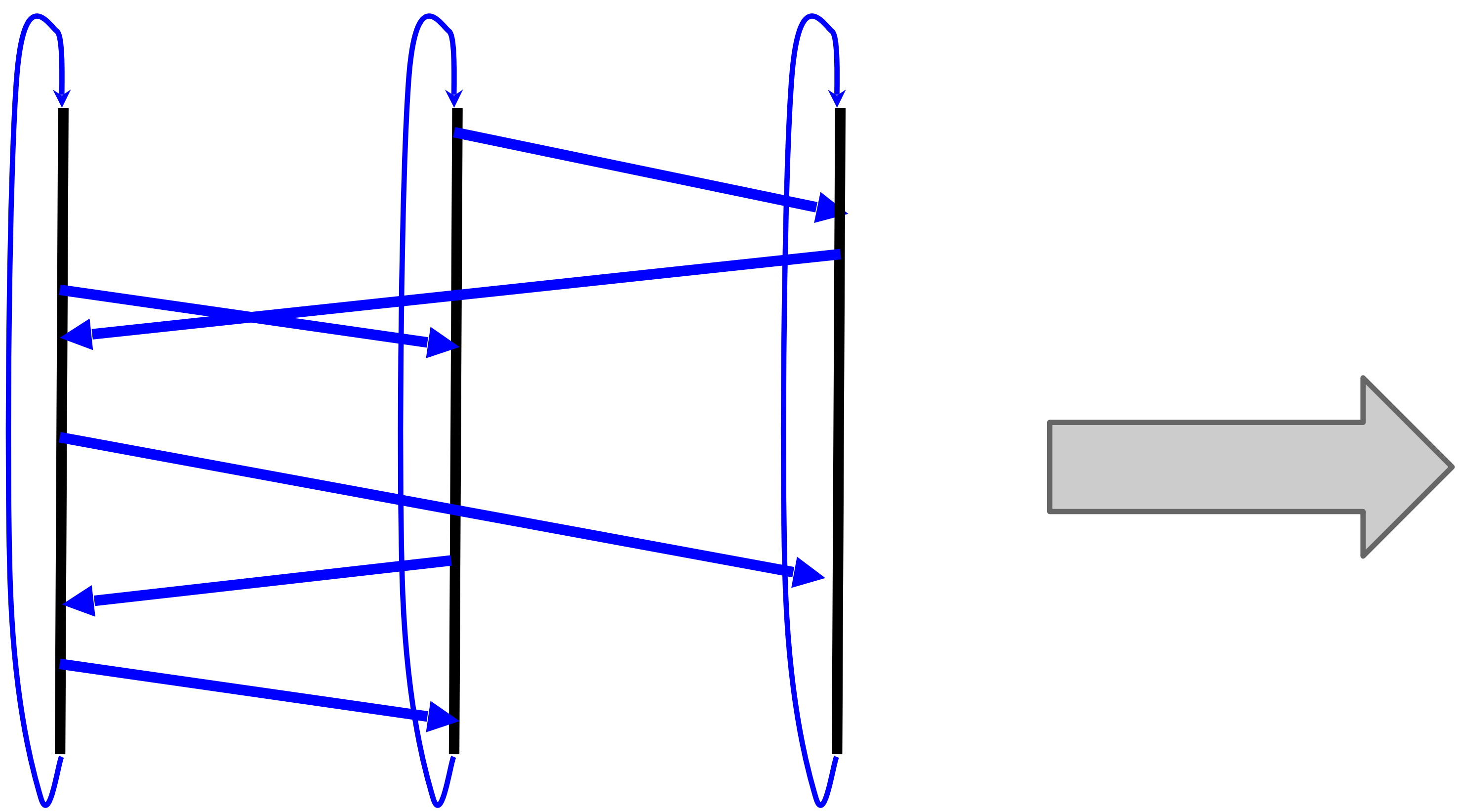
Not needed for multiple aircraft of single type

- can solve with network (next topic)

Necessary for multiple types (our ultimate interest)

- the only way to attack the problem

Programming Framework



Program:

- variables
- constraints
- objective function (most often)

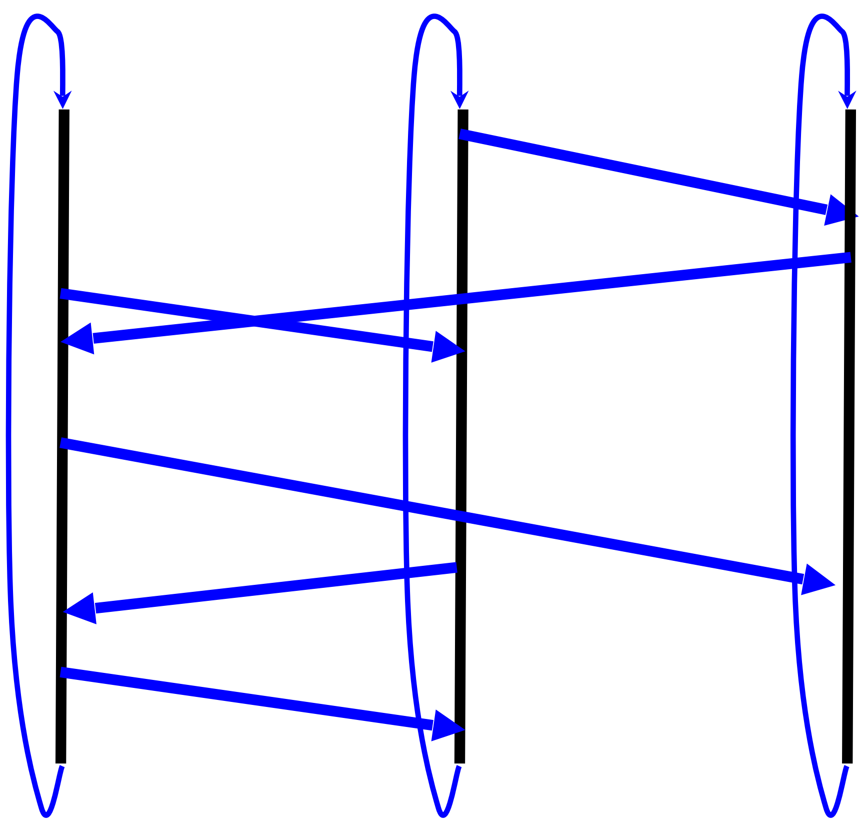
Goal 1 (always)

Solvable instance

Program is feasible

exists at least 1 solution

vars can be assigned values that satisfy all constraints



- variables
- constraints

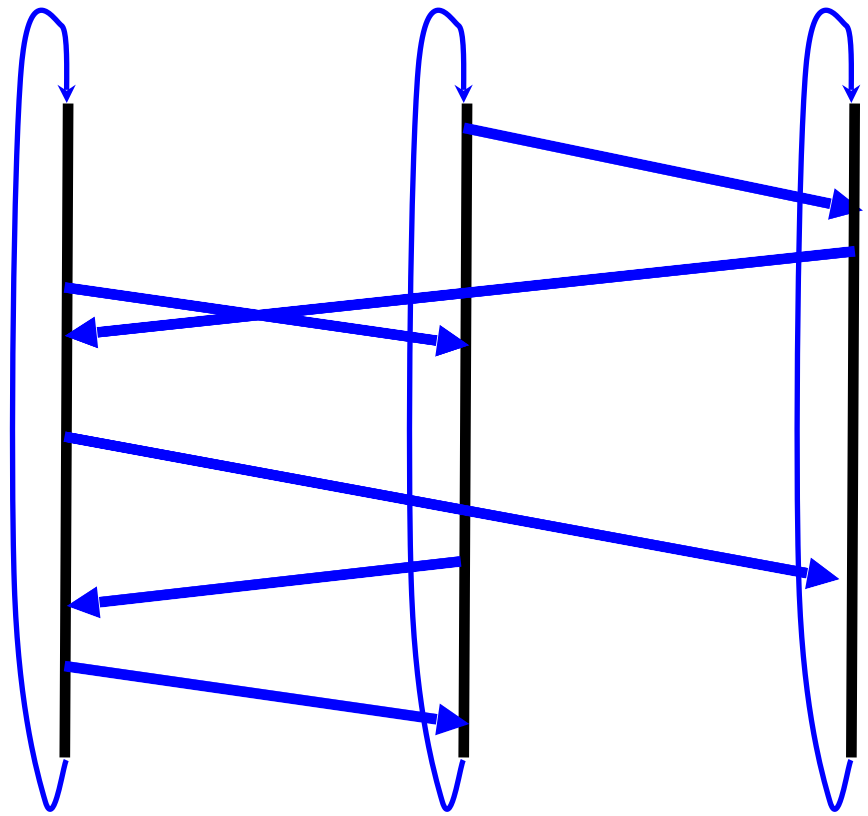
No solution

Program is infeasible

any values assigned to vars violate at least 1 constraint

Goal 2 (most often)

Solution cost → Objective function value = cost



- variables
- constraints
- objective function

Optimal solution → Optimal solution

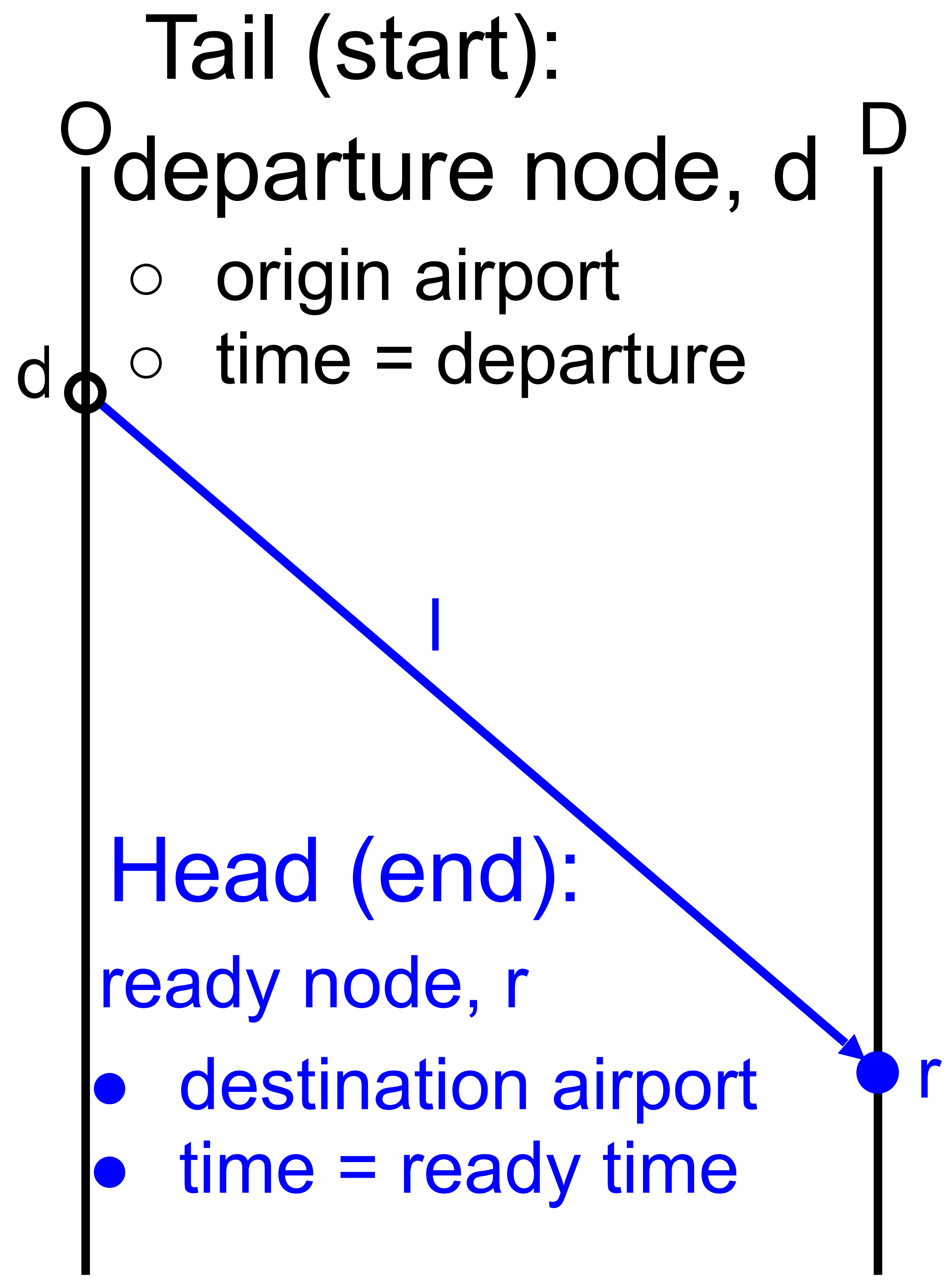
Variable per Arc (Link)

l = ODdr

l = O(l) D(l) d(l) r(l)

x_l = 1, if l is flown

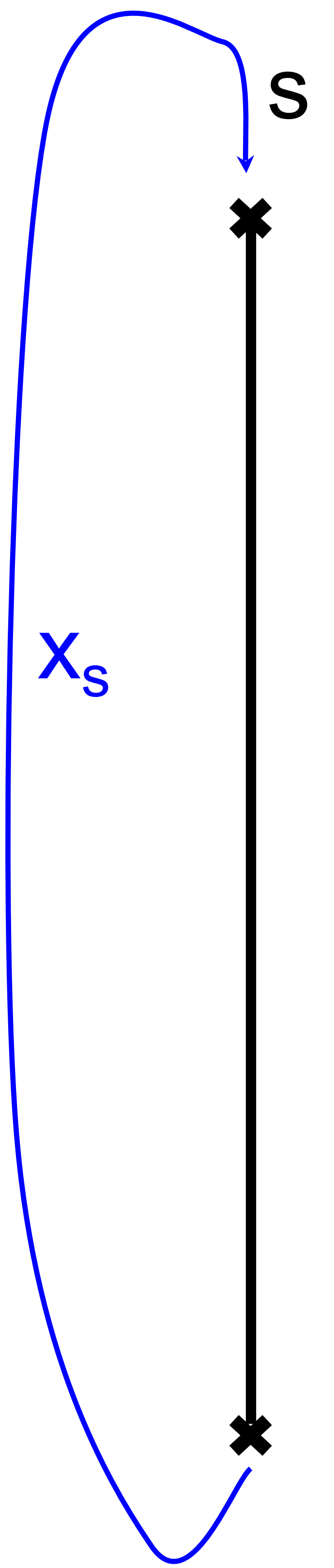
x_l = 0, otherwise



Variable per RON Arc

x_s = number of aircraft overnight at airport (station) s

$x_s \geq 0$



Constraint per Airport

departed + RON

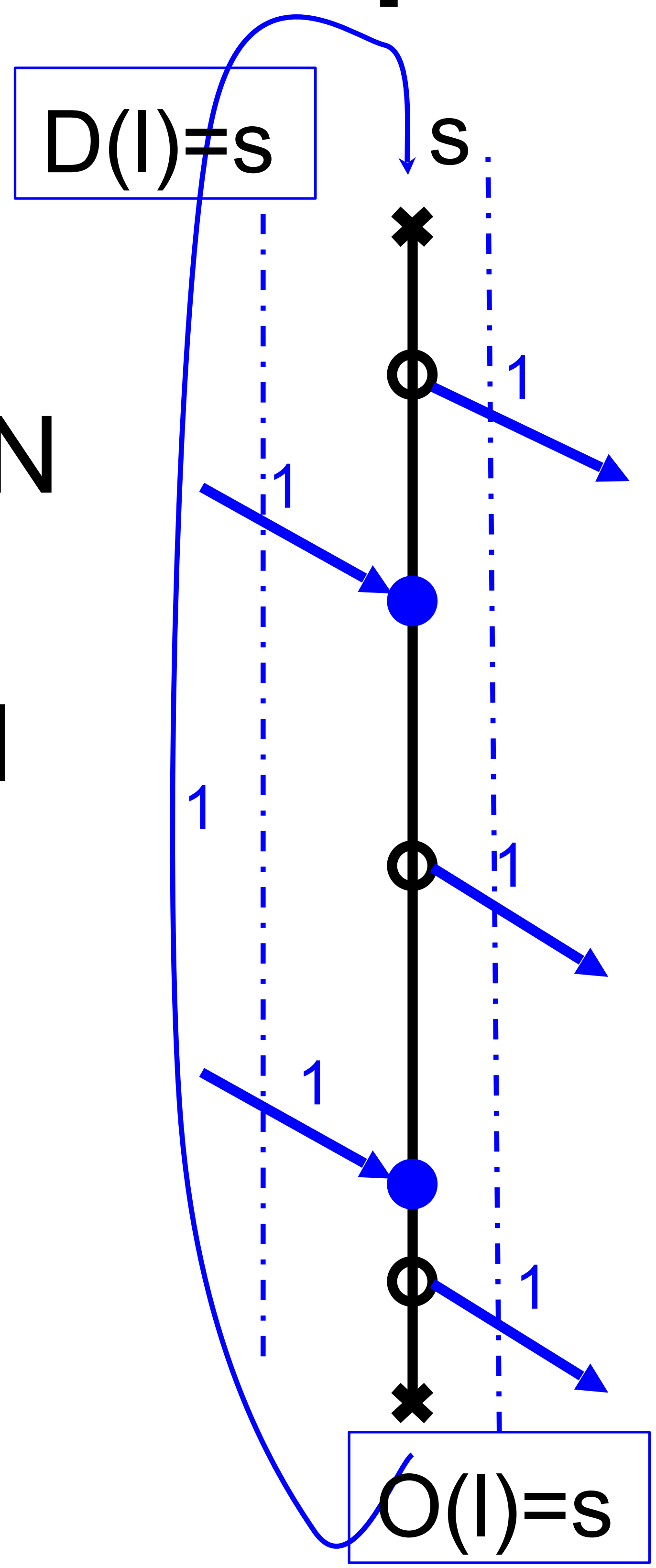
=

RON + arrived

$$\sum_{l:O(l)=s} x_l$$

=

$$\sum_{l:D(l)=s} x_l$$

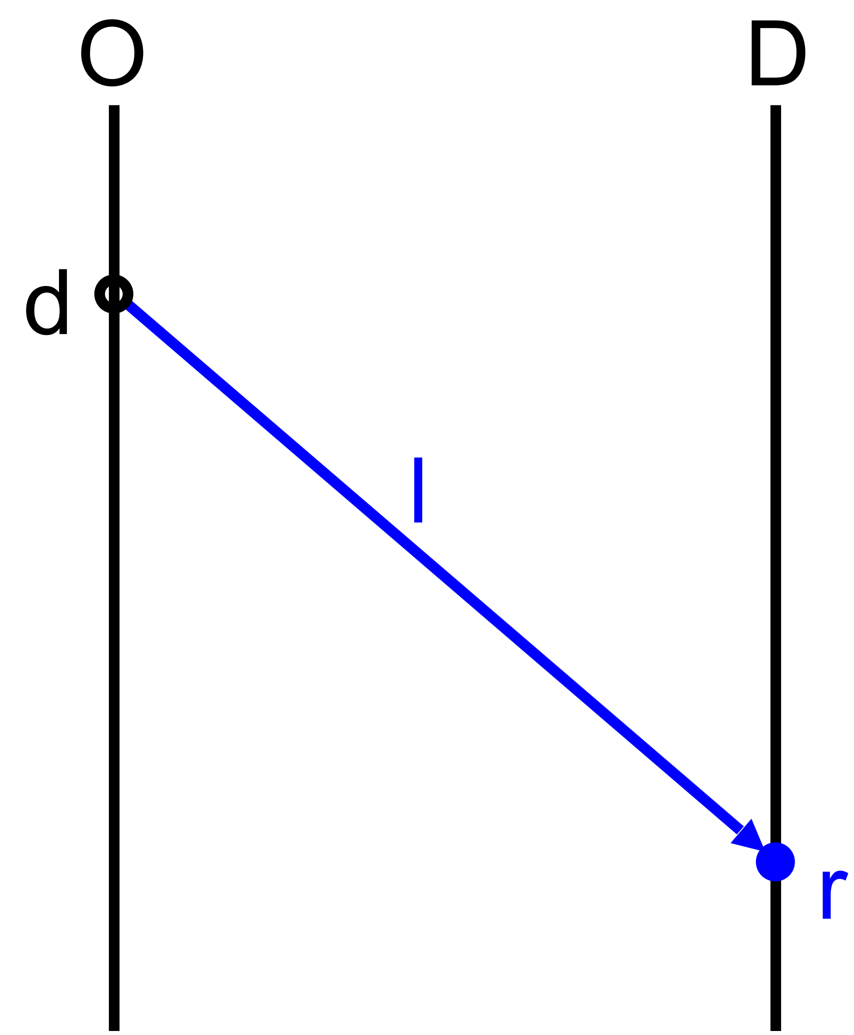


l = ODdr

l = O(l) D(l) d(l) r(l)

x_l = 1, if l is flown

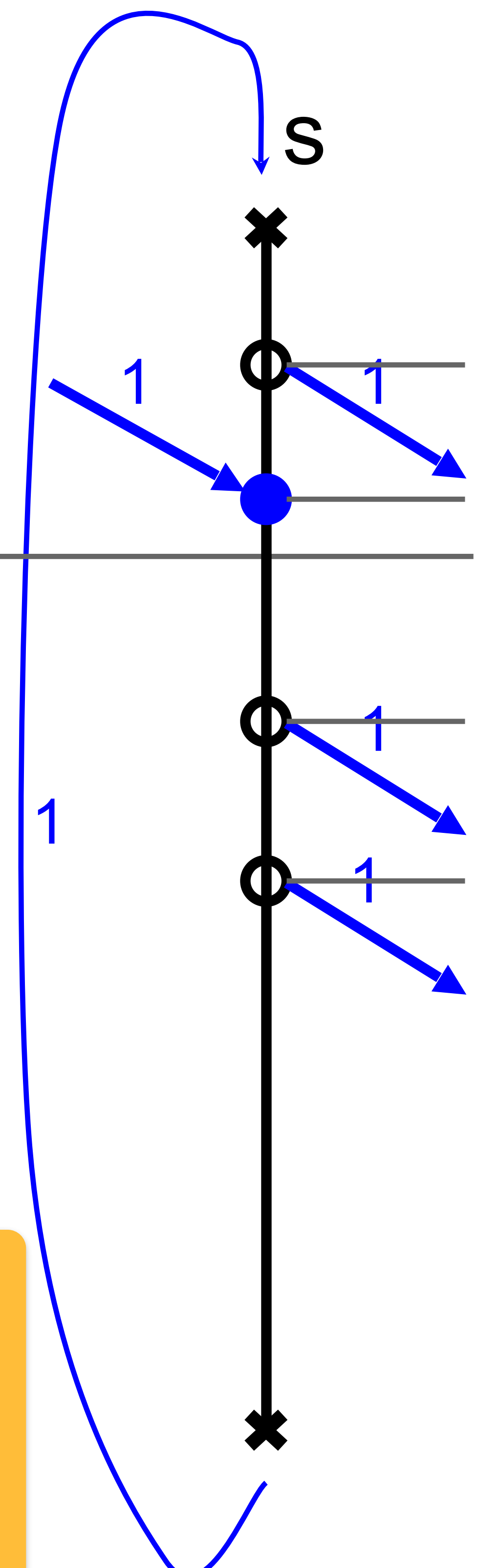
x_l = 0, otherwise



More Constraints

At what times t do these change?

- departed by t
- ready by t



At any time t:

departed by t

at most

ready by t

#departures until t
<= #all that ready until t
and those RON

Constraint per Departure Node

At any departure time t:

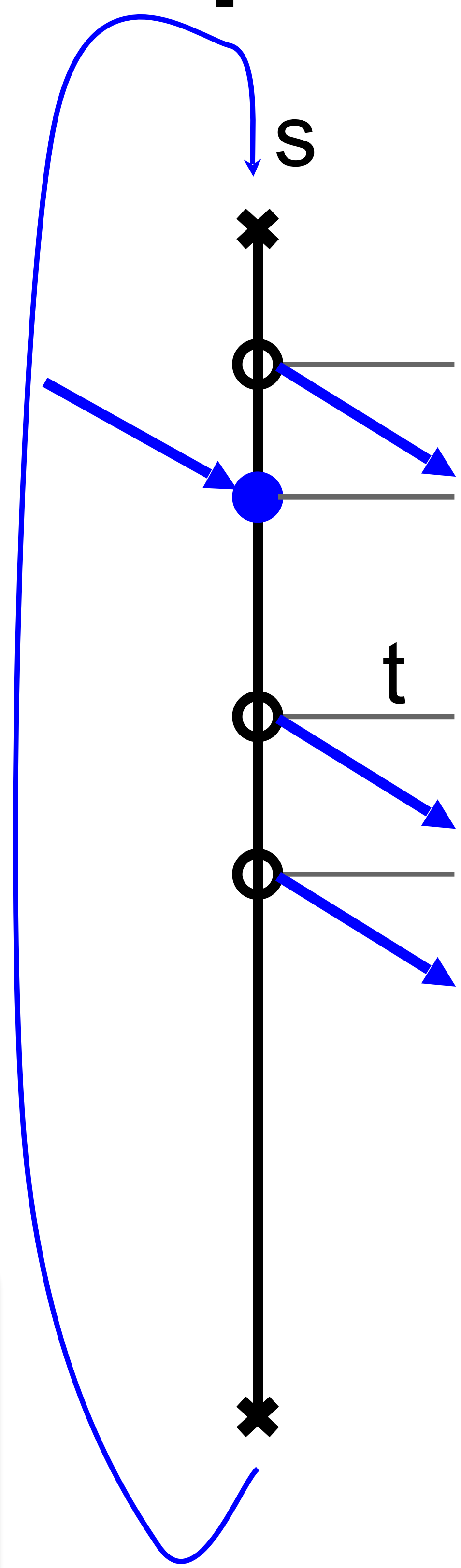
departed by t

at most

ready by t

$$\sum_{l:O(l)=s,d(l)\le t} x_l \le \sum_{l:D(l)=s,r(l)\le t} x_l + x_s$$

#departures until t
<= #all that ready until t
and those RON

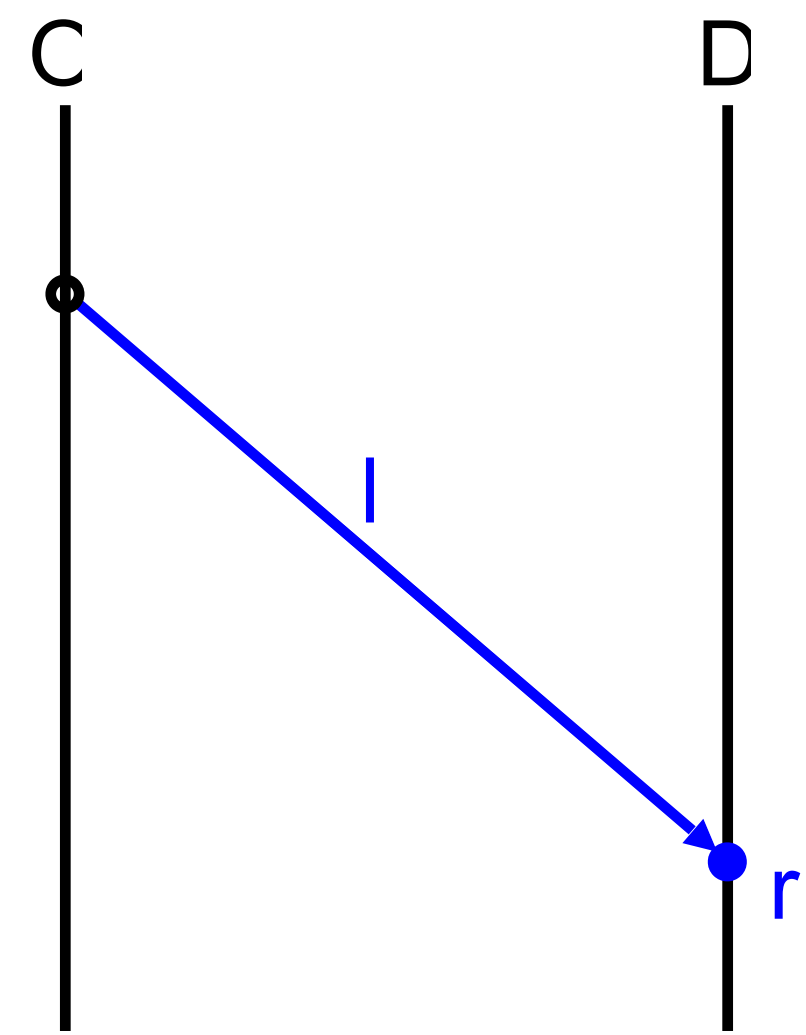


l = ODdr

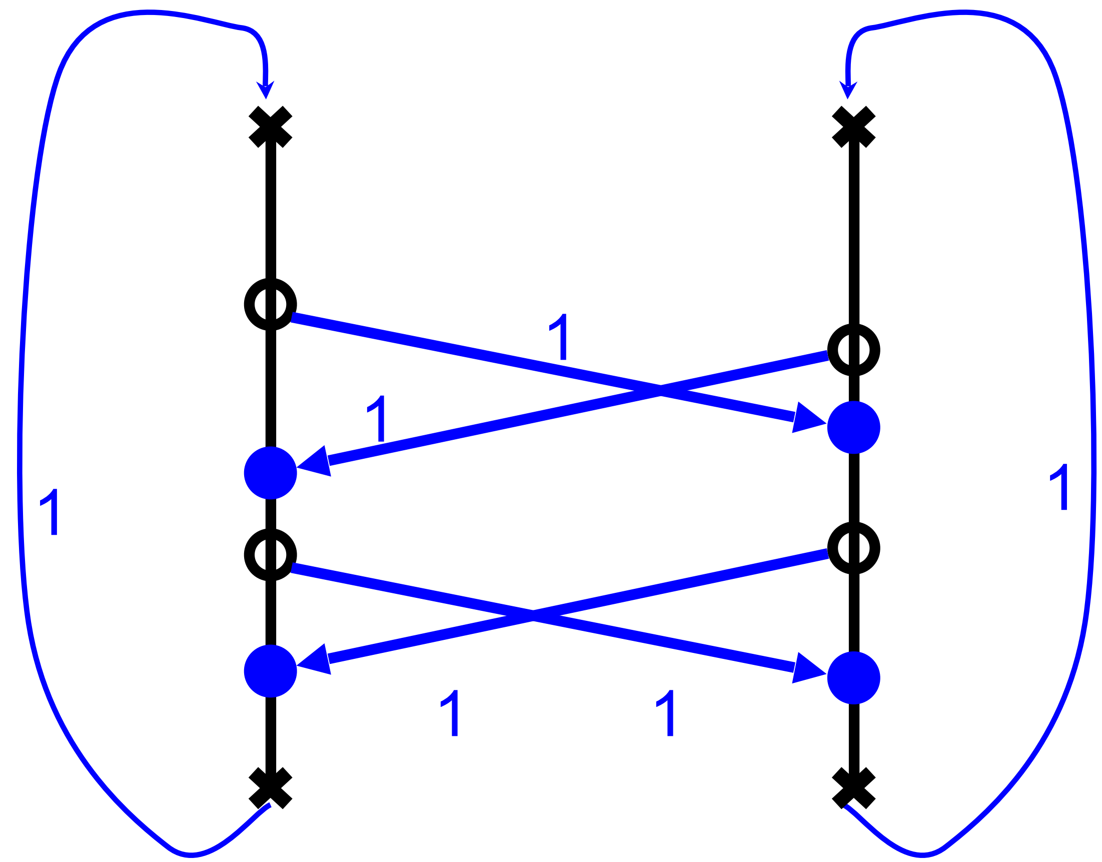
l = O(l) D(l) d(l) r(l)

x_l = 1, if l is flown

x_l = 0, otherwise



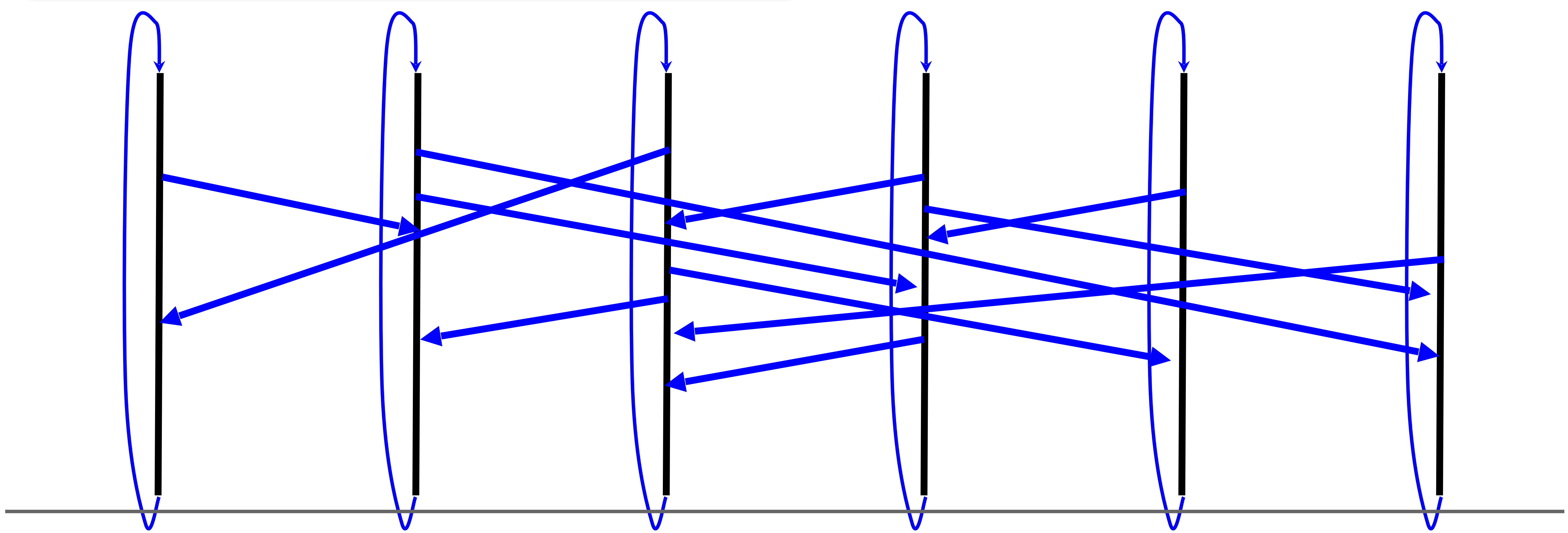
Sizing (Counting) Constraints: RON Arcs



Total number of planes is at most 1

$\sum x_s \leq 1$

We consider the case of a single aircraft ;)!



Fleet Assignment: All Constraints

Per airport s (flow conservation)

$\sum_{l:O(l)=s} x_l \quad = \quad \sum_{l:D(l)=s} x_l$

departed + RON
= RON + arrived

Per departure node t (non-negativity)

$\sum_{l:O(l)=s,d(l)\leq t} x_l \quad \leq \quad \sum_{l:D(l)=s,r(l)\leq t} x_l \; + \; x_s$

#departures until t
<= #all that ready
until t
and those RON

Sizing (sum over RON arcs)

$\sum x_s \leq 1$

$x_s \geq 0$, $x_l = 0 \text{ or } 1$

Total number
of planes
is at most 1

Naming Conventions

Link

Arc

not edge

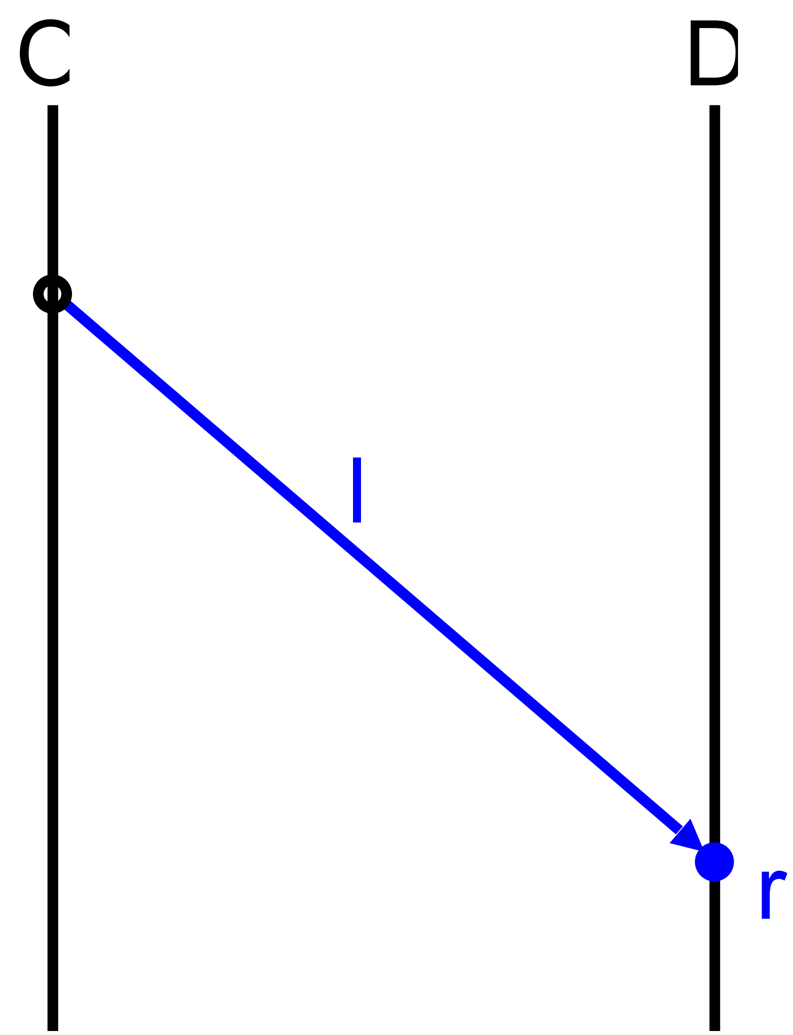
Flight

l = ODdr

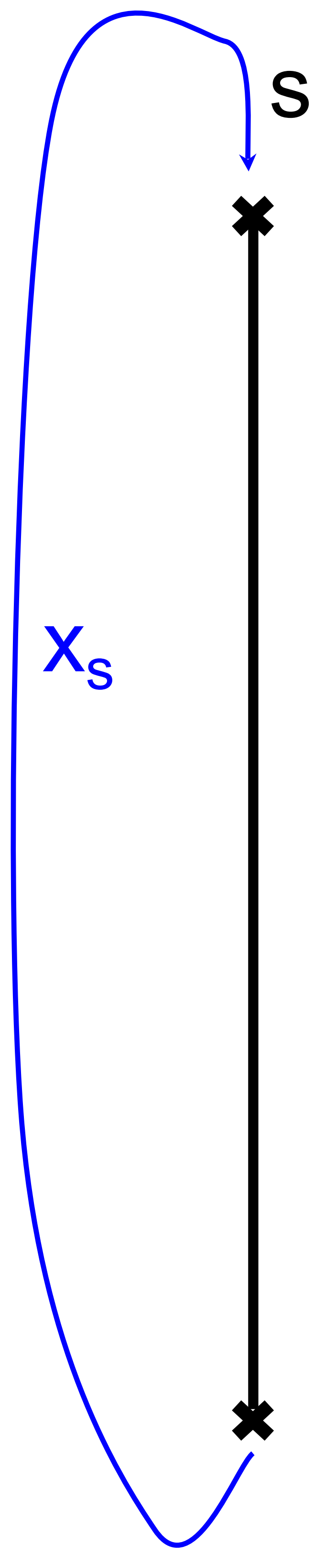
l = O(l) D(l) d(l) r(l)

x_l = 1, if l is flown

x_l = 0, otherwise



Could use
a
(or any other symbol)
in place of
s



Airline with a Single Aircraft

Feasibility

- Is the given schedule realizable?
  = Given the available plane,
  is it enough to implement the schedule?

Solved!
Program feasibility

$$\sum_{l:O(l)=s} x_l \quad = \quad \sum_{l:D(l)=s} x_l$$

$$\sum_{l:O(l)=s,d(l)\leq t} x_l \quad \leq \quad \sum_{l:D(l)=s,r(l)\leq t} x_l + x_s$$

$$\sum x_s \leq 1$$

$$x_s \geq 0 \quad , \quad x_l = 0 \text{ or } 1$$

Airline with a Single Aircraft

Feasibility

- Is the given schedule realizable?
  = Given the available plane,
  is it enough to implement the schedule?

Solved!
Program
feasibility

Optimization

- Maximize the number of flights flown
  - assuming not all flights must be flown

Add objective function:

$$\max \sum x_l$$

Flight Schedule

- Minimum turn times = 30 minutes

Flight No.	Origin	Destin.	Dep. Time	Arrival Time
1	A	B	6:30	8:30
2	B	C	9:30	11:00
3	C	B	16:00	17:00
4	B	A	18:00	20:00

Fleet Assignment: All Constraints

Per airport s (flow conservation)

$$\sum_{l:O(l)=s} x_l \quad = \quad \sum_{l:D(l)=s} x_l$$

Per departure node t (non-negativity)

$$\sum_{l:O(l)=s,d(l)\leq t} x_l \quad \leq \quad \sum_{l:D(l)=s,r(l)\leq t} x_l \; + \; x_s$$

Sizing (sum over RON arcs)

$$\sum x_s \leq 1$$

$$x_s \geq 0 \qquad , \qquad x_l = 0 \text{ or } 1$$

Airline with a Single Aircraft

Feasibility

Solved! Program feasibility

- Is the given schedule realizable?
  = Given the available plane,
  is it enough to implement the schedule?

Optimization

- Maximize the number of flights flown
  - assuming not all flights must be flown

Add objective function:

$$\max \sum x_l$$

Simple Case:
Airline with Single Fleet Type
(= Single Color)
but Several Planes

Fleet (=color) Assignment

Problem void?

Input

- Schedule

For each flight:

  - O, D
  - departure time
  - # of pass
- Fleet
  - number of different-type aircraft
    - e.g.: **2 A310**

Solution: all flights served by the same (one and only) aircraft type

Output

- assignment of plane types (**colors**) to flights

Still many interesting questions!

Feasibility

- Is the given schedule realizable,
  with the given number of aircraft?
- Given the available aircraft,
  are they enough to implement the schedule?
- Counting: How many planes needed to fly all flights?
- Optimization: Minimize the number of planes used
  - Assuming all flights must be flown

Optimization

- Maximize the number of flights flown
  - Assuming not all flights must be flown

Fleet Assignment: All Constraints

Per airport s (flow conservation)

$\sum_{l:O(l)=s} x_l \quad = \quad \sum_{l:D(l)=s} x_l$

departed + RON
= RON + arrived

Per departure node t (non-negativity)

$\sum_{l:O(l)=s,d(l)\leq t} x_l \quad \leq \quad \sum_{l:D(l)=s,r(l)\leq t} x_l \; + \; x_s$

#departures until t
<= #all that ready
until t
and those RON

Sizing (sum over RON arcs)

$\sum x_s \leq N$

Total number
of planes
is at most **N**

$x_s \geq 0 \qquad , \qquad x_l = 0 \text{ or } 1$

Airline with single Fleet Type

Feasibility

Solved! Program feasibility

- Is the given schedule realizable, with the given number of aircraft?
  - Given the available aircraft, are they enough to implement the schedule?
- Counting: How many planes needed to fly all flights?
- Optimization: Minimize the number of planes used
  - assuming all flights must be flown

Optimization

- Maximize the number of flights flown
  - assuming not all flights must be flown

Optimization Problems

Minimize the number of planes used

- Assuming all flights must be flown

Maximize the number of flights flown

- Assuming not all flights must be flown

How to use the program to minimize the number of planes used?

Per airport s (flow conservation)

$\sum_{l:O(l)=s} x_l = \sum_{l:D(l)=s} x_l$

Per departure node t (non-negativity)

$\sum_{l:O(l)=s,d(l)\leq t} x_l \leq \sum_{l:D(l)=s,r(l)\leq t} x_l + x_s$

Sizing (sum over RON arcs)

$\sum x_s \leq N$

$x_s \geq 0$

Minimize the number of planes used

Approach 1: search for min N until feasible

- incremental
- binary

$$\sum_{l:O(l)=s} x_l = \sum_{l:D(l)=s} x_l$$

$$\sum_{l:O(l)=s,d(l)\leq t} x_l \leq \sum_{l:D(l)=s,r(l)\leq t} x_l + x_s$$

$$\sum x_s \leq N \qquad x_s \geq 0$$

Minimize the number of planes used

Approach 2: constraint -> objective function

minimize $\sum x_s$

$\sum_{l:O(l)=s} x_l = \sum_{l:D(l)=s} x_l$

$\sum_{l:O(l)=s,d(l)\leq t} x_l \leq \sum_{l:D(l)=s,r(l)\leq t} x_l + x_s$

~~$\sum x_s \leq N$~~ $x_s \geq 0$

Maximize the number of flights flown

maximize $\sum x_l$

$\sum_{l:O(l)=s} x_l = \sum_{l:D(l)=s} x_l$

$\sum_{l:O(l)=s,d(l)\leq t} x_l \leq \sum_{l:D(l)=s,r(l)\leq t} x_l + x_s$

$\sum x_s \leq N \qquad x_s \geq 0$

Simple case:
Airline with Single Fleet Type
(= single color)

Done!

Feasibility

- Is the given schedule realizable,
  with the given number of aircraft?
- = Given the available aircraft,
  are they enough to implement the schedule?
- = Counting: How many planes needed to fly all flights?
- < Optimization: Minimize the number of planes used
  - assuming all flights must be flown

Optimization

- Maximize the number of flights flown
  - assuming not all flights must be flown

Multiple Aircraft Types

Fleet (=color) Assignment

Input

- Schedule

For each flight:

  - O, D
  - departure time
  - # of pass

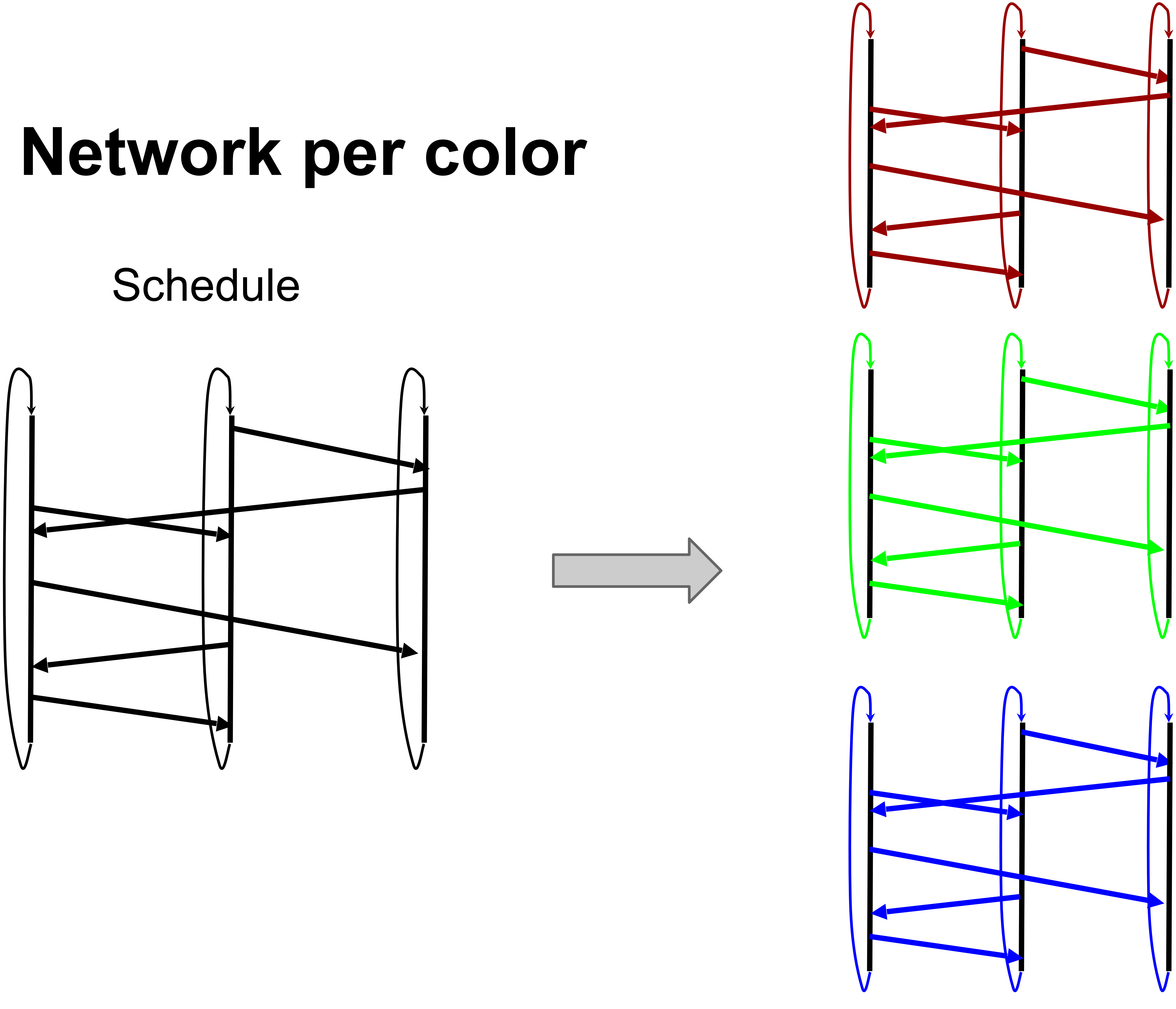
- Fleet
  - number of different-type aircraft
    - e.g.: **2 A310**, **1 A340**, **3 B747**

Output

- assignment of **colors** to flights

Network per color

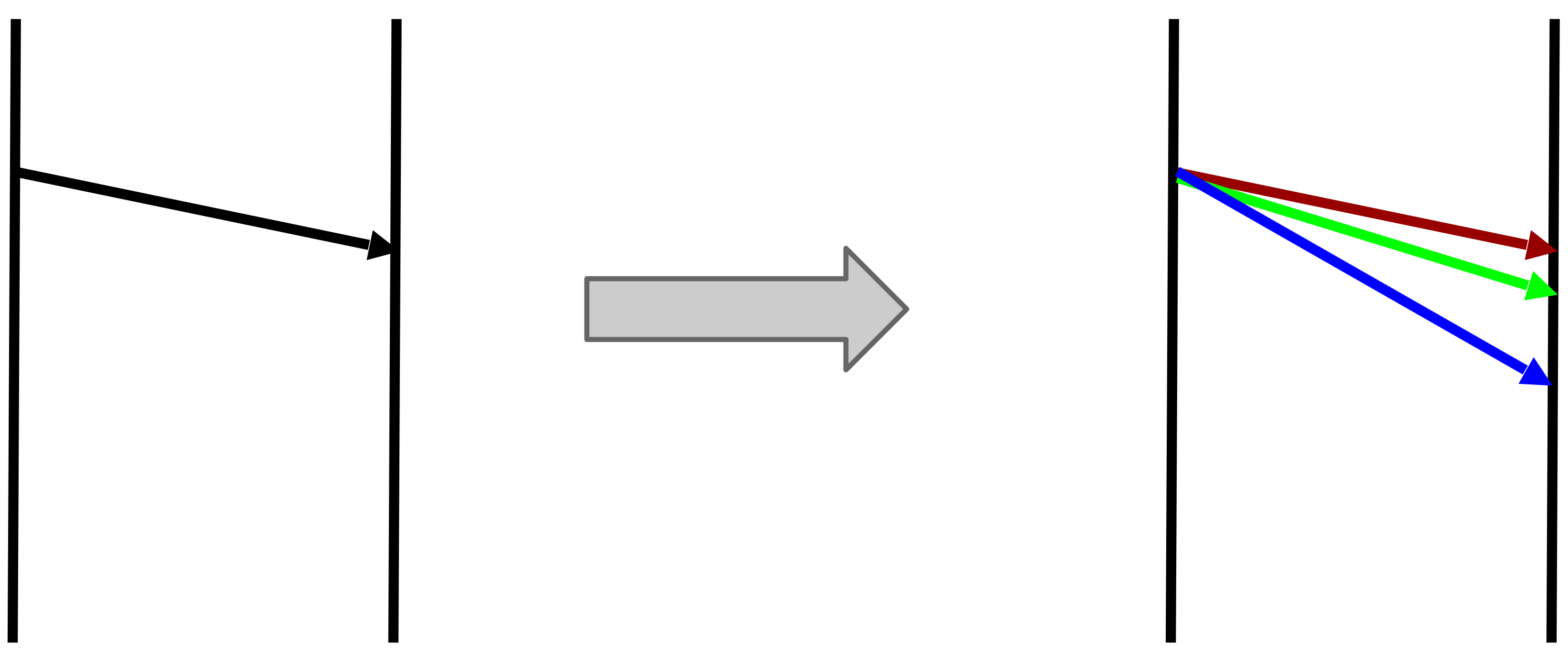
Schedule



Networks are not the same

1 schedule entry

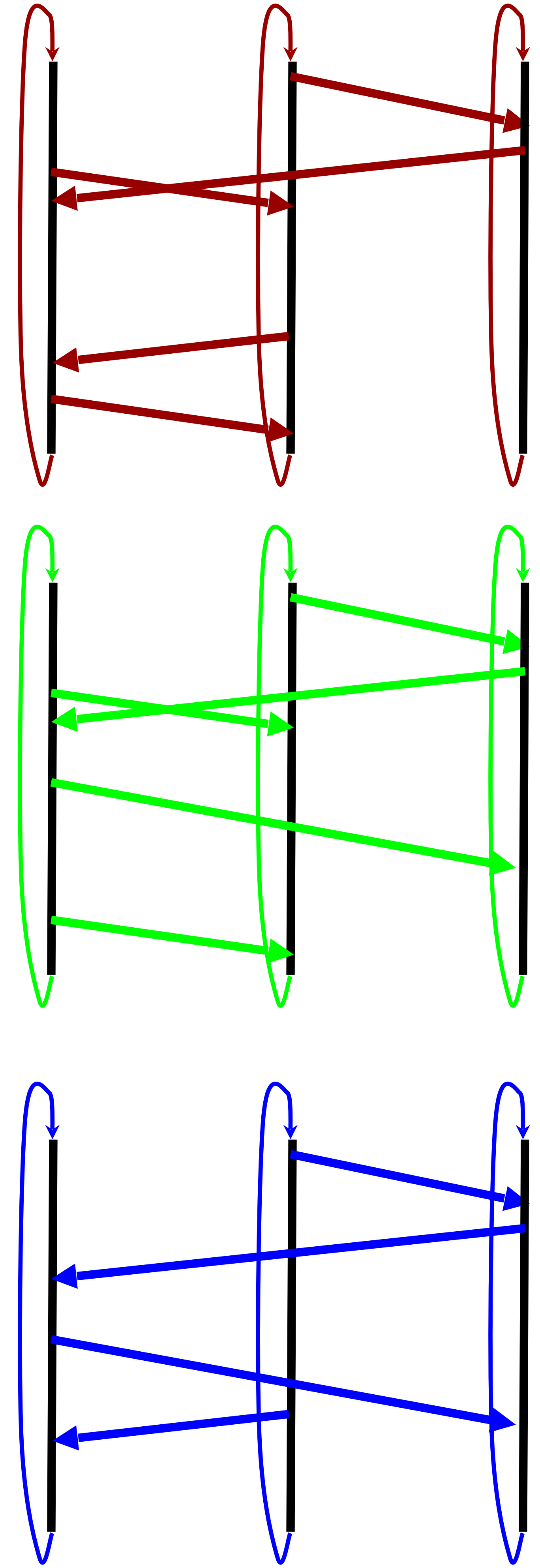
Different ready
times



Networks are not the same

Operational considerations:
(non) existence of arcs
(of certain color)

- seat capacity ≥ expected pax
- |OD| ≤ flight range
- O,D operationally feasible
  - Runway lengths
  - Gates
  - Noise
  - Curfews
  - Maintenance facilities, crew bases



Variable per flight and type (color)

Monochromatic:

$x_l = 1$, if l is flown

$x_l = 0$, otherwise

$x_{la} = 1$, if l is flown by a

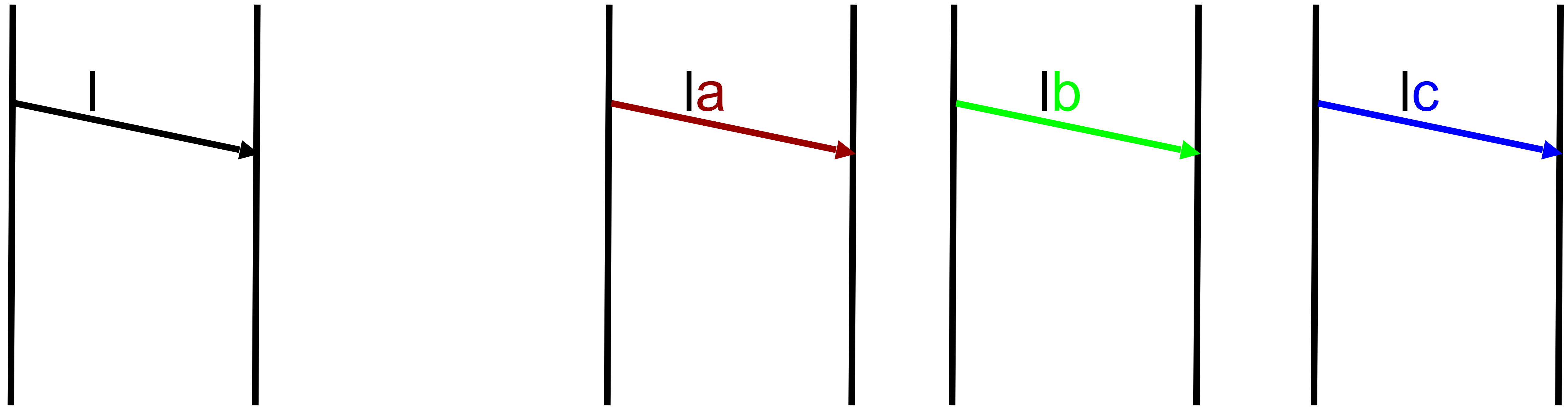
$x_{la} = 0$, o.w.

$x_{lb} = 1$, if l is flown by b

$x_{lb} = 0$, o.w.

$x_{lc} = 1$, if l is flown by c

$x_{lc} = 0$, o.w.



Variable per RON arc per color

x_s = RON at s

x_{sa} = RON at s of type a

x_{sb} = RON at s of type b

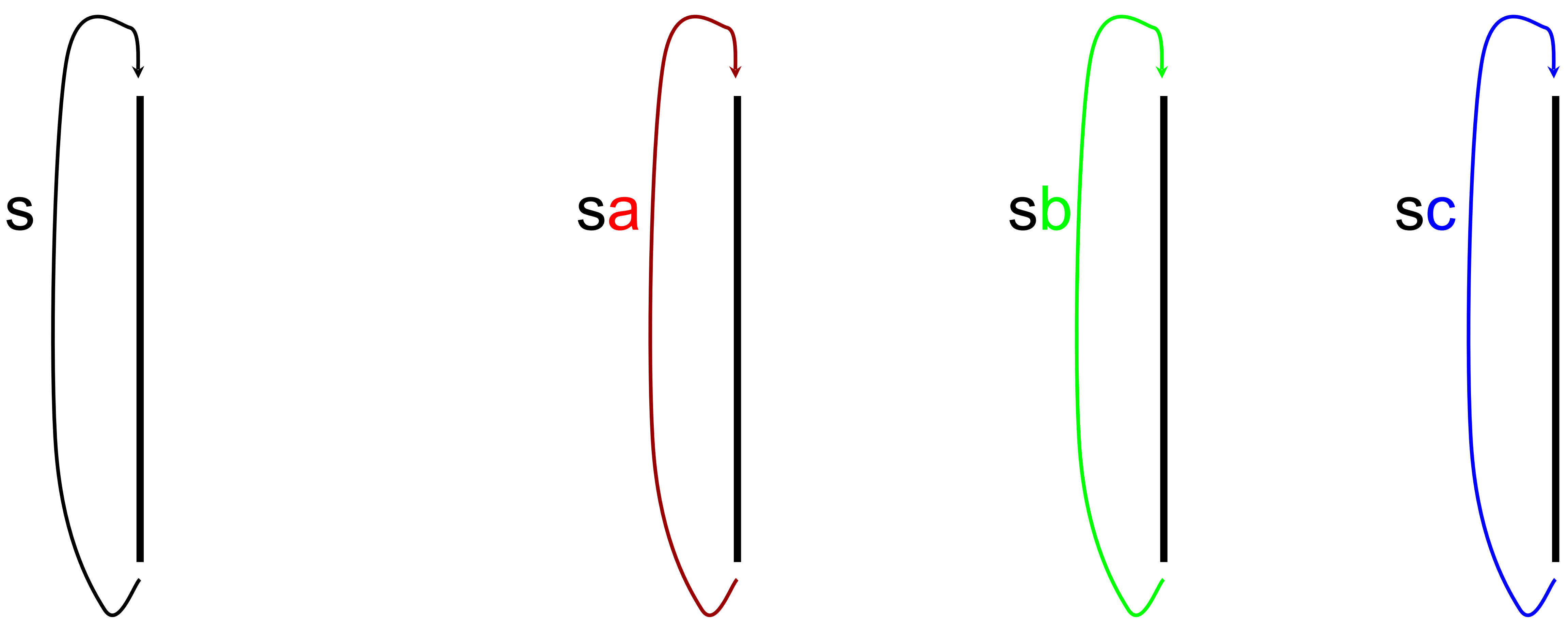
x_{sc} = RON at s of type c

$x_s \geq 0$

$x_{sa} \geq 0$

$x_{sb} \geq 0$

$x_{sc} \geq 0$



Constraints

Flow constraints per color

$\sum_{l:O(l)=s} x_l = \sum_{l:D(l)=s} x_l$

$\sum_{l:O(l)=s,d(l)\leq t} x_l \leq \sum_{l:D(l)=s,r(l)\leq t} x_l + x_s$

$\sum x_s \leq N$

$\sum_{la:O(la)=s} x_{la} = \sum_{la:D(la)=s} x_{la}$

$\sum_{la:O(la)=s,d(la)\leq t} x_{la} \leq \sum_{l:D(la)=s,r(la)\leq t} x_{la} + x_{sa}$

$\sum x_{sa} \leq N_a$

Flow constraints per color

$\sum_{l:O(l)=s} x_l \ = \ \sum_{l:D(l)=s} x_l$

$\sum_{l:O(l)=s,d(l)\leq t} x_l \ \leq \ \sum_{l:D(l)=s,r(l)\leq t} x_l \ + \ x_s$

$\sum x_s \leq N$

$\sum_{lb:O(lb)=s} x_{lb} \ = \ \sum_{lb:D(lb)=s} x_{lb}$

$\sum_{lb:O(lb)=s,d(lb)\leq t} x_{lb} \ \leq \ \sum_{l:D(lb)=s,r(lb)\leq t} x_{lb} \ + \ x_{sb}$

$\sum x_{sb} \leq N_b$

Flow constraints per color

$\sum_{l:O(l)=s} x_l \ = \ \sum_{l:D(l)=s} x_l$

$\sum_{l:O(l)=s,d(l)\leq t} x_l \ \leq \ \sum_{l:D(l)=s,r(l)\leq t} x_l \ + \ x_s$

$\sum x_s \leq N$

$\sum_{lc:O(lc)=s} x_{lc} \ = \ \sum_{lc:D(lc)=s} x_{lc}$

$\sum_{lc:O(lc)=s,d(lc)\leq t} x_{lc} \ \leq \ \sum_{l:D(lc)=s,r(lc)\leq t} x_{lc} \ + \ x_{sc}$

$\sum x_{sc} \leq N_c$

The Glueing Constraint

$x_{la} = 1$, if l is flown by a

$x_{la} = 0$, o.w.

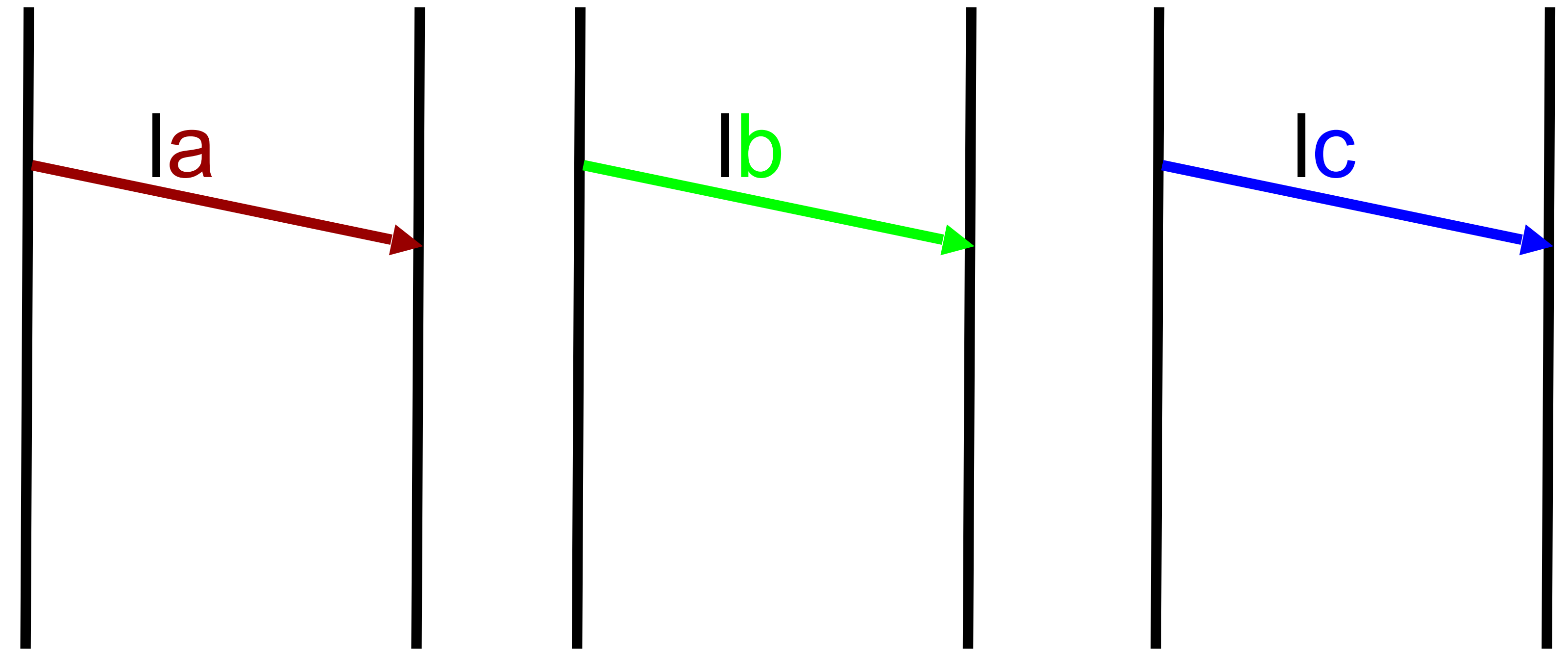
$x_{lb} = 1$, if l is flown by b

$x_{lb} = 0$, o.w.

$x_{lc} = 1$, if l is flown by c

$x_{lc} = 0$, o.w.

$$x_{la} + x_{lb} + x_{lc} = 1$$



Fleet Assignment: Objective function

- -- Cost
  - Fuel
  - Landing fee
  - Aircraft depreciation
- + Revenue

c_{lf} : Per leg basis, for each fleet type f

min cost: minimize $\sum c_{lf} x_{lf}$
or maximize profit

Overall program

minimize $\sum c_{lf}\, x_{lf}$ subject to:

$\sum_{lf:O(lf)=s} x_{lf} = \sum_{lf:D(lf)=s} x_{lf}$ for each airport s and fleet type f

$\sum_{lf:O(lf)=s,d(lf)\leq t} x_{lf} \leq \sum_{lf:D(lf)=s,r(lf)\leq t} x_{lf} + x_{sf}$ for each departure time t and fleet type f

$\sum_{s} x_{sf} \leq N_f$ for each fleet type f

$\sum_{f} x_{lf} = 1$ for each link l

$x_{lf} = 0$ or 1

$x_{sf} \geq 0$

F: fleet types

S: airports

lf: link l in the network for type f from F

sf: RON arc for airport s from S in the network for f

Constraints

F: set of fleet types
lf: flights in the network for f
sf: RONs in the network for f

Operational: (non) existence of arcs (of certain type)

$$\sum_{lf:O(lf)=s} x_{lf} = \sum_{lf:D(lf)=s} x_{lf}$$

$$\sum_{lf:O(lf)=s,d(lf)\leq t} x_{lf} \leq \sum_{lf:D(lf)=s,r(lf)\leq t} x_{lf} + x_{sf}$$

$$\sum_{s} x_{sf} \leq N_f \qquad x_{sf} \geq 0 \qquad x_{lf} = 0 \text{ or } 1$$

Flow: for f in F, per type

$$\sum_{f \text{ in } F} x_{lf} = 1$$ inter-type, gluing

Model Shortcomings

Objective Function Additivity

Not taken into account:

Customer

- Spill (and recapture)
- Revenue per itinerary, not per leg
- Through flight requirements
- etc.

Operational

- Airport fuel management
- Through flight requirements
- etc.

Constraints: Not taken into account

- Max # of aircraft that can stay overnight (easy fix)
- Aircraft routing (separate problem!)
- Crew schedule (separate problem!)
- etc.

Model Variations

Network

- Time-staged network
- Time-expanded network
- Time-line network
- Arrival-time network vs.
- Ready-time network
  - our approach, more accurate

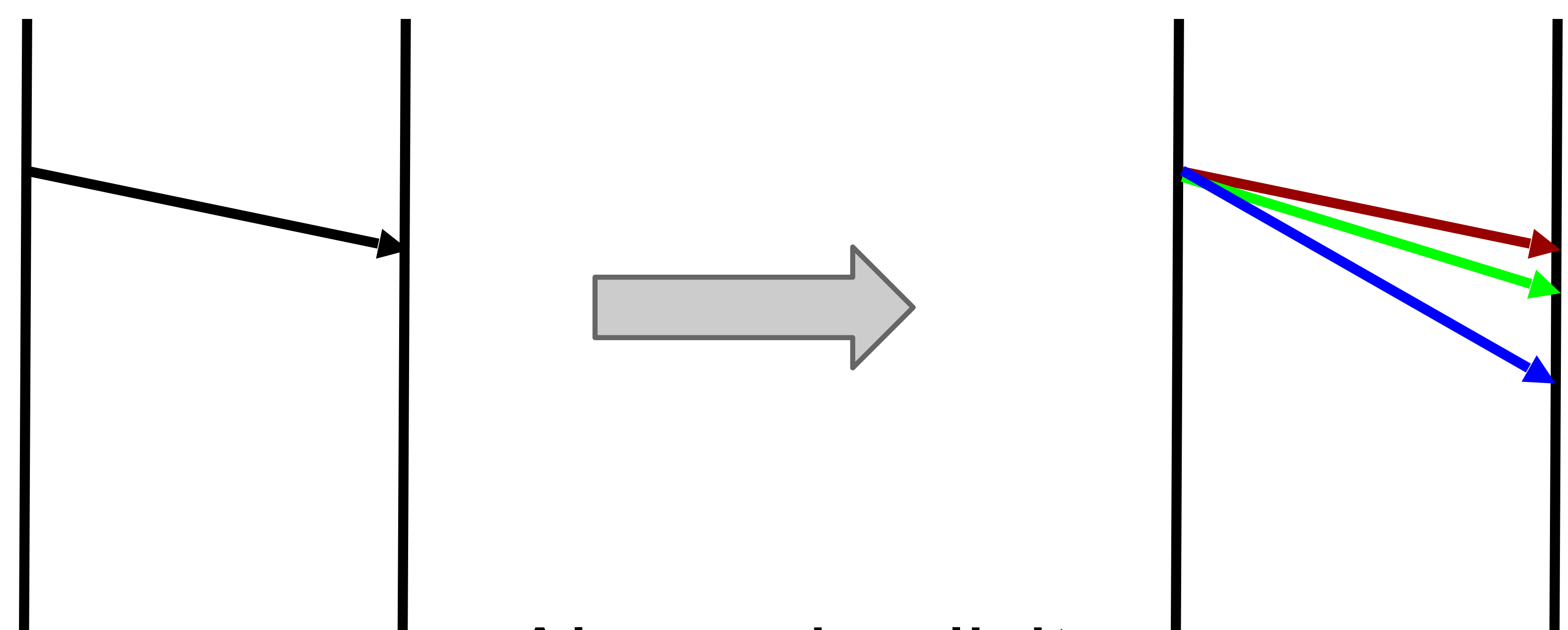
- Time: horizontal or vertical

- Textbook: interconnection nodes, etc.

This course, important difference 1!

schedule is NOT input to fleet assignment

flight, turn times depend on type!



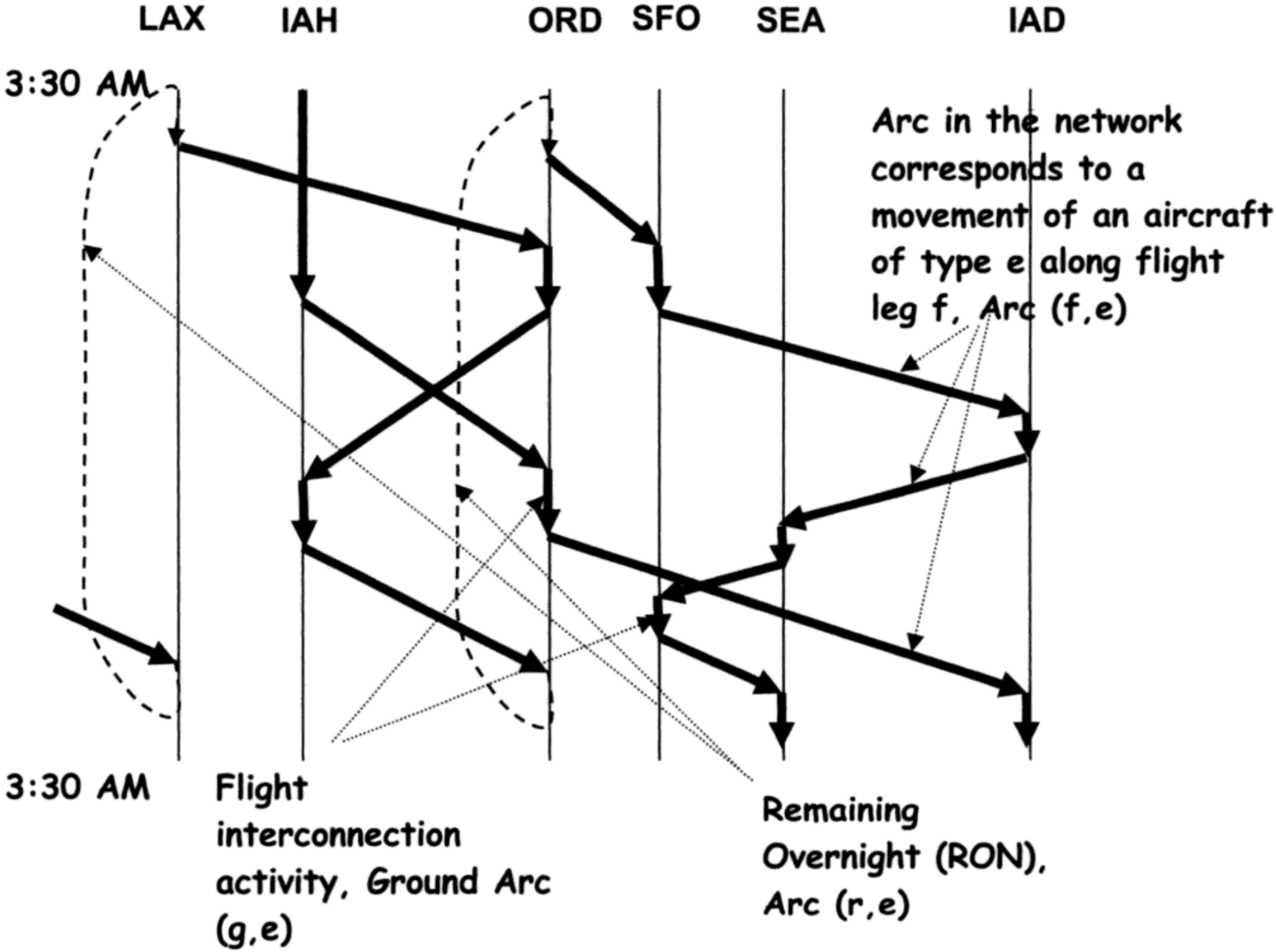
This course, important difference 2!

No ground arcs

- **Conservation via arrival - departure count**

- **Did have RON arcs**
  - **One per type**

Always explicit
Textbook: Interconnection nodes


LAX
IAH
ORD
SFO
SEA
IAD
3:30 AM
Arc in the network corresponds to a movement of an aircraft of type e along flight leg f, Arc (f,e)
3:30 AM
Flight interconnection activity, Ground Arc (g,e)
Remaining Overnight (RON), Arc (r,e)

Interconnection nodes for conservation

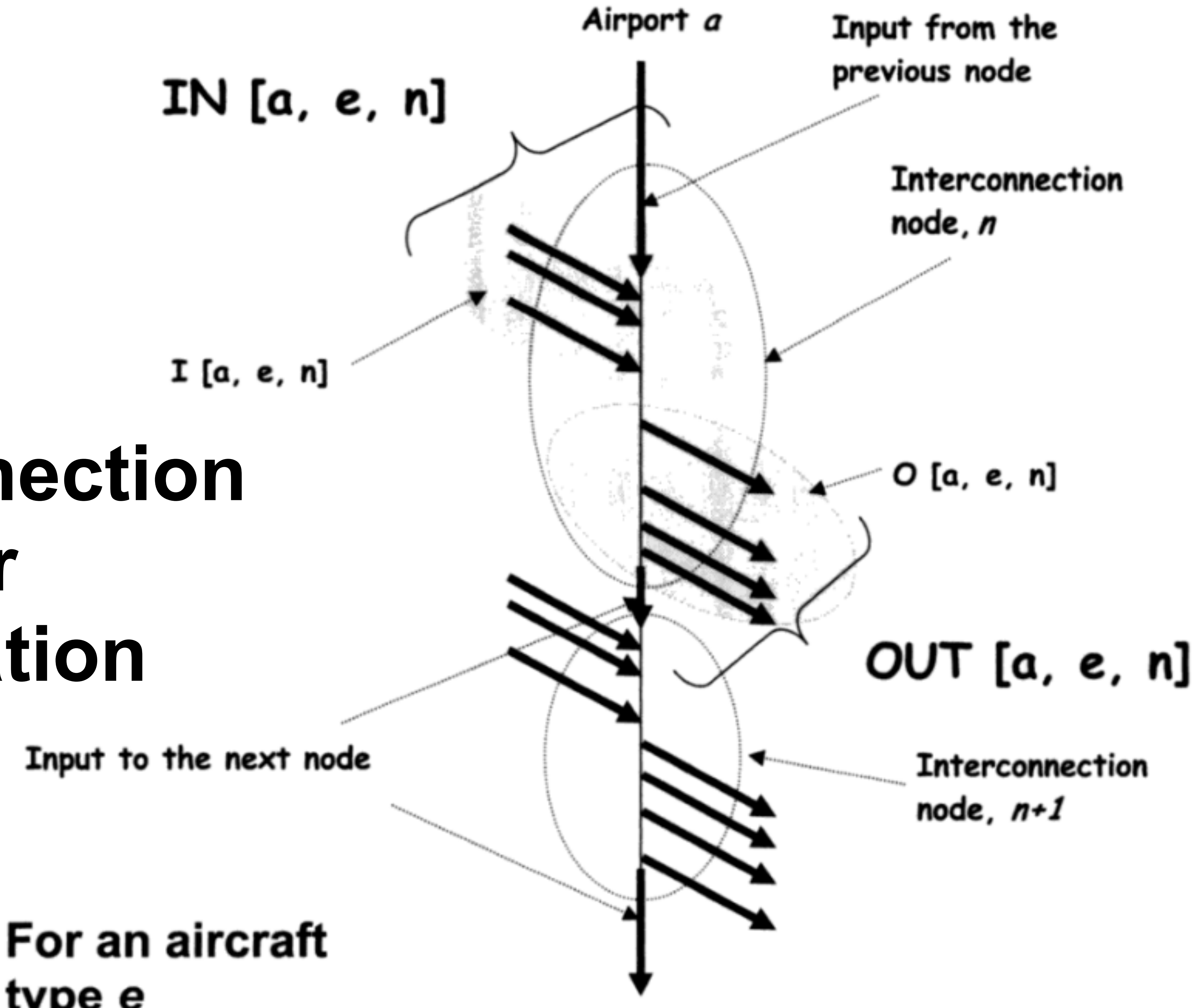
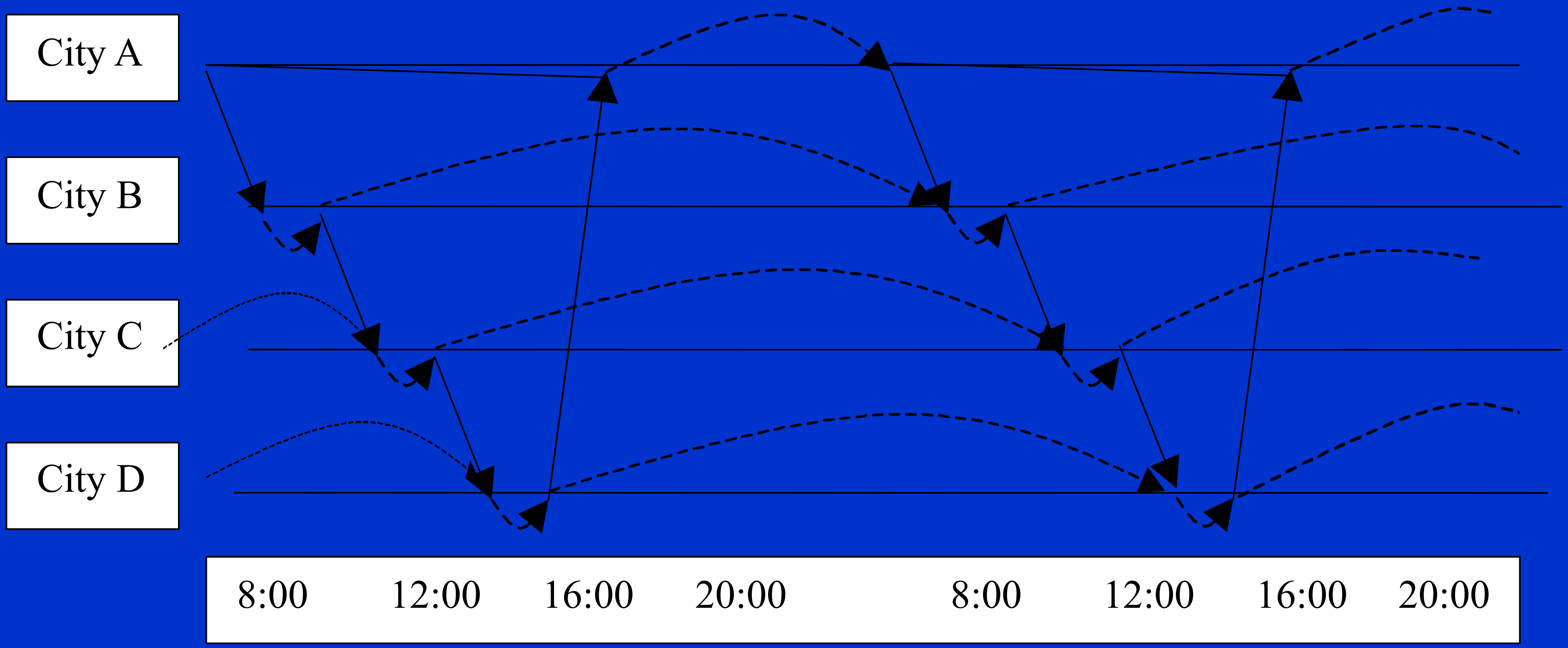


Figure 4.4 Illustration of an interconnection node at a station

Time-Line Network

- Ground arcs



Network Representation

- Topologically sorted time-line network
  - Nodes:
    - Flight arrivals/ departures (time and space)
  - Arcs:
    - Flight arcs: one arc for each scheduled flight
    - Ground arcs: allow aircraft to sit on the ground between flights

NB!

Constraints

- Cover Constraints
  - Each flight must be assigned to exactly one fleet
- Balance Constraints
  - Number of aircraft of a fleet type arriving at a station must equal the number of aircraft of that fleet type departing
- Aircraft Count Constraints
  - Number of aircraft of a fleet type used cannot exceed the number available

Objective Function

For each fleet - flight combination: Cost ≡ Operating cost

- Operating cost associated with assigning a fleet type *k* to a flight leg *j* is relatively straightforward to compute
  - Can capture range restrictions, noise restrictions, water restrictions, etc. by assigning “infinite” costs

FAM Notations

- Decision Variables
  - $f_{k,i}$ equals 1 if fleet type *k* is assigned to flight leg *i*, and 0 otherwise
  - $y_{k,o,t}$ is the number of aircraft of fleet type *k*, on the ground at station *o*, and time *t*
- Parameters
  - $C_{k,i}$ is the cost of assigning fleet k to flight leg *i*
  - N_k is the number of available aircraft of fleet type *k*
  - t_n is the "count time"
- Sets
  - *L* is the set of all flight legs *i*
  - *K* is the set of all fleet types *k*
  - *O* is the set of all stations *o*
  - *CL(k)* is the set of all flight arcs for fleet type *k* crossing the count time

Fleet Assignment Model (FAM)

$$Min \sum_{k \in K} \sum_{i \in L} c_{k,i} f_{k,i}$$

Subject to:

$$\sum_{k \in K} f_{k,i} = 1 \qquad \forall i \in L$$

$$y_{k,o,t^-} + \sum_{i \in I(k,o,t)} f_{k,i} - y_{k,o,t^+} - \sum_{i \in O(k,o,t)} f_{k,i} = 0 \qquad \forall k,o,t$$

$$\sum_{o \in O} y_{k,o,t_n} + \sum_{i \in CL(k)} f_{k,i} \leq N_k \qquad \forall k \in K$$

$$f_{k,i} \in \{0,1\} \qquad y_{k,o,t} \geq 0$$

Hane et al. (1995), Abara (1989), and Jacobs, Smith and Johnson (2000)