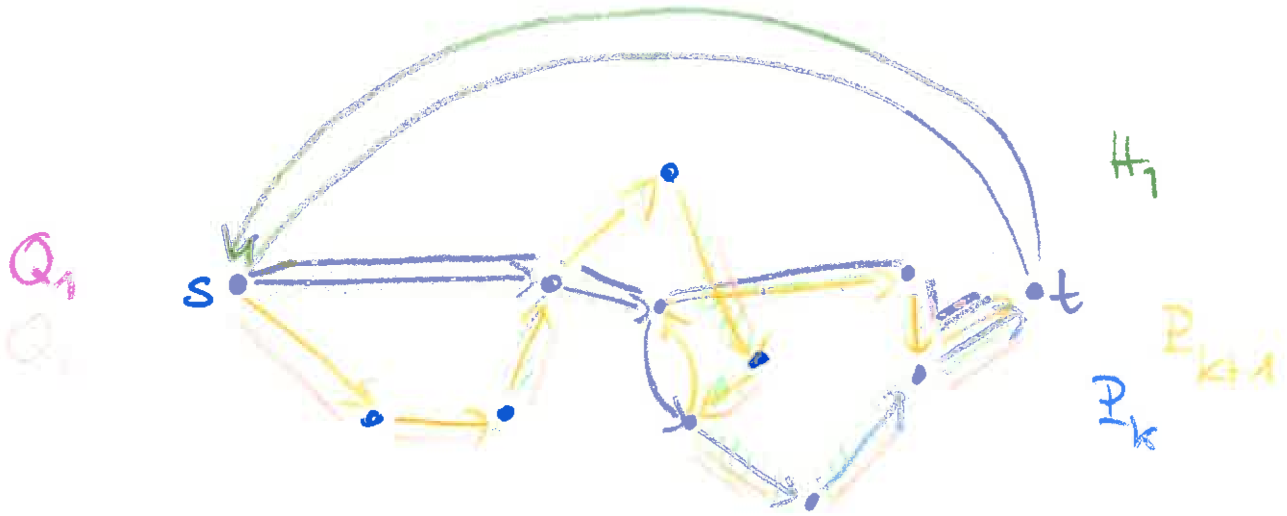
Proof Lemma 8.15:

I

ad (a): Consider the graph G_1 which results from $P_k \dot\cup P_{k+1}$ by deleting pairs of reverse edges.
(Edges, appearing both in P_k and P_{k+1} are taken twice.)



Each edge in $E(G_{f_{k+1}}) \setminus E(G_{f_k})$ must be the reverse of an edge in $P_k \Rightarrow E(G_1) \subseteq E(G_{f_k})$

Let H_1 consist of two copies of (t,s)

$\Rightarrow$ $G_1 + H_1$ is Eulerian

$\Rightarrow$ Each vertex is entered as many times as it is left.

$\Rightarrow$ $G_1 + H_1$ can be decomposed into a set of edge-disjoint circuits

$\Rightarrow$ For each edge in H_1 there is an edge-disjoint s-t-path: Q_1 and Q_2

$E(G_1) \subseteq E(G_{f_k}) \Rightarrow Q_1$ and Q_2 are both f_k-augmenting paths.
P_k is the shortest f_k-augmenting path

$$\Rightarrow \quad |E(P_k)| \le |E(Q_1)|$$
$$|E(P_k)| \le |E(Q_2)|$$

$$\Rightarrow \quad 2|E(P_k)| \le |E(Q_1)| + |E(Q_2)| \le |E(G_1)| \le |E(P_k)| + |E(P_{k+1})|$$

$$\Rightarrow \quad |E(P_k)| \le |E(P_{k+1})|$$

ad (b): By (a) we have also: $|E(P_k)| \leq |E(P_i)| \leq |E(P_\ell)|$ $k<i<\ell$ II

$\Rightarrow$ We consider a pair k, ℓ, such that for $k<i<\ell$ $P_i \cup P_\ell$ does not contain a pair of reverse edges.

As above: Let G_1 be the graph which results from $P_k \dot{\cup} P_\ell$ by deleting pairs of reverse edges.

Again: $E(G_1) \subseteq E(G_{f_k})$:

$E(P_k) \subseteq E(G_{f_k})$

$E(P_\ell) \subseteq E(G_{f_\ell})$

each edge in $E(G_{f_\ell}) \setminus E(G_{f_k})$ must be the reverse of an edge in one of $P_k, P_{k+1}, \dots, P_{\ell-1}$.

But (choice of k and ℓ): only P_k contains the reverse of an edge in P_ℓ

H_1: two copies of (t,s)

$H_1 + G_1$ Eulerian

$\Rightarrow$ two edge-disjoint s-t-paths Q_1, Q_2

Q_1, Q_2 both f_k-augmenting

P_k is shortest f_k-augmenting path

$\Rightarrow |E(P_k)| \leq |E(Q_1)|$

$|E(P_k)| \leq |E(Q_2)|$

$\Rightarrow 2|E(P_k)| \leq |E(Q_1)| + |E(Q_2)| \leq |E(P_k)| + |E(P_\ell)| - 2$

because we deleted at least 2 edges.

$\square$