Average Age of Information in a Multi-Source M/M/1 Queueing Model with LCFS Prioritized Packet Management

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Background: Definition and Appliances

- Internet of Things (IoT) in 5G and beyond
- Cyber-physical control applications
- One key enabler for these services is the freshness of the sensor’s information at the destinations

A status update packet contains
- The measured value of the monitored process
- A time stamp representing the time when the sample was generated

- Generated at random times
- Takes a random time to traverse the network
Background: Age of Information

- The traditional metrics (throughput and delay) can not fully characterize the information freshness.

- AoI (at the destination) is the time elapsed since the last received status update was generated, i.e., the random process

\[ \Delta(t) = t - u(t) \] (1)

  - \( u(t) \) is the time stamp of the most recently received update.

- The most commonly used metrics for evaluating the AoI:
  - Average AoI

![Diagram showing the age of information process](image-url)
Background: Packet management in AoI Analysis


System Model

- Two independent sources, one server, and one sink
- The packets of source $i$ are generated according to the Poisson process with rate $\lambda_i$, $i \in \{1, 2\}$
- The packets are served according to an exponentially distributed service time with mean $1/\mu$
- The load of source $i$ is defined as $\rho_i = \lambda_i / \mu$, $i \in \{1, 2\}$
- The packet generation in the system follows the Poisson process with rate $\lambda = \lambda_1 + \lambda_2$
- The overall load in the system is $\rho = \rho_1 + \rho_2 = \lambda / \mu$
Packet Management Policy

- The queue can contain at most two packets at the same time, one packet of source 1 and one packet of source 2.
- When the system is empty, any arriving packet immediately enters the server.
- When the server is busy, a packet of a source $i \in \{1, 2\}$ waiting in the queue is replaced if a new packet of the same source arrives.
- The fresh packet goes at the head of the queue.

Source 1

Source 2

First packet in the queue

Second packet in the queue
Aol analysis using the SHS technique (1/3)

- Models a queueing system through the states \((q(t), x(t))\)\(^5\)
  - \(q(t) \in Q = \{0, 1, \ldots, m\}\) is a continuous-time finite-state Markov chain that describes the occupancy
  - \(x(t) = [x_0(t)\ x_1(t)\ \cdots\ x_n(t)] \in \mathbb{R}^{1 \times (n+1)}\) is a continuous process that describes the evolution of age-related processes (for instance Aol of source one)

- \(q(t)\) can be presented as a graph \((Q, \mathcal{L})\)
  - A discrete state \(q(t) \in Q\) is a node of the chain
  - A (directed) link \(l \in \mathcal{L}\) from node \(q_l\) to node \(q_l'\) indicates a transition from state \(q_l \in Q\) to state \(q_l' \in Q\)

- A transition occurs when a packet arrives or departs in the system

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Aol analysis using the SHS technique (2/3)

- When a transition \( l \) occurs
  - The discrete state \( q_l \) changes to state \( q'_l \)
  - The continuous state \( x \) is reset to \( x' = xA_l \), \( A_l \in \mathbb{B}^{(n+1)\times(n+1)} \)

- The continuous state \( x \) evolves as a piece-wise linear function
  through the differential equation
  \[
  \dot{x}(t) \triangleq \frac{\partial x(t)}{\partial t} = b_q
  \]
  \( b_q = [b_{q,0} \ b_{q,1} \cdots b_{q,n}] \in \mathbb{B}^{1\times(n+1)} \), \( b_{q,j} \in \{0, 1\} \), \( \forall j \in \{0, \ldots, n\} \), \( q \in Q \)
  - If the age process \( x_j(t) \) increases at a unit rate, we have \( b_{q,j} = 1 \);
    otherwise, \( b_{q,j} = 0 \)

- To calculate the average Aol using the SHS technique
  - The state probabilities of the Markov chain;
    \( \pi_q(t) = \Pr(q(t) = q), \ \forall q \in Q \)
  - The correlation vector between the discrete state \( q(t) \) and the continuous state \( x(t) \);
    \( v_q(t) = [v_{q0}(t) \cdots v_{qn}(t)] \), \( \forall q \in Q \)
Aol analysis using the SHS technique (3/3)

- $L'_q$: set of incoming transitions, and $L_q$: set of outgoing transitions
- Following the ergodicity assumption of the Markov chain $q(t)$,
  - $\pi(t) = [\pi_0(t) \cdots \pi_m(t)]$ converges uniquely to the stationary vector $\bar{\pi} = [\bar{\pi}_0 \cdots \bar{\pi}_m]$ satisfying
    \[
    \bar{\pi}_q \sum_{l \in L_q} \lambda^{(l)} = \sum_{l \in L'_q} \lambda^{(l)} \bar{\pi}_{ql}, \quad \forall q \in Q, \quad \sum_{q \in Q} \bar{\pi}_q = 1, \tag{2}
    \]
  - The correlation vector $v_q(t)$ converges to a nonnegative limit $\bar{v}_q = [\bar{v}_{q0} \cdots \bar{v}_{qn}], \forall q \in Q$, as $t \to \infty$ such that
    \[
    \bar{v}_q \sum_{l \in L_q} \lambda^{(l)} = b_q \bar{\pi}_q + \sum_{l \in L'_q} \lambda^{(l)} \bar{v}_{ql} A_l, \quad \forall q \in Q \tag{3}
    \]
- The average AoI of source 1 is calculated by $^6$
  \[
  \Delta_1 = \sum_{q \in Q} \bar{v}_{q0} \tag{4}
  \]

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The state space of the Markov chain is $Q = \{0, 1, \ldots, 10\}$

<table>
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<th>State</th>
<th>Index of the second packet</th>
<th>Index of the first packet</th>
<th>Index of the packet under service</th>
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<td>1</td>
<td>2</td>
<td>2</td>
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</table>
SHS Analysis for the Proposed System Model (2/6)

- The continuous process is $x(t) = [x_0(t) \ x_1(t) \ x_2(t) \ x_3(t)]$
  - $x_0(t)$: the current AoI of source 1 at time instant $t$, $\Delta_1(t)$
  - $x_1(t)$ encodes what $\Delta_1(t)$ would become if the packet that is under service is delivered to the sink at time instant $t$
  - $x_2(t)$ encodes what $\Delta_1(t)$ would become if the first packet in the queue is delivered to the sink at time instant $t$
  - $x_3(t)$ encodes what $\Delta_1(t)$ would become if the second packet in the queue is delivered to the sink at time instant $t$

- Our goal is to find $\bar{v}_q, \forall q \in Q$ ($\Delta_1 = \sum_{q \in Q} \bar{v}_q$) by solving
  \[
  \bar{v}_q \sum_{l \in \mathcal{L}_q} \lambda^{(l)} = b_q \bar{\pi}_q + \sum_{l \in \mathcal{L}_q'} \lambda^{(l)} \bar{v}_{ql} A_l, \ \forall q \in Q \tag{5}
  \]

- To form the system of linear equations, we need to determine
  - $b_q, \bar{\pi}_q, \forall q \in Q$
  - $\bar{v}_{ql} A_l$ for each incoming transition $l \in \mathcal{L}_q', \forall q \in Q$
Calculating $\bar{\pi}_q$

$$(\bar{\pi}_q \sum_{l \in \mathcal{L}_q} \lambda^{(l)} = \sum_{l \in \mathcal{L}_q'} \lambda^{(l)} \bar{\pi}_q l, \forall q \in Q, \sum_{q \in Q} \bar{\pi}_q = 1,)$$

$\bar{\pi}_0 = \frac{1}{\beta}$, $\bar{\pi}_1 = \frac{\rho_1}{\beta}$, $\bar{\pi}_2 = \frac{\rho_2}{\beta}$, $\bar{\pi}_3 = \frac{\rho_2^2}{\beta}$,

$\bar{\pi}_4 = \frac{\rho_1 \rho_2 (1 + \rho)}{\beta (1 + \rho_1)}$, $\bar{\pi}_5 = \frac{\rho_1^2 \rho_2}{\beta (1 + \rho_2)}$, $\bar{\pi}_6 = \frac{\rho_1^2 \rho_2 (1 + \rho)}{\beta (1 + \rho_1)}$,

$\bar{\pi}_7 = \frac{\rho_1 \rho_2 (1 + \rho)}{\beta (1 + \rho_2)}$, $\bar{\pi}_8 = \frac{\rho_2^2}{\beta (1 + \rho_2)}$, $\bar{\pi}_9 = \frac{\rho_1 \rho_2^2 (2 + \rho)}{\beta (1 + \rho_2)}$, $\bar{\pi}_{10} = \frac{\rho_1 \rho_2^2}{\beta}$,

where $\beta = \rho^2 + \rho (2 \rho_1 \rho_2 + 1) + 1$
SHS Analysis for the Proposed System Model (4/6)

Calculating $b_q$ ($\dot{x} = b_q$)

- $b_{q,1} = 1, \, \forall q \in Q$: the AoI of source 1, $\Delta_1(t) = x_0(t)$, increases at a unit rate with time in all discrete states
- $b_{q,i}$ is equal to 1 if the $i$th packet in the queue is a source one packet

$$b_q = \begin{cases} 
[1 \ 0 \ 0 \ 0], & q = 0, \\
[1 \ 1 \ 0 \ 0], & q = 1, \\
[1 \ 0 \ 0 \ 0], & q = 2, \\
[1 \ 1 \ 1 \ 0], & q = 3, \\
[1 \ 1 \ 0 \ 0], & q = 4, \\
[1 \ 1 \ 1 \ 0], & q = 5, \\
\end{cases}$$

$$b_q = \begin{cases} 
[1 \ 1 \ 0 \ 1], & q = 6, \\
[1 \ 0 \ 1 \ 0], & q = 7, \\
[1 \ 0 \ 0 \ 0], & q = 8, \\
[1 \ 0 \ 1 \ 0], & q = 9, \\
[1 \ 0 \ 0 \ 1], & q = 10 \\
\end{cases}$$
SHS Analysis for the Proposed System Model (5/6)

- Calculating $\bar{v}_{ql}A_l$ for each incoming transition $l \in L'_q, \forall q \in Q$
  - There are 32 transitions
  - For instance transition $l : 3 \rightarrow 6$ in the chain

\[ x' = [x_0 \ x_1 \ x_2 \ x_3]A_l = [x_0 \ x_1 \ 0 \ x_2] \]  \hspace{2cm} (6)

\[ x' = [x_0 \ x_1 \ x_2 \ x_3]A_l = [x_0 \ x_1 \ 0 \ x_2] \Rightarrow A_l = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \]  \hspace{2cm} (7)

\[ \bar{v}_3A_l = [v_{30} \ v_{31} \ v_{32} \ v_{33}]A_l = [v_{30} \ v_{31} \ 0 \ v_{32}] \]  \hspace{2cm} (8)
After solving the system of linear equations we have

\[
\Delta_1 = \frac{\sum_{i=0}^{13} \rho_1^i \psi_i}{\mu \rho_1 (1 + \rho_1) \left( \sum_{j=0}^{11} \rho_1^j \xi_j \right)},
\]

(9)

\[
\psi_0 = \rho_2^7 + 5\rho_2^6 + 12\rho_2^5 + 18\rho_2^4 + 18\rho_2^3 + 12\rho_2^2 + 5\rho_2 + 1,
\]

\[
\psi_1 = 3\rho_2^8 + 22\rho_2^7 + 76\rho_2^6 + 159\rho_2^5 + 222\rho_2^4 + 213\rho_2^3 + 138\rho_2^2 + 56\rho_2 + 11,
\]

\[\cdots\]
Results (1/2)

- Sum average AoI for different values of $\rho$ under different management policies\(^7\) with $\mu = 1$

Results (2/2)

- Sum average AoI for different values of \( \rho \) under different management policies with \( \mu = 1 \)

- Jain’s fairness index: \( J(\Delta_1, \Delta_2) = \frac{(\Delta_1 + \Delta_2)^2}{2(\Delta_1^2 + \Delta_2^2)} \)
Thank You For Your Attention!

Questions?
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