Average Age of Information for a Multi-Source M/M/1 Queueing Model With Packet Management

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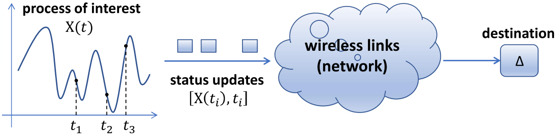
Background: Definition and Appliances

- Internet of Things (IoT) in 5G and beyond
- Cyber-physical control applications
- One key enabler for these services is the freshness of the sensor's information at the destinations



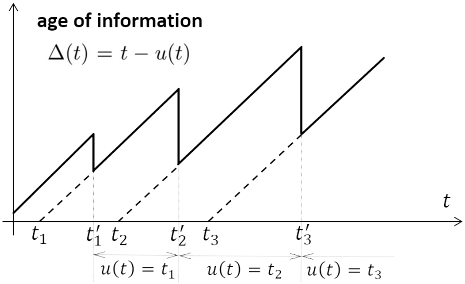
- A status update packet contains
  - The measured value of the monitored process
  - A time stamp representing the time when the sample was generated
- Generated at random times
- Takes a random time to traverse the network

Background: Age of Information

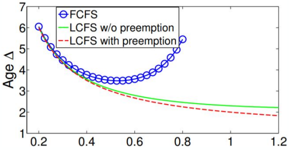
- The traditional metrics (throughput and delay) can not fully characterize the information freshness
- AoI (at the destination) is the time elapsed since the last received status update was generated, i.e., the random process

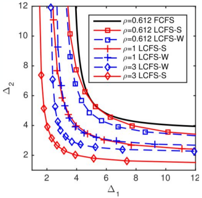
$$\Delta(t) = t - u(t) \tag{1}$$

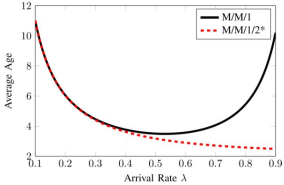
  - $u(t)$ is the time stamp of the most recently received update
- The most commonly used metrics for evaluating the AoI
  - Average AoI

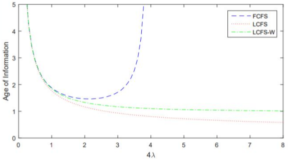


Background: Packet management in AoI Analysis[1] [2] [3] [4]









[1] S. K. Kaul, R. D. Yates, and M. Gruteser, "Status updates through queues," in Proc. Conf. Inform. Sciences Syst. (CISS), Princeton, NJ, USA, Mar. 2123, 2012, pp. 16.

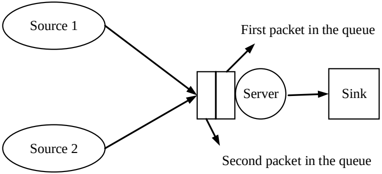
[2] M. Costa, M. Codreanu, and A. Ephremides, "On the age of information in status update systems with packet management," IEEE Trans. Inform. Theory, vol. 62, no. 4, pp. 18971910, Apr. 2016.

[3] R. D. Yates and S. K. Kaul, "The age of information: Real-time status updating by multiple sources," IEEE Trans. Inform. Theory, vol. 65, no. 3, pp. 18071827, Mar. 2019.

[4] A. Javani and Z. Wang, "Age of information in multiple sensing" [Online]. Available: http://arxiv.org/abs/1902.01975, 2019.

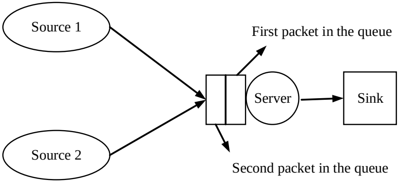
System Model

- Two independent sources, one server, and one sink
- The packets of source i are generated according to the Poisson process with rate λ_i, $i \in \{1, 2\}$
- The packets are served according to an exponentially distributed service time with mean $1/\mu$
- The load of source i is defined as $\rho_i = \lambda_i/\mu$, $i \in \{1, 2\}$
- The packet generation in the system follows the Poisson process with rate $\lambda = \lambda_1 + \lambda_2$
- The overall load in the system is $\rho = \rho_1 + \rho_2 = \lambda/\mu$



Packet Management Policy

- The queue can contain at most two packets at the same time, one packet of source 1 and one packet of source 2
- When the system is empty, any arriving packet immediately enters the server
- When the server is busy, a packet of a source $i \in \{1, 2\}$ waiting in the queue is replaced if a new packet of the **same source** arrives



AoI analysis using the SHS technique (1/3)

- Models a queueing system through the states $(q(t), \mathbf{x}(t))$[5]
  - $q(t) \in \mathcal{Q} = \{0, 1, \ldots, m\}$ is a continuous-time finite-state Markov chain that describes the occupancy
  - $\mathbf{x}(t) = [x_0(t)\ x_1(t) \cdots x_n(t)] \in \mathbb{R}^{1\times(n+1)}$ is a continuous process that describes the evolution of age-related processes (for instance AoI of source one)
- $q(t)$ can be presented as a graph $(\mathcal{Q}, \mathcal{L})$
  - A discrete state $q(t) \in \mathcal{Q}$ is a node of the chain
  - A (directed) link $l \in \mathcal{L}$ from node q_l to node q'_l indicates a transition from state $q_l \in \mathcal{Q}$ to state $q'_l \in \mathcal{Q}$
- A transition occurs when a packet arrives or departs in the system

[5] R. D. Yates and S. K. Kaul, "The age of information: Real-time status updating by multiple sources," IEEE Trans. Inform. Theory, vol. 65, no. 3, pp. 18071827, Mar. 2019.

AoI analysis using the SHS technique(2/3)

- When a transition l occurs
  - The discrete state q_l changes to state q'_l
  - The continuous state $\mathbf{x}$ is reset to $\mathbf{x}'$; $\mathbf{x}' = \mathbf{x}\mathbf{A}_l$, $\mathbf{A}_l \in \mathbb{B}^{(n+1)\times(n+1)}$
- The continuous state $\mathbf{x}$ evolves as a piece-wise linear function through the differential equation $\dot{\mathbf{x}}(t) \triangleq \frac{\partial \mathbf{x}(t)}{\partial t} = \mathbf{b}_q$
  - $\mathbf{b}_q = [b_{q,0}\ b_{q,1} \cdots b_{q,n}] \in \mathbb{B}^{1\times(n+1)}$, $b_{q,j} \in \{0,1\}, \forall j \in \{0,\ldots,n\}, q \in \mathcal{Q}$
  - If the age process $x_j(t)$ increases at a unit rate, we have $b_{q,j} = 1$; otherwise, $b_{q,j} = 0$
- To calculate the average AoI using the SHS technique
  - The state probabilities of the Markov chain; $\pi_q(t) = \Pr(q(t) = q),\ \forall q \in \mathcal{Q}$
  - The correlation vector between the discrete state $q(t)$ and the continuous state $\mathbf{x}(t)$; $\mathbf{v}_q(t) = [v_{q0}(t) \cdots v_{qn}(t)],\ \forall q \in \mathcal{Q}$

AoI analysis using the SHS technique(3/3)

- $\mathcal{L}'_q$: set of incoming transitions, and $\mathcal{L}_q$: set of outgoing transitions
- Following the ergodicity assumption of the Markov chain $q(t)$,
  - $\boldsymbol{\pi}(t) = [\pi_0(t) \cdots \pi_m(t)]$ converges uniquely to the stationary vector $\bar{\boldsymbol{\pi}} = [\bar{\pi}_0 \cdots \bar{\pi}_m]$ satisfying

$$\bar{\pi}_q \sum_{l \in \mathcal{L}_q} \lambda^{(l)} = \sum_{l \in \mathcal{L}'_q} \lambda^{(l)} \bar{\pi}_{q_l}, \ \ \forall q \in \mathcal{Q}, \ \ \sum_{q \in \mathcal{Q}} \bar{\pi}_q = 1, \tag{2}$$

  - The correlation vector $\mathbf{v}_q(t)$ converges to a nonnegative limit $\bar{\mathbf{v}}_q = [\bar{v}_{q0} \cdots \bar{v}_{qn}], \forall q \in \mathcal{Q}$, as $t \to \infty$ such that

$$\bar{\mathbf{v}}_q \sum_{l \in \mathcal{L}_q} \lambda^{(l)} = \mathbf{b}_q \bar{\pi}_q + \sum_{l \in \mathcal{L}'_q} \lambda^{(l)} \bar{\mathbf{v}}_{q_l} \mathbf{A}_l, \ \ \forall q \in \mathcal{Q} \tag{3}$$

- The average AoI of source 1 is calculated by [6]

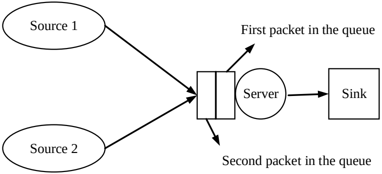
$$\Delta_1 = \sum_{q \in \mathcal{Q}} \bar{v}_{q0} \tag{4}$$

[6] R. D. Yates and S. K. Kaul, "The age of information: Real-time status updating by multiple sources," IEEE Trans. Inform. Theory, vol. 65, no. 3, pp. 18071827, Mar. 2019.

SHS Analysis for the Proposed System Model (1/6)

- The state space of the Markov chain is $\mathcal{Q} = \{0, 1, \cdots, 10\}$

State	Index of the second packet	Index of the first packet	Index of the packet under service
0	-	-	-
1	-	-	1
2	-	-	2
3	-	1	1
4	-	2	1
5	2	1	1
6	1	2	1
7	-	1	2
8	-	2	2
9	2	1	2
10	1	2	2



SHS Analysis for the Proposed System Model (2/6)

- The continuous process is $\mathbf{x}(t) = [x_0(t)\ x_1(t)\ x_2(t)\ x_3(t)]$
  - $x_0(t)$: the current AoI of source 1 at time instant t, $\Delta_1(t)$
  - $x_1(t)$ encodes what $\Delta_1(t)$ would become if the packet that is under service is delivered to the sink at time instant t
  - $x_2(t)$ encodes what $\Delta_1(t)$ would become if the first packet in the queue is delivered to the sink at time instant t
  - $x_3(t)$ encodes what $\Delta_1(t)$ would become if the second packet in the queue is delivered to the sink at time instant t
- Our goal is to find $\bar{v}_{q0}, \forall q \in \mathcal{Q}$ $(\Delta_1 = \sum_{q\in\mathcal{Q}} \bar{v}_{q0})$ by solving

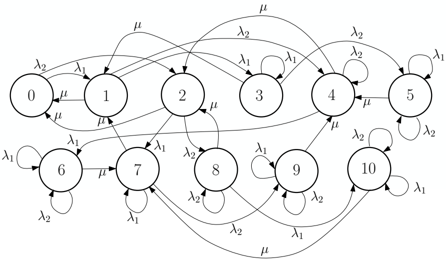
$$\bar{\mathbf{v}}_q \sum_{l\in\mathcal{L}_q} \lambda^{(l)} = \mathbf{b}_q \bar{\pi}_q + \sum_{l\in\mathcal{L}'_q} \lambda^{(l)} \bar{\mathbf{v}}_{q_l} \mathbf{A}_l, \ \ \forall q \in \mathcal{Q} \tag{5}$$

- To form the system of linear equations, we need to determine
  - $\mathbf{b}_q$, $\bar{\pi}_q$, $\forall q \in \mathcal{Q}$
  - $\bar{\mathbf{v}}_{q_l} \mathbf{A}_l$ for each incoming transition $l \in \mathcal{L}'_q, \forall q \in \mathcal{Q}$

SHS Analysis for the Proposed System Model (3/6)

- Calculating $\bar{\pi}_q$

  $(\bar{\pi}_q \sum_{l\in\mathcal{L}_q} \lambda^{(l)} = \sum_{l\in\mathcal{L}'_q} \lambda^{(l)} \bar{\pi}_{ql}, \;\; \forall q \in \mathcal{Q}, \;\; \sum_{q\in\mathcal{Q}} \bar{\pi}_q = 1,)$



$$\bar{\pi}_0 = \frac{1}{\beta}, \qquad \bar{\pi}_1 = \frac{\rho_1}{\beta}, \qquad \bar{\pi}_2 = \frac{\rho_2}{\beta}, \qquad \bar{\pi}_3 = \frac{\rho_1^2}{\beta},$$

$$\bar{\pi}_4 = \frac{\rho_1\rho_2(1+\rho)}{\beta(1+\rho_1)}, \qquad \bar{\pi}_5 = \frac{\rho_1^2\rho_2}{\beta(1+\rho_2)}, \qquad \bar{\pi}_6 = \frac{\rho_1^2\rho_2(1+\rho)}{\beta(1+\rho_1)},$$

$$\bar{\pi}_7 = \frac{\rho_1\rho_2(1+\rho)}{\beta(1+\rho_2)}, \qquad \bar{\pi}_8 = \frac{\rho_2^2}{\beta(1+\rho_1)}, \qquad \bar{\pi}_9 = \frac{\rho_1\rho_2^2(2+\rho)}{\beta(1+\rho_2)}, \qquad \bar{\pi}_{10} = \frac{\rho_1\rho_2^2}{\beta},$$

where $\beta = \rho^2 + \rho(2\rho_1\rho_2 + 1) + 1$

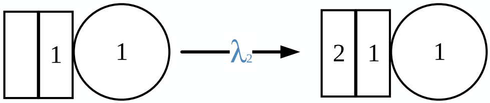
SHS Analysis for the Proposed System Model (4/6)

- Calculating $\mathbf{b}_q$ ($\dot{\mathbf{x}} = \mathbf{b}_q$)
  - $b_{q,1} = 1, \ \forall q \in \mathcal{Q}$: the AoI of source 1, $\Delta_1(t) = x_0(t)$, increases at a unit rate with time in all discrete states
  - $b_{q,i}$ is equal to 1 if the ith packet in the queue is a source one packet

$$\mathbf{b}_q = \begin{cases} [1\ 0\ 0\ 0], & q = 0, \\ [1\ 1\ 0\ 0], & q = 1, \\ [1\ 0\ 0\ 0], & q = 2, \\ [1\ 1\ 1\ 0], & q = 3, \\ [1\ 1\ 0\ 0], & q = 4, \\ [1\ 1\ 1\ 0], & q = 5, \end{cases} \qquad \mathbf{b}_q = \begin{cases} [1\ 1\ 0\ 1], & q = 6, \\ [1\ 0\ 1\ 0], & q = 7, \\ [1\ 0\ 0\ 0], & q = 8, \\ [1\ 0\ 1\ 0], & q = 9, \\ [1\ 0\ 0\ 1], & q = 10 \end{cases}$$

SHS Analysis for the Proposed System Model (5/6)

- Calculating $\bar{\mathbf{v}}_{q_l}\mathbf{A}_l$ for each incoming transition $l \in \mathcal{L}'_q, \forall q \in \mathcal{Q}$
  - There are 32 transitions
  - For instance transition $l : 3 \to 5$ in the chain



$$\mathbf{x}' = [x_0 \;\; x_1 \;\; x_2 \;\; 0] \tag{6}$$

$$\mathbf{x}' = [x_0 \;\; x_1 \;\; x_2 \;\; x_3]\mathbf{A}_l = [x_0 \;\; x_1 \;\; x_2 \;\; 0] \Rightarrow \mathbf{A}_l = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

$$\bar{\mathbf{v}}_3\mathbf{A}_l = [v_{30} \;\; v_{31} \;\; v_{32} \;\; v_{33}]\mathbf{A}_l = [v_{30} \;\; v_{31} \;\; v_{32} \;\; 0] \tag{8}$$

AoI in a Multi-Source M/G/1 Queueing Model (6/6)

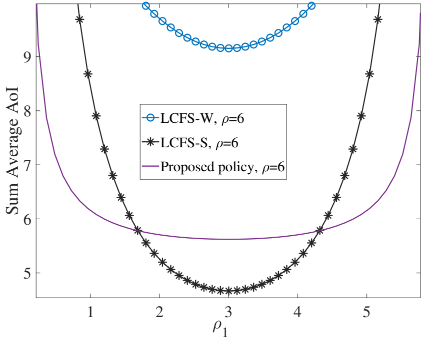
- After solving the system of liner equations we have

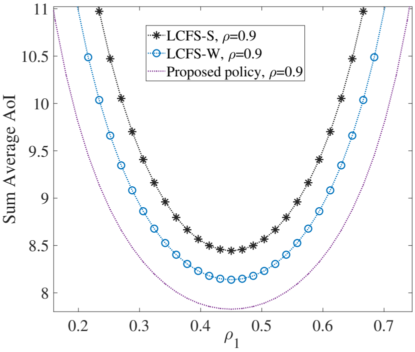
$$\Delta_1 = \frac{\sum_{k=0}^{7} \rho_1^k \eta_k}{\mu\rho_1 (1+\rho_1)^2 \sum_{j=0}^{4} \rho_1^j \xi_j},$$

$\eta_0 = \rho_2^4 + 2\rho_2^3 + 3\rho_2^2 + 2\rho_2 + 1,$ $\eta_1 = 7\rho_2^4 + 15\rho_2^3 + 21\rho_2^2 + 14\rho_2 + 6,$
$\eta_2 = 17\rho_2^4 + 46\rho_2^3 + 64\rho_2^2 + 42\rho_2 + 16,$ $\eta_3 = 15\rho_2^4 + 73\rho_2^3 + 118\rho_2^2 + 78\rho_2 + 26,$
$\eta_4 = 5\rho_2^4 + 52\rho_2^3 + 124\rho_2^2 + 102\rho_2 + 30,$ $\eta_5 = 15\rho_2^3 + 66\rho_2^2 + 79\rho_2 + 24,$
$\eta_6 = 15\rho_2^2 + 31\rho_2 + 11,$ $\eta_7 = 5\rho_2 + 2,$
$\xi_0 = \rho_2^4 + 2\rho_2^3 + 3\rho_2^2 + 2\rho_2 + 1,$ $\xi_1 = 2\rho_2^4 + 6\rho_2^3 + 9\rho_2^2 + 7\rho_2 + 3,$
$\xi_2 = 6\rho_2^3 + 12\rho_2^2 + 10\rho_2 + 4,$ $\xi_3 = 6\rho_2^2 + 8\rho_2 + 3,$
$\xi_4 = 2\rho_2 + 1.$

Results

- Sum average AoI for different values of ρ[7] with $\mu = 1$





[7] R. D. Yates and S. K. Kaul, "The age of information: Real-time status updating by multiple sources," IEEE Trans. Inform. Theory, vol. 65, no. 3, pp. 18071827, Mar. 2019.

Thank You For Your Attention!

Questions?

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