

# Automation for Separation with CDOs: Dynamic Aircraft Arrival Routes

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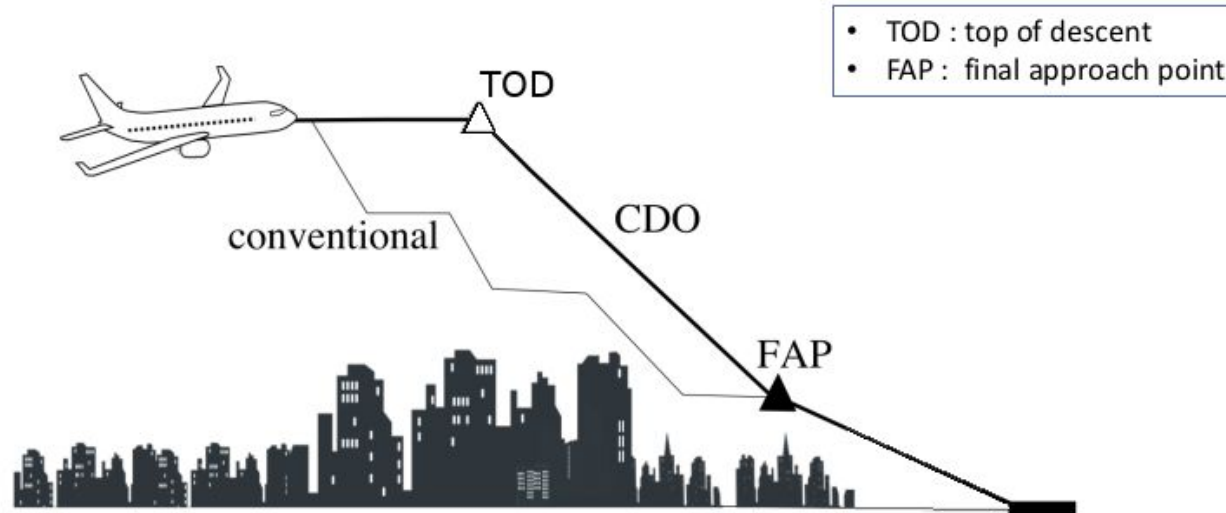
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## Motivation

- ✓ Air transportation grows: pros and cons
- ✓ Increased complexity and environmental effects
- ✓ Terminal Maneuvering Areas (TMAs) - most congested
- ✓ Optimization of arrival and departure procedures is needed
- ✓ Our solution:
  - Automatically separated arrivals to reduce complexity and ATCO's workload
  - **CDOs (Continuous Descent Operations):** promising solution to mitigate environmental effects, according to ICAO and EUROCONTROL: *“CDOs allow aircraft to follow a flexible, optimum flight path that delivers major environmental and economic benefits—reduced fuel burn, gaseous emissions, noise and fuel costs—without any adverse effect on safety”*

## CDOs

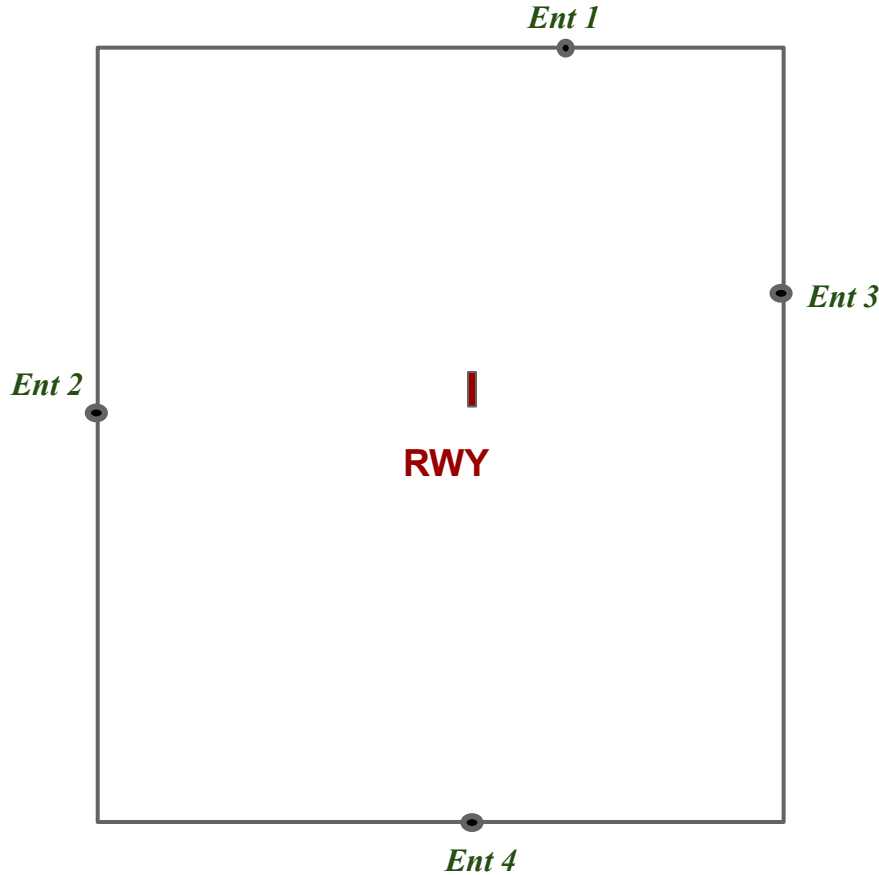
- ✓ CDOs have shown important environmental benefits w.r.t. conventional (step-down) approaches in TMAs



## Previous Work

- ✓ **LiU-LFV**: optimal STARs + time-separated demand-weighted *arrival routes (dynamic, for pre-tactical planning)*
- ✓ **UPC**: CDO-enabled optimized arrival procedures (*engine-idle, low noise*)

**New**: automated time-separated demand-weighted **CDO-enabled** optimized *arrival routes*



## Input

- ✓ Location and direction of the airport runway
- ✓ Locations of the entry points to the TMA
- ✓ Aircraft arrival times at the entry points for a fixed time period
- ✓ Cruise conditions (altitude, true airspeed, distance to entry point + path distance inside TMA) and aircraft type for CDO profile generation



## Output

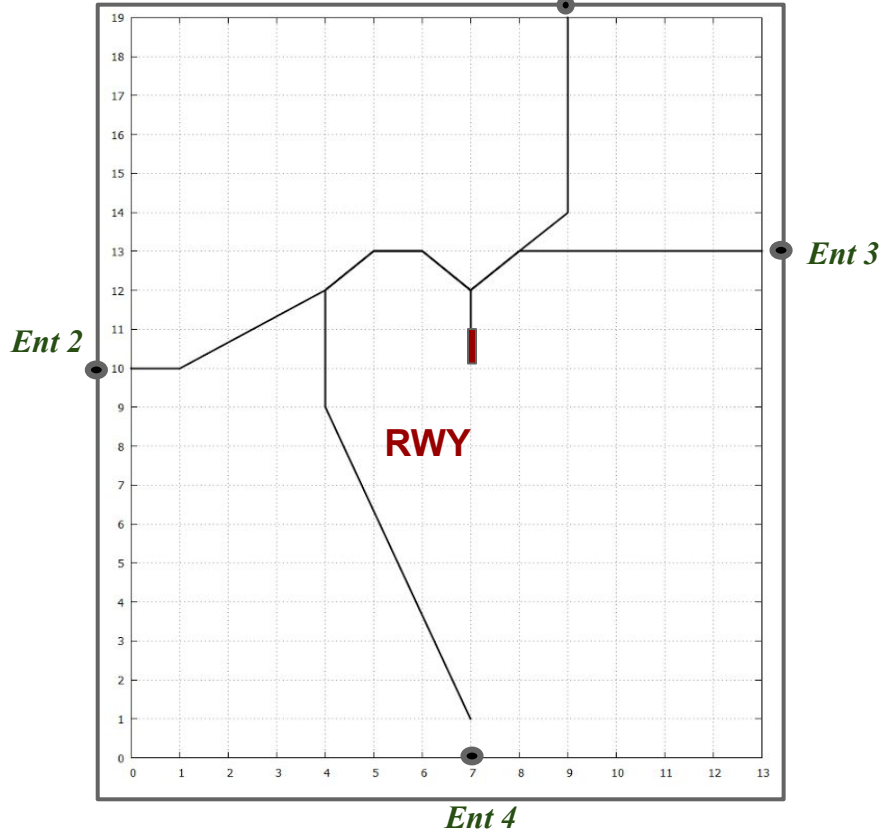
Optimal arrival tree  
that merges traffic  
from the entries to the runway  
ensuring safe aircraft separation  
for the given time period



## Output

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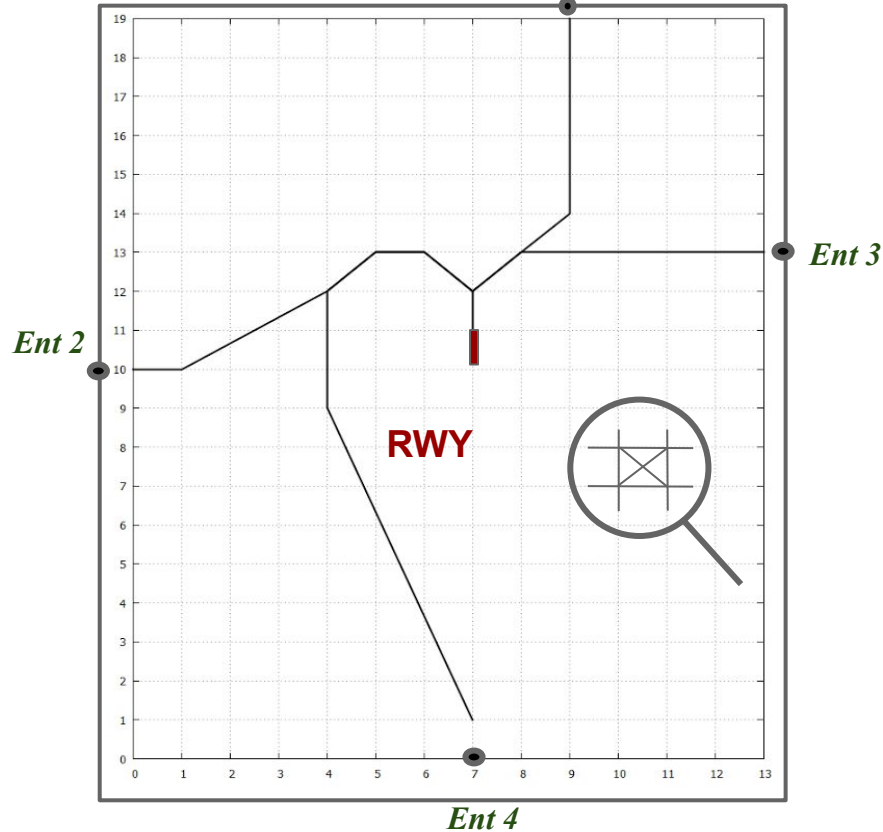
=  
a set of time-separated **CDO-enabled**  
aircraft trajectories  
optimized w.r.t. the traffic demand  
during the given period

*Ent 1*

## Grid-Based MIP Formulation

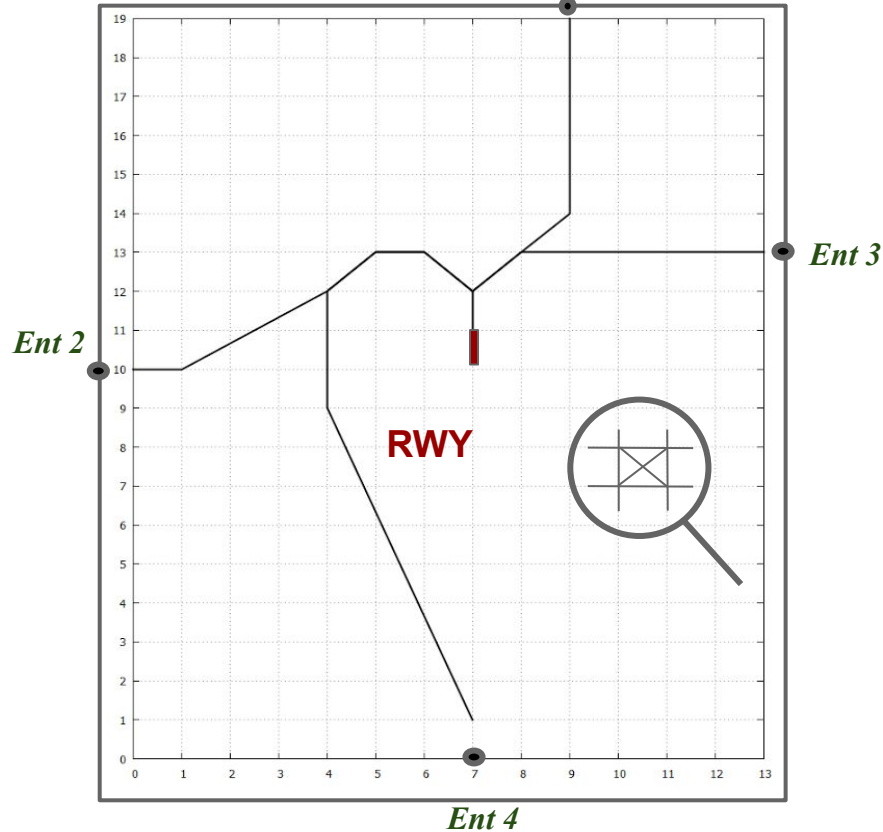
- ✓ Square grid in the TMA
- ✓ Snap locations of the entry points and the runway into the grid
- ✓ Grid cell side of the length  $l$  (*separation parameter*)



*Ent 1*

## Grid-Based MIP Formulation

- ✓ Square grid in the TMA
- ✓ Snap locations of the entry points and the runway into the grid
- ✓ Grid cell side of the length  $l$  (*separation parameter*)
- ✓ Every node connected to its 8 neighbours



## Grid-Based MIP Formulation

- ✓ Square grid in the TMA
- ✓ Snap locations of the entry points and the runway into the grid
- ✓ Grid cell side of the length  $l$  (*separation parameter*)
- ✓ Every node connected to its 8 neighbours
- ✓ Problem formulated as MIP

*Based on flow MIP formulation for Steiner trees*

## Operational Requirements

- ✓ No more than two routes merge at a point
- ✓ Merge point separation
- ✓ No sharp turns
- ✓ Temporal separation of all aircraft along the routes
- ✓ All aircraft fly energy-neutral CDO:  
idle thrust, no speed brakes (noise avoidance)
- ✓ Smooth transition between consecutive trees when switching

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# MIP Formulation

## VARIABLES

$x_e$  - decision variable - indicates whether edge  $e$  participates in arrival tree

$f_e$  - gives the flow on edge  $e = (i, j)$ , non-negative

## OBJECTIVES

$$\begin{array}{ll}
 \text{Total path length:} & \min \sum_{e \in E} \ell_e f_e \\
 \text{Total tree weight:} & \min \sum_{e \in E} \ell_e x_e
 \end{array}
 \quad \Longrightarrow \quad
 \min \beta \sum_{e \in E} \ell_e x_e + (1 - \beta) \sum_{e \in E} \ell_e f_e$$

## MIP formulation: Constraints

- ✓ Flow constraints
- ✓ Degree constraints
- ✓ Turn angle constraints
- ✓ Auxiliary constraints to prevent crossings
- ✓ Temporal separation of all aircraft along the routes
- ✓ Realistic CDO speed profiles
- ✓ Consistency between trees of different time periods

## MIP formulation: Constraints

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## MIP formulation: Constraints

### ✓ RE: Flow constraints

$$\sum_{k:(k,i) \in E} f_{ki} - \sum_{j:(i,j) \in E} f_{ij} = \begin{cases} \sum_{b \in \mathcal{P}} w_b & i = r \\ -w_i & i \in \mathcal{P} \\ 0 & i \in V \setminus \{\mathcal{P} \cup r\} \end{cases}, \text{ where } \mathcal{P} \text{ set of entry points}$$

$w_b$  - number of a/c entering TMA from the entry point  $b \in \mathcal{P}$

$$x_e \geq \frac{f_e}{Q} \quad \forall e \in E$$

$$f_e \geq 0 \quad \forall e \in E \quad \text{where } Q \text{ is a large number (e.g., } Q = |\mathcal{P}|)$$

$$x_e \in \{0, 1\} \quad \forall e \in E$$



T. Andersson, T. Polishchuk, V. Polishchuk, C. Schmidt. Automatic Design of Aircraft Arrival Routes with Limited Turning Angle. ATMOS 2016, Aarhus, Denmark.



## MIP formulation: Constraints

### ✓ RE: Degree constraints

$$\sum_{k:(k,i) \in E} x_{ki} \leq 2 \quad \forall i \in V \setminus \{\mathcal{P} \cup r\}$$

- maximum indegree

$$\sum_{j:(i,j) \in E} x_{ij} \leq 1 \quad \forall i \in V \setminus \{\mathcal{P} \cup r\}$$

- maximum outdegree

$$\sum_{k:(k,r) \in E} x_{kr} = 1$$

- runway  $r$  has only 1 in-going edge

$$\sum_{j:(i,j) \in E} x_{ij} = 1 \quad \forall i \in \mathcal{P}$$

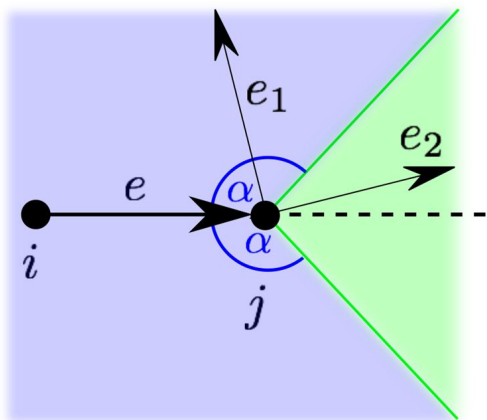
- only 1 out-going edge for entry points



T. Andersson, T. Polishchuk, V. Polishchuk, C. Schmidt. Automatic Design of Aircraft Arrival Routes with Limited Turning Angle. ATMOS 2016, Aarhus, Denmark.

## MIP formulation: Constraints

### ✓ RE: Turn angle constraint



$$c_e x_e + \sum_{f \in \Gamma_e} x_f \leq c_e \quad \forall e \in E$$

for each edge  $e = (i, j)$  used in the arrival tree, all outgoing edges at  $j$  must form an angle of at least  $\alpha$  with  $e$ .



T. Andersson, T. Polishchuk, V. Polishchuk, C. Schmidt. Automatic Design of Aircraft Arrival Routes with Limited Turning Angle. ATMOS 2016, Aarhus, Denmark.

# MIP formulation: Constraints

## ✓ RE: Auxiliary Constraints to Prevent Crossings

For all points except last column, last row, entries and rwy:

$$x_{i,i+1+n} + x_{i+1+n,i} + x_{i+n,i+1} + x_{i+1,i+n} \leq 1$$

$$\forall i \in V' \setminus \{\mathcal{P} \cup r\} : i+1+n, i+n, i+1 \notin \{\mathcal{P} \cup r\}$$

$$V' = V \setminus \{\text{last row}\} \setminus \{\text{last column}\}$$

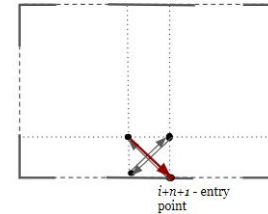
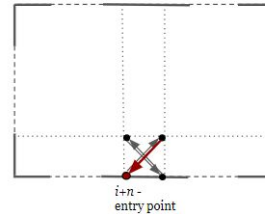
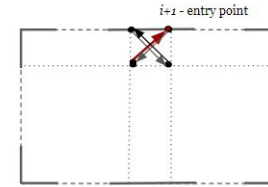
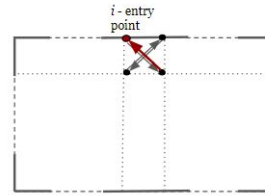
For different entry point locations:

$$x_{i,i+1+n} + x_{i+n,i+1} + x_{i+1,i+n} \leq 1 \quad \forall i \in \mathcal{P}$$

$$x_{i,i+1+n} + x_{i+1+n,i} + x_{i+1,i+n} \leq 1 \quad \forall i : i+1 \in \mathcal{P}$$

$$x_{i,i+1+n} + x_{i+n+1,i} + x_{i+n,i+1} \leq 1 \quad \forall i : i+n \in \mathcal{P}$$

$$x_{i+1+n,i} + x_{i+n,i+1} + x_{i+1,i+n} \leq 1 \quad \forall i : i+n+1 \in \mathcal{P}$$



J. Dahlberg, T. Andersson Granberg, T. Polishchuk, C. Schmidt, L. Sedov. Capacity-Driven Automatic Design of Dynamic Aircraft Arrival Routes. DASC 2018, London, UK.

# MIP formulation: Constraints

## ✓ RE: Temporal Aircraft Separation

**More variables**  $y_{a,j,t}$  - binary, shows a/c  $a$  at node  $j$  at time  $t$   
 $x_{e,b}$  - binary: edge  $e$  in the route from entry point  $b$

Forward the information on the times at which  $a$  arrives at nodes along the route from  $b$  to the rwy

Connect to  $x_e$ :  $x_{e,b} \leq x_e \forall b \in \mathcal{P}, \forall e \in E$

$$\sum_{j:(j,k) \in E} x_{(j,k),b} \times y_{a,j,t} = y_{a,k,t+u}$$

Set:

$$y_{a,b,t_a^b} = 1 \quad \forall b \in \mathcal{P}, \forall a \in \mathcal{A}_b$$

$$y_{a,b,t} = 0 \quad \forall b \in \mathcal{P}, \forall a \in \mathcal{A} \setminus \mathcal{A}_b, \forall t \in T$$

$$y_{a,b,t} = 0 \quad \forall b \in \mathcal{P}, \forall a \in \mathcal{A}_b, \forall t \in T \setminus \{t_a^b\}$$

$\forall b \in \mathcal{P}, \forall a \in \mathcal{A}_b, \forall k \in V \setminus \mathcal{P}, \forall t \in \{0, \dots, \bar{T} - u\}$

Not linear  $\Leftrightarrow$  linearize ...

Time separation:

$$y_{a,j,t} \leq \sum_{\substack{k \in V: \\ (k,j) \in E}} x_{(k,j)} \quad \forall b \in \mathcal{P}, \forall a \in \mathcal{A}, \forall j \in V \setminus \mathcal{P}, \forall t \in T$$

$$T = \{0, \dots, \bar{T}\}$$

$$\sum_{\tau=t}^{t+\sigma-1} \sum_{a \in \mathcal{A}} y_{a,j,\tau} \leq 1 \quad \forall j \in V, \forall t \in \{0, \dots, \bar{T} - \sigma + 1\}$$

$\sigma$  - separation parameter



J. Dahlberg, T. Andersson Granberg, T. Polishchuk, C. Schmidt, L. Sedov. Capacity-Driven Automatic Design of Dynamic Aircraft Arrival Routes. DASC 2018, London, UK.

## MIP formulation: Constraints

- ✓ Flow constraints
  - ✓ Degree constraints
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- ✓ Temporal separation of all aircraft along the routes
  - ✓ Realistic CDO speed profiles
  - ✓ Consistency between trees of different time periods

## MIP Formulation: Realistic CDO Speed Profiles

- ✓ The state vector  $\mathbf{x}$  represents the fixed initial conditions of the aircraft: TAS  $\mathbf{v}$ , altitude  $\mathbf{h}$  and distance to go  $\mathbf{s}$
- ✓ To achieve environmentally friendly trajectories, **idle thrust** is assumed and **speed-brakes** use is **not allowed** throughout the descent → energy-neutral CDO
- ✓ The flight path angle is the only control variable in this problem → control vector  $\mathbf{u}$

$$\mathbf{x} = [v, h, s]$$

$$\mathbf{u} = [\gamma]$$

## MIP Formulation: Realistic CDO Speed Profiles

- ✓ A point-mass representation of the aircraft reduced to a “gamma-command” is considered, where vertical equilibrium is assumed → Dynamic constraints  $f$
- ✓ Path constraints  $h$  are enforced to ensure that the aircraft airspeed remains within operational limits, and that the maximum and minimum descent gradients are not exceeded
- ✓ Terminal constraints  $\psi$  fix the final states vector

### Dynamic constraints

$$f = \begin{bmatrix} \dot{v} \\ \dot{h} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} \frac{T_{idle} - D}{m} - g\gamma \\ v\gamma \\ v + w \end{bmatrix}$$

### Path constraints

$$h = \begin{bmatrix} v_{CAS,min} - v_{CAS} \\ v_{CAS} - VMO \\ M - MMO \\ \gamma \\ \gamma_{min} - \gamma \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

### Terminal constraints

$$\psi = \begin{bmatrix} v - v_f \\ h - h_f \\ s - s_f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

## MIP Formulation: Realistic CDO Speed Profiles

- ✓ The trajectory is divided in two phases: the latter part of the cruise phase prior the top of descent (TOD) and the idle descent
- ✓ The original cruise speed is not modified after the optimization process, so the two-phases optimal control problem can be converted into a single-phase optimal control problem
- ✓ BADA V4 is used to model the aircraft performance

$$J = \frac{f}{v_{cruise}} + \int_{t_0}^{t_f} (f_{idle} + CI) dt$$



Sáez, R., Dalmau, R., & Prats, X. (2018, Sep). Optimal assignment of 4D close-loop instructions to enable CDOs in dense TMAs. Proceedings of the 37th IEEE/AIAA Digital Avionics Systems Conference (DASC)



## MIP formulation: Constraints

- ✓ Flow constraints
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# MIP formulation: Constraints

## ✓ NEW: Integration of CDO-enabled Realistic Speed Profiles

**Substitute:**  $y_{a,j,t}$  with  $y_{a,j,p,n,t}$  - binary, indicates whether a/c  $a$  using speed profile  $p$  occupies the  $n$ -th vertex  $j$  at time  $t$ .  
and the corresponding equations with

$$\begin{aligned} \sum_{p \in \mathcal{S}(a)} y_{a,b,p,1,t_a^b} &= 1 & \forall b \in \mathcal{P}, \forall a \in \mathcal{A}_b \\ y_{a,b,p,k,t_a^b} &= 0 & \forall b \in \mathcal{P}, \forall a \in \mathcal{A}_b, \forall p \in \mathcal{S}(a) \\ & & \forall k \neq 1 \in \mathcal{L} \\ y_{a,b,p,1,t} &= 0 & \forall b \in \mathcal{P}, \forall a \in \mathcal{A}_b, \forall p \in \mathcal{S}(a) \\ & & \forall t \in T \setminus \{t_a^b\} \\ y_{a,b',p,k,t} &= 0 & \forall b' \neq b \in \mathcal{P}, \forall a \in \mathcal{A}_b, \forall p \in \mathcal{S}(a) \\ & & \forall k \in \mathcal{L}, \forall t \in T \\ y_{a',b,p,1,t_a^b} &= 0 & \forall b \in \mathcal{P}, \forall a' \neq a \in \mathcal{A}_b, \\ & & \forall p \in \mathcal{S}(a) \\ y_{a,j,p,k,t} &\leq \sum_{\substack{i \in V: \\ (i,j) \in E}} x_{(i,j)} \forall j \in V \setminus \mathcal{P}, \forall a \in \mathcal{A}, \forall p \in \mathcal{S}(a), \end{aligned}$$

Compute  $l(b)$  - path from  $b$  to the rwy

$$\ell(b) = \sum_{(i,j) \in E} x_{(i,j),b}$$

For each a/c  $a$  arriving from  $b$  we pick the speed profile from  $\mathcal{S}(a)$  that has the length  $l(b)$ :

$$y_{a,b,\ell(b),1,t_a^b} = 1 \text{ and } y_{a,b,p,1,t_a^b} = 0 \forall p \neq \ell(b)$$

$l(b)$  var, not a parameter  $\implies$  aux vars and constraints

...

Separation constraint:

$$\sum_{\tau=t}^{t+\sigma-1} \sum_{a \in \mathcal{A}} \sum_{p \in \mathcal{S}(a)} \sum_{k \in \mathcal{L}} y_{a,j,p,k,\tau} \leq 1 \quad \forall j \in V, \\ \forall t \in \{0, \dots, \bar{T} - \sigma + 1\}$$

$\sigma$  - separation parameter

## MIP formulation: Constraints

✓ **NEW**: Consistency between trees of consecutive time periods

**Define:**  $x_{ij}$  and  $x_{ij}^{old}$  - edge indicators for current and previous periods

$U$  - limits the number of differing edges

$$ax_{ij} \leq x_{ij} - x_{ij}^{old} \quad \forall (j, i) \in E$$

$$ax_{ij} \leq x_{ij}^{old} - x_{ij} \quad \forall (j, i) \in E$$

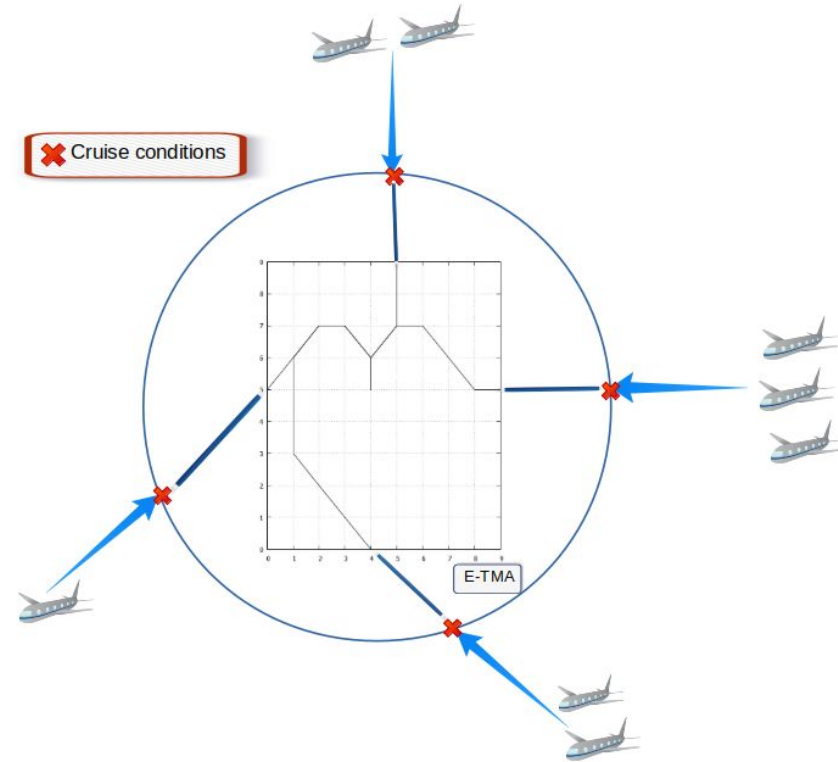
$$\sum_{(i,j) \in E} ax_{ij} \leq U$$

## Experimental Study: Stockholm Arlanda Airport

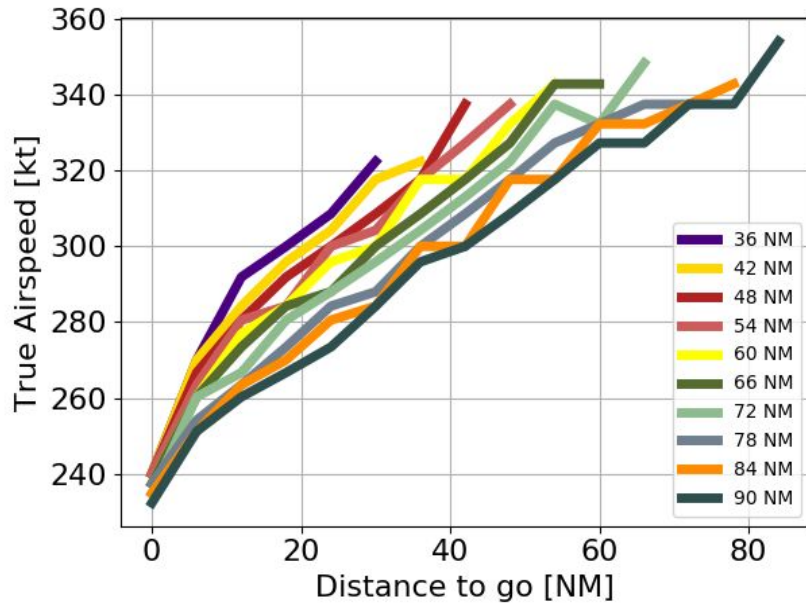
- ✓ Data: Stockholm Arlanda airport arrivals during one hour of operation
- ✓ Source: EUROCONTROL DDR2, BADA 4
- ✓ High-traffic scenario on October 3, 2017, time: 15:00 - 16:00
- ✓ Solved using GUROBI
- ✓ Run on a powerful Tetralith server, provided by SNIC, LIU: Intel HNS2600BPB nodes with 32 CPU cores and 384 GiB RAM

## CDO speed profiles inside TMA

- ✓ Cruise conditions are obtained from DDR2
- ✓ TOD position and descent phase are optimized
- ✓ Same time at the entry point for different path lengths inside TMA



## CDO speed profiles inside TMA

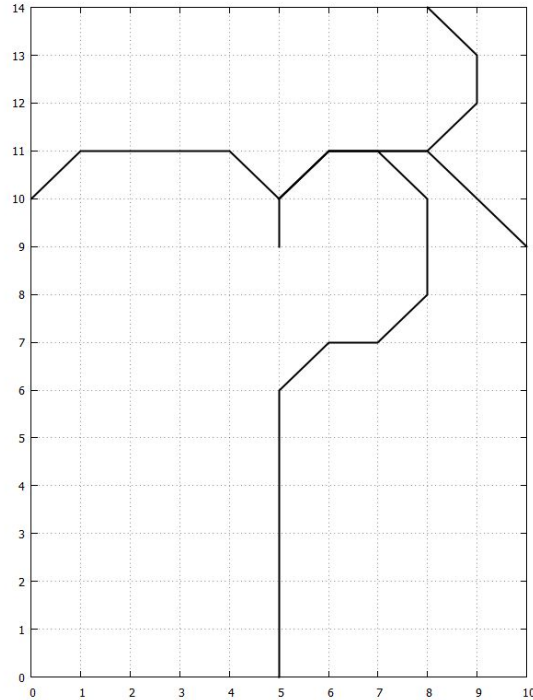


Example of A320 speed profiles for different path lengths inside TMA

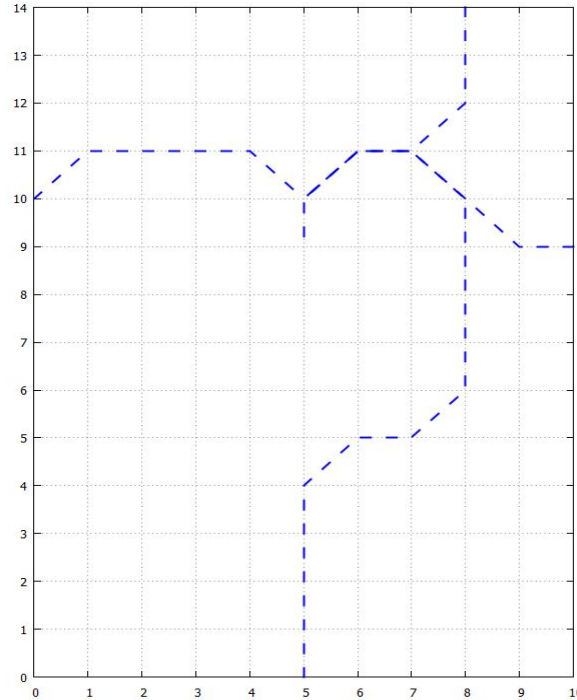
- ✓ A set of realistic alternative speed profiles for different possible route lengths inside TMA
- ✓ Generated for all a/c types arriving to Arlanda during the given period
- ✓ Used as input to MIP

# Results: Stockholm Arlanda Airport

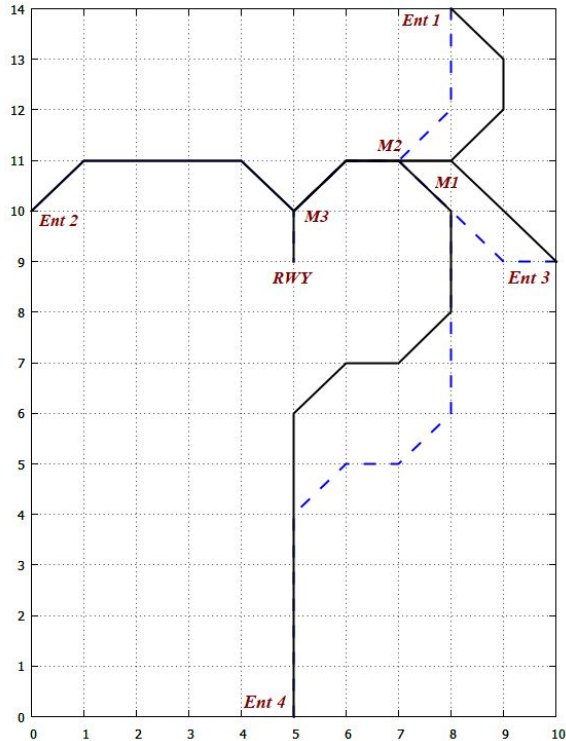
**Tree 1:** time: 15:00 - 15:30 (10 a/c)



**Tree 2:** time: 15:30 - 16:00 (7 a/c)



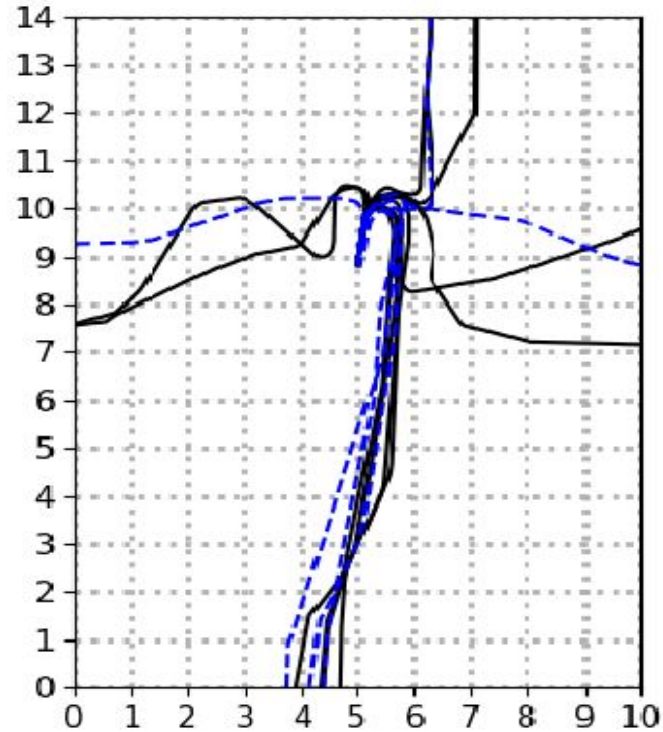
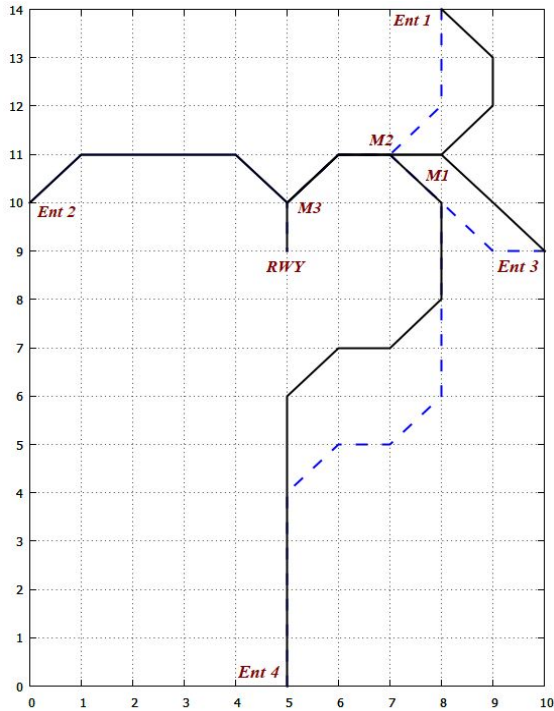
## Results: Stockholm Arlanda Airport



- ✓ **Tree 1:** time: 15:00 - 15:30 (10 a/c)
- ✓ **Tree 2:** time: 15:30 - 16:00 (7 a/c)
- ✓ Optimized for 30 min intervals (longer periods may be sub-optimal. Note: time within TMA 5-18 min)
- ✓  $U = 23$  provides consistency between the trees
- ✓ Separation: 2 min, ~6 nm
- ✓ 17 out of 22 arrivals scheduled
- ✓ 5 filtered out, because of:
  - Initial violation of separation at entry points
  - Potential overtaking problem
  - In general, about 10-15% are not scheduled

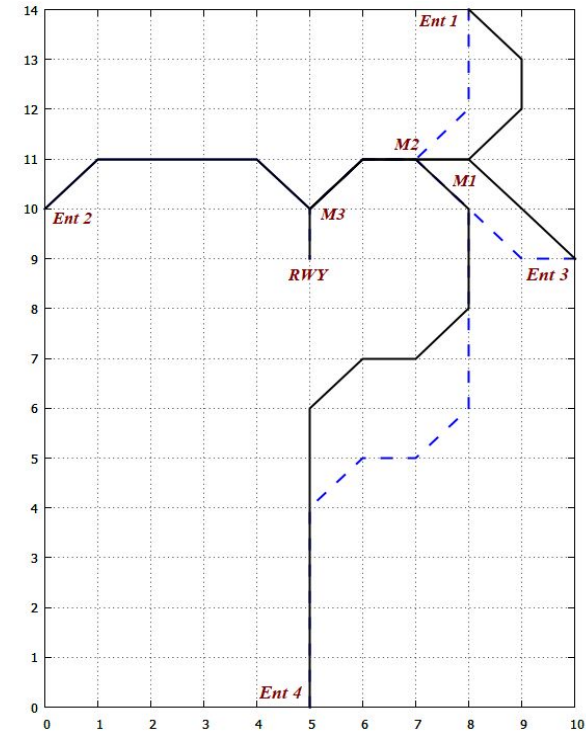


# Comparison against historical trajectories (Open Sky Network)

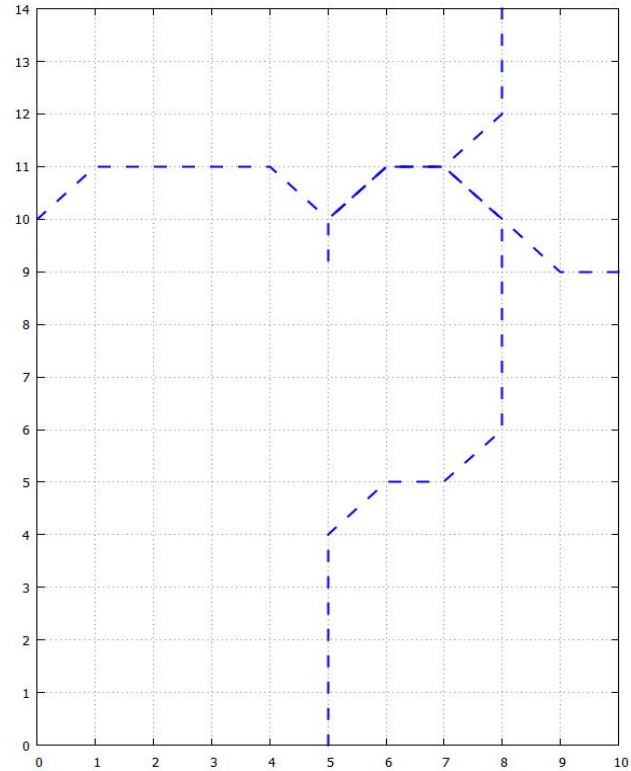
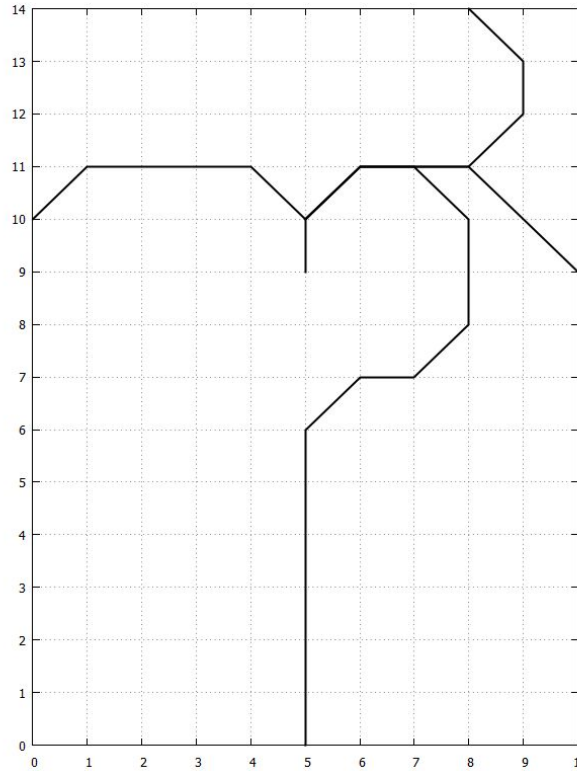


# Time Schedule

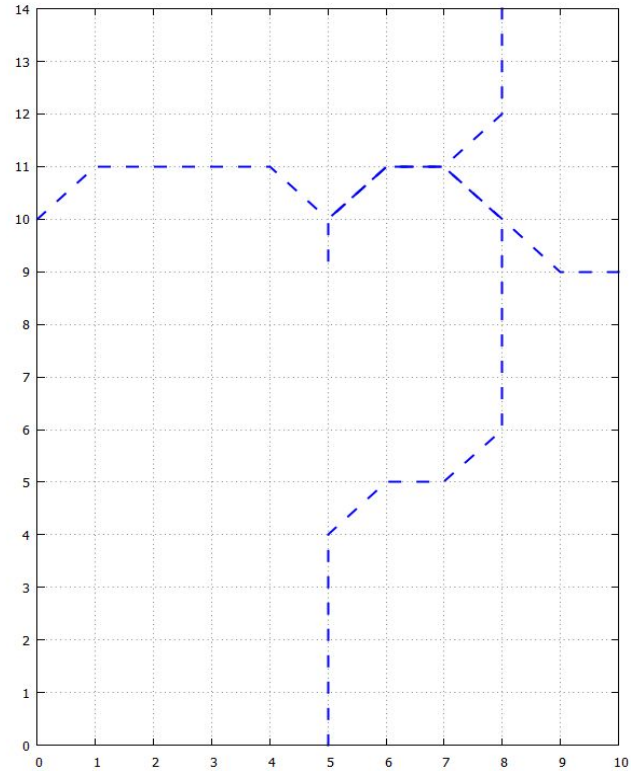
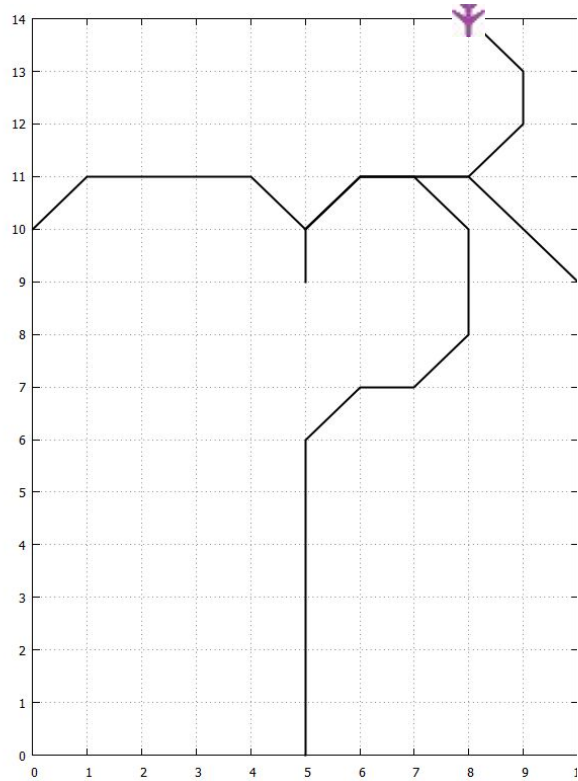
Arrivals		Simulated time [min]			
Aircraft	Entry point	Entry	M1	M2	M3
a1	Ent1 (North)	3	9	11	15
a2	Ent2 (West)	8	-	-	13
a3	Ent3 (East)	13	15	16	18
a4	Ent4 (South)	4	-	18	22
a5	Ent4	18	-	30	32
a6	Ent2	17	-	-	25
a7	Ent1	17	20	21	23
a8	Ent1	21	24	25	27
a9	Ent2	19	-	-	29
a10	Ent3	28	30	32	34
a11	Ent4	34	45	46	48
a12	Ent3	41	43	44	46
a13	Ent2	32	-	-	37
a14	Ent1	39	-	42	44
a15	Ent1	49	-	55	59
a16	Ent4	53	-	-	-
a17	Ent2	57	-	-	-



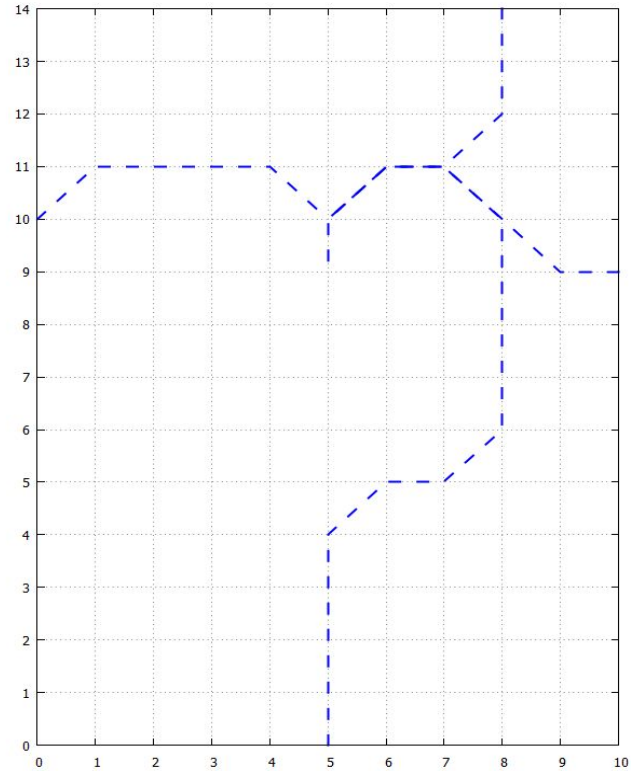
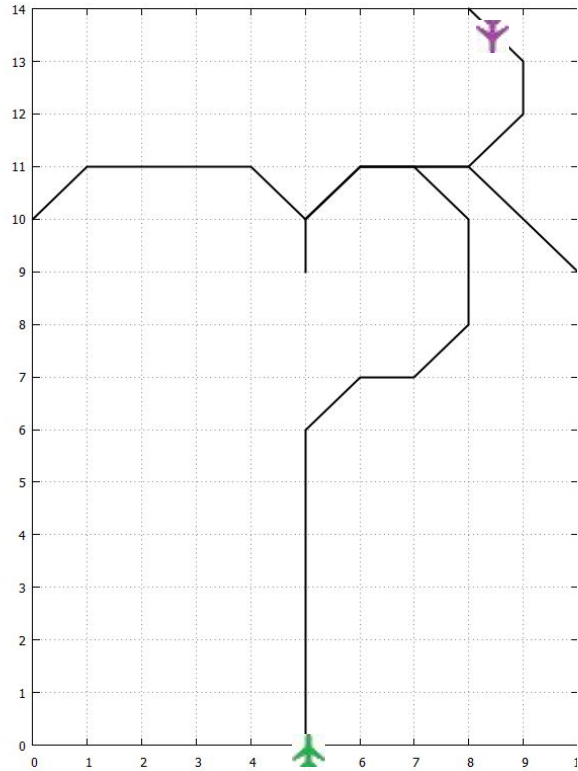
t = 15:00



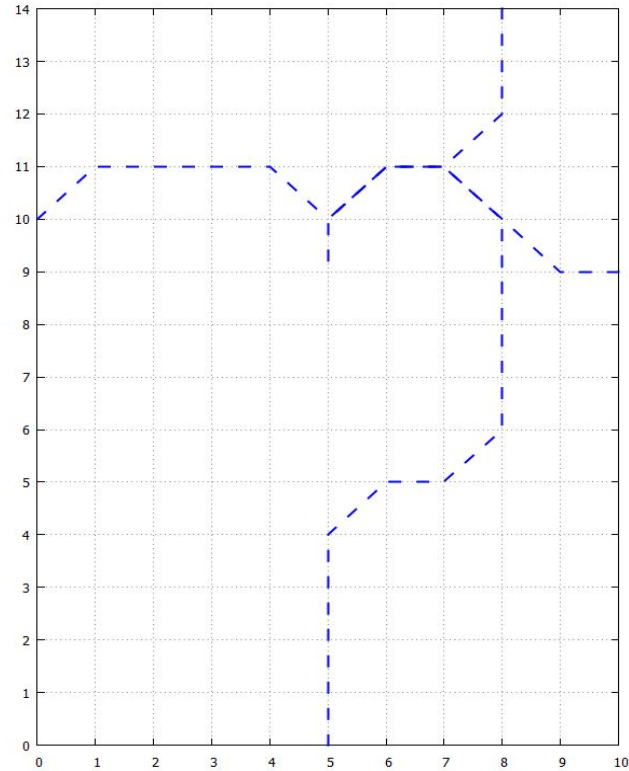
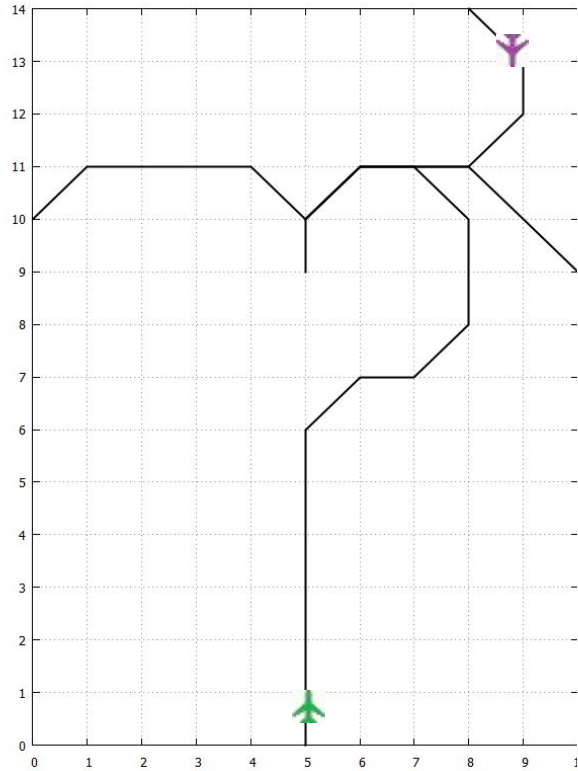
t = 15:03



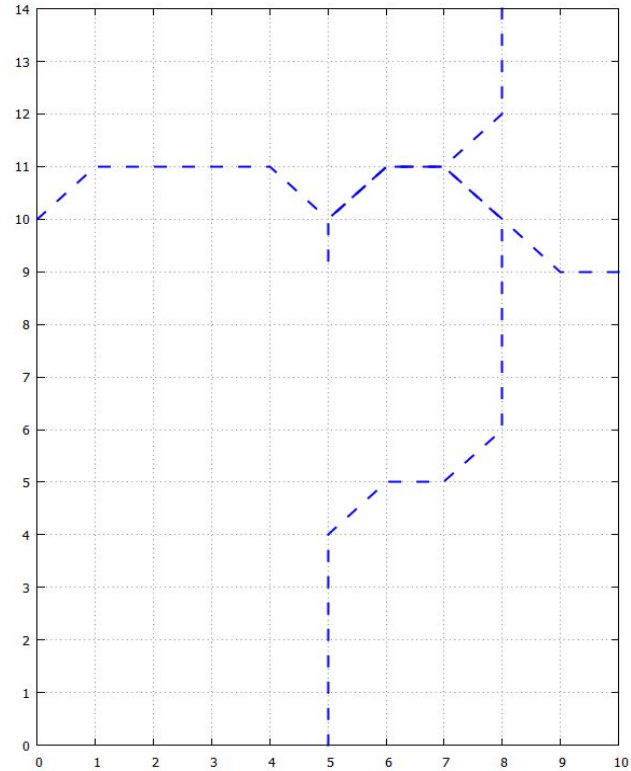
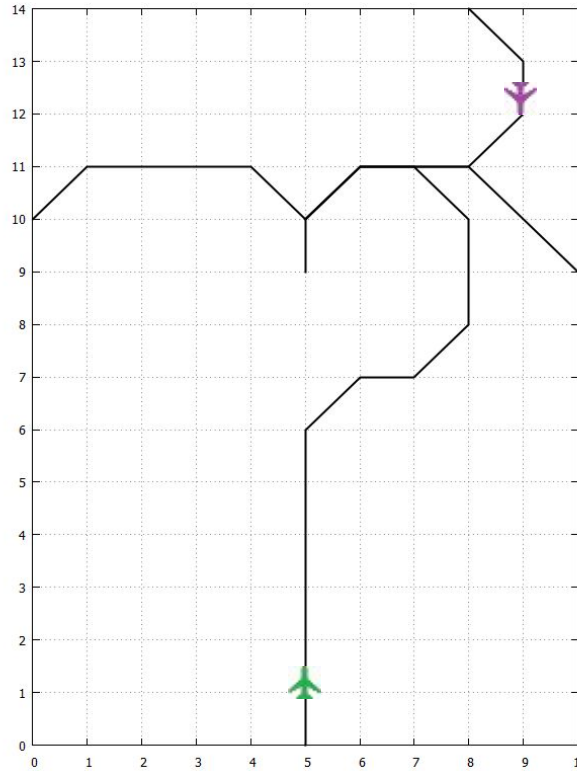
t = 15:04



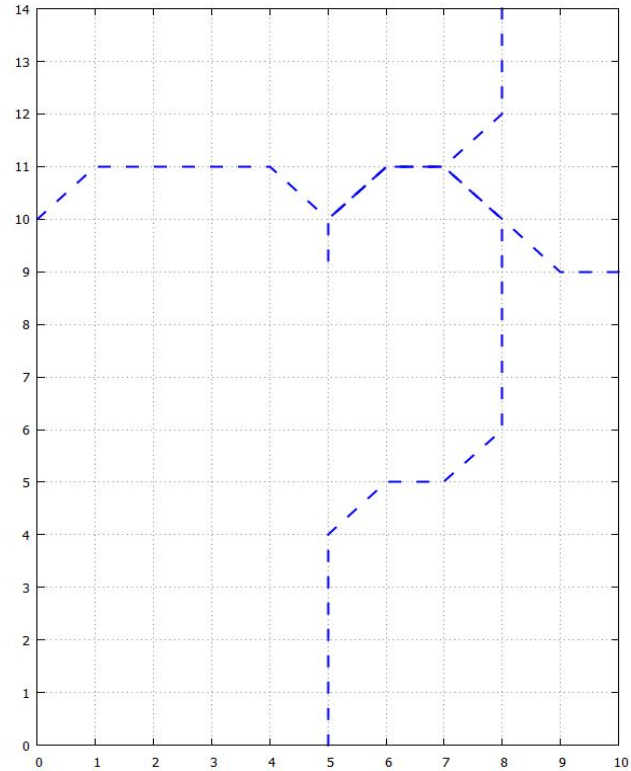
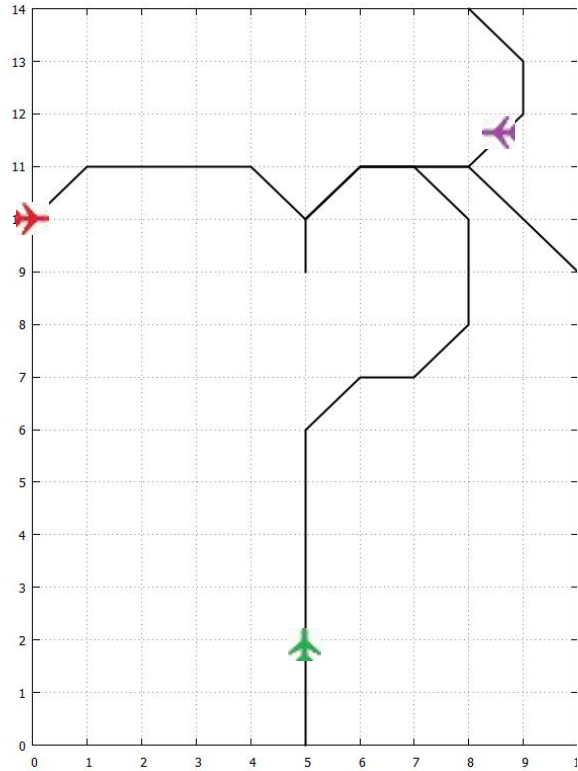
t = 15:05



t = 15:07

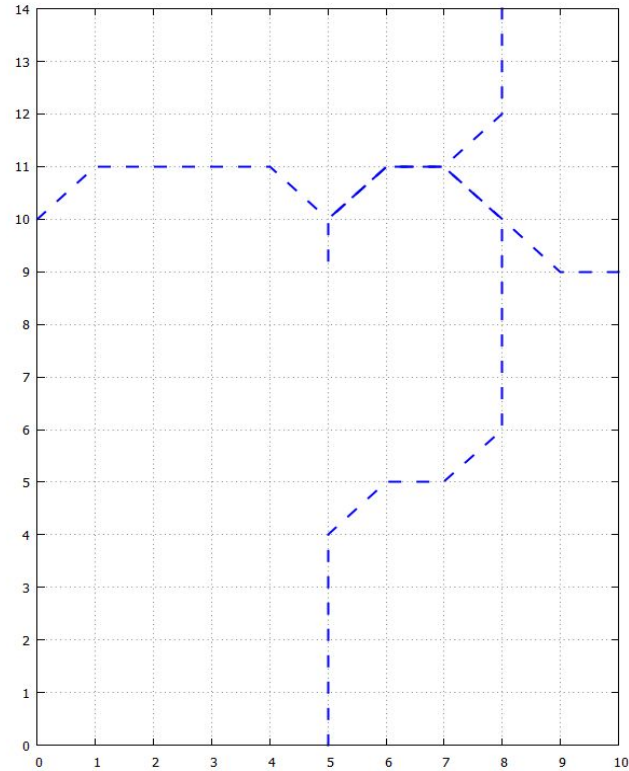
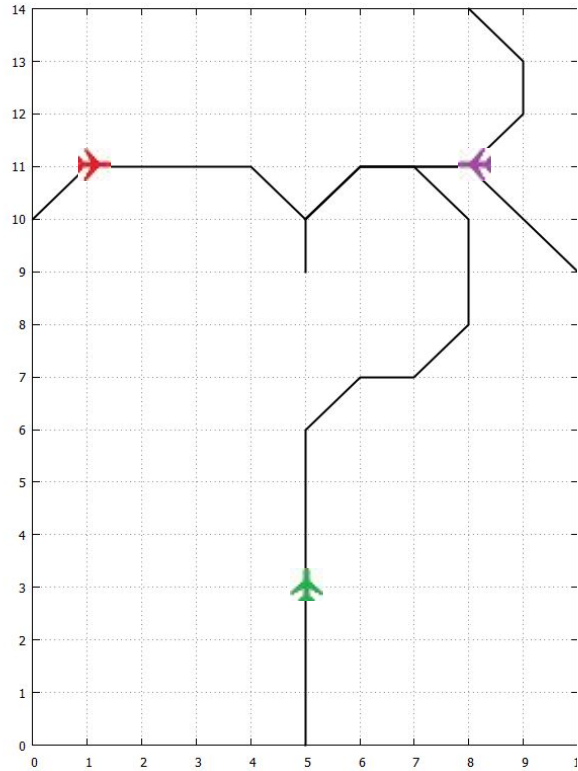


t = 15:08

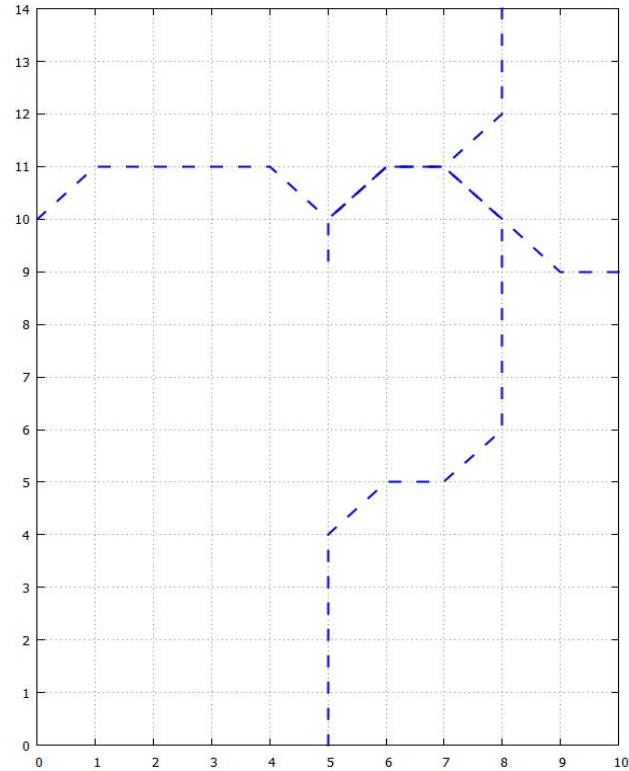
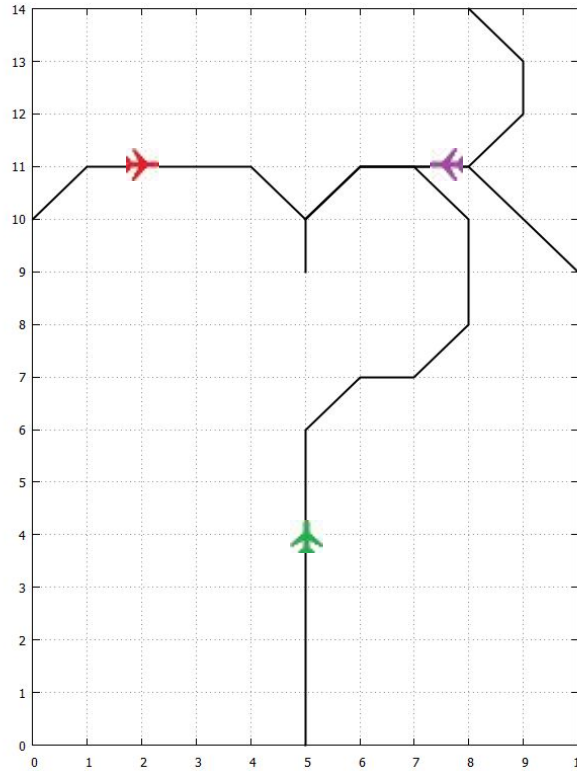




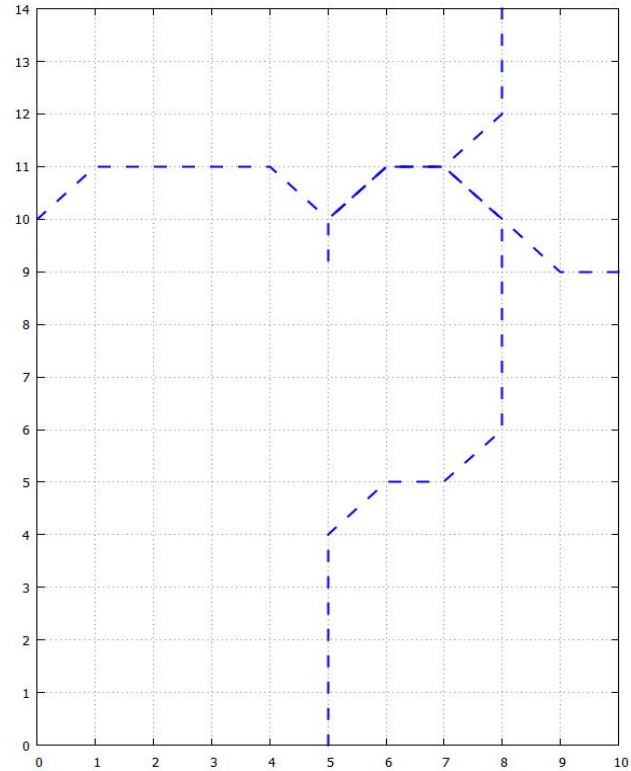
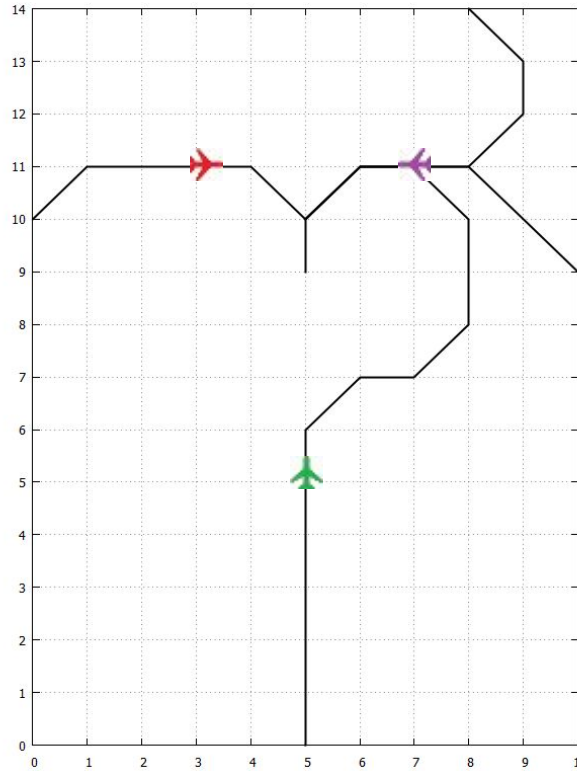
t = 15:09



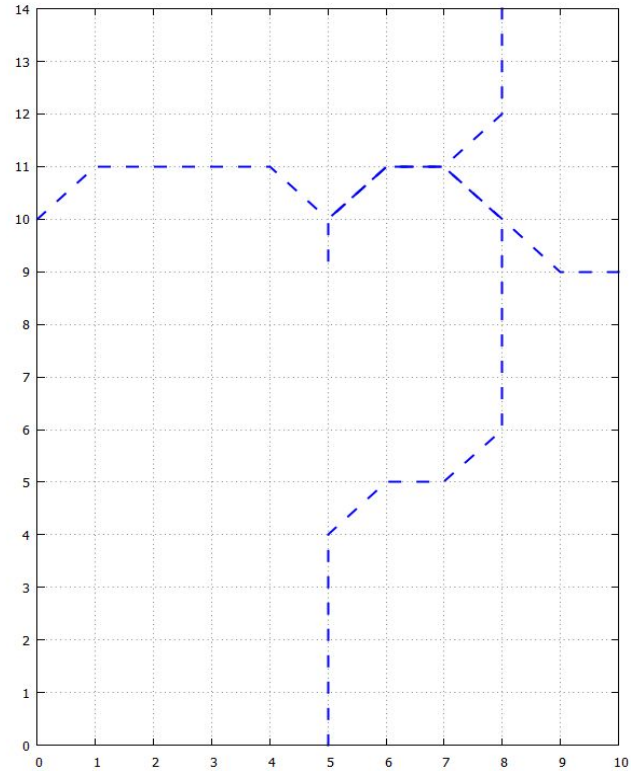
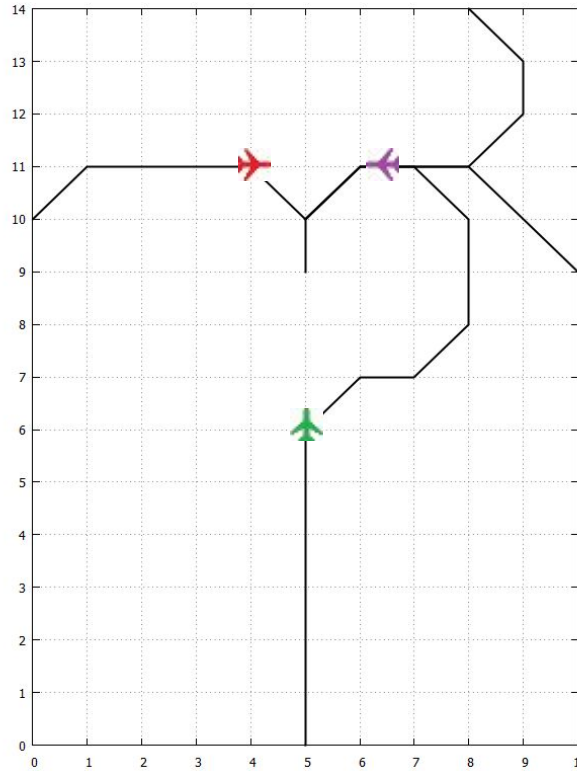
t = 15:10



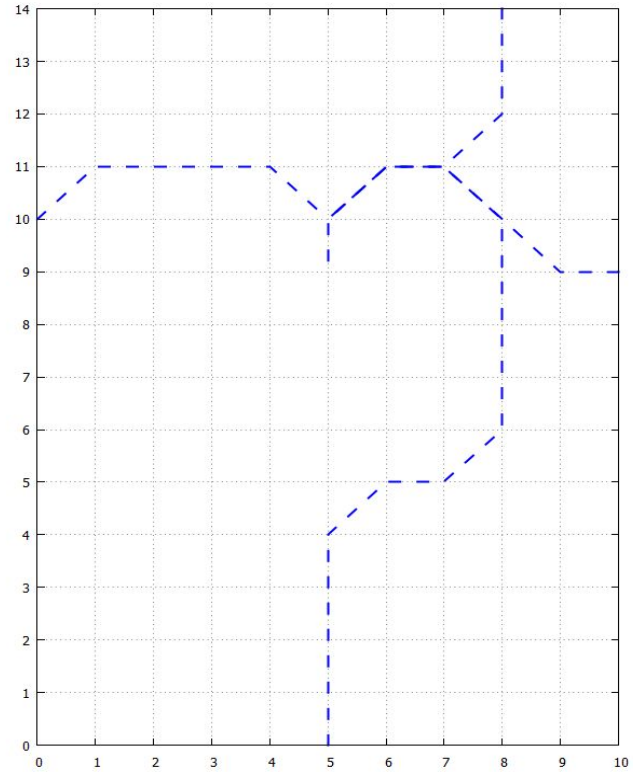
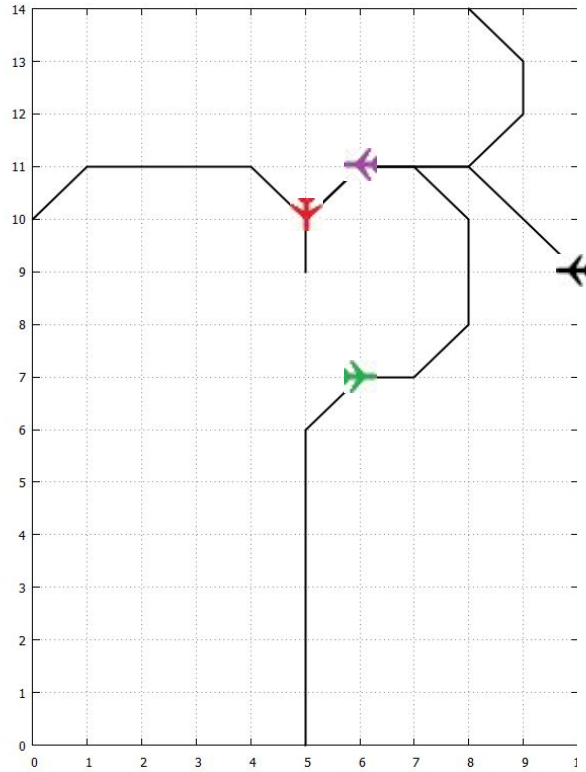
t = 15:11



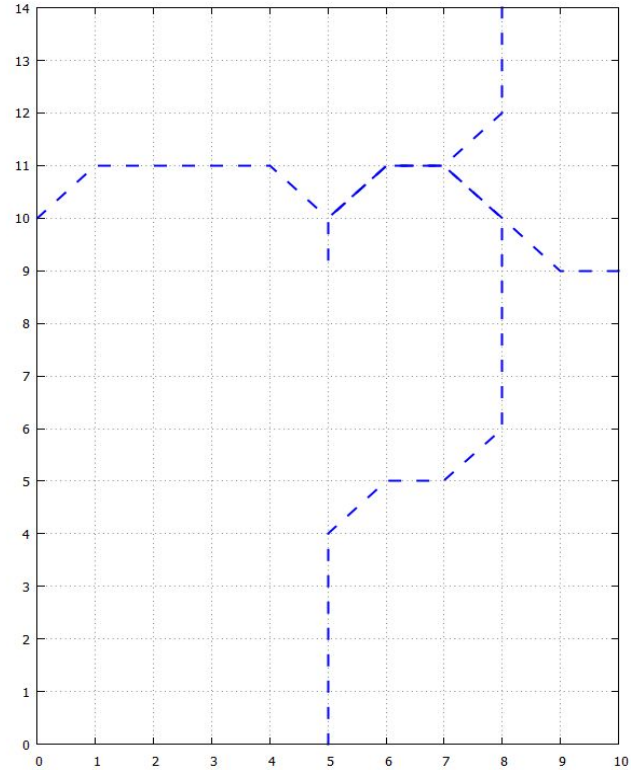
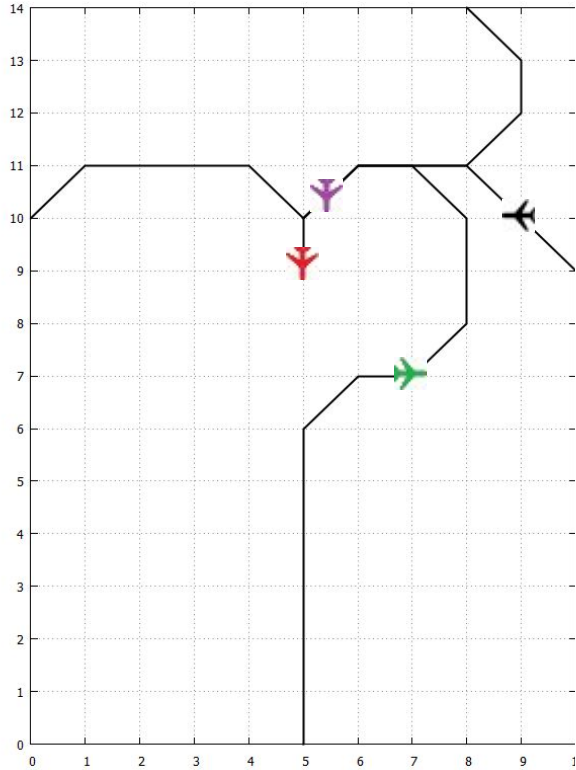
t = 15:12



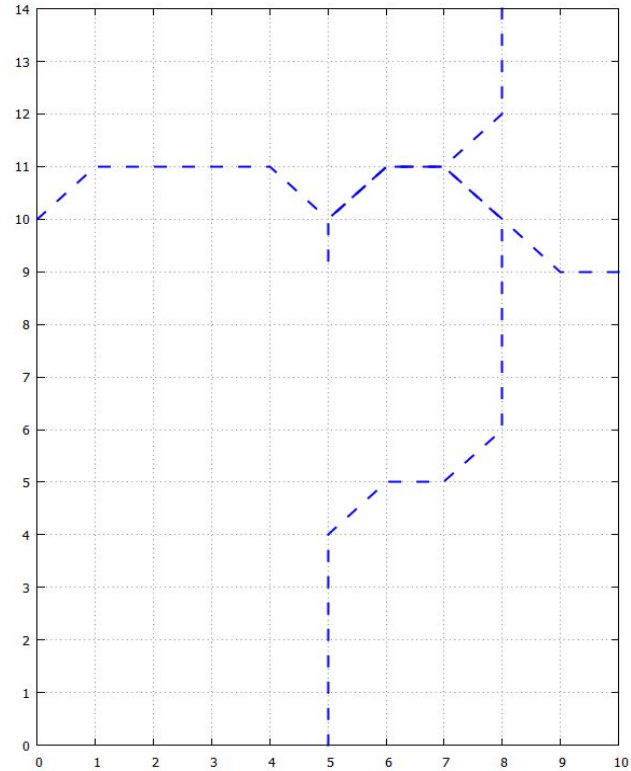
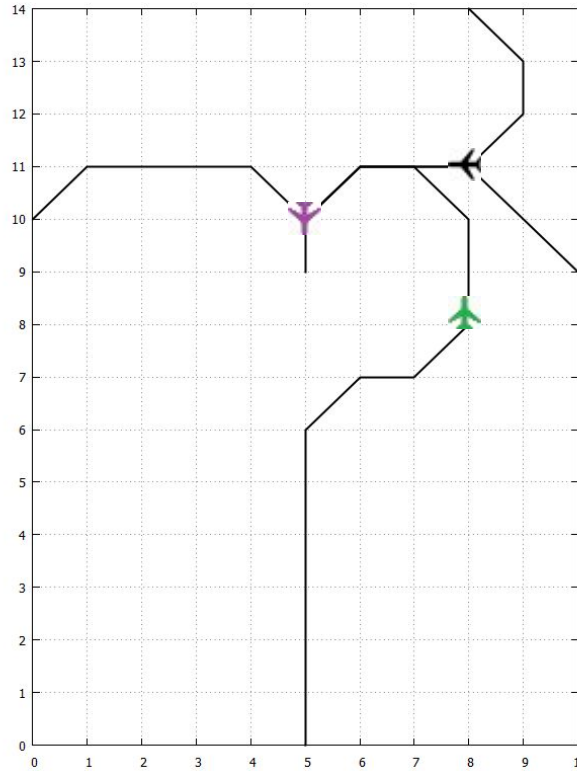
t = 15:13



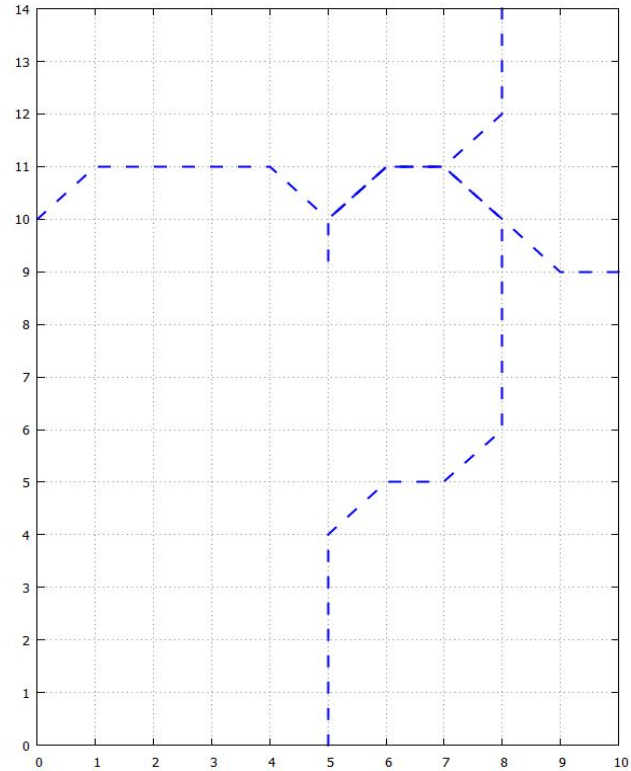
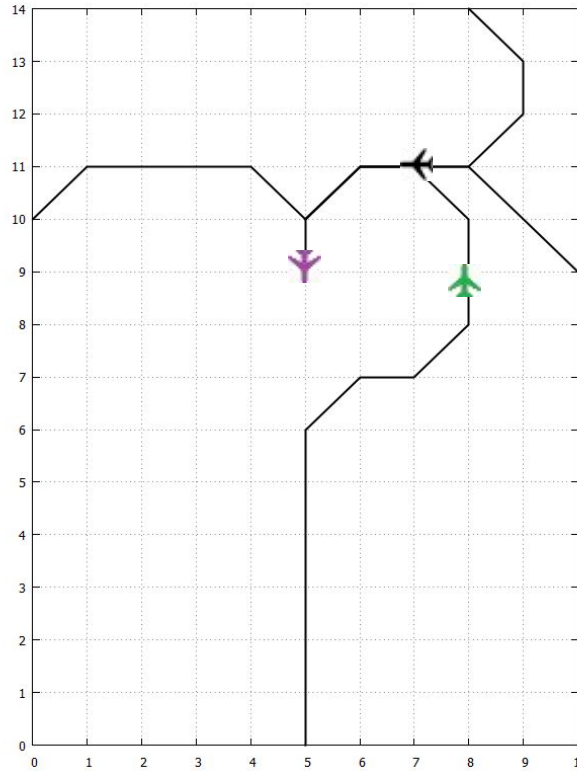
t = 15:14



t = 15:15

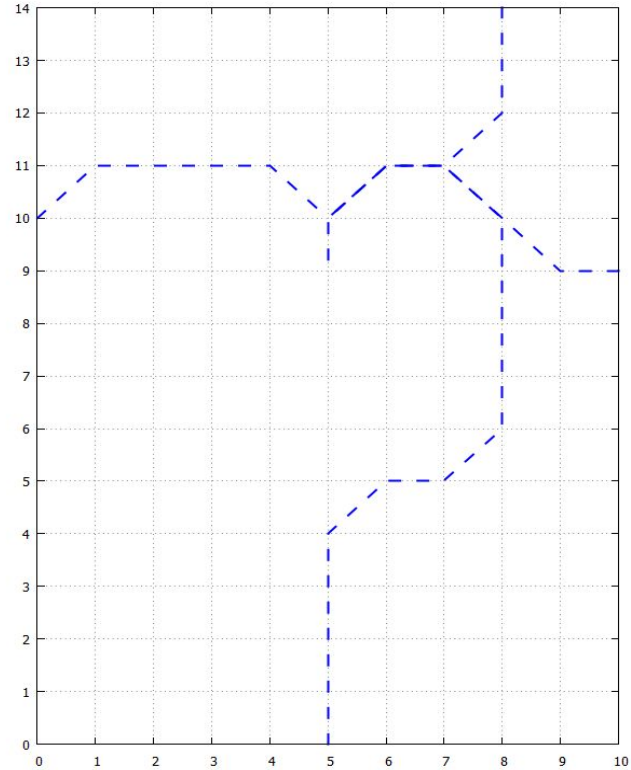
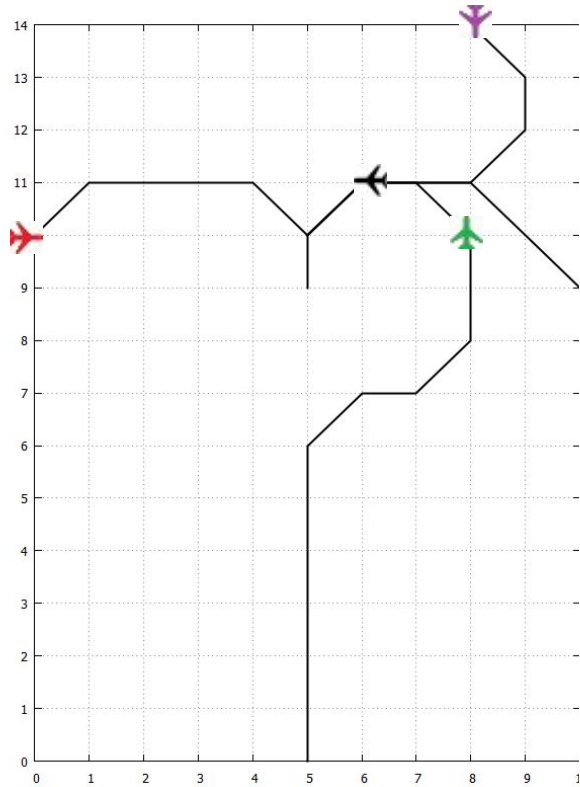


t = 15:16

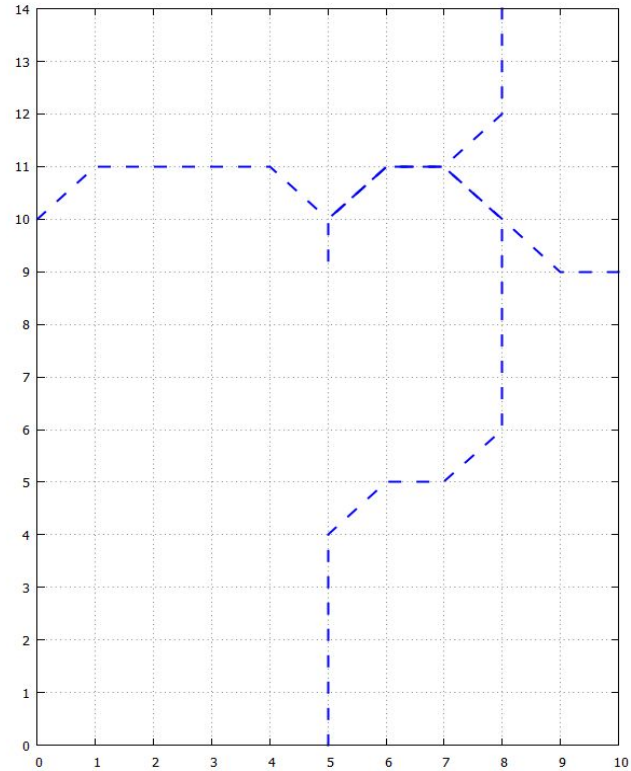
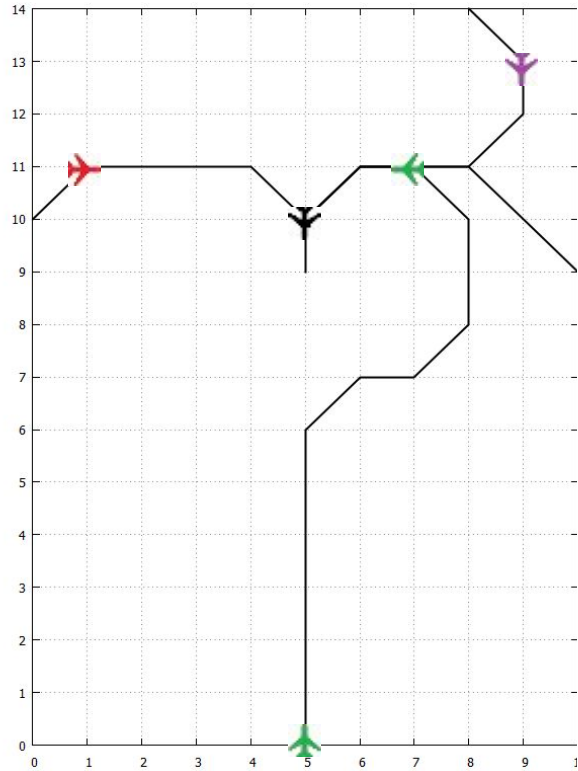




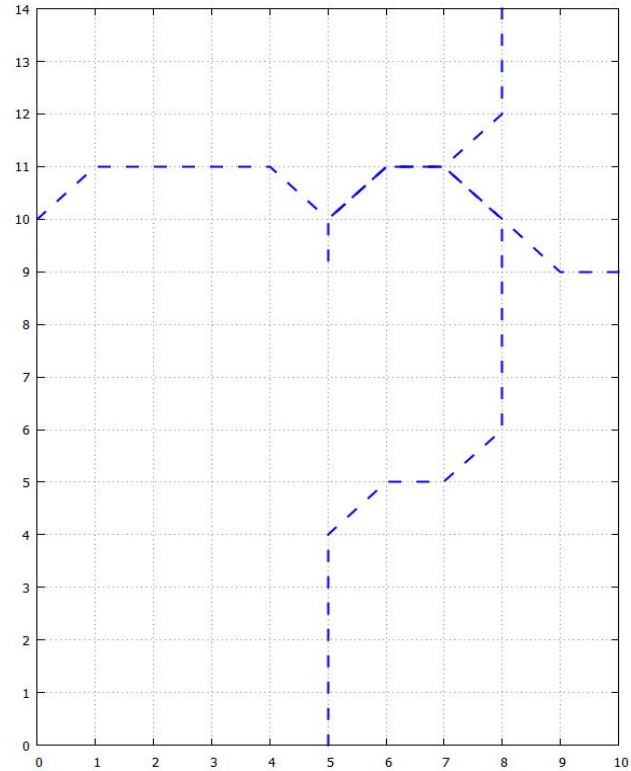
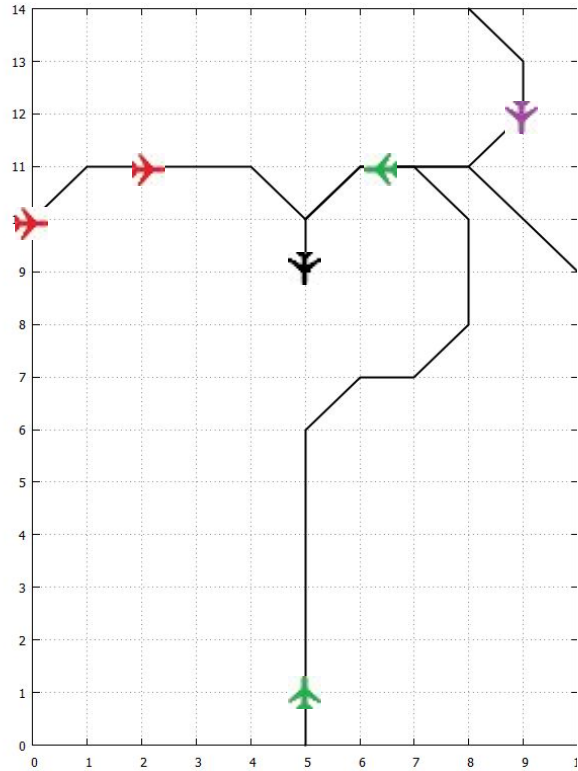
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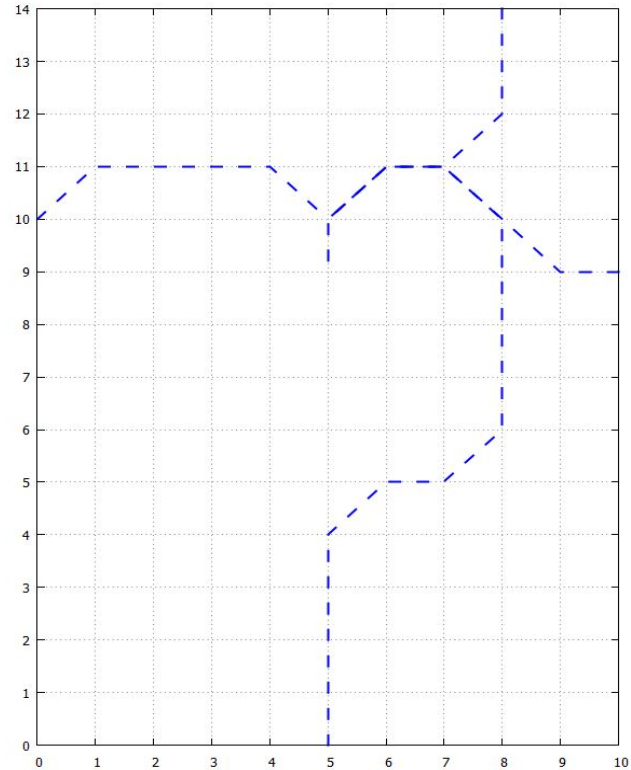
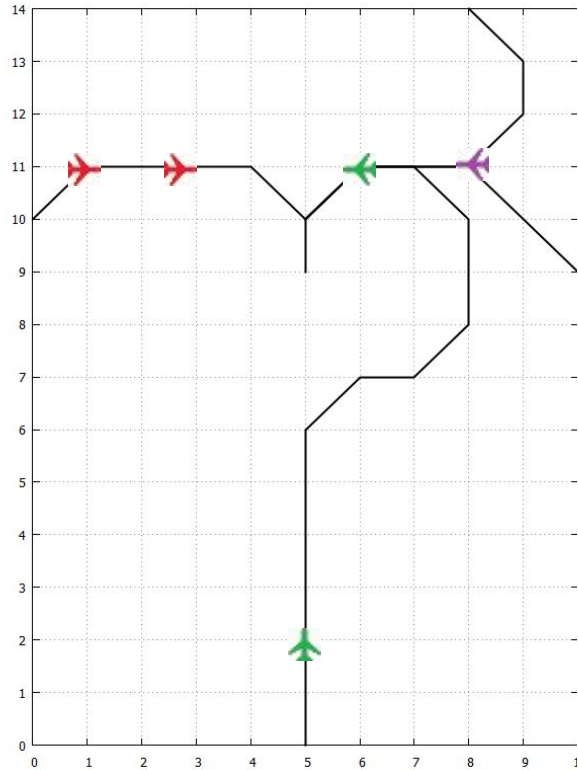
t = 15:18



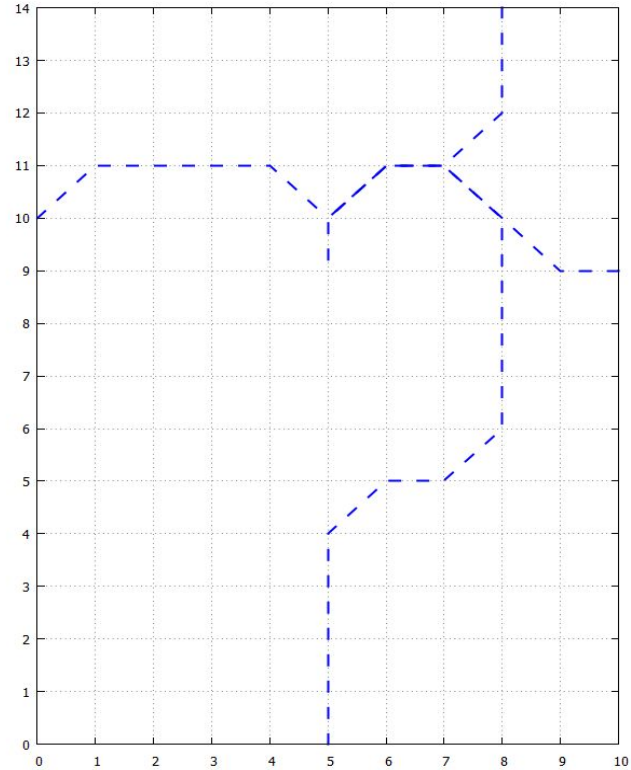
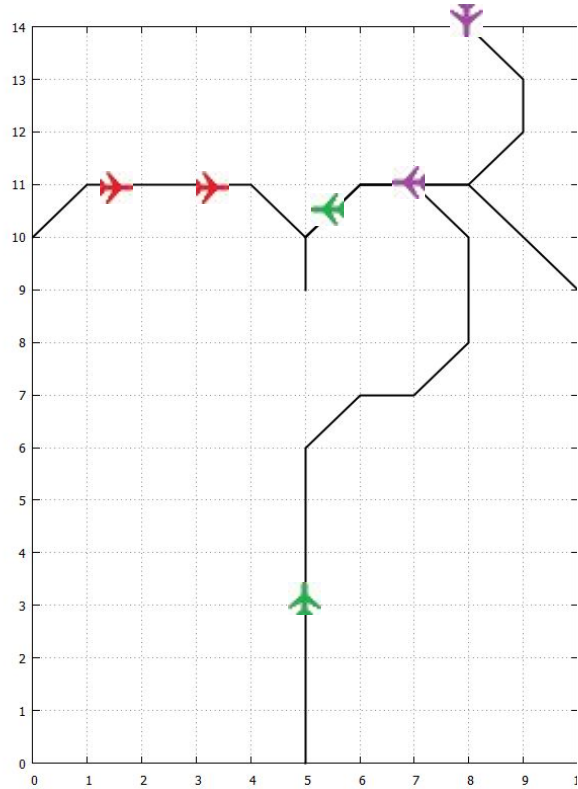
t = 15:19



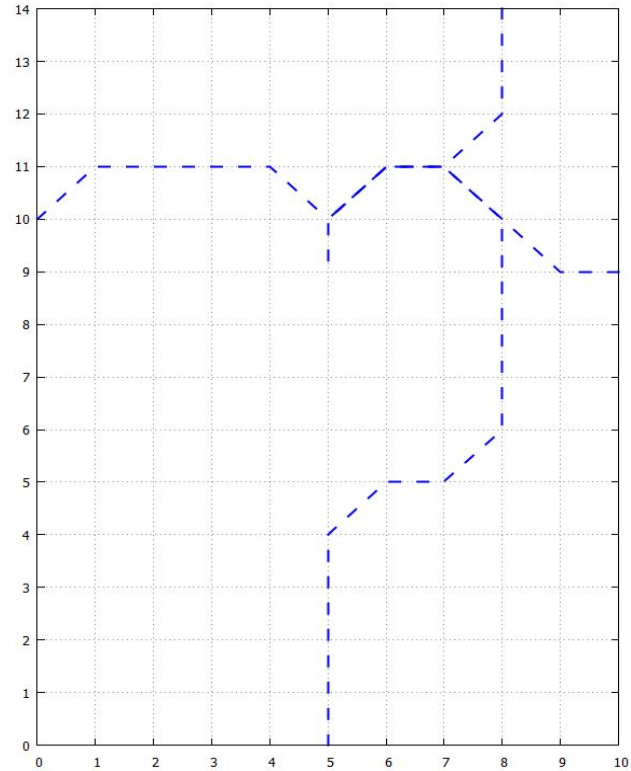
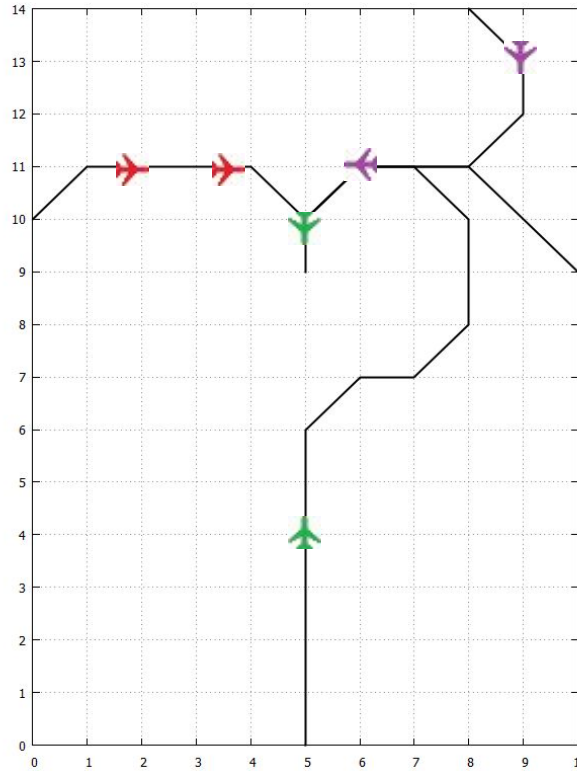
t = 15:20



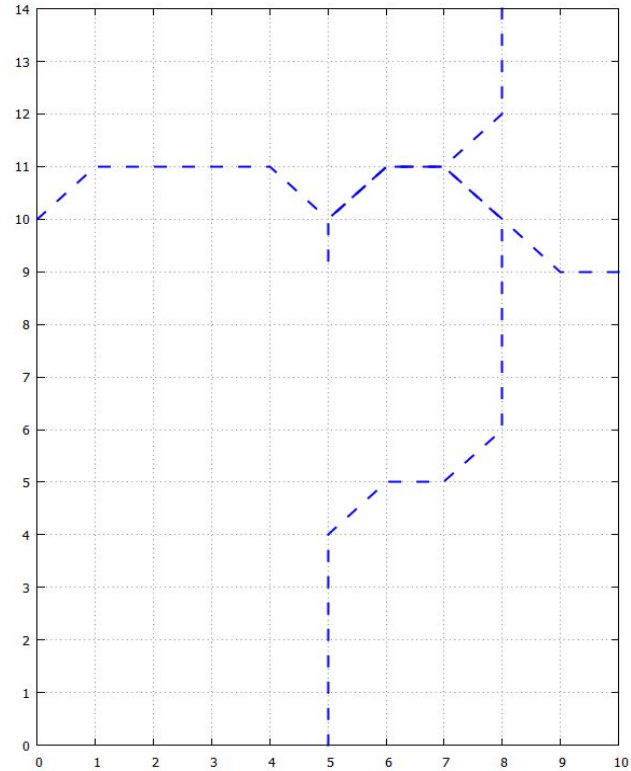
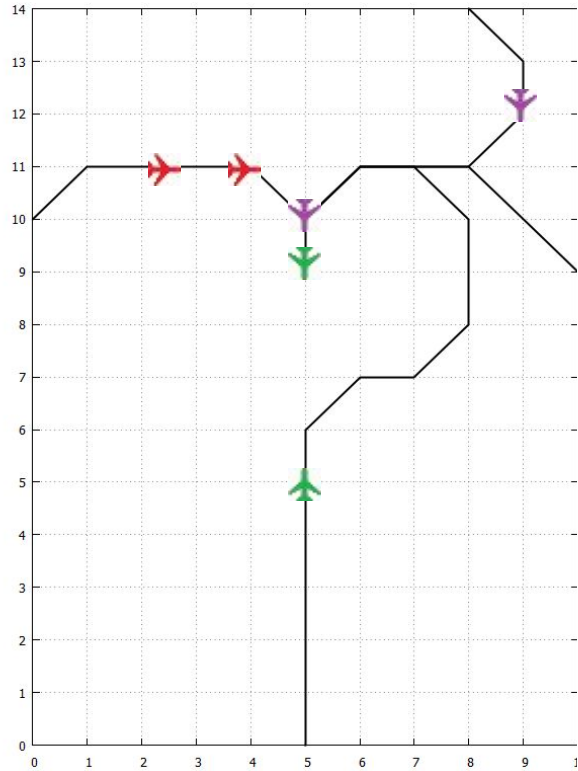
t = 15:21



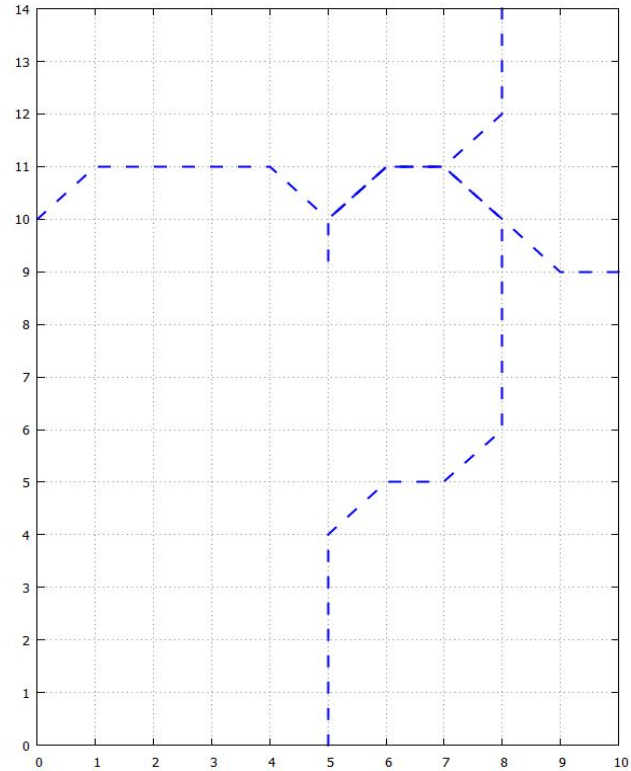
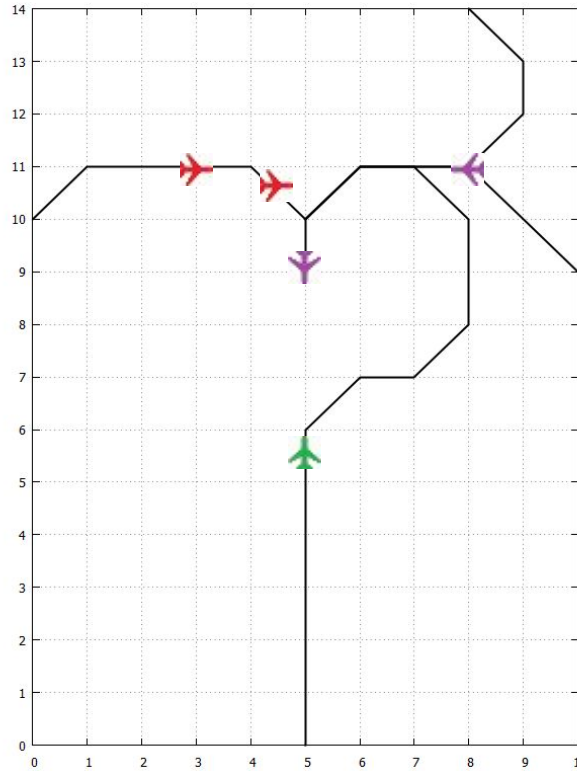
t = 15:22



t = 15:23

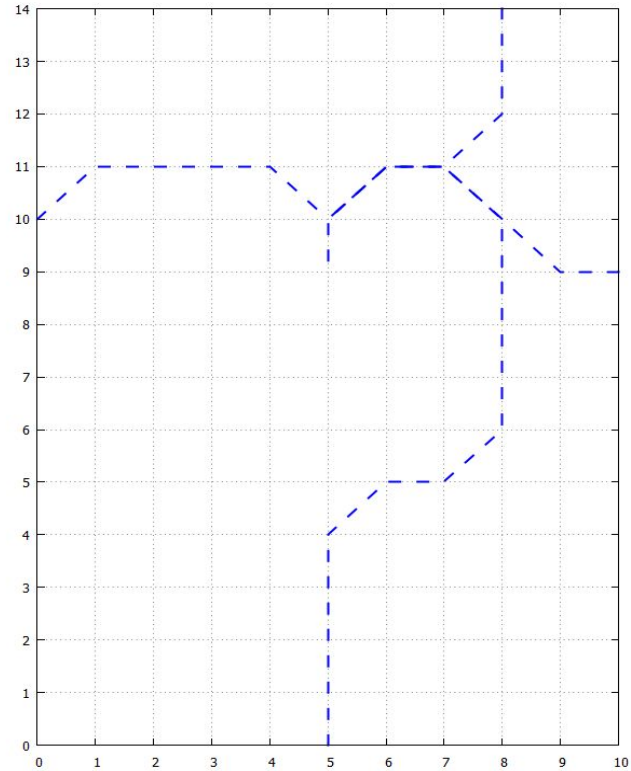
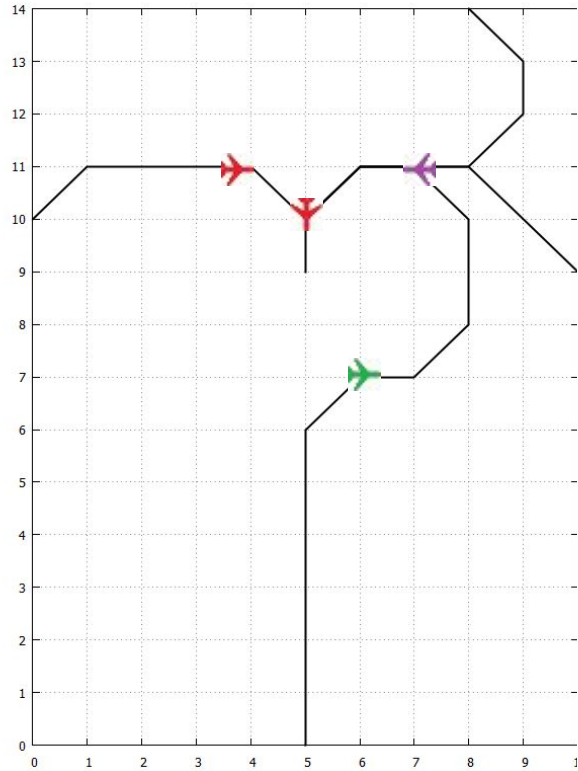


t = 15:24

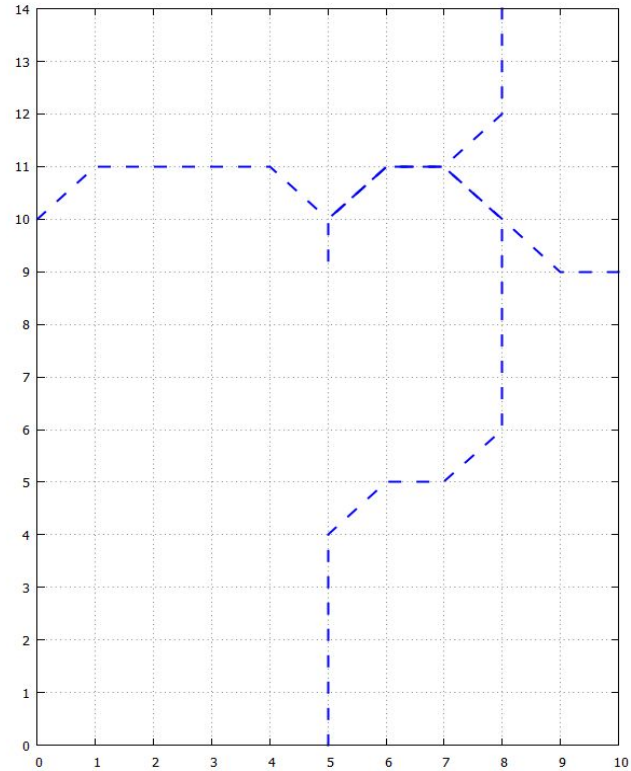
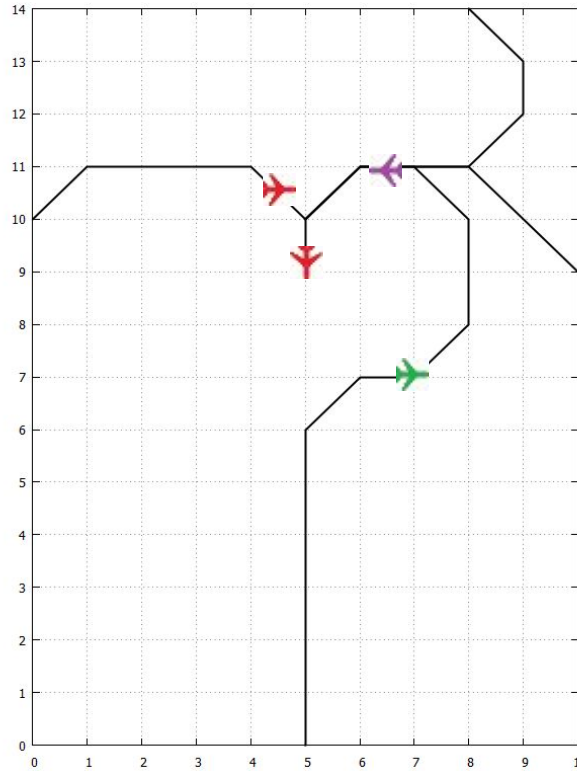




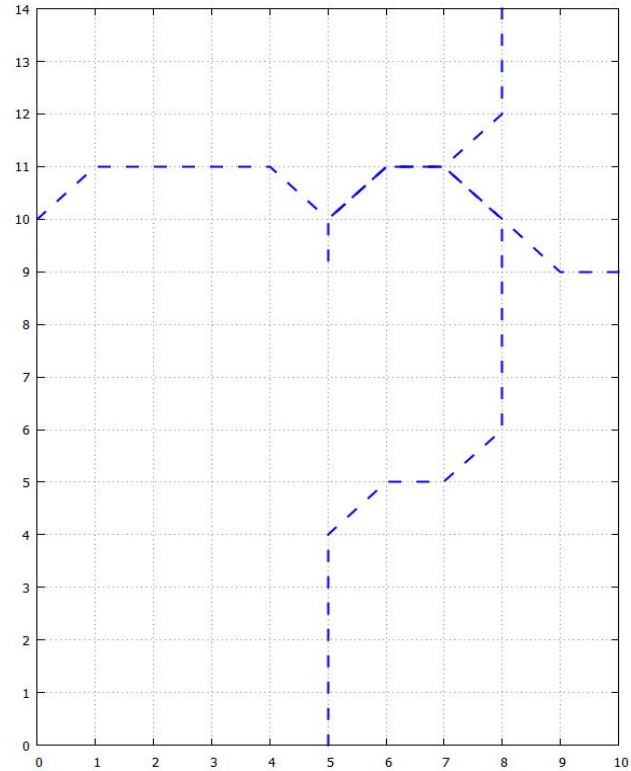
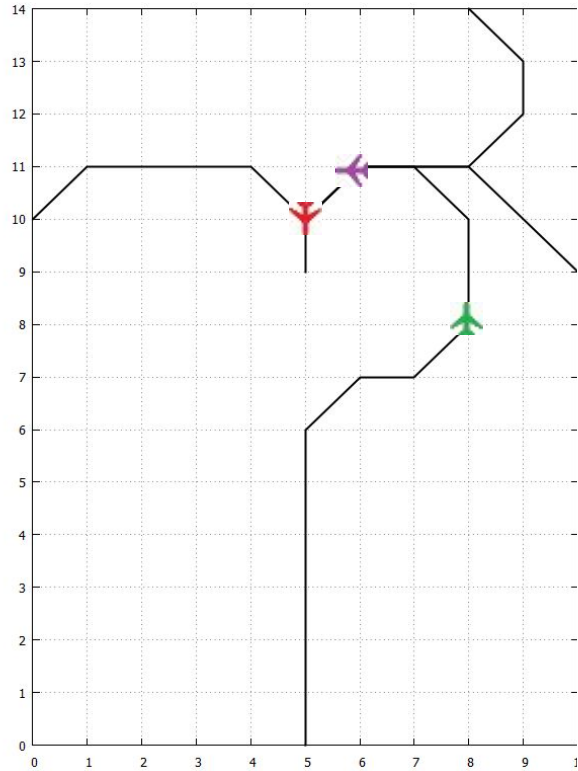
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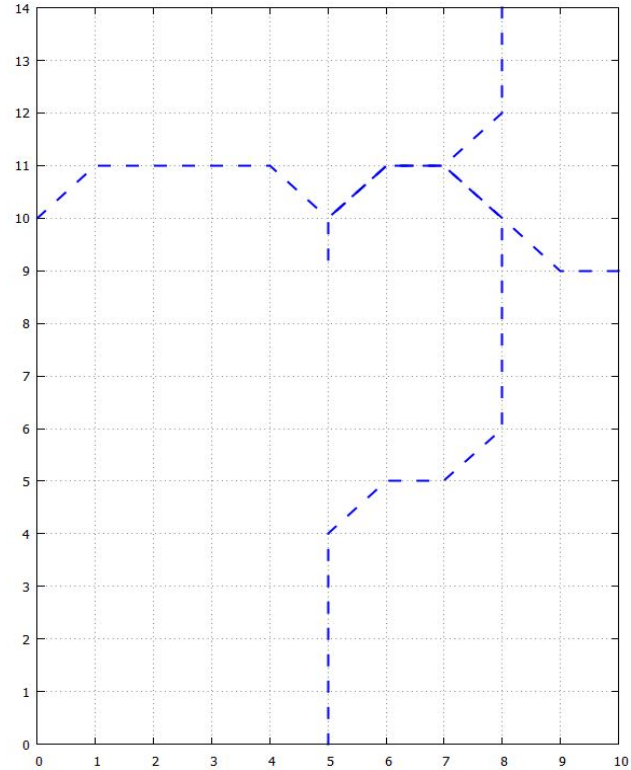
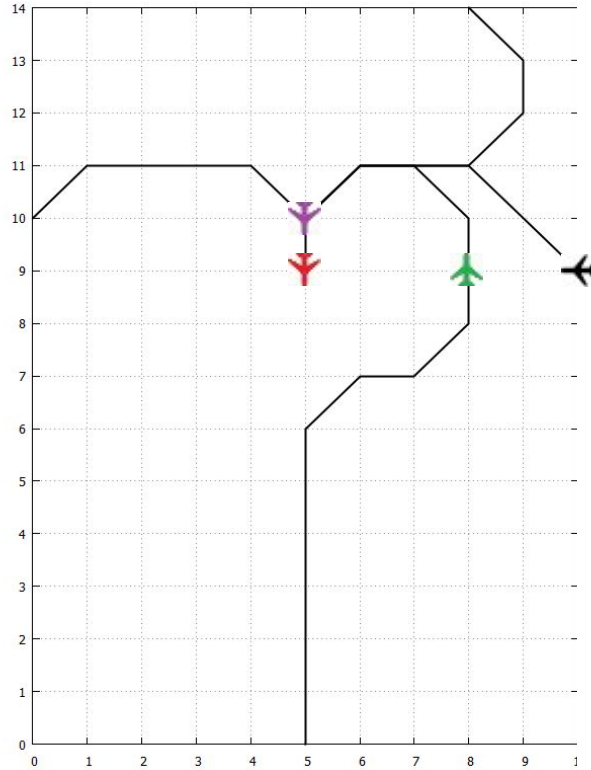
t = 15:26



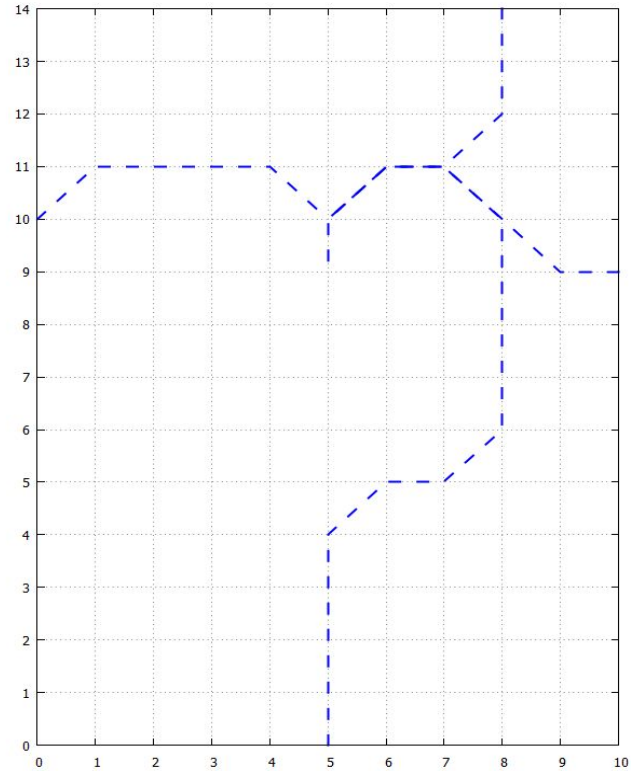
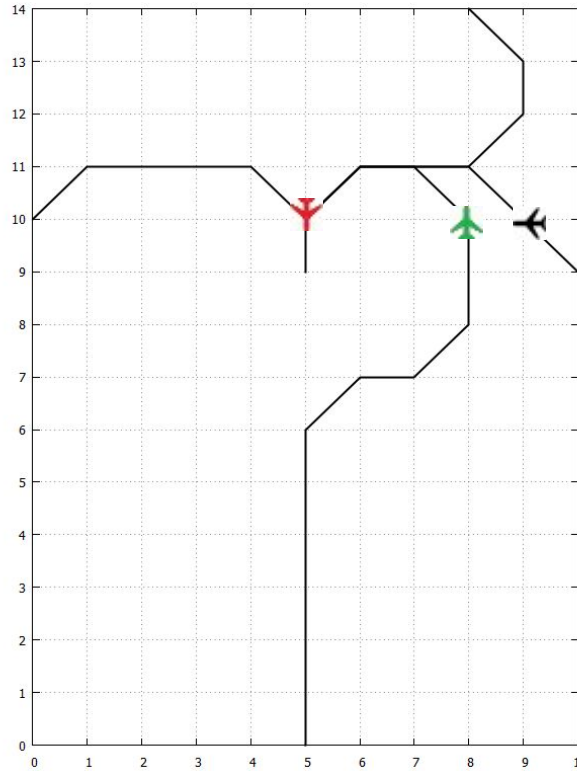
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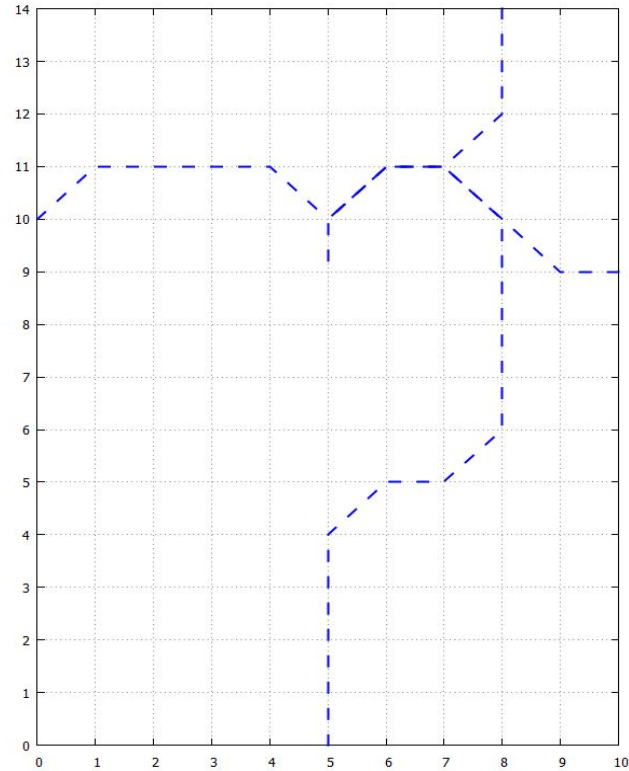
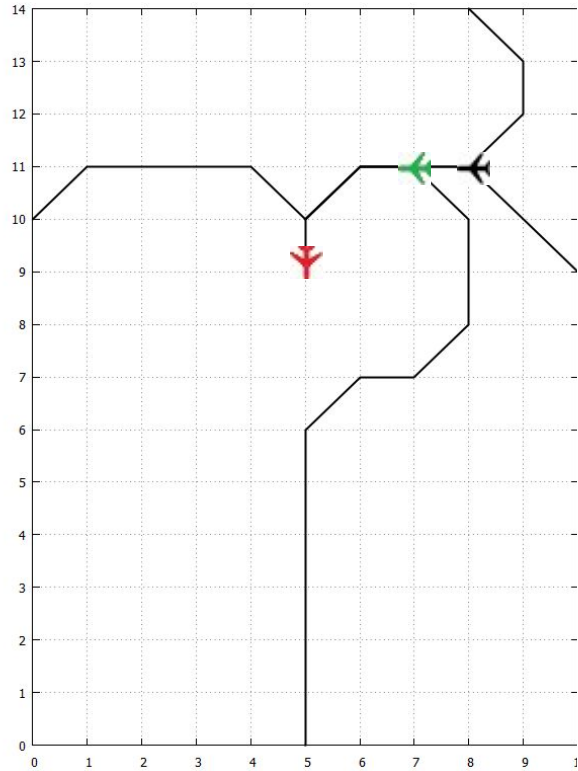
t = 15:28



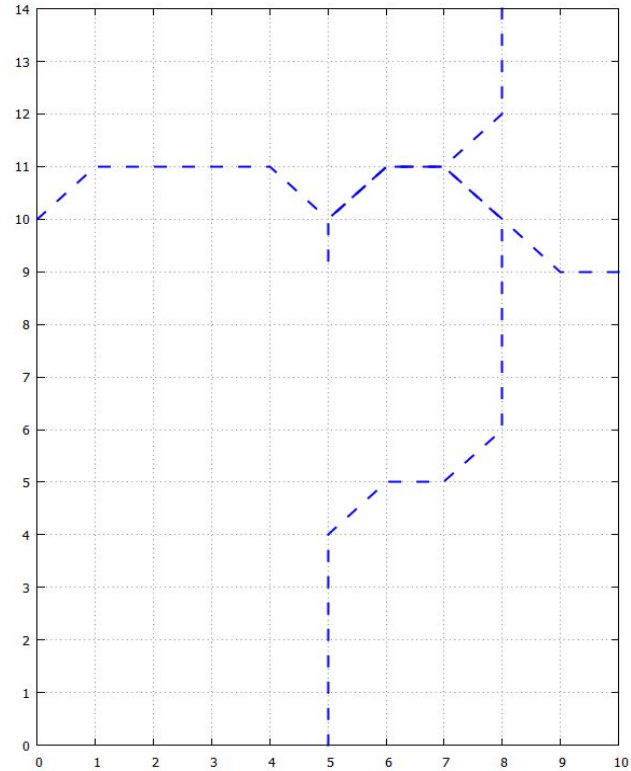
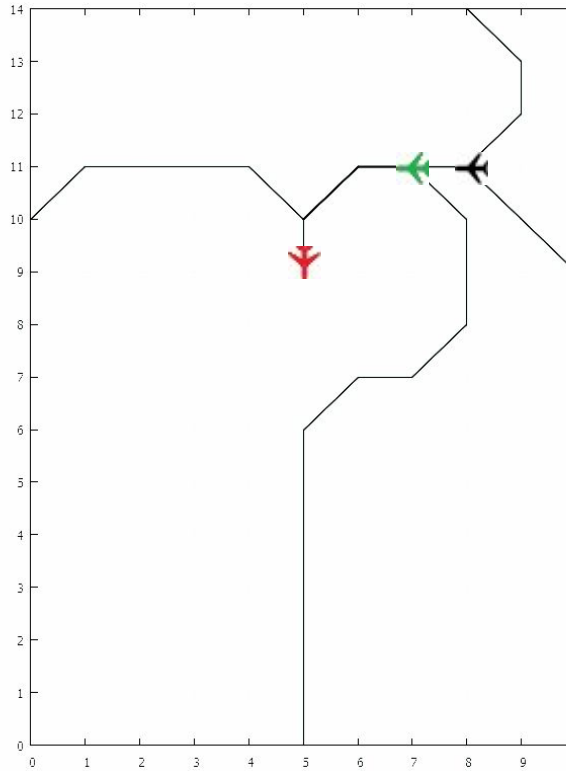
t = 15:29



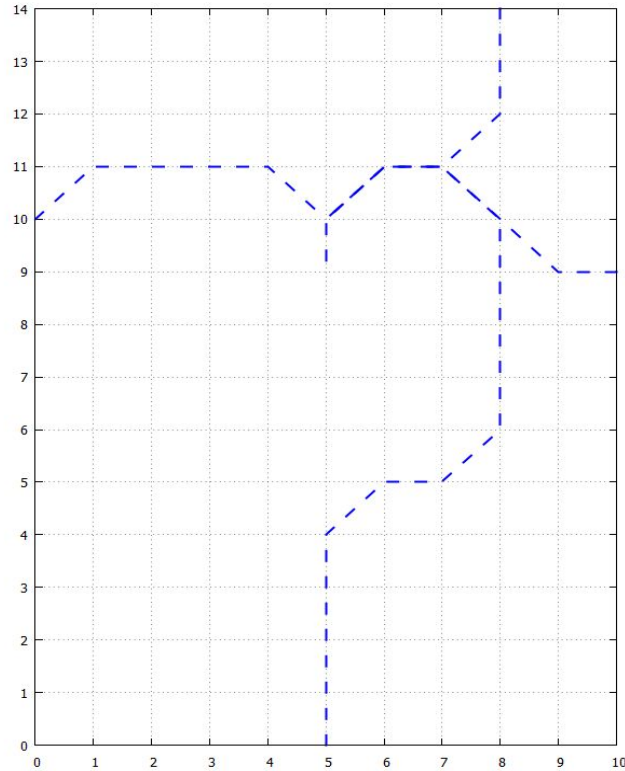
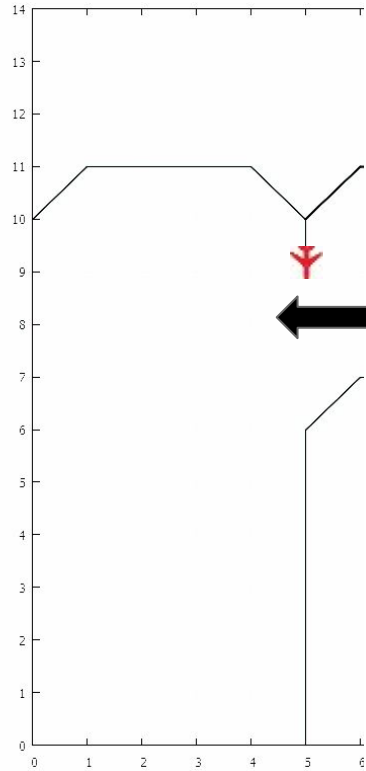
t = 15:30



**t = 15:30**

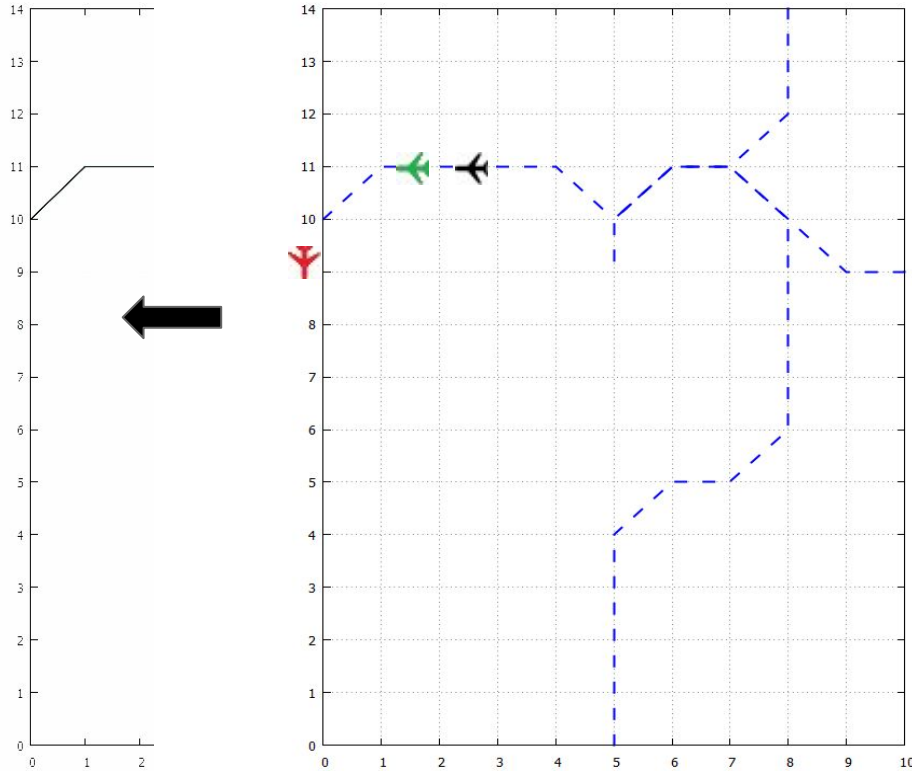


**t = 15:30**

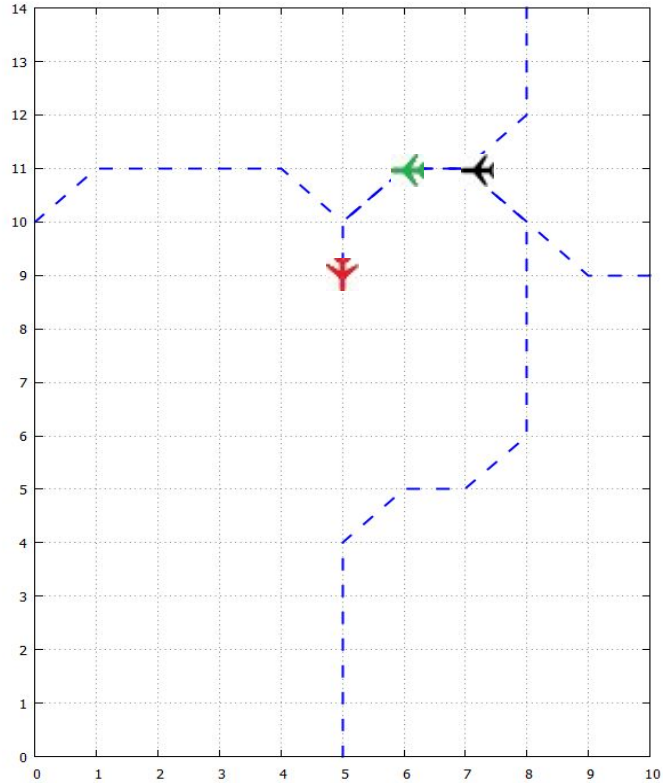




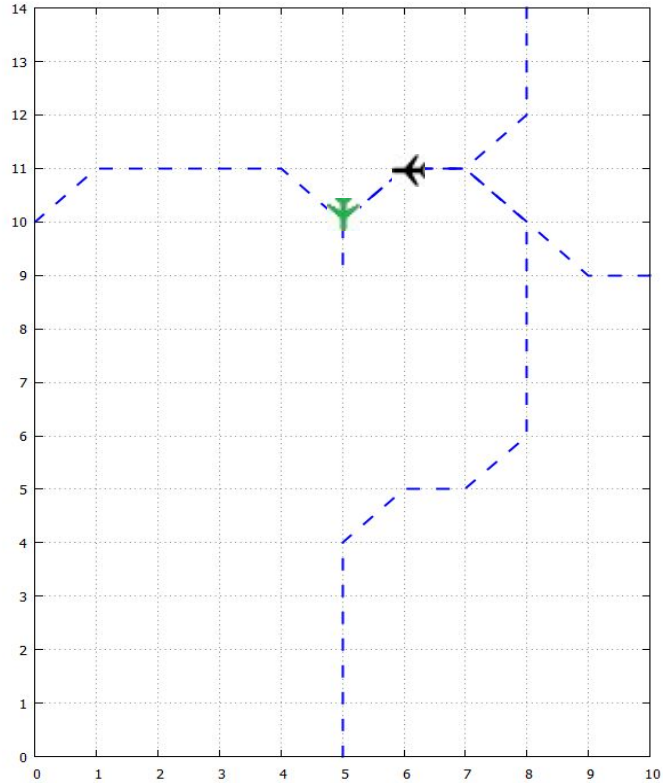
**t = 15:30**



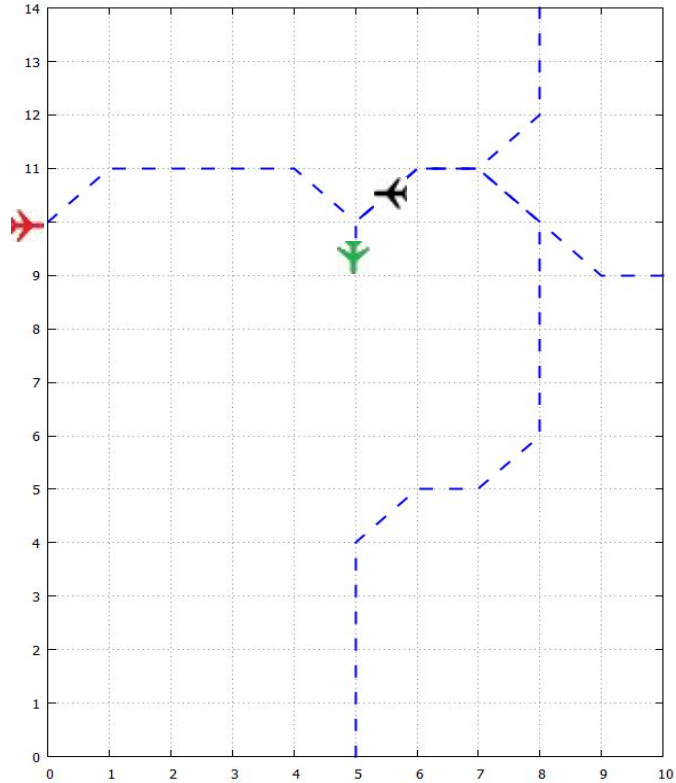
**t = 15:30**



**t = 15:31**



**t = 15:32**



## Conclusions

- ✓ Flexible optimization framework for dynamic route planning inside TMA
- ✓ Automated space and time separation
- ✓ Environmentally-friendly speed profiles (CDO)
- ✓ Applicable to any other realistic speed profiles
- ✓ May be used for TMA capacity evaluation

## Future Work

- ✓ Account for uncertainties due to variations in arrival times
- ✓ Solve overtaking problem (allow non-optimal profiles, or route stretching)
- ✓ Consider fleet diversity
- ✓ Elaborate on implementation possibilities, link to the future operational enablers (data links, technologies) for air-ground synchronisation (EPP)

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*Thank you!*

*Questions?*